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WORKS OF PROF. JOHN C. TRACY

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PLANE SURVEYING

A TEXT-BOOK AND POCKET MANUAL

BY
JOHN CLAYTON TRACY, C.E.
ASSISTANT PROFESSOR OF STRUCTURAL ENGINEERING
SHEFFIELD SCIENTIFIC SCHOOL
OF
YALE UNIVERSITY

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PREFACE

This book has been written to meet the requirements of the new methods of teaching surveying, especially of those methods developed in connection with summer courses where much of the instruction formerly given in the class-room has been transferred to the field. In plan it is a text-book and pocket manual combined, while in scope its aim is—not to cover the whole field of surveying, but—to treat with great thoroughness fundamental principles and methods. As a text-book it deals with the theory of surveying, while as a manual it gives many practical suggestions and directions which are usually left for oral instruction. Oral instruction is peculiarly inadequate in field work where the students are more or less widely scattered, while directions and suggestions if given in the class-room are frequently misunderstood or forgotten before they can be applied in the field. The problem is not solved by issuing a printed collection of exercises in which successive steps are outlined, for students are apt to follow the outline blindly and thus to defeat the end in view. It would seem better to provide a book in which explicit directions for methods of procedure are supplemented by the same explanations and comments as the instructor endeavors to give orally.

With more than one hundred students in the field at one time, the need of such a book in my own classes has become more and more evident. For several years my assistant instructors have helped me to keep a card index of the questions asked, the mistakes made, and the difficulties encountered by students in successive classes. Thus not only were the needs of students ascertained, but those which are most common were emphasized by duplicate cards. This card index led to the preparation, five years ago, of a pocket manual for use in my own classes. Yearly editions of this manual have been issued, each containing changes and additions suggested by

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the ever-growing card index, until now it is offered in its present form for general use. Though intended primarily for students, it is hoped that it may prove helpful to men in actual work, especially to those who are striving in the early years of practice to perfect themselves in the art of surveying.

In attempting to produce a book which shall be useful to the beginner, I have probably gone into details more minutely than seems necessary to the experienced surveyor, who wonders "why something is not left to the common sense of the student." Yet, a long experience in teaching impels me to the belief that nothing should be taken for granted, that anything worth telling the student is worth putting in print, and that after all, common sense in surveying is closely related to a knowledge of common things—little details which some one must tell the student for the first time. If put in print where they will be seen again and again, many of these practical details can be learned just as well before as after graduation, while if left to be picked up in practice many of them may not be learned at all.

A more definite idea of the general plan and scope of the book may be gained by reading the short introductory notes at the heads of the various chapters. Attention is called, however, to the following features:

1. Great care has been taken to make the book convenient for ready reference, so that the student may turn quickly to any part of the subject.

A purely progressive order of arrangement being impracticable, the plan has been followed of grouping closely related chapters and topics.

2. Each branch of field work in Part I is treated under three general heads, viz.: (1) the use of the instrument; (2) the general method of procedure; (3) the practical details of field work.

By this arrangement, general methods of procedure are not obscured by directions for the use of the instrument, or *vice versa*, while that part of the work which can be studied in the class-room is separated from that which is better studied in the field.

3. The time-honored system of explaining the general methods of surveying under the head of compass surveying has been abandoned, and the whole theory of horizontal control is explained in connection with transit surveying.

This innovation is justified by the fact that many engineers seldom, if ever, have occasion to use the compass.

4. Field work and office work are treated in separate parts

of the book, but the important relations between the two are emphasized throughout.

This division of the subject, adopted five years ago in the first preliminary edition of this book, corresponds to a similar division of the work in practice.

5. General statements of important principles and methods are almost invariably supplemented by special applications or practical illustrations, given as a rule in fine print.

This is for the special benefit of those who are studying the subject with little if any help from a teacher.

6. When there are two or more methods of doing the same thing, they are given one after the other and then compared.

For example, see Chapter XII for four methods of running transit lines; Chapter XIII for different methods of locating details; pages 141 and 150 for five methods of locating boundaries; and page 258 for five methods of setting grade-stakes.

7. A great deal of practical information is given without obscuring general methods.

For example, in explaining any method of procedure a concise outline of the essential steps is given first, followed by practical suggestions in fine print.

8. In the chapters on field notes, an attempt has been made to give a systematic treatment of a very important but much neglected part of field work.

Not only have numerous forms and examples been given throughout the book, and especially in Chapter XV, but different *methods* of keeping notes are *explained and compared*. Surprisingly little on this part of the subject is to be found in standard text-books.

9. Throughout the book special effort has been made to give a systematic and practical discussion of the subject of errors.

Consistent accuracy rather than *exceptional* accuracy should be the first aim; hence stress is laid on the *relative importance* of errors from different sources, as well as upon methods of eliminating errors.

10. Methods of checking are emphasized, and wherever possible they are included in methods of procedure so that work may be checked as it progresses rather than after it is finished.

For example, see methods of procedure on pages 152–156.

11. No space has been wasted in describing and illustrating visible parts of instruments. In Part III, however, a critical study is made of the construction and adjustment of instruments and an unusually complete chapter is given on the care of instruments.

This part of the subject should be studied *after* the student has become accustomed to the use of the instruments; hence it is treated at the end of the book instead of at the beginning.

12. The method of calculating bearings in Chapter XXX is original, but it is such a simple and natural application of algebraic signs to angles, that the same general method is probably used by many engineers.

13. It is hoped that chapters on various branches of surveying, such as stadia, compass, and plane-table work, will be found unusually complete. Attention is called to Chapters XVII and XXIII on special work in transit surveying and leveling, and to the treatment of office computations and mapping in Part II. No attempt, however, has been made to cover such subjects as city surveying and mining surveying, since they cannot be treated adequately in a book of this kind. There is an opportunity for specialists to write books of reference on these two subjects, following the general plan of our best books on railway surveying and topographic surveying.

14. Through the kindness of Mr. Charles W. Reinhardt of the *Engineering News*, and of Prof. C. E. Sherman of Ohio State University I have been able to include in Chapters III and XLII valuable extracts from their admirable books on lettering.

I have found it impracticable in many cases to acknowledge my indebtedness to other writers on surveying, so much of the subject having long been the common property of all engineers. Certain portions of the chapters on errors, and on the construction, use, and adjustment of instruments were first suggested by Prof. I. O. Baker's excellent book, *Engineers' Surveying Instruments.* For the most part, however, I have drawn upon data gathered from a great variety of sources during the past fifteen years.

I am indebted to many engineers and to members of the faculty of the Sheffield Scientific School for valuable suggestions offered during the preparation of this book. I am under special obligations to Prof. C. S. Hastings for his helpful criticism of the chapters on errors, to Mr. C. S. Reinhardt for assistance in the preparation of some of the cuts, and to Mr. R. S. Kirby for numerous suggestions concerning office-work, as well as for valuable help in the preparation of illustrative examples in Part II. Grateful acknowledgment is also made of the help received from my colleague, Mr. C. S. Farnham, *and from Mr. C.* A. Wheeler of the Connecticut Agricultural

College. Mr. Farnham, besides contributing knowledge gained from practical experience, has been of great assistance in the preparation of the manuscript. Mr. Wheeler has read the entire proof with great care, and has made important corrections and additions.

Acknowledgments are due to Professors Ives and Hilts for the use of Table III, reprinted from their Problems in Surveying; to Professors Merriman and Brooks for Tables VI, IX, and X, reprinted from their Hand Book for Surveyors; and to Professor Searles for Tables VIII, XI, XIV, XV, XVI, and XVIII. Table XVII is from Carhart's Field Book for Civil Engineers.

JOHN C. TRACY.

NEW HAVEN, CONNECTICUT,
August, 1907.

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INTRODUCTION

To the Student:

The theory of plane surveying is surprisingly simple, but the application of the theory in practice is more difficult. There is not much to understand, but there is a great deal to remember. Success depends more upon training and judgment than upon a knowledge of theory. You can hardly expect in a college course to acquire the skill which comes from long-continued repetition, or the judgment which comes from wide experience. On the other hand you should realize that the habits formed when first learning the use of surveying instruments, and the methods of field work acquired in college are apt to cling to you long after graduation. It is the object of this book not only to explain the theory of surveying but to help you to acquire those habits and methods of work that are in accord with the best practice. This does not mean that you can read over the practical suggestions and directions for field work once for all and then remember them ever after. On the contrary, you should take the book into the field with you and consult it constantly. Study it first for general principles and for a few of the most important directions, and then, as you have opportunity, read and reread portions of the text in order to pick up the fine points of good field work. For example, you are a rodman and are waiting for the levelman to set up. Now is a good time to reread the suggestions for the rodman, page 261. Or if a member of the transit party with little to do, read portions of Chapters IX, XII, XIII, and XIV on transit surveying. In this way much time can be used to good advantage which would otherwise be wasted.

Learn different methods of doing the same thing and study the advantages and disadvantages of each method. Study also the different sources of error, for field work is a continual fight against errors. In practice the surveyor aims to save time and expense. Working thus under pressure, it is a great advantage to be able in any given case to choose the most suitable method, and to know what sources of error may be ignored and what must be eliminated. Learn to check work while it is in progress, for a considerable portion of a surveyor's work is in making sure that the remaining portion is correct.

Finally, do not work too much by rule, and do not form the "handbook habit." The sooner you are able to work without help from this book the better the book will serve its purpose. Throughout the course maintain an attitude of questioning. Do not accept what an instructor says or what the book tells you as gospel truth. Constantly ask yourself '*Why* ' and do not rest satisfied until you know *why*. This is the spirit which will enable you to get the most out of the course.

J. C. T.

PLANE SURVEYING

PART I

FIELD WORK

CHAPTER I

INTRODUCTION

FUNDAMENTAL PRINCIPLES AND GENERAL METHODS

1. The Earth's Surface is understood in geodesy to be the surface of a globe as determined by the mean level of the ocean. The actual surface of the land is, for the most part, above the earth's surface. It is not necessary, however, to make this distinction in ordinary surveying except when dealing with elevations. The earth's surface is generally spoken of as a sphere; it is more nearly an oblate spheroid of revolution.

The longer axis is in the plane of the equator, the shorter from pole to pole. The lengths as generally used in geodesy are: long axis 41 852 124 ft., short axis 41 710 242 ft.—a difference of 141 882 ft., or about 26.87 miles. The average radius of the earth's surface is therefore 3956.5 miles.

2. Sea Level is understood to be the mean between high and low tides. It can be determined at any place by means of self-registering gages, the observations extending over a long period of time.

3. Horizontal and Vertical Planes. At any given point on the earth's surface there can be but one vertical line, i.e., a line to the earth's center as indicated by a plumb-line. A plane perpendicular to this line is a *horizontal plane.* A plane passed through this line is a *vertical plane.* At any given point there can be but one horizontal plane; there can, however, be an indefinite number of vertical planes.

Remark. *A level surface* is not a horizontal plane, indeed it is not a *plane at all but a surface* which is at every point perpendicular to a plumb-line at that point. If the plumb-line at every point pointed to the zenith, a level surface would conform to the earth's surface (an oblate spheroid), but owing to the irregular distribution of mass there are many deviations of the plumb-line, and a level surface follows all such deviations. These deviations of the plumb-line are not taken into account in plane surveying.

4. Surveying. Making such observations and measurements as will determine the relative positions of points on the earth's surface.

Measurements may be either angular or linear.

5. Plane Surveying. Surveying in which all measurements are either made or reduced parallel to a horizontal or vertical plane.

In *plane* surveying the curvature of the earth's surface * is not taken into account, in *geodetic* surveying it is; hence the necessity for the two terms *plane* and *geodetic*. For ordinary surveys, extending over small areas, the methods of plane surveying involve no appreciable errors. The limit is roughly placed at about 100 square miles, although plane surveys of much greater extent are often made when great accuracy is not required.

The difference in length between the arc on the earth's surface and a straight line is only about 0.05 ft. in 11½ miles, 0.5 ft. in 23 miles, and 1.0 ft. in 34½ miles.

6. Horizontal and Vertical Measurements. There are four kinds of measurements used in plane surveying:

HORIZONTAL	VERTICAL
1. Horizontal Distances	3. Vertical Distances
2. Horizontal Angles	4. Vertical Angles

Remark. In exceptional cases linear measurements are sometimes made along an inclined surface, but these measurements are afterwards reduced to a horizontal or vertical basis. It is important that the student should understand at the outset the distinctions as illustrated in Fig. 6.

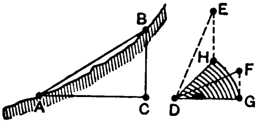

FIG. 6.

Illustration. Let A be a point at the bottom of a hill and B a point at the top. The distance AB as understood in surveying is not the air-line distance AB but its *horizontal projection* AC. Again, let H, D, and G be points in the same horizontal plane, E a point in a vertical line through H, and F a point in a vertical line through G. The angle EDF as understood in surveying is not the actual angle between the two lines ED and DF, but the *horizontal projection* of that angle, i.e., the angle HDG, which lies in a horizontal plane through D, the vertex.

BC, EH, and FG are *vertical distances* and BAC, EDH, and FDG are *vertical angles*. If A is at the sea-level, BC is the elevation of B above the mean surface of the earth; if both A and B are above the sea-level, then BC is simply the *difference in elevation* of A and B.

---

* About 0.7 ft. in *one* mile or $0.667D^2$ miles = curvature in *feet*, where D is the distance in miles, i.e., $\text{Curvature} = \frac{\text{Square of distance}}{\text{Mean diameter of earth}}$.

7. Units of Measurement. (*a*) LINEAR UNITS. In the United States the unit of length is the *foot.* The surveyor uses *decimal* parts of a foot instead of inches and fractions of an inch. For example, in surveying 50 ft. 9 in. is usually given as 50.75 ft. It is seldom necessary to give measurements closer than the nearest thousandth of a foot; more often the nearest hundredth or even tenth of a foot will answer every purpose.

Angular units. Angles are measured in *degrees, minutes,* and *seconds.* 60 sec. = 1 min.; 60 min. = 1 deg. One degree is $\frac{1}{360}$ part of the circumference of a circle. Usually angles are measured to the nearest minute or half-minute.

(*a*) *Changing decimals of a foot to inches.* For the student who is accustomed to fractions of an inch but not to decimals of a foot it is helpful to remember that 0.01 ft. is approximately $\frac{1}{8}$ inch. A difference, for example, between two measurements of 0.01 ft. seems almost inappreciable to the average student until he realizes that it is $\frac{1}{8}$ inch. Having fixed this relation in mind he can readily convert decimals of a foot to inches and *vice versa.*

Illustration. 0.04 ft. $= 4 \times \frac{1}{8}'' = \frac{1}{2}''$. 0.07 ft. $= 7 \times \frac{1}{8}'' = \frac{7}{8}''$. 0.1 ft. $= 10 \times \frac{1}{8}'' = 1\frac{1}{4}''$. These values are only approximately correct, but near enough for most purposes, as seen from the following table:

Inches	Approximate	To five places
$\frac{1}{8}$	0.01 ft.	0.01042
$\frac{1}{2}$	0.04 "	0.04167
1	0.08 "	0.08333
$1\frac{1}{4}$	0.10 "	0.10417

(*b*) OTHER UNITS. In certain kinds of work it is convenient for the surveyor to use feet and inches, and it is often necessary to change inches to a decimal of a foot and *vice versa.* A useful table for this purpose is given in the back of this book. The foot is one third of the length of the *standard yard.* The standard yard is a length fixed by a standard bar at 62° Fah. This standard length was originally an arbitrary one. The standard bar is kept in the Standards Department of the Board of Trade in London. In some countries, France for example, the *meter* is the standard of length. One meter = 3.2808333 ft. The unit of length in Spanish measures is the *vara,* several values of which seem to be in use, viz.: California, 1 vara = 33 inches; Texas, 1 vara = $33\frac{1}{3}$ inches; Mexico, 1 vara = 32.9931 inches.

In land surveying Gunter's chain gives rise to other units, viz., the *chain* and the *link.* 1 chain = 66 ft., 1 link = 66 ÷ 100 = 0.66 ft. or 7.92 inches. This unit is based upon the fact that ten square chains equal an acre, i.e., $10 \times 66^2 = 43\,560$ square ft. = 1 acre. To reduce links to feet: Subtract as many units as there are hundreds of links and take $\frac{2}{3}$ of the remainder.

Illustration. 50 links $= (50 - 0.50) \times \frac{2}{3} = 33$ ft.

8. Seven Methods of Locating a Point. The object of surveying is to determine the *relative* positions of points. To locate a point its position with respect to at least two other points must be known. Let A and B be two points of reference, and P a point to be located with respect to A and B. The five most common methods are:

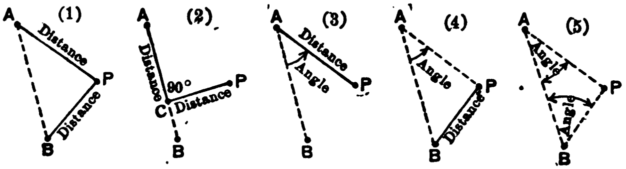

Fig. 8a.

1. By the two distances AP and BP.

2. By the distances AC and CP when the angle at C is 90°. (CB may be used in place of AC.)

3. By the angle at A and the distance AP. (Or the angle at B and the distance BP.)

4. By the angle at A and the distance BP. (Or the angle at B and the distance AP.)

5. By two angles, one at A, the other at B.

Notice that in the first two methods no angles are measured, and that in the fifth method no distances are measured, the distance from A to B being known.

Compare the fourth method with the third, since in each method one angle and one distance are measured.

Two other methods used for locating a point are:

6. By the intersection of two straight lines between four known points A, B, C, and D.

7. By two angles taken at the point to be located, and formed by lines to three known points E, F, and G.

In these two methods no distances are measured, although of course the distances between the given points of reference are known. The seventh method is the only one of all seven methods in which an angle is measured *at the point* to be located.

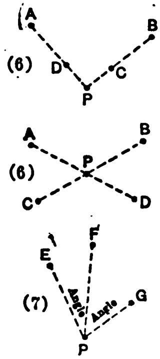

Fig. 8b.

Remark. A large part of surveying consists in locating two or more points or stations with respect to each other, and then locating boundary-lines, houses, roads, streams, and other details with respect to these stations. It is simply performing on a large scale many of the familiar operations of the drafting-room, using the tape in place of a scale, and a transit in place of a protractor.

9. Surveying,—General Methods. The work of the surveyor may be divided into three parts: (1) *Field Work;* (2) *Office Work;* (3) *Care and Adjustment of Instruments.*

(*a*) Field Work. The primary operations in field work are: (1) *Measuring distances* (Chaps. IV and V); (2) *Measuring angles* (Chaps. VIII–X); and (3) *Recording field notes* (Chaps. III and XV). Some of the things which the surveyor has to do and which involve these three primary operations may be outlined as follows:

(4) *Establishing points and lines of reference* with respect to which all details of the survey are located (Chaps. XI and XII).

(5) *Locating details* of the survey with respect to the reference-points and lines referred to in (4) (details such as boundary-lines, streets, buildings, fences, rivers, bridges, and other important features of the tract surveyed) (Chaps. XIII and XIV).

(6) *Staking out* roads, boundaries, buildings, masonry structures, and other details, the reverse of (5) (Chap. XVII).

(7) *Establishing* parallel lines, perpendiculars, measuring to inaccessible points, measuring inaccessible distances, running straight lines through obstacles, and doing other special work depending upon geometrical or trigonometrical constructions (Chaps. VII and XVIII).

(8) *Determining the relative elevations* of points (Leveling, Chaps. XIX–XXII).

9. *Making topographic surveys* in which horizontal and vertical measurements are combined (Chap. XXVII).

(*b*) Office Work. The indoor work of the surveyor consists chiefly of (1) *drawing*, and (2) *computing*. The drawing consists mainly in plotting measurements to scale, and therefore, to a large extent, in reversing on paper the operations in the field. The computing is of two kinds (1) that made for purposes of plotting, and (2) that made in order to obtain necessary data and information concerning the tract surveyed. Some of the things done in the office, which are treated in detail in Part *II, may be outlined* as follows:

(1) *Working up field notes* preparatory to plotting.

(2) *Plotting reference points and lines* which form the skeleton of the map.

(3) *Plotting details* by measurements from the reference points and lines referred to in (2).

(4) *Finishing up the map,* tracing, lettering, and blue-printing.

(5) *Calculating bearings,* latitudes and departures, unmeasured distances and angles, adjusting measurements, balancing errors, and making other computations necessary for plotting.

(6) *Calculating areas,* dividing up land; calculating angles and distances to be laid off in the field, and making computations to obtain all other data which may be desired.

(7) *Preparing level notes* for plotting.

(8) *Plotting profiles,* contour lines, and other level notes.

(9) *Finishing up profiles.*

(10) *Calculating grades,* cut and fill, and making other earthwork computations.

(*c*) THE CARE AND ADJUSTMENT OF INSTRUMENTS implies a thorough acquaintance with the instruments themselves; for the most part this must be gained in the field,—not from books. There is much, however, concerning the construction of surveying instruments which is not evident from inspection, and there are certain directions for their care and adjustment which should also be given. This information will be found in Part III.

10. Important Questions in Surveying which arise in connection with the field work are: (1) Degree of accuracy required; (2) Methods to be used; (3) Instruments to be used; (4) Errors, their sources, relative importance, and prevention; (5) Speed; (6) Systematizing the work; (7) Justifiable cost of the survey. These questions are closely related to each other, and depend to a large extent on the purpose for which a survey is made.

11. The Purpose of a Survey. The following illustrations indicate how the purpose of a survey helps to determine the degree of accuracy required and to decide other questions suggested in the preceding article.

Illustrations. A particular survey may be for the purpose of making a rough map, plotted to small scale, say 1 inch = 200 feet. The points will not be plotted nearer than $\frac{1}{100}$th of an inch; hence it is useless to take measurements nearer than to the nearest two feet.

Another survey may be for a map also to small scale, but exact measurements are wanted and will be printed on the map. Accurate measurements must be made accordingly.

A survey is made of a farm worth $20 an acre. The value of the land does not warrant great expense, probably not more than 10 to 25 cents per acre: but very accurate work costs money; hence very accurate work cannot be done, and the method used would be a compass survey or some other quick method.

A survey is made of a city lot worth $1000 a front foot. Every fraction of a foot is valuable; hence time and expense are not spared in getting an accurate survey.

A surveyor stakes out a dirt road; he sets the stakes as nearly on line and gives the corresponding grades as nearly as he can without wasting time in splitting hairs. If, however, he is giving curb-line and grade for a permanent pavement, the work must be done with greater precision.

12. Errors in Surveying. It is essential that the student, early in his course, should learn to distinguish between the different kinds of errors in surveying, to recognize their relative importance, and to use the customary methods of prevention, correction, and distribution. This subject is at once so large and so important that the whole of the next chapter is devoted to it.

13. Accuracy in Surveying. As indicated in Art. 11, p. 6, the purpose of a survey and the value of the land largely determine the accuracy needed in the field work. In addition, the surveyor may be handicapped by rough country, too short a time for work, too small a party, poor instruments, bad weather, and many other unfavorable conditions.

(*a*) *Limits of error*, therefore, cannot be given once for all. Before beginning a survey a surveyor should, if possible, establish limits of error for himself, i.e., how close to read angles, how accurately to measure distances. In this he is guided by his own experience and that of others. He should know what is considered good work under given conditions. Rules for limits of error, however, cannot be applied blindly. The best surveyor is not he who is extremely accurate in all his work, but he who does it just accurately enough for the purpose without waste of time or money.

(*b*) *Consistent accuracy.* In fixing limits of error for angles and distances, have the two consistent. There is no use in measuring angles accurately and corresponding distances roughly, or *vice versa.* Measure one as carefully or as roughly as the other.

Not all the work of a survey is done with the same precision. Transit lines, since they form the basis of all measurements, are usually run with care, for an error in angle or distance affects the whole survey, but an error in the angle or in the distance to a corner of a building, for example, affects only the location of that corner and not the whole survey. If land is valuable, boundaries are usually located with more care than buildings and other details for obvious reasons. *Make the precision of each step in the field work correspond to the importance of that step.*

14. Speed in surveying depends to a large extent on skill in handling instruments. Equally important are:

(1) Planning the survey and laying out the work. A little foresight in the choice of methods, choice of instruments, choice of stations, and the use of common sense in deciding similar questions will save time.

(2) Working with consistent accuracy. Much time is wasted, not only by students but by experienced surveyors, in needless refinements.

(3) Systematizing the work in order to avoid delays and to save unnecessary operations.

Speed is not necessarily the result of hurrying; it is more apt to be the result of economizing time along the lines suggested in this article.

15. System in surveying not only economizes time but diminishes the chances of error. There are many things in handling instruments and in other operations of field work which can be done always in the same way. If one is systematic, much of this work becomes second nature, and mistakes are thus avoided. Even when a mistake has been made one will often become conscious of it because of the violation of some fixed habit and so can find the error and correct it. For similar reasons system is equally valuable in recording notes and in office computations, and much of the value of a young surveyor's services lies in his knowledge of his employer's system, and in his ability to work in that system.

16. Equipment. The equipment needed both for field work and for office work is described in Part III.

CHAPTER II

ERRORS

In the first part of this chapter the aim is to define the different kinds of error, and to show how these errors affect measurements; also to explain how the most probable or best value of a quantity may be obtained, and how the precision of measurements may be judged. In the second part of the chapter a brief review of the method of least squares is given in so far as that method is likely to be used in plane surveying.

Methods of preventing or eliminating errors, and the relative importance of errors from different sources, will be treated more in detail in subsequent chapters.

17. Error. No measurement in surveying can be exact. The difference between a measurement and the true value of the quantity measured is the *true* error of that measurement. *The true error of a measurement is never known,* for the true value of the quantity measured is never known. How then can a surveyor judge of the precision of his measurements, and how can he determine the most probable or best values of the quantities measured? These are the main questions to be considered in any discussion of errors.

18. Sources of Error. (1) Natural; (2) Instrumental; (3) Personal.

(1) *Natural.* Temperature, wind, refraction, action of gravity, obstacles to measurement.

(2) *Instrumental.* Imperfections in the construction and adjustment of instruments. Expansion, contraction, and other possible changes in the instruments.

(3) *Personal.* Imperfections of human sight and touch, Actual mistakes.

19. Three Classes of Errors. (1) *Mistakes;* (2) *Constant errors;* (3) *Accidental errors.* If a measurement of a quantity is greater than the true value of that quantity, the error is *plus;* if less than the true value, the error is *minus.* The total error is not due to any one cause, but it is the *algebraic* sum of errors due to different causes.

Mistakes are errors which have their source in the mind of the observer.

Constant errors are those which occur from well-understood causes and can be eliminated.

Accidental errors are those which remain after mistakes and constant errors have been eliminated.

An accidental error is as likely to be plus as minus, but a constant error under given conditions always has the same sign and the same magnitude.

Thus, for example, if a tape is ¼″ too long because of high temperature, this is a constant error, because under the same conditions, i.e., the same temperature, the tape will always be ¼″ too long. Under *changed* conditions, i.e., low temperature, the tape may become too short; then the sign of the error is changed. This illustrates how a *constant* error from a given source may be plus or it may be minus, but under the *same* conditions it cannot be plus *or* minus. Suppose, on the other hand, that an observer in measuring between two points makes an error, in reading the tape, so small that he cannot detect it. This *accidental* error is as likely to be plus as minus, and if he repeats the measurement he is liable to make a similar error and that, too, is as likely to be plus as minus.

If a measurement is repeated a number of times under precisely the same conditions, the *constant* error is the same for each measurement, both in sign and magnitude. It is not reduced or affected in any way by taking the mean of all the measurements. The accidental error of each measurement, on the contrary, is as likely to be plus as minus, and the accidental error of the mean, therefore, is likely to be less than the accidental error of any one measurement. Theoretically, that part of the total error due to accidental errors would entirely disappear from the means of an indefinitely large series of observations, taken under precisely the same conditions; that part due to constant errors would still remain.

(*a*) *Illustrations.* A few illustrations of the three classes of errors will help to make the distinctions clear:

Mistakes. Errors due to carelessness, inexperience, and mental confusion, such as mistakes in reading the tape, the vernier, or the level-rod; *keeping* the wrong count of tape-lengths; turning the wrong tangent screw; *sighting at the wrong* station.

Constant errors. Errors due to such well-understood sources as temperature, sag, pull, imperfect graduation, and imperfect adjustments of instruments.

Accidental errors. Errors due to imperfections of human sight and touch which render it impossible to make exact readings, to know when the cross-hair exactly bisects a signal, to handle instruments delicately; sudden and imperceptible changes in an instrument itself; indeterminate *variations* in pull, temperature, and certain other sources of constant errors.

(*b*) *Remark.* When mistakes are very small it is sometimes difficult to distinguish them from accidental errors. It is to be noticed also that while temperature is a source of constant error, *variations* of temperature too small to be measured may be classed as accidental errors; the same is true of small variations in some of the other sources of constant errors.

Notice also that constant errors from different sources tend to balance each other when some are plus and some are minus; as, for example, when the error in chaining due to high temperature tends to balance error due to sag. Again, constant errors from the same source may tend to balance each other in the mean of measurements taken under *different* conditions, as, for example, when, in chaining, part of the measurements of a line are made when the temperature is above the standard temperature for the tape, and part when it is below.

(*c*) *Cumulative and compensating errors.* When the value of a quantity is found by adding together the measurements of several smaller quantities, any source of constant error becomes a source of *cumulative* errors, because, under given conditions, a constant error always has the same sign; on the other hand, any source of accidental error becomes a source of *compensating* errors, since for each of the separate measurements the sign is as likely to be plus as minus. *Compensating* errors tend to balance each other, while the cumulative errors from any one source affect the total result in the same way.

(*d*) *Illustrations.* Any constant error in the length of a tape becomes cumulative when the tape is applied several times in measuring a line, i.e., when the tape-lengths are added together, for the total error due to that cause will vary in direct proportion to the number of tape-lengths. On the other hand, errors which occur in marking where the end of the tape comes each time that it is stretched are compensating, for the chances are that if one tape-length is marked too long the error will be balanced by another tape-length marked too short.

Errors due to the settling of the instrument in leveling are cumulative, because the effect is always the same: it makes the calculated elevations of succeeding points too high. Errors due to the difficulty of telling when the cross-hair exactly bisects the target are compensating.

20. Discrepancies. In ordinary surveying most of the distances, angles, and elevations are measured but once. An important measurement, however, is often repeated, and if the discrepancy is too large the quantity is measured a third time.

A large discrepancy between two measurements of the same quantity indicates a *mistake* in the work, but it does not follow that because the discrepancy is small the *error* is small. The discrepancy is not an error, it is simply the difference between two measurements each of which contains an error that may be very large. This is an important point too often overlooked.

(*a*) *Illustration.* A surveyor measures a line, the true length of which is 400 ft., with a tape which, unknown to him, is ½ ft. too short. The first time he gets 402 ft. and the second time 402.05. The discrepancy is only 0.05 ft., indicating that he has made no serious mistake, but the true error is 2 ft. for the first measurement and 2.05 ft. for the second.

21. Elimination of Constant Errors. The discrepancy between the two measurements in the preceding illustration did not indicate the true error, because of a *constant* error in the length of the tape which no amount of repeating the measurement under the same conditions could reveal. The surveyor should have known that his tape was ½ ft. too short, and have corrected his measurements accordingly; then the discrepancy would have indicated more nearly the precision of his work. This illustrates why it is the first business of the surveyor to eliminate constant errors as far as practicable. A mistake, as revealed by a discrepancy, may then be corrected by again repeating the measurement. Constant errors and mistakes having been eliminated only accidental errors remain, and these may be reduced by taking the arithmetical mean.

As pointed out on p. 11, under certain conditions constant errors become *cumulative,* while accidental errors are *compensating;* hence the obvious importance of eliminating constant errors in chaining, in leveling, and in all similar work involving the summation of separate measurements.

22. Prevention and Elimination of Errors. (*a*) *Mistakes* are best discovered by systematic checks. *Checking* work, an essential part of all surveying, consists in comparing results with known conditions.

Illustrations. The discrepancy between duplicate measurements should be small; the sum of two or more measurements should equal another measurement, as, for example, when the sum of two angles should equal a third angle; the sum of two or more measurements should fulfill some known condition, as, for example, the interior angles of a triangle should add up 180°.

(*b*) *Constant errors* are prevented or eliminated partly by systematic methods of field work, partly by calculation and correction.

Illustrations. Errors due to imperfect adjustment can be eliminated by certain systematic methods in the use of instruments, such, for example, as the method of double reverse or making backsights and foresights equal in leveling. Reading both verniers on a transit eliminates error due to eccentricity. Each of these examples illustrates how *by taking observations in pairs* a plus error from a certain source of constant error may be offset by a minus error from the same source.

Errors in chaining due to temperature can be calculated and corrected; *so can errors* due to sag and pull, though it is better to prevent them by *taking proper precautions.* (See Chapter V.)

(*c*) *Accidental errors* cannot be eliminated, but they follow a mathematical law,—the law of probability,—according to which they may be greatly reduced by comparing the results of a series of measurements. (See p. 15.)

23. Relative Importance of Errors from Different Sources. It is evident that constant and cumulative errors as a rule are more important than accidental and compensating errors, and that the more measurements there are of a quantity the less the mean is affected by accidental errors. Beyond this it is difficult to make any general statement. Any error large enough to be *appreciable* should usually be avoided or corrected but appreciable is a relative term, and an error which is appreciable under certain conditions is inappreciable under other conditions. Thus for example, the error in chaining due to changes in temperature is an appreciable error provided the precision required warrants the use of a thermometer, otherwise it may be considered inappreciable. Errors in plane surveying so small that they are ignored would often be considered of great magnitude in geodetic surveying.

(*a*) *Remark* The more the surveyor knows about the sources and nature of errors the more likely he is to judge correctly of their relative importance. He must know that some sources of constant or cumulative errors are more important than others, and that certain errors, though they may be compensating, are not to be ignored. A more thorough treatment of these questions is given in the chapters on *Errors in Linear Measurement*, *Errors in Angular Measurement*, and *Errors in Leveling*.

24. Probable Values and the Adjustment of Measurements. Probable values are found from observed values. When the relations between observed values are complicated a knowledge of the "theory of least squares" is necessary. Some simple cases which frequently occur in surveying may be given, however, without entering upon that subject.

There are two general cases, viz.: (1) When the measurements are of one and the same quantity; (2) When the measurements are of different but related quantities.

(*a*) *Several measurements of the same quantity.* The most probable value of the quantity is the arithmetical mean of all the measurements, *provided the measurements are made with equal care.*

Illustrations. Three measurements of a line made with equal care are 160.01, 160.1, and 160.03. The most probable value is (160.01 + 160.1 + 160.03) ÷ 3 = 160.047. Four measurements of an angle are 30° 29′, 30° 31′, 30° 31′, and 30° 30′. The most probable value is (30° 29′ + 30° 31′ + 30° 31′ + 30° 30′) ÷ 4 = 30° 30′ 15″.

(*b*) *Measurements of several related quantities.* When quantities are related because of *two or more* conditions which they must fulfill, the problem of finding the most probable values of these related quantities properly belongs to the "theory of least squares." When, however, there is only *one* condition fixing the relations between the quantities, the most probable values, as a rule, can be easily found. Two cases under this head which often arise in surveying are: (1) when the sum of several measurements should equal an *exact* quantity, and (2) when the sum should equal some other *measurement.* In the first case the difference between the sum and the *exact* quantity is a true error; in the second case the difference between the sum and a *measured* quantity is a *discrepancy.*

(*c*) Case. I. *When the sum of several measurements should equal an exact quantity distribute the true error equally among all the measurements.* The results will be the most probable values of the measurements.

Illustration. The three angles of a triangle add up 179° 59′. The *exact* quantity fixing the relations between the measurements is 180°. The *true* error is 1′ or 60″. Hence add to each of the three angles one third of 60″, or 20″.

In the same way the most probable values of the interior angles of any polygon may be found by finding the true error of their sum according to the rule on p. 119, and distributing this error equally among all the angles. Notice, however, that the true error of any one angle is not known.

(*d*) Case II. *When the sum of several measurements should equal some other measurement distribute the amount of the discrepancy equally among all of the measurements, including that one to which the sum should be equal.* In this case the check measurement (i.e., the total measurement) cannot be exact, and, if all the measurements are made with the same care, it is likely to contain an error equal to that contained in each of the smaller measurements. *If, however, the correction is added to each of the smaller measurements, it must be subtracted from the total measurement and vice versa.*

(*e*) *Illustration.* If in the illustration of Case I the three angles had been the three parts of a *fourth* angle instead of the three angles of a triangle, and if the measured value of the *fourth* angle were 180°, then 180° − 179° 59′ = 1′ = 60″ is the discrepancy. The correction is 60″ ÷ 4 = 15″ instead of 20″, and this correction is added to each of the three smaller angles but subtracted from the total angle. The most probable value of the latter is, therefore, 180° − 15″ = 179° 59′ 45″.

(*f*) *Illustration.* The measured value of an angle is 40°. When divided into two angles the measurements are respectively 24° 30′ and 15° 29′. The discrepancy is 40° − (24° 30′ + 15° 29′) = 1′ = 60″. Three measurements of equal care are involved, hence the correction is 60″ ÷ 3 = 20″ and the most probable values are 24° 30′ 20″, 15° 29′ 20″, and 39° 59′ 40″.

(*g*) *Remark.* The essential difference between Case I and Case II is *that in Case I the quantity* used as a check has a true value which cannot be changed, while in Case II the check value is itself a measurement, and liable, therefore, to contain an error equal to that contained in each of the smaller measurements. The method in Case II does not hold for a number of linear measurements of different lengths.

25. The Theory of Errors or Method of Least Squares.* The "method of least squares" is a method of determining the most probable or best result of observations. It is based on the assumption that accidental errors will entirely disappear from the mean of an indefinitely large series of independent observations of the same quantity made under precisely the same conditions, and that therefore *if constant errors are eliminated from these observations* the arithmetical mean is the most *probable value* of that quantity. In practice, however, (1) an *indefinitely* large number of observations cannot be made, (2) no two observations can be made under *exactly* the same conditions, and (3) constant errors cannot be *entirely* eliminated. It follows, therefore, that the most *probable value* of a quantity is strictly a *theoretical* value, and that the greater the number of observations, and the more completely constant errors are eliminated from these observations, the nearer the most *probable* value of a quantity will approach the *true* value as a limit. It never reaches this limit (or if it did, there would be no means of knowing it), and therefore, no matter how accurate the observations may be, the probable value always contains an error.

(*a*) *A residual* is the difference between the value of a quantity as obtained by a single observation and the most probable value of that quantity as obtained by a number of observations. In any given case there are as many residuals as there are observations.

(*b*) *The probable error* of a quantity is a magnitude such that the chances are even that the true error contained in that quantity is greater or less than the probable error.

Illustration. A series of measurements of the length of a line gives for the mean or most probable value 528.91 ft., and the probable error of that mean, determined by a method to be explained later, is ±0.034 ft. This means that it is an even wager that the true error contained in 528.91 ft. does not exceed ±0.034 ft. Notice that this is quite different from saying that the true error does not exceed ±0.034 ft.

26. Use made of Most Probable Values and Probable Errors. *Most probable values* are used in surveying for the adjustment of measurements, and *probable errors* to indicate the precision of measurements.

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* It is the purpose of this article to give only so much of the theory of errors as may be needed in the simple cases which arise in plane surveying. For a more comprehensive treatment of the subject consult one of the standard works on the method of least squares.

Most probable value. The simplest and most common cases of adjusting measurements by finding the arithmetical mean have already been given on p. 13.

(*a*) *Probable error of the mean.* Suppose that a second series of measurements of the length of the line in the preceding article gives 528.94 ft. for the mean and ±0.065 ft. for the probable error of that mean. The results of the two sets of measurements are:

Most probable value	Probable error
528.91	±0.034
528.94	±0.065

The probable error of the mean of all the measurements in the first series is approximately half that of the mean as determined by the second series. This indicates that the precision of the first result (528.91) may be regarded as twice that of the second (528.94).

(*b*) The *probable error of a single observation* is found from the residuals of a number of observations, as explained in the next article. If it is necessary to take a number of observations in order to determine the probable error of a single observation, why not be content to determine the probable error of the mean of all the observations and stop there? Of what use is the probable error of a single observation? The answer is that, having determined the probable error of a single observation under given conditions, this probable error indicates the precision which may be expected in any single observation made under the same conditions.

Illustration. In measuring an angle a number of times the probable error of a single observation is found to be $\pm 8''$. (See p. 19.) This means that for any other angle measured by the same observer with the same transit under practically the same conditions a probable error may be expected of about $\pm 8''$. If this is well within the allowable error, the observer is justified in proceeding with single observations of all angles that can be measured under the given conditions.

(*c*) *Probable errors and weights.* Probable errors afford a means of finding the weights which should be given different sets of observations, for it has been found that the weights of observations vary inversely as the squares of their probable errors.

Illustration. On p. 19 the probable error of the mean of the first set of observations is $2.8''$, and of the second set $4.6''$. Hence the weights are as $\frac{1}{(2.8)^2}$ to $\frac{1}{(4.6)^2}$, or about as 3 to 1.

27. To Determine Probable Errors. The formulas given below for calculating probable errors are based on the following assumptions:

(1) That small errors are more frequent than large ones.
(2) That positive and negative errors are equally frequent.
(3) That very large errors do not occur.

Additional points to be kept in mind are: (1) Constant errors must be eliminated as far as practicable; (2) Observations should be made under as nearly the same conditions as possible. (3) At least three observations should be made. If there are only two observations, the *discrepancy* will indicate the precision as truly as will the probable error. The more observations there are the more significance may be attached to the probable error, although a small number of observations will give surprisingly satisfactory results if the other conditions already mentioned are fulfilled.

(*a*) *Remark.* The student is warned against attaching importance to probable errors unless he is reasonably sure that the above conditions are fulfilled, especially that pertaining to the elimination of constant errors. Since the method of least squares is based on the arithmetical mean as the most probable value, and since, as pointed out on p. 10, constant errors cannot be eliminated by taking the mean of observations, the more completely the constant errors are eliminated, or corrected, the more applicable is the method of least squares. If, for example, a tape is too long, all measurements made with it will be too short, and there is no magic in least squares which will correct the results. The probable error in this case loses its significance unless each of the separate measurements from the mean of which the probable error is derived is first corrected for the constant error in the length of the tape.

28. Formulas for Probable Errors.*

Let n = the number of the observations;
v = the difference between any one observation and the mean of all the observations, i.e., the *residual error* (p. 15);
E = the probable error of a single observation;
E_0 = the probable error of the mean of all the observations;
Σ = a symbol signifying *sum* of.

Then, from the theory of least squares,

$$E=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}, \quad \text{or} \quad \frac{0.6745}{\sqrt{n-1}}\sqrt{\Sigma v^2}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$E_0=\frac{E}{\sqrt{n}}=0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}, \quad \text{or} \quad \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}. \quad (2)$$

Let Σp = sum of the weights and R_0 the probable error of the general or weighted mean; then

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* See any treatise on the *Theory of Least Squares.*

$$R_0=0.6745\sqrt{\frac{\Sigma pv^2}{(n-1)\Sigma p}}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let R_s = the probable error of the sum or difference of several independent quantities z, z_1, z_2, . . . z_n.

$r, r_1, r_2, \ldots r_n$ = the probable errors of $z_1, z_2, \ldots z_n$, respectively.

$$R_s=\sqrt{r^2+r_1^2+r_2^2 \ldots +r_n^2}. \quad . \quad . \quad . \quad . \quad (4)$$

The probable error of a product Az, where A is a known quantity, z an observed quantity, and r the probable error of z, is

$$R_p^2=A^2r^2, \quad \text{or} \quad R_p=Ar. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let SS_1 = an area found by multiplying together the measured sides S and S_1;

r and r_1 = the probable errors of S and S_1 respectively. Then the probable error of the computed area SS_1 is

$$R_a=\sqrt{S^2r_1^2+S_1^2r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

29. Abbreviated Methods. In applying the formulas of the preceding article approximate values will often be all that are required. For this purpose the two following equations may be substituted for equations (1) and (2) respectively:

$$E=\frac{0.8453\Sigma v}{\sqrt{n(n-1)}}. \quad . \quad (7) \qquad E_0=\frac{0.8453\Sigma v}{n\sqrt{n-1}}. \quad . \quad (8)$$

When there are only two observations and D is the discrepancy between them,

$E=0.47D$ or roughly $\frac{1}{2}D$ and $E_0=0.337$ or roughly $\frac{1}{3}D$. (9)

In formulas (1) and (2) work may be saved by using the coefficients given in the following table:

n	Single Observations, $\frac{0.6745}{\sqrt{n-1}}$	Series of Observations, $\frac{0.6745}{\sqrt{n(n-1)}}$	n	Single Observations, $\frac{0.6745}{\sqrt{n-1}}$	Series of Observations, $\frac{0.6745}{\sqrt{n(n-1)}}$
			11	0.2133	0.0643
2	0.6745	0.4769	12	0.2034	0.0587
3	0.4769	0.2754	13	0.1947	0.0540
4	0.3894	0.1947	14	0.1871	0.0500
5	0.3372	0.1508	15	0.1803	0.0465
6	0.3016	0.1231	16	0.1742	0.0435
7	0.2754	0.1041	17	0.1686	0.0409
8	0.2549	0.0901	18	0.1636	0.0386
9	0.2385	0.0795	19	0.1590	0.0365
10	0.2248	0.0711	20	0.1547	0.0346

A table of squares and square roots greatly facilitates the work. Other tables similar to that given above will be found in standard text-books on least squares.

30. Illustration of the Method of Least Squares Applied to Angles.*

First Observer				Second Observer			
No.	Angle	v	v^2	No.	Angle	v	v^2
1	60° 20′ 20″	4	16	1	60° 20′ 50″	14	196
2	30	6	36	2	21 00	24	576
3	10	14	196	3	20 40	4	16
4	20	4	16	4	20	16	256
5	40	16	256	5	30	6	36
6	10	14	196	6	10	26	676
7	20	4	16	7	20	16	256
8	40	16	256	8	21 00	24	576

First Observer:

8)190 $\Sigma v^2 = 988$

Mean = 60° 20′ 24″ −

$$E_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}$$

$= 0.0901 \times (\pm 31.4) = \pm 2.8''$

$E = E_0\sqrt{n} = \pm 2.8'' \times 2.8 = \pm 8.0''$

Second Observer:

8)162 170 $\Sigma v^2 = 2588$

Mean = 60° 20′ 36″ +

$$E_0 = \frac{0.6745}{\sqrt{n(n-1)}}\sqrt{\Sigma v^2}$$

$= 0.0901 \times (\pm 50.9) = \pm 4.6''$

$E = E_0\sqrt{n} = \pm 4.6'' \times 2.8 = \pm 12.9''$

Illustration. The results given above were obtained by two different observers using different transits. The probable error of the mean in the first case is ±2.8″, while in the second it is ±4.6″. This indicates that the relative precision of the work of the two observers is as 2.8 to 4.6, or nearly as 5 to 3.

To find the most probable value of the angle by comparing the mean values of the two series of observations it is necessary to weight their mean values. Since weights are inversely proportional to the squares of probable errors, p. 16, the weights are as $\frac{1}{2.8^2}$ to $\frac{1}{4.6^2}$, or nearly as 3 to 1.

The most probable value is $60° \, 20' + \frac{24'' \times 3 + 36'' \times 1}{4} = 60° \, 20' \, 27''$.

The probable error of the above value may be obtained as follows:

$24'' = z_1$ and $36'' = z_2$, $\pm 2.8'' = r_1$ and $\pm 4.6'' = r_2$, $\frac{3}{4} = A_1$ and $\frac{1}{4} = A_2$.

$$\text{From (5)} \begin{cases} \text{Prob. error of } A_1z_1 \text{ or } 24 \times \frac{3}{4} = A_1r_1 = (\frac{3}{4} \times \pm 2.8'') = \pm 2.1'' = R_p, \\ \text{`` `` ``} \; A_2z_2 \text{ or } 36 \times \frac{1}{4} = A_2r_2 = (\frac{1}{4} \times \pm 4.6'') = \pm 1.15'' = R_{p1}. \end{cases}$$

The probable error of the sum of $(24'' \times \frac{3}{4}) + (36'' \times \frac{1}{4})$ is, from (4),

$$R_s = \sqrt{R_p^2 + R_{p1}^2} = \sqrt{2.1^2 + 1.15^2} = \pm 2.4''.$$

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* For an illustration of the method of least squares applied to linear measurements, see p. 59.

CHAPTER III

FIELD NOTES

In this chapter are given the general directions and suggestions which hold good for *all* kinds of field notes. In Chapter XV will be found special directions for keeping the *notes of a transit survey*, and also some illustrations of the forms in common use. Forms of notes for stadia surveying, topographic surveying, compass surveying, and leveling are given in the corresponding chapters on those subjects.

31. Field Notes. Field notes are written records of field work, *made at the time the work is done;* notes made afterwards, or notes copied from field notes, may be useful, but they are not field notes. Field notes must be as complete and accurate as possible, for the most careful and reliable field work will be of little or no value if the record of that work is unreliable. That is why engineers often prefer to keep the field notes themselves rather than trust them to a less experienced assistant. The student must realize at the beginning of the course that not only is recording field notes as important as any part of the field work, but it is perhaps the most difficult thing he will be called upon to do, and will require all the alertness, care, accuracy, and intelligence which he can command.

Note. To keep field notes satisfactorily one must have had long experience in practical work. It is not expected that the student can do it at the outset, indeed comparatively few men in actual practice are good note-keepers, but this is all the more reason why instructor and student alike should pay special attention to perfecting the student's ability in note-keeping.

32. Methods of Keeping Notes. Methods of note-keeping are different for different kinds of work, and will be given in detail as the work may require. It must be remembered, however, that there may be many methods equally good for a given kind of work, that an engineer must choose the one best adapted to his needs, and that as a rule of the many good methods in common use, only one or two illustrations are given in this book for each kind of work.

33. Field Notes—General Suggestions.

(1) Use a good note-book that will stand hard usage.

(2) Use a hard sharp pencil—3 H or 4 H; never a pencil soft enough to blur.

(3) The notes should read from left to right, and from the bottom up as in working drawings. In printing vertically hold the right-hand edge of the book towards you.

(4) Trust nothing to memory; when in doubt as to whether or not an item will be needed, put it in.

(5) Constantly ask yourself what will be needed in your notes in the office work. To answer this you must know the purpose of the work; if it is a survey and a map is to be plotted from the notes, what the map is to be used for, and what, therefore, should go on the map; if leveling, what is to be done with the data obtained.

(6) If a page of notes becomes illegible, make a copy of it while the data are fresh in mind, but mark it "COPY," and preserve the original page.

(7) The notes of a survey should be preceded by a title-page, and if there are many pages, there should be an index. At the top of each page print a descriptive title so that one can tell at a glance what the notes on that page are about.

(8) Notes should be signed each day by the note-keeper.

(9) It is usually desirable to at least indicate all the numerical work of the calculations and reductions made in the field. If the numerical work itself is done in the note-book, it should not be allowed to obscure the main notes, and hence it is often best to put such computations on separate pages and cross-reference them. This does *not* mean that numerical work is to be *scribbled* in the note-book at random; on the contrary, it should be kept in as neat and systematic a manner as any other portion of the notes.

34. The Three Parts of Field Notes. Field notes like working drawings may be subdivided into three parts:

(1) *Numerical values.* Records of all measurements (angles and distances).

(2) *Sketches.* Records of outlines, relative locations, topographic features.

(3) *Explanatory notes.* All notes of an explanatory nature

which make clear those sketches and numerical values that might otherwise be obscure or misunderstood. Also data that may be of future value, such as names of the surveying party, any special or unusual conditions under which the work was done, equipment, weather, and date.

35. Numerical Values.

(1) *Make plain figures.* Words can be guessed at, but not figures.

(2) *Make large figures.* The tendency of the beginner is to make his numerals too small.

(3) *Never write one figure on top of another.* A constant source of annoyance and error. If, for example, a 5 is written and then a 7 on top of it, how is one to know which is correct?

(4) *Do not try to change one figure into another;* for example, a 3 into a 5 or a 2 into a 3. The reason is obvious.

(5) *Erasing.* Avoid erasing. Draw a line through an incorrect value and write the correct value directly above or near by. This will show that a mistake was noticed in the field and corrected, thus giving double weight to the corrected value, while erasing or changing a figure leaves the impression of "doctoring" the notes. When it becomes necessary to alter a sketch or to make any part of the notes clearer erasing is justifiable, provided nothing is lost which would give added weight to the notes.

(6) *Measurements or sketches.* If measurements are put directly on the sketches, make it clear where they belong. The use of dimension-lines, as in working drawings, is often necessary. When the space on the sketch is too small for figures, a measurement may be placed to one side, and an arrow drawn to indicate where the measurement should go.

(7) *Recording measurements.* In recording measurements as they are being taken in the field there are two sources of trouble: (1) Measurements omitted because not taken. (2) Measurements misunderstood as they are called out.

(1) The note-keeper must therefore be on the alert to see that every measurement that should be recorded is taken and that every measurement taken is recorded.

(2) The note-keeper should *repeat,* loudly enough to be distinctly heard, all measurements which others may call out to him to record. Measurements not taken in the presence of the note-keeper should be given to him in writing. For example,

if the note-keeper is busy taking transit notes and the chainmen are at work elsewhere, they should report all measurements to him in writing at the first opportunity.

36. Sketches.

(1) *Scale.* Sketches are almost never made to scale; in fact it is usually necessary to exaggerate certain portions of the sketch for the sake of clearness.

(2) *Make sketches large, open, and clear.* Use the pages of the note-book generously—they are cheap compared with time wasted in deciphering crowded notes.

(3) *Use of straight-edge.* Sketches are made free-hand, but frequently time may be saved and the sketch greatly improved by using a straight-edge for the straight lines. A small triangle carried in the pocket will be found useful.

(4) *Angles.* The protractor is seldom used in the field; angles are either guessed at or no attempt is made to draw them true size.

(5) *Exaggeration of details,* or portions of a sketch drawn larger size at one side of the main sketch, or on another page, is often necessary.

(6) *What a sketch should show* must be determined on the spot by the note-keeper, and for the most part before he begins the sketch.

(7) *When to make a sketch.* In general, if measurements cannot be easily described, or if the description (in words) of a series of measurements would take more time or space than would be required for a sketch, make a sketch. Always make a sketch when it will settle beyond question any doubt which otherwise might arise in the interpretation of the notes.

37. Explanatory Notes.

(1) *Object.* To make clear that which is not perfectly evident from numerals and sketches, and to record such information concerning important features of the ground covered and the work done as might be of possible use later. In addition, to give for future reference names of party, date, weather, equipment, and special conditions under which the work was done.

(2) *Lettering.* The common practice is to print notes, using some good style of free-hand lettering. Compactness and legibility are gained by such lettering well done. In many cases ordinary handwriting can be used provided it is as legible as

good free-hand printing. One method used for free-hand lettering is given in the next article.

(3) *Place the notes* in vacant spaces where they will not interfere with or obscure numerals, dimension-lines, or sketches.

(4) *When needed.* Ask yourself in a given case if the dimensions you have put down, or the sketch you have made, could possibly need oral explanation should some one else use the notes. If in doubt, put the explanation in writing in the notes themselves.

38. Free-hand Lettering. The style of lettering recommended for field notes is the "single-stroke" lettering. It is a surprisingly easy system to learn, and it is especially adapted to rapid note-taking because of its simplicity. In this article the essential features of this system are given in so far as they apply to field notes. For a more thorough treatment of single-stroke lettering the student is referred to an excellent little book on that subject by Mr. Charles W. Reinhardt.*

The student is urged to practice until he can letter almost as rapidly as he can write. When he can do this and do it well he has acquired something of great value, not only for recording field notes but for other lines of work. Even a few hours' intelligent practice, in which the *direction and sequence* of the strokes are carefully observed, will yield results well worth attaining.

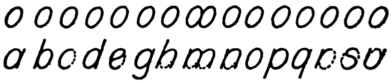

FIG. 38 (*a*).

(*a*) In Fig. 38 (*a*) it is seen that fifteen of the lower-case letters of the alphabet can be formed with a small ellipse as a basis; the other nine can be formed with straight lines. This method of forming letters, however, is not recommended, but is inserted here as introductory to a method somewhat similar. In Fig. 38 (*b*) this other method is shown.

(*b*) The methods given in this article for forming inclined letters hold good also for upright letters. Inclined letters, however, can be made more rapidly and with greater uniformity by the average man. In Fig. 38 (*b*) the arrows in the first and fourth rows indicate the directions of the strokes; in the second and fifth lines the completed letters are shown; in

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* Mr. Reinhardt's book on lettering (Engineering News Publishing Co., New York City) was the first to indicate the sequence and direction of the *separate strokes by arrows*, and it has done much to improve free-hand *lettering among draftsmen* throughout the country.

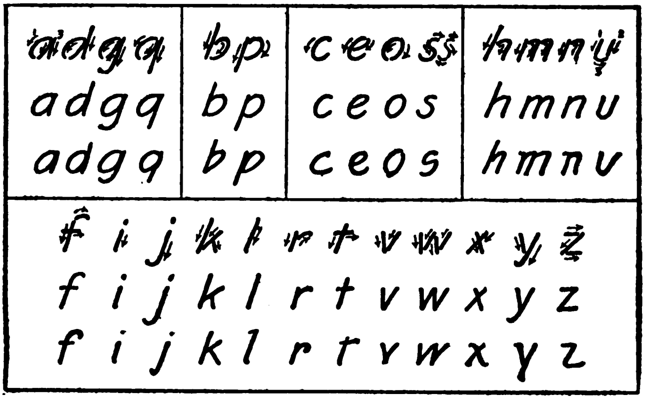

FIG. 38 (*b*).

the third and sixth lines *incorrect* methods of forming letters are illustrated. The main points to keep in mind concerning the different letters are as follows:

(1) The *i*, *j*, *l*, and *c* are the only letters made with one continuous stroke.

(2) The strokes in each letter are as far as possible *downward*. The slope of the main stem with few exceptions is about 1 to 2½. [For height of letters see p. 29, Art. 38 (*f*).]

(3) The order or sequence of strokes is from left to right, except in *s*, *u*, *k*, *x*, and *z*.

(4) In the first group, *a*, *d*, *g*, and *q*, the only difference between the letters is in the third stroke. In each of these letters the second or middle stroke is flatter than the first in order that there may be a clear space between the second stroke and the vertical stem. In order to avoid the effect shown in the third row which comes from not doing this, it may be well for the beginner to exaggerate, somewhat, the "hump" in the first stroke and then make the second stroke almost straight.

(5) In the letters *b* and *p* the oval is wider at the top than at the bottom.

(6) The basis of each of the letters in the third group, c, e, *o*, and *s*, is an inclined ellipse. The e is made with two strokes;

if made with one it is apt to have a different slant from the other letters, as shown in the third line. Notice that the first stroke of the *s* is started in the opposite direction from that naturally expected. Some draftsmen prefer to make the *s* in two strokes instead of three as shown in the supplementary *s*. The *o* is made with two strokes each of which should begin and end almost horizontally in order that the two strokes may meet in a smooth curve and not in a sharp apex; this also gives proper width to the *o* without making it round.

(7) In the letters of the fourth group, *h*, *m*, *n*, and *u*, the first effort should be to get the vertical strokes parallel. The second stroke in *h*, *m*, and *n* should not be started too far up the first stroke, and it should make almost a sharp turn at the upper corner. The first stroke should not begin with a curve. Notice that the third stroke of the *u* is at the bottom, the other two strokes being made first to secure parallelism and proper width of letter.

(8) In the letters *g*, *q*, *b*, *p*, *j*, *l*, and *y* the portion above or below the main body of the letter is a little less than one half the total height of the letter; in the letter *t* it is still less.

(9) In the letter *k* the third stroke does *not* start at the intersection of the other two. Make the second stroke strike pretty near the bottom of the first and begin it well to the right so that the letter will not be too narrow. Notice that the second stroke of the *r* is *upward*.

(10) In the letters *v*, *w*, *x*, and *y* make the strokes in one direction almost vertical, and the strokes in the opposite direction at a considerable slant, so that the main axis of the letter may have the slope corresponding to that of other letters. The intersection of the two strokes in *x* is a little more than halfway up.

(11) In the letters *k*, *v*, *w*, *x*, *y*, and *z* make the angles at the intersection of lines clear and sharp.

(12) In the letters *g*, *j*, and *y* the stroke extending below the line is straight until just at the end it turns sharply to the left, forming a *small* hook.

(13) The cross-strokes in the *f* and *t* are on a line with the tops of most of the other letters.

(*c*) Good numerals are even more important than good letters in field notes: they must be made unmistakable. The main points to be kept in mind are:

(1) Make figures somewhat larger than lower-case letters. Reinhardt gives the height equal to that of capital letters, but this is rather large for field notes. [See Art. 38 (*f*).]

(2) In the 2 the down curve is *reversed* and the base is *straight*. Avoid the effect shown in the third line. Some draftsmen prefer an abrupt turn or angle in the down-stroke as shown in the supplementary 2.

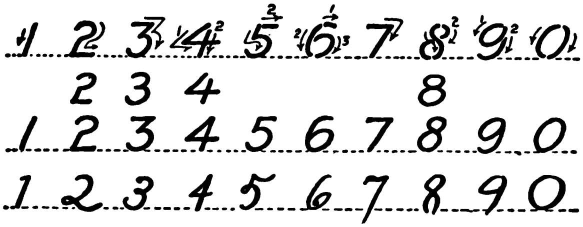

FIG. 38 (*c*).

(3) Of the two kinds of 3's that with its upper line horizontal and straight is perhaps the easier to make. The lower loop should begin with a sharp turn *upward*, and its height should be more than half the total height of the numeral.

(4) Of the two kinds of 4's that shown in the top row seems to be preferred. The first stroke begins at about 45°, and should extend more than half-way down the figure before turning into the horizontal, otherwise the top of the 4 will be too small as shown in the third line.

(5) In the 5 the first stroke should turn abruptly about two fifths of the way down, and extend almost horizontally for quite a little distance; it really curves slightly *above* the horizontal to form the upper portion of a loop similar to that of the 3. The second is *straight* and horizontal; make it fairly long.

(6) In the 6 the tendency is to make the lower loop too small.

(7) The 7 is formed by two straight lines,—no curves,—with the down-stroke at about 45°.

(8) In pencil the 8 is best made with two strokes. Sometimes it is convenient to form it with two ovals, the upper one being the smaller. In inking, the pen is lifted where the second stroke crosses the first.

(9) The 9 is an inverted 6, but the upper loop is nearly finished with one stroke. The tendency is to make the loop too small.

(10) The 0 is made with two strokes each of which begins and ends almost horizontally to give smoothness to the curve and width to the whole numeral without making it round.

Note. **The student may find that in keeping field notes in pencil some of the numerals made in Fig. 38 (*c*) with two or more strokes can be made in one stroke. In making figures in ink, however, he will get better results to follow the strokes as given.**

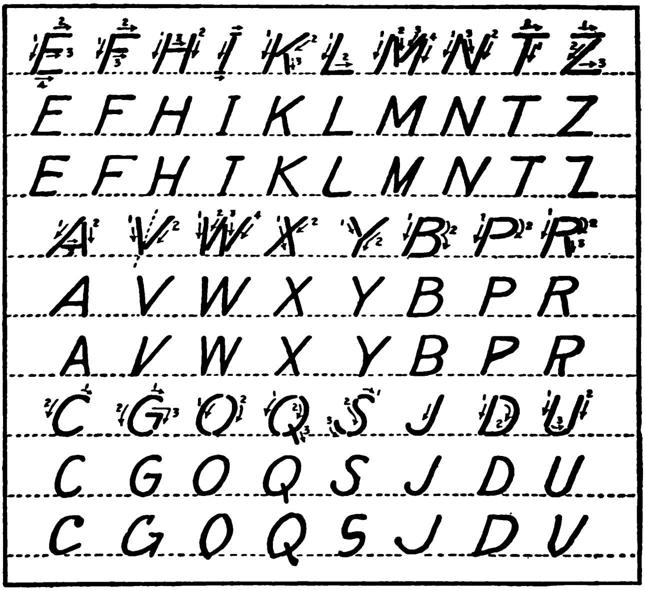

FIG. 38 (*d*).

(*d*) *Capital letters* are shown in Fig. 38 (*d*). Many of the directions already given for the lower-case letters apply to the capital letters also. Some additional points are as follows:

(1) The height of capital letters is not quite double that of lower-case letters. [See Art. 38 (*f*).]

(2) The width of most of the capital letters is about equal to the height of lower-case letters.

(3) The third or horizontal stroke in E, F, and H is slightly more than half-way up. This is true also of the middle horizontal line of B and the corresponding lines of P and of R.

(4) The second stroke in L and in T is comparatively short.

(5) In A, V, W, X, and Y the chief difficulty is in getting the *axis* of the letter at the standard slope. This is accomplished by making certain lines almost vertical, and opposite lines at a correspondingly greater slant, as shown in the figure. The horizontal stroke in *A* is quite near the bottom.

(6) The width of M and of W is greater than the width of other letters. The second and third strokes of the M meet in a point *above* the bottom line of the letter.

(7) The letters C, G, O, Q, and S are based on the ellipse. The tendency is to make these letters too wide.

(8) In practicing lettering observe carefully the order and direction of the strokes, which are for the most part *downward* and from *left to right,* and notice what errors to avoid as illustrated in the lines 3, 6, and 9.

(*e*) *Spacing the letters.* Crowd the letters close together and thus compress words into small space, otherwise one of the chief advantages of lettering will be lost. The tendency is to make letters too wide and too far apart. To make narrow compressed letters seem large and clear increase the height. Thus, for example, in Fig. 38 (*e*) the third line takes no more space lengthwise than the second, but the words are clearer because the height of the letters is greater. The first line illustrates how space may be wasted by not compressing the letters.

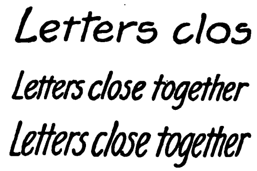

FIG. 38 (*e*).

(*f*) *Height of letters.* A good height for lower-case letters is $\frac{1}{16}''$, the height of numerals being slightly greater. This makes the height of capital letters a little less than $\frac{1}{8}''$. The effect when these heights are used may be judged by the field notes shown in Chapter XV.

39. Final Suggestions on Taking Notes. Remember that the extra time required in making field notes plain will probably be saved when the notes are plotted. Never let anything appear in the notes that can be misunderstood or misinterpreted, especially numerals. Usually there will be more to the notes for a given portion of the work than at first appears; the sketch will grow more complicated and there will be unexpected dimensions to be recorded; make a generous allowance for this in starting a page of notes so that they will not become crowded. In level notes and in most of the forms of transit notes it is better to omit every other line, thus leaving room for corrections. Be neat and systematic in keeping notes for field work, and the habits thus formed will be of great value in many other lines of work.

40. Special Directions for Class Work.

(1) A "party note-book" will be furnished for each party. Each student is expected to keep the notes of the party of which he is a member "copied up" in his own note-book, this book to be a *duplicate* of the party book. Copy *everything*, including mistakes and their corrections. Do not copy the notes of other parties.

(2) Pages should be numbered to correspond to those in the party note-book.

(3) In beginning the notes of a survey allow a few pages for a title and an index.

(4) In a closed survey the notes should be preceded by a sketch showing all transit lines with the letters or numbers of each station. (See p. 390.) *This sketch should be completed as fast as stations are established.*

(5) Print a descriptive title at the top of each page. This title may extend across the tops of two opposite pages.

(6) Each page in the party note-book should be dated and signed by the note-keeper as the work progresses.

(7) The notes in the party note-book should be inspected each day by the instructor in charge of the party. Students should not copy notes on any page which is not marked "Inspected" and signed by the instructor.

(8) Do not "scribble" in either the party note-book or your own note-book: carry a scribbling-pad in the pocket.

CHAPTER IV

CHAINING

In this chapter are given general directions for the use of the steel tape; many of them apply equally well to the use of the surveyor's chain. For a *description* of different chains and tapes, and for the methods of repairing and standardizing tapes, consult the chapter on "Chains and Tapes" in Part III.

41. Chaining. A term which originally meant measuring with a chain, but as now used it denotes measuring with either the chain or the tape.

42. Importance of Chaining. Linear measurement is the basis of all surveying. No matter how accurately angles may be measured the survey can be no more accurate than the chaining. Not only students but chainmen in actual work are apt to look upon chaining as drudgery calling for no special ability. As a matter of fact wide-awake, careful chainmen are scarce and are valued accordingly. It is not easy to avoid mistakes, nor is it a simple thing to chain a line when great accuracy is required. To chain a line twice and get the same result is no indication of great accuracy unless certain constant errors have been eliminated. (See Chapter V.) It is worth while, therefore, to study the subject of chaining carefully, especially anything that will help to avoid mistakes and eliminate errors.

43. Care of the Steel Tape. Steel tapes are easily broken. The chief danger is in pulling on the tape when there is a loop or kink in it; consequently guard against this. Do not jerk the tape needlessly, or step on it when it is on soft ground, or allow vehicles to run over it, or bend it into sharp corners. If the tape gets wet, wipe it dry before putting it away.

44. To Do Up a Steel Tape in a Figure 8. If a steel tape is gathered in in five-foot lengths *without being allowed to turn over*, it will fall of itself into concentric folds, and after the tape is all in, these folds may be crossed and tied at the center, forming a figure 8. This is a very convenient form, and in some respects it is better than winding the tape on a reel. Any quick method of bringing the five-foot marks together in one hand may be used. The following is recommended: Stretch the tape straight on the ground its entire length. Stand at one end squarely facing the tape, so that you look along its length. Take the end of the tape in the right hand, *palm of the hand up,* and swing the right hand behind you, *pulling the tape through the left hand,* held palm up. As each five-foot mark comes along, the left hand seizes it and lays it over the palm of the right hand *without permitting the tape to turn over.* The right hand closes on it and swings behind for another five-foot length, and thus the tape is gathered in in five-foot lengths. The swing behind is about as long as the average man can take readily. Once the knack is acquired it is hardly necessary to watch for the five-foot marks, and the whole thing is done very rapidly.

45. To Undo a Steel Tape (when it is in a figure 8). Hold in one hand and let out in five-foot lengths, *one at a time,* the reverse of doing it up. Attempting to let it out all at once will almost surely result in an annoying tangle.

46. To Do Up a Chain. Beginning at the *middle,* fold up the *doubled* chain, taking hold of two links at a time with one hand and laying each pair obliquely across the others so that the chain will have an hour-glass shape, easy to strap up and carry.

47. To Undo a Chain. Take the handles together in the left hand and the rest of the chain in the right hand, allowing a few links nearest the handle to drop off. Throw the chain in a direction opposite to that in which the chaining is to be done. The chain should straighten out doubled. As the head chainman, taking one end of the chain, proceeds along the line every link of the chain passes the rear chainman, who is thus able to detect and straighten all links that are bent.

(*a*) *Remark.* In doing up the chain some surveyors prefer to begin at the end of the chain instead of the middle, folding up two links at a time as before. In this case the chain may be undone by simply laying it down at the starting-point, the head chainman taking the handle which is *on top* and walking towards the further end of the line, while the rear chainman allows the chain to slip through his hands examining the links as before. This method saves the wear and tear on the chain that comes from throwing it out.

48. To Read a Tape. The easiest tape to read is one that is graduated continuously, the numbers starting from zero at one end and increasing in one direction only. If all tapes were thus numbered and if each foot of the tape were subdivided, many mistakes would be avoided and the somewhat elaborate precautions outlined in the succeeding articles would be unnecessary. Many tapes, however, are numbered from each end towards the middle to correspond to the numbering on chains. This gives rise to the common mistake of reading from the wrong end, as, for example, reading 40 ft. instead of 60 ft. When every fifth foot of the tape is numbered, even though the numbers run in one direction only, the mistake of reading on the wrong side of a number is often made, as, for example, 41 ft. instead of 39 ft. Unless every foot is numbered, therefore, make this an inviolable rule: *Read the number on each side of the required reading.* For example, in the mistake just mentioned the chainman noticed that the reading was 1 ft. from the mark numbered 40, and jumped to the conclusion that the reading was 41; if he had read the number on the other side (i.e., 35) he would have seen that the reading was between 35 and 40 and thus have corrected his mistake.

The use of tapes in which only the end foot is subdivided gives rise to other mistakes which are avoided only by taking precautions similar to those outlined in Art. 50 (*a*).

49. To Read a Chain. If the chain is a 66-ft. chain (Gunter's chain) measurements are made in chains and links, but as each link is $\frac{1}{100}$ of a chain the links are recorded as decimal parts of a chain. Thus, 8 chains and 39 links is 8.39 chains, and 8 chains and 8 links is 8.08 chains, *not* 8.8 chains.

When 100-ft. chains are used measurements are kept in feet as usual. Mistakes of reading in the wrong direction should be avoided by reading the number each side of the required reading.

(*a*) *Remark.* Chains are heavy and clumsy, difficult to stretch tight and straight, change their length rapidly from wear, and are at the best only good for comparatively rough measurements. Tapes are now made of such strength and durability that even in rough country they will hold their own with chains, and it is only a question of time when they will probably replace them altogether.

50. General Method of Chaining. The two chainmen are called *head chainman* and *rear chainman.* In ordinary work, however, whichever chainman happens to be nearer the point towards which the chaining is being done acts as head chainman. The method of chaining depends upon whether the distance to be measured is less or greater than the length of the tape.

(*a*) *Measurements less than the length of the tape.* If the tape is graduated continuously from zero, every foot being subdivided into tenths, no special directions are required. Whichever chainman has the zero end of the tape holds it at one point and the other chainman reads the tape at the other point. In many tapes, however, although the foot at each end of the tape is subdivided, there are no subdivisions in the intermediate feet. Suppose it is desired to measure a distance from A to B which is less than the length of such a tape. One chainman holds the zero end at A, the other chainman notes which even foot-mark is just beyond B, and holds that mark at B, calling out what it is. The tape is tightened and the zero is no longer at A. The first chainman *mentally subtracts a foot* and adds the tenths, reading the tenths *not from the end of the tape* but from the *first* foot-mark. The important thing to notice is that the final reading is made by the chainman who is at the *end* of the tape. As he calls out the final reading, the other chainman must listen to make sure that a foot has been subtracted from the reading which he first gave, and if it has been, he calls out "All right."

(*b*) *Illustration.* The distance from A to B is 60.8 feet. One chainman holds the zero at A. The other chainman finding that the 61 foot-mark falls just beyond B, holds that mark at B and calls out "61." The first chainman pulls back the tape until it is tight, finds that A is eight tenths from the first foot-mark (not from the end of tape or zero), mentally subtracts a foot from 61 and calls out "60.8." The other chainman, noticing that a foot has been subtracted from his own reading of 61, calls out "All right."

(*c*) *Note.* Students are urged to follow out *precisely* the directions just given. There are other methods, of course, but unless some one method is adopted and always used, there is sure to be trouble. If the chainman at the end of the tape always gives the final reading, no question arises as to whether the tenths are to be added or subtracted. Suppose, for example, in the preceding illustration the chainman at the end simply calls out "Eight tenths"; if the other chainman has to make the final reading, he may think that it is eight tenths from the *end*, or eight tenths "off," and will then read 60.2 instead of 60.8. Remember, then, that it is *the chainman at the end of the tape* who makes the final reading, and that it is understood that the other chainman *never subtracts a foot* before calling out the even foot-reading.

(*d*) *Remark.* It will be noticed that nearly every precaution in reading the tape thus far mentioned would be unnecessary if every foot on the tape were numbered, and if each foot were subdivided. The first cost of such a tape is greater, but in the end this is more than made up by the time saved and the mistakes avoided.

(*e*) *Measurements greater than the length of the tape.* In chaining between two points when the distance is greater than the length of tape, three things must be kept in mind: (1) The chaining must be done in as direct a line as possible: (2) The point where the end of the tape comes each time must be carefully marked; (3) The tape must be stretched straight and tight each time. Any method may be used that accomplishes these things without loss of time. It is customary for the rear chainman to line the head chainman in every time he moves ahead, although in very accurate work this is done from the transit. Chaining-pins are ordinarily used to mark the points at the end of the tape; for accurate work stakes may be set and a tack or a knife-mark on the stake used to indicate the exact point. The chainmen should agree upon some set of signals similar to those used in the following illustration.

(*f*) *Illustration.* Required to chain from *A* to *B*, distance 634.3 feet with a 100-foot tape. Rear chainman (R.C.) holds the zero end of the tape at *A* and motions the head chainman (H.C.) into line with a pole held or set at *B*. R.C. sees that the tape is straight and tight, and calls out "Stick," or "All right," and H.C. "sticks" the pin at the end of the tape and calls out "Stuck." R.C. *drops* his end of the tape and H.C. moves on, dragging the tape after him and keeping himself approximately in line by looking back at *A* over the pin already set. When the zero end of the tape reaches the pin, R.C., who is following, calls out "Chain," a signal for H.C. to stop. R.C. then holds the zero at the pin and the process is repeated. The short distance from the last pin to the station (34.3 feet) is measured according to the method of paragraph (*a*). Counting the last pin that was set, the rear chainman, if he gathered up the pins as he went along, should have six pins, which tells him that up to that point the tape was stretched six times, or it is 600 feet.

51. End Graduation. It is sometimes uncertain just what points of a tape to take for its ends. For example, the foot at each end may or may not include the handle. To find out, compare with some intermediate foot of the tape.

52. Setting the Chaining-pins. Stick the pin in the ground slanting, at right angles to the line, so that it slides under the tape, with its *center* opposite the graduation-point on the tape. On the whole this is better than sticking a pin with *its edge* against the end of the tape or opposite a graduation.

53. Chaining on a Slope. In measuring a slope there are two ways of getting the *horizontal* distance, which is the distance usually required in surveying: (1) To measure along the slope and correct this measurement by calculation; (2) To take such precautions that the tape, instead of resting on the ground, will be *horizontal* every time it is stretched for a measurement.

(*a*) *First method. The calculated correction for slope.* If c is the difference between the length on a slope and the horizontal projection, d the difference in elevation of the two ends of the slope, and s the length of the slope, then approximately $c=\frac{d^2}{2s}$.

Or if the angle x which the slope makes with the horizontal is known, then the horizontal projection of the slope is $s \cos x$.

(*b*) *Second method. Chaining with tape horizontal.* Usually it is easier and better to chain down-hill than to chain up-hill. The up-hill end of the tape is held on the ground; the down-hill end is held high enough off the ground to make the tape horizontal, and the point on the ground under the down-hill end is found by using the plumb-bob. The process is repeated in a series of steps, the tape corresponding to a tread, and the plumb-line to a riser. Chaining up-hill is the reverse of this process, but it is more difficult. In either case there is but one good way of holding the down-hill end of the tape. Stand with one side of the body towards the other chainman, the tape passing across in front so that when it is stretched the hand and arm holding the tape are pulled against the other side of the body. In this way the weight of the body resists the pull, steadies the end of the tape, and leaves one hand free to manipulate the plumb-line.

(*c*) *Practical suggestions.* Use short lengths of the tape, the steeper the slope the shorter the length; on very steep slopes use ten-foot lengths or shorter. Dropping pebbles, chain-pins, or plumb-bob from the raised end of the tape to find the point underneath will give only approximate results. The transit-pole, held vertical by means of a plumb-line, is sometimes used. Accurate results may be obtained by driving stakes in line, short distances apart, and measuring between them, using the plumb-line; this will often require more than two chainmen. Better results still will follow if a tripod is placed over the down-hill stake to support the end of the tape and hold the plumb-line steady. The tripod belonging to the transit or level, detached from the instrument, will answer the purpose, or, if much of such chaining is to be done, construct two rough tripods. Four men with two tripods can chain a steep slope accurately and quickly; while three of the men are measuring between two stakes, a fourth man can be setting the tripod which is not in use over the next down-hill stake.

54. Precautions to Insure Accuracy. In ordinary work no account is taken of small errors in the length of the tape, and temperature, pull, and sag are neglected. Ordinary chain-pins are used; the rear chainman lines the head chainman in, and slopes are measured by holding the tape horizontal judged by the eye alone. In very accurate work the length of tape is carefully tested; the temperature noted during chaining and the correction applied; the pull kept standard with the spring-balance; the tape supported at short intervals to eliminate sag, or else the sag correction is determined; some device, as a fine scratch on top of a stake, marks the chain-lengths in place of the chain-pins. The chainmen are lined in from the transit, or else stakes are set in line for the use of the chainmen; measurements are made horizontal by stakes set on a level, or if on a grade, the correction for the slope is determined. For work between ordinary work and extremely accurate work precautions must be adopted to correspond. (See the next chapter.)

55. Errors. Some of the things which chainmen should know concerning errors in chaining are:

1. Sources and relative importance of errors.
2. Methods for eliminating or correcting errors.
3. Customary limits of error for different kinds of work.
4. Field-work requirements corresponding to different limits of error.

These subjects are more fully discussed in the next chapter.

56. Summary. (From preceding pages.)

(*a*) *Some things a chainman should do.*

1. Find the exact points which mark the ends of a tape.
2. Find by what method the tape is graduated, and note how it must be read.
3. Agree on a set of signals with the other members of the party.
4. Find by trial what an 8- or 12-pound pull feels like.
5. See that the tape is straight and exert a steady standard pull while measuring.
6. Find out the limit of error and work accordingly.
7. Keep the tape horizontal and use short lengths on steep slopes.
8. Line in and stick the chain-pins correctly.
9. Keep the right count of tape-lengths.
10. Unless every foot on the tape is numbered make sure of a reading by looking at the number on each side of it.
11. When only the end foot is subdivided the chainman at the *end* of the tape should invariably make the final reading.
12. The head chainman should keep himself approximately "in line" by looking backward over the line. He should lift his end of the tape to straighten it, and then let it down slowly on the line to be sure that it remains straight. In accurate work, after the pin is stuck, he should lift the chain as before and let it down again to test the position of the pin; frequently

this test will show that the pin has not been set in exactly the right place.

13. When calling out a measurement to a note-keeper make sure that he gets it by having him repeat it after you.

14. In extremely accurate work use the spring-balance and thermometer, and eliminate sag by supporting the tape. In such work it is more necessary than ever to compare the tape with some standard length.

15. Be constantly on the alert to preserve the tape from injury. When not in use, stretch it beside a fence or in some out-of-the-way place where it will not be stepped on. Keep in touch with the work so as to notice any measurements inadvertently omitted, and to anticipate directions from the head of the party

(*b*) *Some things a chainman should not do.*

1. Don't jerk the tape, or pull on it when it is kinked, or let vehicles run over it, or bend it sharply into corners, or step on it. Don't lose the handles: tie them on.

2. Don't split hairs lining in.

3. Don't forget to subtract a foot from the reading if it ought to be subtracted. Don't read from the wrong end of the tape.

4. Don't disturb the chaining-pin, or let it be pulled over.

5. Don't let go of the tape until the other chainman signals.

6. Don't stick the pin until the other chainman signals.

7. Don't pull up the pin until sure that it will be no longer needed.

8. Don't hit the leg of the transit when measuring near it.

9. The rear chainman should not get in the line of sight when the transitman is lining in the head chainman.

10. If the tape is in a leather or metallic box do not keep pulling the tape out and winding it up; if much work is to be done, it may be better to detach the box.

CHAPTER V

ERRORS IN LINEAR MEASUREMENTS

In this chapter sources of error in chaining are pointed out, the relative importance of errors from different sources is discussed, and methods of correction or elimination are given. Limits of precision are suggested for different kinds of work, and field-work requirements are indicated for different limits of precision. A general classification of errors is given on p. 9, and it is necessary in studying this chapter to keep clearly in mind the difference between *accidental* and *constant* errors, and between *cumulative* and *compensating* errors. The distinction between *discrepancy* and *error* is also an important one, since the discrepancy between duplicate measurements may or may not indicate the precision of the work (see Art. 20, p. 11).

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57. Sources of Error in Chaining.

(1) Error in the length and in the graduations of the tape.*

(2) Errors due to temperature, and to other natural sources.

(3) Errors in manipulation, — tape not horizontal, not stretched tight, imperfect alignment, sag, pull, and marking tape-lengths.

(4) Personal errors. Mistakes in reading the tape. Blunders, —such, for example, as occur in counting tape-lengths.

58. Error Expressed by a Ratio. $\frac{1}{500}$, $\frac{1}{1000}$, $\frac{1}{12000}$, etc., mean an error of 1 part in 500, or 1 part in 1000, or 1 part in 12 000, and so on.

Illustration. An error of 0.05 feet in a measurement of 714.85 feet is: $0.05 \div 714.85 = \frac{1}{14297}$, or about $\frac{1}{14300}$.

59. Error in the Length of the Tape. *Cumulative* and important. *Plus* or *minus* according to whether the tape is too short or too long. If a tape is *too short*, the measured distance between two fixed points will be *longer* than the true distance (error plus); and contrariwise if the tape is *too long*, the measured distance will be *shorter* than the true distance (error minus).

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* The error in the length of a steel tape is relatively small and remains so, but in a chain it often becomes large from connecting rings being stretched, links being bent, or joints becoming worn.

(*a*) *Illustration.* The *true* distance between two points is 400 feet. If, however, this distance is measured with a 100-ft. tape which is 0.02 ft. too short (i.e., length = 99.98 ft.), the result will be 400 ÷ 0.9998 = 400.08 ft. If, on the other hand, the tape is 0.02 ft. too long (i.e., length = 100.02 ft.), the result will be 400 ÷ 1.0002 = 399.92 ft.

(*b*) *To apply the tape correction.* The tested length of a 100-ft. tape multiplied by 0.01 gives the length of the tape in terms of the standard (i.e., in terms of 100 ft.). Thus a tape 0.1 ft. too long is 100.1 ft. long, or 1.001 times what it should be. Call this constant for any tape *C*, then use the

Rule. Multiply all measurements by C to reduce to true measurements.

(*c*) *Illustration.* In the preceding illustration the constant for the tape which is 0.02 ft. too short is 0.9998, and for the tape which is 0.02 ft. too long it is 1.0002; hence

Measured Distance	×	Constant	=	True Distance.
400.08	×	0.9998	=	400 ft.
399.92	×	1.0002	=	400 ft.

In establishing one point at a required distance from another point the rule is reversed. *Rule. Divide the required distance by C: the result is the actual (false) distance to measure off.*

(*d*) *Illustration.* It is required to establish one point 400 ft. from another point with a tape 0.02 ft. too short (i.e., $C = 0.9998$); then the *false* distance which must be measured off is 400 ÷ 0.9998 = 400.08 ft.

60. Error due to the Tape not being Horizontal. Error is *cumulative, plus,* and important. Eliminated by the method of Art. 53.

61. Error when the Tape is not Stretched Tight and Straight. (Ends in line, tape lying on surface of ground, but not stretched tight.) *Cumulative, plus,* and important.

62. Error due to Temperature. Error is *cumulative,—plus* when the temperature is below the standard temperature for the tape, *minus* when it is above. The length of a 100-ft. steel tape may be changed more than ½ inch by an extreme change in temperature from 20° to 90° Fah. In accurate work this source of error must, as far as possible, be eliminated by noting the temperature and applying the correction. In ordinary work temperature may be neglected. Base-lines are usually measured on a cloudy day (sometimes at night), so that the tape may be nearer the temperature of the atmosphere. The temperature of a tape lying in the sun is difficult to obtain.

(*a*) *Note.* If the coefficient of expansion of a steel tape is not known, use 0.0000065 per 1° Fah. or 100 × 0.0000065 = 0.00065 ft. change per 1° Fah. for a tape 100 ft. long. Thus 70 × 0.00065 = 0.0455 ft. for extremes of 70° change for a 100-foot tape. The following approximate rate of change is, perhaps, the easiest to remember. *The length of a 100-ft. steel tape changes $\frac{1}{100}$ of a foot for every 15° (Fah.) change of temperature.*

(*b*) *Remarks.* The error due to temperature may be quite large, and in certain lines of work it is too often neglected by surveyors. A city block, for example, 400 ft. to 500 ft. long, measured with the same tape may be found apparently 2″ or 3″ shorter in summer than in winter, if it is in a climate where the thermometer ranges from 0° to 90° (Fah.).

(*c*) *An experiment.* In order that the effect of even a small *change in temperature* may be perfectly evident, and incidentally to show the effect also of the sun's rays, the students in the author's classes are required to make the following experiment: About noon on a clear day a 100-ft. steel tape is stretched across supporting stakes, set on line 20 ft. apart, with their tops on a level. A 12-pound pull is kept by means of a spring-balance, and a fine scratch is made on the top of the stake at each end of the tape. At about 5 P.M. of the same day the tape is again stretched under the same conditions. The scratches made at noon are often found to be $\frac{1}{8}''$ or more farther apart *apparently*, than when they were made. This shrinkage of $\frac{1}{8}''$ in the tape corresponds to a fall of 15° in temperature, but the change in the atmosphere may have been much less than this. A large part is due to the sun, which at noon shone on the tape with much greater intensity than at five o'clock. Still more striking results may be obtained by repeating the experiment with the tape stretched in contact with the warm ground or a hot pavement.

63. Error in Alignment. Error is *cumulative* and *plus*,—relatively unimportant. Much time is often wasted in splitting hairs when "lining in" with the transit. Setting pins as much as six inches, first one side and then the other of the line, affects measurements but little. In ordinary work the chainmen line themselves in. The rear chainman moves his hand right or left to line the head chainman in: a vigorous motion, move quite a distance; a slight motion, a small distance. Much time can be saved if the head chainman will line himself in, approximately, by looking backward over the head of the rear chainman to the point or station from which the measurement is being made. In accurate work the transitman "lines in" the head chainman, in which case the rear chainman must keep out of the line of sight. Range-poles are sometimes used in lining in.

64. Error due to Sag. Error is *cumulative* and *plus*. There are three ways of eliminating error due to sag. (1) Put an extra pull on the tape which will stretch it enough to offset the sag—quite difficult to do. (2) Calculate the correction from the equation of the curve and apply the correction.* If, how-

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* The following formulas for sag and pull are derived in Johnson's Surveying, p. 505:

c = excess (in inches) in length of curve over the linear distance between supports; d = distance between supports in inches; w = weight of one inch of tape in pounds; P = pull in pounds.

$$c=\frac{d}{24}\left(\frac{wd}{P}\right)^2.$$

C_p = correction for pull in inches; l = length of tape in inches; S = cross-sectional area of tape in square inches; E = modulus of elasticity. (If not known for the tape it may be taken at 28,000,000)

$$C_p=\frac{Pl}{SE}.$$

The pull required to balance sag is

$$P_n=\sqrt[3]{\frac{w^2d^2SE}{24}}.$$ (This pull is called normal tension.)

ever, work is important enough for sag to be taken into account it will usually warrant the expense of (3) providing supports near enough together to practically eliminate sag.

65. Errors due to Uneven Pull. If the pull is always too large or always too small the error becomes cumulative otherwise it may be compensating. Pull is unimportant except in very accurate work; ordinarily guessed at; it should be strong but steady, from 8 to 12 lbs. One can guess nearer if he actually experiments with a spring-balance a few times. An ordinary 100-ft. steel tape stretches less than $\frac{1}{100}$ of an inch per pound of pull.

66. Errors in Marking Tape-lengths. *Compensating,* except for very short lines; important. When chain-pins are used, observe the directions on p. 35 for setting pins. In accurate work the error is greatly reduced by making fine scratches to mark the tape-lengths. Disturbing pins after they are set must be classed as a blunder. One of the greatest sources of error in marking tape-lengths is in plumbing down from an end of the tape held off the ground.

67. Errors in Reading the Tape. *Compensating.* Errors in reading the tape are not likely to be important unless large enough to be classed as actual mistakes or blunders. In very accurate work, however, such as the measurements of base-lines, they become important, and some device provided with a microscope or magnifying-glass is often used for observing graduation-marks on the tape. If the tape is graduated continuously throughout its length, important sources of mistakes are avoided. Otherwise chainmen forget to subtract the one foot from the first reading (see Art. 50, p. 34), or they read from the wrong end of the tape, especially if the distance is only a little greater or less than half the length of the tape. Thus, 39 ft. instead of 61 ft., or 55 ft. instead of 45 ft. (see p. 33).

Another common mistake is to read on the wrong side of a number, as, for example, 41 instead of 39 (see p. 33). This mistake is most likely to occur when the numerals on the tape are placed at the five-foot points only, but even when using a tape that has every foot numbered it is best to follow the rule on page 33 and read the number on each side of the given point.

Blunders in keeping count of tape-lengths are very common. When pins are used such blunders may be avoided by keeping tally with the pins (p. 35).

68. Sources of Errors in Chaining.—Summary.

1.	Incorrect length of tape	Cumulative	+ or −
2.	Tape not stretched *horizontally*	Cumulative	+
3.	Tape not stretched tight and straight, but both ends in line	Cumulative	+
4.	Error due to temperature	Cumulative	+ or −
5.	Error in alignment	Cumulative	+
6.	Error due to sag	Cumulative	+
7.	Variation in pull	Compensating	±
8.	Error in setting chain-pins	Compensating	±
9.	Disturbing pins after they are set	Blunder	
10.	Errors in reading the tape	Mistake	
11	Incorrect count of tape-lengths	Blunder	

69. Relative Importance of Sources of Error. As an aid to the discussion of this subject a table has been prepared showing for each source of error the conditions which will cause an error of 0.01 ft. (or about ⅛ in.) in a distance of 100 ft. Some of the values given will differ for different tapes, but they are approximately correct for the average 100 ft. steel tape.

	Conditions.	Error.
1.	*Length of tape.* A difference of ⅛ in. from standard length	0.01 ft.
2.	*Tape not horizontal.* One end 1.4 ft. higher than the other end	0.01 "
3.	*Tape not stretched tight.* Center of tape 0.7 ft. or 8⅜ in. out of line	0.01 "
4.	*Temperature.* For every fifteen degrees change	0.01 "
5.	*Alignment.* One end of tape in line, the other 1.4 ft. out of line	0.01 "
6.	*Sag.* The middle of the tape 0.61 ft. or 7⅜ in. below the ends	0.01 "
7.	*Pull.* For every 15 pounds pull	0.01 "
8.	*Marking tape-lengths.* Error of ⅛ in. in marking or plumbing	0.01 "
9.	*Reading tape.* Error of ⅛ in. in reading	0.01 "

(*a*) From inspection of this table it is evident that an error of 0.01 ft. in 100 ft. is much more likely to occur from some sources than from others. It is quite likely that the temperature will vary 15°, but not at all likely that the pull will vary 15 pounds. Chainmen are more likely to chain along a slope without realizing that one end of the tape is 1.4 ft. lower than the other than they are to allow one end of the tape to get 1.4 ft. out of line when chaining between two points.

A steel tape, unknown to the surveyor, may be ⅛″ too long or too short, and though it may not occur to him to test the length of the tape, yet he is very careful not to make an error of ⅛ in. in marking tape-lengths (an error less important because compensating).

In ordinary work certain errors like that due to temperature are unimportant, because it is not necessary to take them into account in order to gain the required precision, and for that class of work they are said to be inappreciable (see Art. 23); but for city work and other accurate work these same errors become appreciable and important.

(*b*) In judging of the relative importance of errors the following additional points are to be kept in mind:

1. Cumulative errors are more important than compensating errors, but, as shown above, not all cumulative errors are equally important.

2. In a short line a compensating error may occur only once or twice and thus fail to be compensating. For example, if the length of a line is a little less than two tape-lengths, any error in marking the end of the first tape-length is not compensating. The more tape-lengths there are in a line the more likely are such errors to be truly compensating, and the more times a line is measured the more likely are accidental errors to disappear from the mean. (See p. 13.)

3. One cumulative error sometimes balances another cumulative error. For example, a strong pull may offset sag, or high temperature may offset a slight shortage in the length of a tape.

4. Other things being equal, it is most important to guard against those errors which are most likely to occur.

70. Limits of Error. The allowable error in chaining is often determined by the time and money spent on a survey and not by the importance of obtaining accurate results. Assuming, however, that the surveyor can proceed regardless of time or expense, he should for any particular piece of work determine the limit of error within which the chainmen must come. The purpose of a survey determines this, but different parts of the same survey may require different degrees of accuracy. For example, a survey is made of a city lot with buildings on it, to be plotted to a scale of 1″=20 ft. Unless the dimensions of the buildings are to be put on the map or kept for record, no great care is necessary in measuring them, for $\frac{1}{20}$ of an inch on the map equals a foot, and an error of less than $\frac{1}{2}$ ft. will hardly show. On the other hand, the lengths of boundary-lines should be determined as accurately as possible, *because these* lengths should be written on the map (by numerals and dimension lines) regardless of scale, and if incorrect they may give rise to boundary disputes. In general, then, *the greatest pains should be taken in measuring distances to be preserved in records or on maps in actual figures, while distances that will be scaled from a map, or distances needed simply to plot a map, may be measured less carefully.* It is one thing to know the precision required, it is another thing to obtain it. Some conditions affecting accuracy of the work are: (1) Correctness and fineness of the graduations on the tape; (2) Nature of the ground chained, hilly or level, open or covered with trees and underbrush; (3) Speed, i.e., time spent as determined by the importance of the work or the allowable cost; (4) Weather—clear or cloudy, very hot or very cold, calm or windy; (5) The effectiveness of devices for stretching and supporting the tape, and for measuring temperature; (6) The ability of the chainmen, and their willingness to take pains.

71. Probable Error of a Line N Tape-lengths Long. (*a*) A line N tape-lengths long may be regarded as the *sum* of N measurements. If the probable error of each measurement (a tape-length long) is the same, and this probable error is represented by r, then from formula (4), p. 18, $R_s = \sqrt{Nr^2}$ or $R_s = r\sqrt{N}$. Thus it is seen that the probable error of the total observed length of a line may be found by multiplying the probable error of one tape-length by the square root of the number of tape-lengths (i.e., length of the line in tape-lengths). The more completely the constant errors are eliminated, and the more completely the other assumptions of "least squares" are fulfilled, the nearer this formula is to the truth. Its use, therefore, is chiefly confined to work of great precision.

(*b*) In ordinary chaining the accuracy of the work is judged in most cases by the discrepancy between two measurements of the line made with the same care. In such a case constant errors do not affect the discrepancy. (Why?) And since this discrepancy is largely due to accidental errors, it will *not* vary in proportion to the length of the line. On the other hand, the number of tape-lengths is usually too small to warrant the application of the theory of least squares without reservation. All that can be said is that if the discrepancy between two measurements for a hundred feet (or tape-length) is a *certain amount d, it* is likely that the discrepancy for any other length will be $D = d\sqrt{N}$, where N is the number of tape-lengths. This is considerably nearer the truth than to say $D = dN$.

(c) *Remark.* In rough chaining constant errors are likely to predominate, but in work of great precision, where pains have been taken to eliminate constant errors, accidental errors are likely to predominate. Hence in rough work the true or unknowable error is likely to vary more nearly as the *length* of the line, but as the precision becomes greater and greater the true error will vary more and more nearly as the square root of the number of tape-lengths. Notice that this applies to the *true* or unknowable errors and not to discrepancies.

72. To Determine Coefficients of Precision. (a) When the chaining is done under such conditions as to warrant the use of the method of least squares, the value of r may be determined by measuring a line of average length a number of times under the conditions which will prevail in the measurement of other lines; then from the formulas on p. 17 determine the error of a single observation. Substitute this value for R_s in the formula $R_s = r\sqrt{N}$, substitute for N its value, and solve for r. (See p. 59 for a complete illustration.)

Another method is to make duplicate measurements of several lines of different lengths, and find r from the formula $r = 0.4769\sqrt{\frac{\Sigma pd^2}{n}}$, where d = difference between two measurements of same line, Σ = sum of, n = number of lines, and p is the weight for any one line, which varies inversely as its length, or $p = \frac{1}{l}$. (See Merriman's *Method of Least Squares*, p. 103.)

(b) For ordinary chaining where the use of the method of least squares is hardly justified, the value of d in $D = d\sqrt{N}$ may be obtained by measuring a line of average length a number of times under the prevailing conditions of field work, and substituting for D the mean of the discrepancies, solve for d.

(c) *Illustration.* The average length of lines in a survey will be, say, about 400 ft. The chaining is to be done under favorable conditions. Four measurements of a line, made as nearly as possible with the same speed and under the same conditions that will prevail in the survey, give 400.1, 399.95, 400.05, 400.15. Comparing the first measurement with each of the other three, the second with the succeeding two, the third with the fourth, six discrepancies are obtained and the mean is

$$(0.15 + 0.05 + 0.05 + 0.1 + 0.2 + 0.1) \div 6 = 0.1083.$$

Then $0.1083 = d\sqrt{4}$, or $d = 0.0542$. For any other line the discrepancy may be expected to be about $0.0542\sqrt{N}$, or for a line say 900 ft. long $0.0542\sqrt{9} = 0.1026$ ft.

73. Customary Limits of Error. (a) The method suggested in the preceding illustration for comparing measurements of different lines is for guidance only, and in determining the coefficient d it is absurd to carry results to several decimal places as was done in that illustration. When it is not worth while to determine d in the field it may be assumed as 0.01, 0.02, 0.03, or some other quantity which accords with experience. In order to give some definite idea of what constitutes good chaining under favorable conditions, coefficients are suggested in Table I, on the next page, for different classes of work. For hilly, rough country they must be modified, especially if underbrush, bad weather, and other unfavorable conditions prevail. Under such conditions coefficients called fairly good in the table may become good or even excellent, and the same is true, also, when the purpose of the survey does not require very accurate results.

The impossibility of fixing any standard for chaining owing to varying conditions is of course apparent, and the coefficients given are for guidance only; they should be modified, when necessary, to accord with the dictates of common sense and experience. It is assumed that the duplicate measurements, from which the discrepancy in any given case is determined, are made under approximately the same conditions —same chainmen, same tape, same temperature, and same speed.

The upper part of the table suggests coefficients, and the lower part gives the allowable discrepancies for lines of different lengths and for different coefficients.

(*b*) *Illustration.* A line is chained twice under favorable conditions. A steel tape and plumb-bob are used, the tape being kept horizontal by the eye. The two measurements are 828.91 and 828.88. The discrepancy is 0.03. In Table I the nearest value for 800 ft. is 0.028, which corresponds to a *coefficient* of 0.01. From the upper part of the table it is seen that 0.01 is *excellent* for the conditions under which the chaining was done. If a thermometer and spring-balance had been used and other precautions taken to correspond, it is seen from the table that 0.01 is considered *good* or *fairly good.*

74. Limits of Precision Expressed by a Ratio. (*a*) The method of expressing errors by ratios (see p. 39) is used in two ways to indicate limits of precision, viz.: (1) To express the ratio of the probable error of a single measurement or the probable error of the mean of several measurements to the length of the line chained; (2) To express the ratio of a discrepancy to the length of the line chained. For reasons given in Art. 71 the ratio of the discrepancy to the *length* of the line may be considered to indicate closely the variation to be expected *in comparatively short lines*, but as the lines become longer the discrepancy will vary more nearly as the *square root* of the length. Table II has been made out on the assumption that the conditions for chaining are favorable; for unfavorable conditions the values must be modified. The warning against the injudicious use of Table I holds good for this table also. For lines less than 1000 ft. long the values in the two tables correspond closely (see Table III), and hence it makes little difference which is used, although Table II is in the more convenient form. For lines over 1000 ft. use Table I.

TABLE I

COEFFICIENTS OF PRECISION FOR CHAINING

Conditions: Fairly open, level ground. Measurements made with	Coefficients for different results		
	Excellent	Good	Fairly Good
Chain, plumb-bob, chain-pins, average speed.	0.03 to 0.04	0.05 to 0.08	0.09 to 0.1
Steel tape, plumb-bob, chain-pins, average speed.	0.01 to 0.02	0.02 to 0.04	0.04 to 0.05
Steel tape, spring-balance, thermom., sag eliminated	0.002 to 0.005	0.005 to 0.01	0.01 to 0.02

ALLOWABLE DISCREPANCIES IN CHAINING

Length	Discrepancies for different values of coefficient "*d*"									
100	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
300	0.017	0.035	0.052	0.07	0.09	0.10	0.12	0.14	0.16	0.17
400	0.020	0.040	0.060	0.08	0.10	0.12	0.14	0.16	0.18	0.20
600	0.024	0.049	0.073	0.10	0.12	0.15	0.17	0.20	0.22	0.24
800	0.028	0.057	0.085	0.11	0.14	0.17	0.20	0.23	0.25	0.28
1000	0.032	0.063	0.095	0.13	0.16	0.19	0.22	0.25	0.28	0.32
1200	0.035	0.069	0.104	0.14	0.17	0.21	0.24	0.28	0.31	0.35
1400	0.037	0.075	0.112	0.15	0.19	0.22	0.26	0.30	0.34	0.37
1600	0.040	0.080	0.120	0.16	0.20	0.24	0.28	0.32	0.36	0.40
1800	0.042	0.085	0.127	0.17	0.21	0.26	0.30	0.34	0.38	0.42
2000	0.045	0.089	0.134	0.18	0.22	0.27	0.31	0.36	0.40	0.45
3000	0.055	0.110	0.163	0.22	0.27	0.33	0.38	0.44	0.49	0.55
4000	0.063	0.126	0.190	0.25	0.32	0.38	0.44	0.51	0.57	0.63
5000	0.071	0.141	0.212	0.28	0.35	0.42	0.49	0.57	0.64	0.71
6000	0.077	0.155	0.232	0.31	0.39	0.46	0.54	0.62	0.70	0.77
7000	0.084	0.167	0.251	0.33	0.42	0.50	0.59	0.67	0.75	0.84
8000	0.089	0.179	0.268	0.36	0.45	0.54	0.63	0.71	0.80	0.89
9000	0.095	0.190	0.285	0.38	0.47	0.57	0.66	0.76	0.85	0.95

Determine from the upper part of the table the coefficient to be used. Enter the column in the lower part of the table headed by that coefficient. Run the eye down the column until opposite the length of the line chained. The result is the discrepancy in feet which may be reasonably expected. For 0.002, 0.005, etc., use columns 0.02, 0.05, etc., adding a *decimal place*.

TABLE II
ALLOWABLE DISCREPANCIES IN CHAINING

Discrepancy of two measurements of the same line, given for different lengths					Ratio of the discrepancy to length of measured line	Ratio of the probable error of either measurement*	Ratio of the probable error of arithmetical mean*	Results
300	400	600	800	1000				
Measurements made with *chain*, plumb-bob, chain-pins, and average speed								
0.06	0.08	0.12 ft.	0.016	0.2	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{15000}$	Excellent
0.12	0.16	0.24 "	0.32	0.4	$\frac{1}{2500}$	$\frac{1}{5000}$	$\frac{1}{7500}$	Good
Measurements made with *steel tape*, plumb-bob, chain-pins, and average speed								
0.012	0.016	0.024 ft.	0.032	0.04	$\frac{1}{25000}$	$\frac{1}{50000}$	$\frac{1}{75000}$	Excellent
0.024	0.032	0.048 "	0.064	0.08	$\frac{1}{12500}$	$\frac{1}{25000}$	$\frac{1}{37500}$	Good
0.06	0.08	0.12 "	0.16	0.20	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{15000}$	Fair
Measurements made with steel tape, spring-balance, thermometer, sag eliminated, accurate alignment								
0.003	0.004	0.006 ft.	0.008	0.01	$\frac{1}{100000}$	$\frac{1}{200000}$	$\frac{1}{300000}$	Excellent
0.006	0.008	0.012 "	0.016	0.02	$\frac{1}{50000}$	$\frac{1}{100000}$	$\frac{1}{150000}$	Good
0.012	0.016	0.024 "	0.032	0.04	$\frac{1}{25000}$	$\frac{1}{50000}$	$\frac{1}{75000}$	Fairly good
0.018	0.024	0.036 "	0.048	0.06	$\frac{1}{16000}$	$\frac{1}{32000}$	$\frac{1}{48000}$	Passable

* Too much importance must not be attached to probable errors of only two observations.

(*b*) *Illustrations.* Two measurements made with a steel tape under ordinary conditions are 651.2 and 651.15. The discrepancy is 0.05. Nearest value in Table II for 600 ft. for given conditions is 0.048, or ratio is $\frac{1}{12500}$, which is considered *good*. In very accurate work this would not be considered as even passable.

TABLE III

COMPARISON OF RESULTS FROM TABLE I WITH THOSE FROM TABLE II

Length	300		600	
Tables	I	II	I	II
	Chain, plumb-bob, chain-pins, average speed			
Excellent	0.05 -0.07	0.06	0.07 -0.10	0.12
Good	0.09 -0.14	0.12	0.12 -0.20	0.24
	Steel tape, plumb-bob, chain-pins, aver. speed			
Excellent	0.017-0.035	0.012	0.024-0.049	0.024
Good	0.035-0.07	0.024	0.049-0.10	0.048
Fair	0.07 -0.09	0.06	0.10 -0.12	0.12
	Steel tape, spring-balance, thermometer, and corresponding precautions			
Excellent	0.003-0.009	0.003	0.005-0.012	0.006
Good	0.009-0.017	0.006	0.012-0.024	0.012
Fair	0.017-0.035	0.012	0.024-0.049	0.024

Length	1000		5000	
Tables	I	II	I	II
	Chain, plumb-bob, chain-pins, average speed			
Excellent	0.09 -0.13	0.20	0.21-0.28	1.0
Good	0.16 -0.25	0.40	0.35-0.57	2.0
	Steel tape, plumb-bob, chain-pins, aver. speed			
Excellent	0.032-0.063	0.04	0.07-0.14	0.2
Good	0.063-0.13	0.08	0.14-0.28	0.4
Fair	0.13 -0.16	0.20	0.28-0.35	1.0
	Steel tape, spring-balance, thermometer, and corresponding precautions			
Excellent	0.006-0.016	0.01	0.014-0.035	0.05
Good	0.016-0.032	0.02	0.035-0.071	0.1
Fair	0.032-0.063	0.04	0.07 -0.14	0.2

75. Comparison of Tables II and III. In Table III discrepancies for 300, 600, 1000, and 5000 feet taken from Table I are compared with values taken from Table II. It is seen that for lengths up to, say, 1000 ft. there is not much difference, but beyond that point the discrepancies from Table II increase much more rapidly than do those from Table I. This is of course to be expected, since in Table II discrepancies vary as the length of the line, while in Table I they vary as the square root of the length. For example, in good chaining under ordinary conditions the apparent error for 300 ft. should not exceed from 0.035 to 0.07 (Table I) or 0.024 (Table II); for 600 ft. the values are closer, 0.049 to 0.1 (Table I) and 0.048 (Table II); for 1000 ft., 0.063 to 0.13 (Table I) and 0.08 (Table *II). For 5000 ft.*, or a little less than a mile, the value from Table II has increased to 0.4 ft., which is 0.12 ft. greater than 0.28, the maximum value from Table I. The rougher the measurements the more striking is the difference between the two tables for lengths over 1000 ft., as shown by the first, second, and fifth lines in the above table for length 5000.

76. Requirements Corresponding to Different Limits of Precision. (*a*) In ascertaining the requirements for any given limit of precision the sources of error summarized in Art. 68, p. 43, must be considered. It is necessary to know what errors may be neglected and to what extent the others must be eliminated or corrected. This is the only way in which the surveyor can be reasonably sure that the *discrepancy* or the *probable error*, as the case may be, really indicates the precision of his work. Some of the questions he must settle are, how accurately to read the tape, to mark the tape-lengths, to keep the chain horizontal; to what extent sag must be eliminated, and within what limits the temperature must be known. In a general way the answers to these questions are indicated in Tables I and II. As a further aid, the following table of *approximate* values has been prepared. The table shows what amounts per 100 ft. correspond to different limits of precision for each of the different sources of error.

TABLE I.

	Errors per 100 ft. from different sources corresponding to different ratios of precision				
Ratio of precision	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{20000}$	$\frac{1}{50000}$	$\frac{1}{100000}$
Error in 100 ft.	0.02	0.01	0.005	0.002	0.001
	ft. or in.	ft. or in.	ft. or in.	ft. or in.	ft. or in.
1. Error in *length* of the tape.	0.02 $\frac{1}{4}$	0.01 $\frac{1}{8}$	0.005 $\frac{1}{16}$	0.002 $\frac{1}{40}$	0.001 $\frac{1}{80}$
2. Dif. in *elevation* of ends of tape.	2.0 24	1.4 16$\frac{3}{4}$	1.0 12	0.63 7$\frac{9}{16}$	0.44 5$\frac{3}{8}$
3. *Center* of tape out of line.	1.0 12	0.7 8$\frac{3}{8}$	0.5 6	0.3 3$\frac{5}{8}$	0.24 2$\frac{7}{8}$
4. Change in *temperature*	30°	15°	7.5°	3°	1.5°
5. One *end* of tape out of line.	2.0 24	1.4 16$\frac{3}{4}$	1.0 12	0.63 7$\frac{9}{16}$	0.44 5$\frac{1}{4}$
6. *Sag* at the middle of tape.	0.86 10$\frac{3}{8}$	0.61 7$\frac{3}{8}$	0.43 5$\frac{1}{8}$	0.27 3$\frac{1}{4}$	0.19 2$\frac{1}{4}$
7. Variation in the *pull*.	30 lbs.	15. lbs	7.5 lbs.	3 lbs.	1.5 lbs.
8. Error in *marking* tape-lengths.	0.02 $\frac{1}{4}$	0.01 $\frac{1}{8}$	0.005 $\frac{1}{16}$	0.002 $\frac{1}{40}$	0.001 $\frac{1}{80}$
9. Error in *reading* the tape.	0.02 $\frac{1}{4}$	0.01 $\frac{1}{8}$	0.005 $\frac{1}{16}$	0.002 $\frac{1}{40}$	0.001 $\frac{1}{80}$

(*b*) The chief use for Table IV is to indicate the precautions which are necessary in order to attain certain degrees of precision. Thus, for example, it is seen that a variation of 15° in temperature will cause an error of $\frac{1}{8}$″ per 100 ft. corresponding to $\frac{1}{10000}$, and a sag of $7\frac{3}{8}$″ will cause the same error. If one end of a 100-ft. tape is 12″ out of line, it will cause an error of 0.005 ft., or $\frac{1}{16}$″ (or $\frac{1}{20000}$), while it will take a variation in pull of $7\frac{1}{2}$ lbs. to cause the same error. An error of $\frac{1}{4}$″ in the length of the tape, in marking tape-lengths, or in reading the tape, is equivalent to $\frac{1}{5000}$. An idea of the extreme precautions necessary for a precision of $\frac{1}{100000}$ is gained from the last column, where a sag of $2\frac{1}{4}$″ will in itself cause an error equal to this limit. A variation of $1\frac{1}{2}$° in temperature or $1\frac{1}{2}$ lbs. of pull will cause the same amount; hence the need of a thermometer and spring-balance.

(*c*) *Combining errors.* It is comparatively easy to estimate the approximate error for a single tape-length which may be expected from each source of error under given conditions, but it is not a simple problem to combine these errors in order to determine the total error which may be expected in a line a number of tape-lengths long. Thus, for example, a surveyor may know: (1) That the error in the length of his tape does not exceed, say, $\frac{1}{16}$″; (2) That the maximum error in keeping his tape horizontal will not exceed 1.4 ft.; (3) That the center of his tape will never be out of line more than 4″; (4) That the variation in temperature will not exceed 15°; (5) That one end of the tape lined in by eye will never be more than 12″ out of line; (6) That sag will not exceed 7″; (7) That the variation in pull will not exceed 3 lbs.; (8) That the error in marking each tape-length will not exceed $\frac{1}{16}$″; and (9) That the tape will be read to the nearest 0.01 ft. From the table the errors in feet per 100 ft. are: (1) = 0.005, (2) = 0.01, (3) = 0.002, (4) = 0.01, (5) = 0.005, (6) = 0.01, (7) = 0.002, (8) = 0.005, (9) = 0.01.

Assuming that these values correspond to the same degree of care in chaining and that all have the same algebraic sign, their sum (0.059) is the maximum possible error that could occur in 100 feet—a precision of about $\frac{1}{1700}$. It is evident, however, that to add the errors from all nine sources, and then multiply the result by the number of tape-lengths in the length of a line, will not give the total error for the following reasons: (1) Constant errors from some sources may be plus, while constant errors from other sources may be minus, and hence even constant errors may tend to balance each other; (2) While that part of the total error due to constant or cumulative errors varies as the number of tape-lengths, that part due to accidental or compensating errors varies as the *square root* of the number of tape-lengths.

The term error as used above means the *true* or unknowable error as distinguished from the discrepancy between duplicate measurements. As already pointed out the discrepancy is not affected by constant errors (p. 11).

The problem of how far the constant error from each source must be eliminated in order that the discrepancy between two measurements of a line shall indicate the true error within prescribed limits (assuming no mistakes are made) is seen to be a complicated one. Even could proper algebraic signs be given to constant errors from different sources, it would still be difficult to assign values which would represent the same care in the corresponding operations of chaining. In practice, time and expense usually limit the precautions to be taken in eliminating constant errors, and where the work is important enough to warrant it the aim is to eliminate or correct the error from each source as far as it is possible for human skill to do it under the prevailing conditions. In ordinary work, however, the practical question is not how to attain the *greatest* precision but how to attain the *required* precision, and this means that the chainmen should know what errors to slight and what not to slight. Chainmen too often waste time and effort in eliminating errors which are relatively unimportant, and ignore at the same time errors which are much more serious. A careful study of this chapter and especially of Art. 69, p. 43, and of Table IV should help to correct such a tendency.

CHAPTER VI

CHAINING. SPECIAL WORK

IN this chapter methods are given for chaining between points when they are not visible from each other, for referencing points, for making a chain survey, for measuring base-lines, and for doing similar work which may arise in practice, but which is not a part of the routine of chaining.

Purely geometrical and trigonometrical problems in chaining are treated in the next chapter.

77. Obstacles to Chaining. In addition to the two examples here given, other obstacle problems with geometrical solutions are given in Chapter VII.

(*a*) *To chain over a high wall or other similar obstacle.* Drive two nails through a board, the heads projecting on one side and the points on the other. Lay the board across the wall with the points of the nails up, and bring both nails in line. Suspend a plumb-line from the head of each nail and thus establish two points on the ground, one each side of the wall, a known distance apart (i.e., distance of nails apart). A transit-pole with plumb-lines tied to it can be used in place of the board and nails.

(*b*) *To chain between two points when woods or underbrush intervene so that neither point can be seen from the other.* Starting at one of the points A, chain along a straight line AC as nearly as can be guessed at in the direction of the other point B. Measure the distance CB that this line strikes to one side or the other of B. Assume any distance on AC as AD, and from similar triangles $DE = \frac{AD \times CB}{AC}$. Lay off DE, thus establishing the point E. If this is done with reasonable care, the line AE, when produced, should pass through B, or near enough, at least, to show where a line should be cleared for the final chaining.

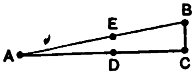

FIG. 77 (*b*).

78. Referencing a Point. (*a*) To fix its position with respect to two or more points, so that if it is lost or destroyed it may be easily restored. There are three ways of doing this with the tape, corresponding to three of the methods of locating a point in Art. 8, p. 4: (1) By two distances; (2) By offset; (3) By two intersecting lines.

(*b*) *Illustration.* (1) Let *A* in Fig. 78 (*b*) be a tack in a stake. Two distances from the corners of a building near by were measured, thus referencing *A*. If the stake is destroyed, another may be easily reset, *provided the intersection of the two reference-lines is sharp and definite.* The point *B*, for example, cannot be referenced accurately from the same two corners because it is too nearly in a straight line between them for the intersection to be definite.

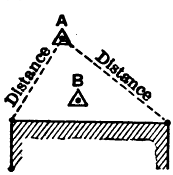

FIG. 78 (*b*).

(*c*) *Illustration.* (2) In Fig. 78 (*c*) the point *C* was referenced by measuring a distance along a fence from a fence-corner, and then another distance out at right angles. This method is more difficult and less accurate in field work than that of the preceding illustration.

(*d*) *Illustration.* (3) A third method of referencing a point is to establish two other points in the same straight line, one each side of the given point, and measure the distance from the given point to each of these reference-points. Or, better still, establish four reference-points so that the given point may be determined without measurement by the intersection of two straight lines. (See Fig. 8 (*b*), p. 4.)

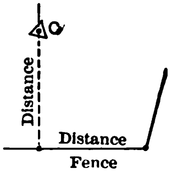

FIG. 78 (*c*).

79. To Chain between Two Points when Neither can be Seen from the Other. (*a*) *Illustration.* Let *A* and *D* be two points with the brow of a hill between them. One chainman, *B*, goes as far back as he can and still see *D*. The other chainman, *C*, goes as far down hill as he can and still see *A*. *B* and *C* are both out of line, but they proceed to line each other in alternately. *B* lines *C* in with *D*. *C* then lines *B* in with *A*. *B* again lines *C* in with *D*, and *C* again lines *B* in with *A*. Thus the process is carried on until both *B* and *C* are in line. Poles are set at *B* and *C*, and these enable the chainmen to measure in a straight line between *A* and *D*. The method of Art. 77 (*b*) can also be used.

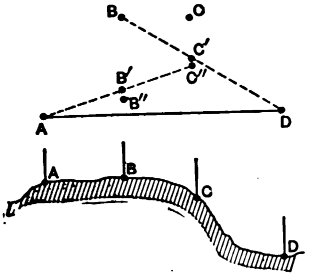

FIG. 79 (*a*).

(*b*) *Practical suggestions.* Poles may be set at *A* and *D* to begin with. The chainmen themselves can hold either poles or plumb-lines. Time will be saved if when one chainman is lining the other in he will send him a little farther in the direction in which he is moving than appears necessary. Thus in Fig. 79 (a) *B* moves *C* to *C''* instead of to *C'*, and *C''* then moves *B* to *B''* instead of to *B'*, and so on.

(c) *Remark.* When a valley intervenes between two points a similar method can be employed to establish points on line in the valley. In most cases, however, since one point can be seen from the other, a pole is set on line with the transit. When a transit is not at hand still another method is as follows: One chainman holds a plumb-line over one point and, placing his eye so that the string covers a rod held at the other point (on the opposite hill), lines in a second rod held in the valley until it too is covered by the string.

80. To Make a Chain Survey. It is sometimes necessary to make a complete survey using the tape only. In many cases this can be done with as great accuracy as the purpose of the survey requires. In Fig. 80 is shown the survey of a house and lot, in which no angles were measured. A station *S* was chosen from which all four corners of the lot could be seen. The four sides of the lot and the four dotted lines were measured. From these eight measurements the boundaries of the lot were plotted. Had it not been for the house it would have been simpler to divide the quadrilateral by diagonals into two triangles instead of four.

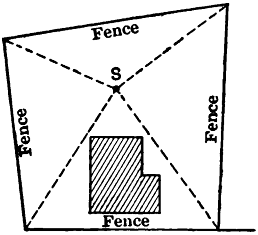

FIG. 80.

(a) *Question.* What is the best method of locating the house by linear measurements only?

(b) *Question.* If desired, how can the angle at each of the four fence-corners be found from the eight original measurements?

(c) *General method for a chain survey.* Establish a system of triangles to serve as reference-lines. This network of triangles should be as simple as possible and planned with an eye to locating details. Measure the three sides of each triangle. Locate details with respect to the most convenient reference-lines by any of the methods for linear measurement given in Chapter XIII.

(d) *Remark.* The transit is used so generally that students are apt to think that a survey cannot be made with a tape alone. For short lines and small areas, however, the tape will often answer every purpose. Even horizontal angles can be measured with it (p. 63), though this is not often necessary. The reference-lines are laid out in triangles to save measuring angles, and this is the chief difference between a chain survey and a transit survey.

81. Locating Houses, Fences, Streets, Streams, and Other Details with the Tape. Not only in chain surveys, but in transit surveys as well, the chainmen are often called upon to locate details of the survey with reference to transit lines by the use of the tape alone. The methods of locating such details by linear measurements only are fully discussed in Chapter XIII.

82. To Measure a Base-line, *or any other straight line with more than ordinary accuracy.* There are several devices for measuring base-lines with great accuracy (see Chapter XVI). The following method in which a steel tape is used should give results accurate enough for most purposes; i.e., with reasonable care the *discrepancy* between two measurements of the same line should not exceed $\frac{1}{100000}$. The tape is supported by stakes, the pull is kept at standard with a spring-balance, and a thermometer gives the temperature for which correction is made. Thus the three chief sources of error are as far as possible avoided. The work may be outlined as follows:

(1) Drive stakes at short intervals along the line so that when a tape is stretched across the tops it will be horizontal. The distances between the stakes should be such that the end of the tape will come on a stake each time that the tape is stretched.

(2) Stretch the tape, using a spring-balance at one end to keep the pull at 8 lbs. or 12 lbs. or whatever the standard for the tape may be, mark the end of each tape-length with a fine scratch on top of the stake, and from a thermometer reading correct the *total* measurement for temperature (see Art. 62, p. 40).

(3) Repeat the measurement a number of times and take for the final result the arithmetical mean of all the measurements, discarding, of course, any measurement which is obviously incorrect.

(1) *Practical suggestions.* Use large stakes, say 2″×4″ for intermediate stakes and 4″×4″ or 6″×6″ for the two end hubs. All stakes must be *firmly* set. They may be lined in with the transit and the tops sawed off to a level, or nails may be driven in the sides of the stakes, the tape resting on the nails. The latter method is not recommended for work of great precision. A zinc plate upon which to make the scratches is sometimes put on the top of each stake. In important work the two end stakes or hubs should be made more or less permanent and the ends of the base-line *carefully* referenced (p. 54). This is sometimes done by setting the hub in concrete and building a fence around it to keep it from being disturbed during construction. To keep the tape in position on the tops of supporting stakes drive two small nails part way into each stake so that there will be one on each side of the tape when it is stretched.

(2) When it is impracticable to set stakes on a level they may be set with their tops (or supporting nails) on a known slope, and the measured length is then corrected for the slope. Occasionally it may be necessary

to divide a base-line into several sections, each section at a different level, the plumb-line being used as in ordinary chaining but with more precautions to insure accuracy.

Remark. The author's classes have sometimes used the following method for measuring a base-line through thick underbrush. The line was first cleared by cutting the brush to within a foot or so of the ground. Tripods, belonging to a number of levels and transits, were set in line, 20 ft. apart, with their tops on a level. These acted as supports for the tape, keeping it three feet or so above the ground. As fast as tripods were left behind they were taken up and set on line in advance of the tape. If care is taken to prevent any movement of the tripods while they are being used, excellent results can be obtained.

Such a large number of tripods will seldom be available in practice, but rough tripods can be easily constructed and used in their places. For an example of a more elaborate apparatus for supporting and stretching the tape, used for very precise measurements, see *Engineering News*, Feb. 1, 1894, July 12, 1894, Sept. 14, 1902; also Transactions A. S. C. E., Aug. 1902, and Johnson's Surveying, p. 498.

(3) Ideal conditions for base-line measurements are (1) level ground, (2) cloudy weather, and (3) the same temperature as that for which the tape is standard. Needless to say all three of these conditions seldom exist in practice. It is difficult to ascertain the temperature of a tape lying in the sun, hence a cloudy day or even the night-time is preferable for accurate measurements.

(4) A finely divided scale may be used to measure the decimal of a foot in the end measurement.

(5) Rough devices with turnbuckles for keeping the tape steady and at a given pull are easily constructed and add greatly to the accuracy of measurements. These devices are usually frameworks which can be quickly moved and set up to hold the tape, thus taking the place of the chainmen. In the absence of such a device each chainman can brace his hands by a stake held obliquely with one end on the ground to resist the pull. Whatever happens the supporting stakes should not be disturbed.

(6) In chaining a base-line it is desirable to chain in opposite directions and to proceed in such a manner that in marking where the end of the tape comes one is not influenced by scratches made in previous measurements.

Illustration. The supporting stakes for a base-line *AB* are 20 ft. apart. The base-line is 420 ft. ± long. A 100-ft. steel tape is used. Five different measurements of the line can be made without having two scratches on the same stake.

$$\dot{A}\ \dot{2}\ \dot{4}\ \dot{5}\ \dot{3}\ \dot{1}\ \dot{2}\ \dot{4}\ \dot{5}\ \dot{3}\ \dot{1}\ \dot{2}\ \dot{4}\ \dot{5}\ \dot{3}\ \dot{1}\ \dot{2}\ \dot{4}\ \dot{5}\ \dot{3}\ \dot{1}\ \dot{B}$$

(1) Starting at *A* the scratches fall on stakes marked 1; (2) starting at *B* they fall on stakes marked 2; (3) starting at *A* with a measurement of 80 ft., the intermediate measurements being 100 ft., scratches fall on stakes marked 3; (4) starting from *B* with a first measurement of 80 ft. they fall on stakes marked 4; (5) finally, starting from *A* with 60 ft. (intermediate measurements 100 ft.) scratches fall on stakes marked 5.

83. To Find the Probable Error. Suppose that in the preceding illustration when the corrections for temperature have been made, and all other constant errors have been eliminated as far as possible, the five measurements are those given in the first column on the next page. It is required to find the probable error of a single measurement and the coefficient of precision for chaining (see p. 46). This is done by the method of least squares, using the formulas on p. 17. It will be noticed that the algebraic sum of the residuals is not zero as it should be. This is because the values of *v* were taken only to the nearest unit in the third decimal place, which is sufficient for all practical purposes. The work is given in detail below.

Measurement.	v	v^2
419.862	0.002	0.000004
419.863	0.003	0.000009
419.857	0.003	0.000009
419.861	0.001	0.000001
419.858	0.002	0.000004
5)4.301		$\Sigma v^2=0.000027$
419.8602 = mean		

$$E_0=0.6745\sqrt{\frac{\Sigma v^2}{n(n-1)}}.$$

$$=0.6745\times\sqrt{\frac{0.000027}{20}}$$

$$=0.00078$$

The probable error of the mean is 0.00078 or about $\frac{1}{540000}$.

The probable error of a single observation is $E=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}$,

or in this case $0.6745\sqrt{\frac{0.000027}{4}}=0.00175$ or about $\frac{1}{240000}$.

To determine coefficient of precision. The probable error of the total observed length $R_s=r\sqrt{N}$ (see p. 45) or $0.00175=r\sqrt{N}$ $=r\sqrt{4}+$ or $r=0.0008+$. Hence the probable error of a line say 900 ft. measured under exactly the same conditions may be expected to be about $0.0008\sqrt{9}=0.0024$ ft.

CHAPTER VII

CHAINING—PROBLEMS

There are certain problems which occasionally arise in chaining but which are of too infrequent occurrence to be called a part of the routine field work. These problems have been grouped for reference in this chapter. Since many of them are simply geometrical constructions, it will be helpful to remember that in working them out in the field the tape or chain is used very much as the scale and compasses are used in similar constructions in the drafting-room. Indeed many of the problems can be worked out just as well in the classroom, but each student should do enough of them in the field to gain confidence in the different methods, and in his ability to carry them out.

---

84. To Construct a Right Angle with a Tape by the 3–4–5 Method. If the lengths of the sides of a triangle are in the ratios of 3:4:5, one angle of the triangle must be 90°. Hence to construct a right angle, form a triangle with the tape, using, for example, any one of the following combinations:

[3–4–5], [6–8–10], [9–12–15], [12–16–20], [15–20–25], [18–24–30], [21–28–35], and so on.

(*a*) *Illustration.* In Fig. 84 (*a*) one chainman holds the 0 and the 48-ft. mark of a 50-ft. cloth tape together, a second man holds the tape at the 36-ft. mark, while a third holds it at the 20-ft. mark. When stretched the tape will form a right-angled triangle. (Why?)

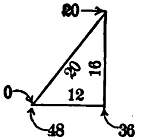

Fig. 84 (*a*).

There are two objections to the above method: (1) three chainmen are required, and (2) if a steel tape is used it cannot be bent sharply enough to form the vertices of the triangle. Another method of forming the triangle often used is shown in the following illustration:

(*b*) *Illustration.* In Fig. 84 (*b*) it is desired to construct a right angle at a point *c* in the line *ab*, one side of which shall coincide with *ac*. Establish a point *e* 30 ft. from *c*. The 0 end of a 100-ft. tape is held at *e*, the 100-ft. end at *c*, and the 50-ft. and 60-ft. marks are brought together, the portion of the tape not used forming a loop. The tape stretched tight forms a right-angled triangle with sides 30, 40, and 50 ft. long. By fastening the ends of the tape at *e* and *c* one man can do all the work.

Question. If a 50-ft. tape is used and the 25-ft. and 30-ft. marks are brought together, what should be the distance from *c* to *e*?

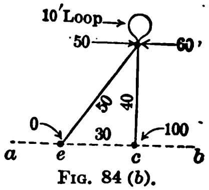

Fig. 84 (*b*).

85. Perpendiculars and Parallels. The methods of establishing perpendiculars and parallels with the tape are based upon familiar geometrical constructions. It is to be remembered, however, that straight lines and arcs of circles cannot be drawn upon the ground with the same ease that they can be drawn on paper; the ground is often so rough or sloping that they cannot be drawn at all. In many cases, therefore, chainmen must exercise ingenuity in applying geometrical principles. The difference between methods of the drafting-room and methods of field work in making the same geometrical construction is brought out in the following illustrations. *It is important not only to know geometrical constructions, but also to know how the tape should be used in making them.*

(*a*) *To erect a perpendicular at any point in a line.* The 3–4–5 method of the previous article may be used. Two other methods are indicated in Figs. 85 (*b*) and 85 (*c*).

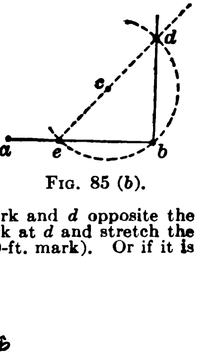

FIG. 85 (*b*).

(*b*) *Illustration. Geometrical construction.* Required to erect a perpendicular at *b* to the line *ab*. From *any* point *c* describe an arc of a circle of radius *cb*, intersecting *ab* at *e*. Find *d* in line with *ec*.

Field work. The field work with a 100-ft. tape could be done as follows: Establish *c* 50 ft. from *b* by holding 0 at *b*. Keeping the 50-ft. mark at *c*, find *e* by swinging the 0 end of the tape and with 0 at *e*, 50-ft. mark still at *c*, stretch the tape until the 100-ft. mark determines *d*.

(*c*) *Illustration.* The difference between using the compasses and the tape is illustrated in Fig. 85 (*c*). With the compasses everything is done with intersecting arcs as indicated in the figure. With the tape a better way is as follows: With the 25-ft. mark at *b*, establish *c* opposite the 0 mark and *d* opposite the 50-ft. mark. Keeping 0 at *c*, hold the 100-ft. mark at *d* and stretch the tape, establishing *e* at the middle of the tape (50-ft. mark). Or if it is a steel tape, bring the 45-ft. mark and 55-ft. mark together to avoid bending, thus establishing a point on the perpendicular 45 ft. from both *c* and *d*. If a check is desired, find another point on the opposite side of the line in exactly the same manner, or if this be impossible, another point on the same side, using different distances.

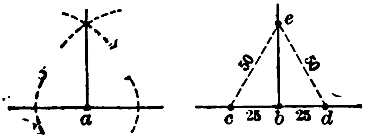

FIG. 85 (*c*).

(*d*) *Remark.* In each of the above illustrations notice that *one* point of the tape was fixed during the entire work in order to save unnecessary settings of the tape. In the geometrical constructions which follow it is *not enough simply* to recognize the method, but consider carefully how each one could be carried out in the field, setting and stretching the tape the smallest number of times.

To let fall a perpendicular to a given line from any given point.

(*e*) In Fig. 85 (*e*) from the given point *P* swing an arc intersecting the given line in two points. The required perpendicular through *P* passes half-way between the two points of intersection.

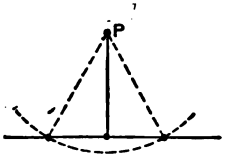

Fig. 85 (*e*).

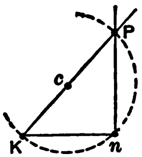

Fig. 85 (*f*).

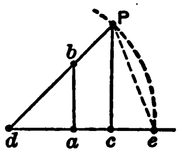

Fig. 85 (*g*).

Question. If an arc could not be drawn on the ground, how would the tape be used?

(*f*) In Fig. 85 (*f*) the construction is that of Fig. 85 (*b*) worked backward. Choose any convenient point on the line as *K*, find the point *c* half-way between *P* and *K*, and with *c* as a center and *cK* as a radius cut the given line at *n*, the foot of the required perpendicular.

(*g*) In Fig. 85 (*g*) a perpendicular is first erected at any convenient point as *a*, and a line through *P* constructed parallel to it. See Arts. 85 (*h*), 85 (*i*), and 85 (*j*). Or any two points as *d* and *b* may be assumed in line with *P*, and *c*, the foot of the required perpendicular, found from similar triangles $dc=\frac{da\times dP}{db}$.

A second method. With any point on the given line as *d* swing an arc through *P* intersecting the given line at *e*. To find *c*, since $dP=de$, then $ce=\frac{\overline{Pe}^2}{2de}$.

Question. How could the methods of Fig. 85 (*g*) be best carried out with a tape?

Question. What methods of erecting or letting fall a perpendicular are best adapted when the perpendicular is near or at the end of a line and the line itself cannot be produced?

To run a line through a given point parallel to a given line.

(*h*) In Fig. 85 (*h*) a perpendicular *Pp* is let fall from the given point to the given line, and at some other point in the line as *r* another perpendicular *rs* of equal length is erected.

(*i*) In Fig. 85 (*i*) choose any convenient point on the given line as *c* and find *a* half-way between *P* and *c*. Assume a point *e* and make *ab* equal *ea*.

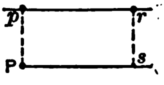

FIG. 85 (*h*).

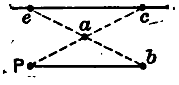

FIG. 85 (*i*).

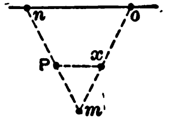

FIG. 85 (*j*).

(*j*) In Fig. 85 (*j*) assume any convenient line as *mn* through *P*, and make *mo* equal to *mn*. Find *x* when $mx = mP$.

Remark. Other methods will suggest themselves to the student. The method of Fig. 85 (*i*) is quick and accurate and easily checked by the method of Fig. 85 (*h*). A check on the method of Fig. 85 (*h*) is the equality of the diagonals.

86. Laying off or Measuring Angles.—Angles may be laid off by means of a steel tape with considerable accuracy, the error often not exceeding 5′. The trigonometrical methods of the chapter on *Plotting Angles*, Part II, are used. Since measurements are in feet instead of in inches and since the tape is used instead of drawing-instruments, the operations in the field are somewhat different from those in the drawing-room, as, for example, when erecting perpendiculars, but the trigonometrical methods are of course the same.

(*a*) *Tangent method.* (See p. 458.) The base by which the tangent of the angle is to be multiplied may be any convenient number of feet, say 50′ or 100′. Perpendiculars are erected by one of the methods of Art. 85.

(*b*) *Chord method.* (See p. 459.) The unit radius may be any convenient number of feet, say 50′ or 100′. As circles cannot often be drawn on the ground, many of the practical suggestions for using the compasses do not hold good for the tape. Use the tape in a manner similar to that shown in Fig. 86 (*b*), one portion of the tape stretched to form the chord of known length, the other portion forming the required side of the angle.

Illustration. To lay off an angle of 47° 8′ clockwise from *AB*, using 100-ft. tape. Chord for unit radius is 0.7996. For 50 ft. it is 39.98 ft. Find *c* 50′ from *A* on *AB*. Holding 0 mark at *c* and 100 mark at *A*, bring 50-ft. mark and 39.98 together. This will give *e*, a point on the required side.

Remark. The chord method is usually the most satisfactory one to use in the field.

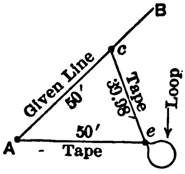

FIG. 86 (*b*).

(*c*) *Sine-cosine method.* For any unit radius the sine and cosine distances can be found and used in the same way that the two distances were used in the preceding example.

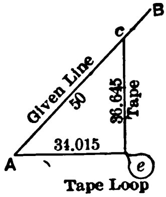

FIG. 86 (*c*).

Illustration. To lay off an angle of 47° 8′. Sine = 0.7329, cosine = 0.6803. Find *c* 50′ from *A* as before. sin × 50 = 36.645, cos × 50 = 34.015. When the tape is stretched as shown in the figure it will form a *right angle* at *e*, a point on the required side.

(*d*) *Measuring angles.* Angles between two given lines may be measured with the tape by reversing the methods outlined in this article.

87. Obstacles to Chaining which prevent chainmen from measuring directly between two points give rise to a set of problems in which distances are found by indirect measurements. In this article these problems are all solved by geometrical methods.*

Obstacles to chaining are of two kinds: (1) *obstacles to measurement* and (2) *obstacles to alignment.* For any problem likely to occur in practice there are usually several geometrical solutions; it will suffice to give two for each problem.†

(*a*) *To measure between two points when both are accessible.*

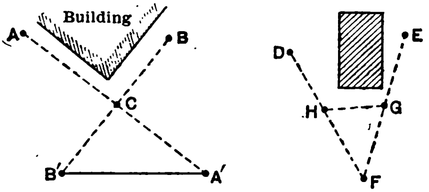

FIG. 87 (*a*)

First method. Assume *C*, make *CB′* = *BC* and *CA′* = *AC*; then *A′B′* = *AB*.

Second method. Establish *F* at any point equal distances from *D* and *E*. Make *FH* = *FG* any convenient distance. Then *DE* = (*HG* × *DF*) ÷ *HF*. (See Fig. 87 (*b*), p. 65, for a third method.)

(*b*) *To measure between two points when one is inaccessible.*

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* Many of the same problems can be more easily solved by trigonometrical methods, provided a transit is used. (See Chapter XVIII.)

† *The student with a good knowledge of geometry should be able to devise other methods.* For a very complete collection of obstacle problems *and solutions see Gillespie's* Surveying, Vol. 1, pages 76–99.

First method. Lay off AC any convenient length perpendicular to the required distance AB, and lay off DC perpendicular to BC. Then $AB = AC^2 \div AD$.

Second method. Assume I anywhere in line with the required distance FG. Assume H anywhere. Make $HJ = HI$ and $HK = HG$. Establish L in the line FH and at the same time in the line JK produced. Then $KL = FG$.

(*c*) *Other methods.* Assuming that the point B in Fig. 87 (*b*) is not a fixed point but any point in line on the opposite bank of the river, how could an equilateral triangle be constructed and used for finding the distance AB? How could a right triangle with a 60° angle at C be used for finding the same distance?

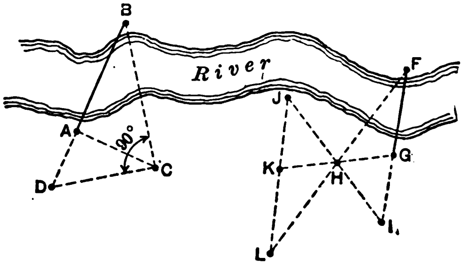

FIG. 87 (*b*).

(*d*) *To measure to an inaccessible point high above ground,* i.e., to measure the height of an object. Set a transit-pole in the ground some distance from the object and with the eye some feet away from the pole sight across the top of the pole to the inaccessible point. This line of sight is the hypothenuse of two similar triangles; the unknown height is the vertical side of one triangle, and the transit-pole the vertical side of the other. The two bases of the triangles can be measured and the proportion formed. This method will of course give only approximate results.

(*e*) *To measure between two points when both are inaccessible.*

First method. AB, Fig. 87 (*e*), is the required distance, A and B being both inaccessible. Assume any line CC' and find its middle point D. Assume E in AC, and F in BC. Make $DE' = DE$, and $DF' = DF$. Find A' at the intersection of AD and $E'C'$, and find B' at the intersection of BD and $F'C'$. Then $A'B' = AB$. (Why?)

Second method. GH, Fig. 87 (*e*), is the required distance. Assume I anywhere. Find the distances GI and HI by either method of Fig. 87 (*b*). Assume J anywhere in GI. Lay off $IK = (HI \times JI) \div GI$. Measure JK. Then $GH = (JK \times GI) \div JI$.

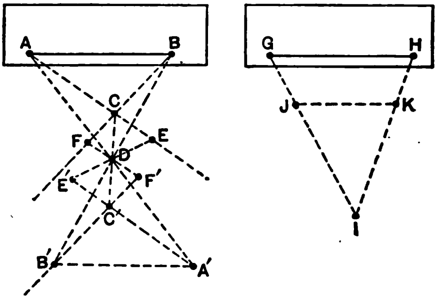

FIG. 87 (*e*).

(*f*) *To prolong a straight line through an obstacle.*

First method. Let AB be the line to be prolonged in Fig. 87 (*f*).

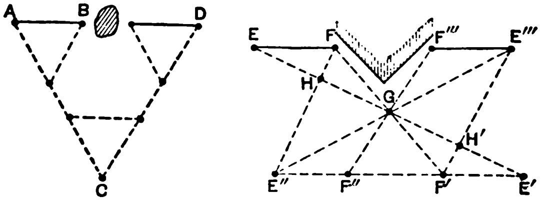

FIG. 87 (*f*).

Construct an equilateral triangle ACD by constructing a smaller equilateral triangle at each apex A, C, and D. This is another method also of measuring between two points.

Second method. EF in Fig. 87 (*f*) is to be prolonged. Assume G. Make $GF' = FG$, $GE' = EG$, $GF'' = GF'$, $E''F'' = E'F'$, $GF''' = GF''$, and $GE''' = GE''$. (This method is a double application of that shown in Fig. 87 (*a*).) If more convenient, instead of finding F'' assume E'' *anywhere* in line with $F'E'$, find H at intersection of $E''F$ and EG, make $GH' = HG$, and find E''' at intersection of $E''G$ and $F'H'$. A second point could be determined in a similar manner.

CHAPTER VIII

READING ANGLES

THIS chapter is divided into two parts to correspond to two distinct steps in reading an angle: (1) reading the *limb*, and (2) reading the *vernier*. In the first part are shown several different graduations of the limb, and in the second part several forms of verniers. To study all these graduations at the outset is likely to lead to confusion. Begin with the simplest forms, as, for example, a limb graduated continuously (page 70), and a vernier reading to minutes (page 78).

Observe the common mistakes as illustrated by the incorrect readings in the book, and then practice with a transit graduated to correspond. When this much is thoroughly understood it is an easy matter to proceed with a transit more finely graduated.

88. The Two Horizontal Plates. One of the horizontal plates carries the graduated circle, the other plate carries the vernier. The uprights (or standards) which support the telescope are attached to the vernier plate, and the zero of the vernier serves as an indicator. Hence the telescope cannot be revolved right or left without taking the indicator with it, and the *horizontal* angle through which it is turned is measured by the arc on the graduated limb passed over by the indicator. If the indicator is opposite the zero of the limb when the telescope points to the backsight, and stops exactly opposite a graduation-mark when the telescope is turned to the foresight, the angle is read *without* the use of the vernier; when, as is more often the case, the indicator stops *between* graduation-marks the vernier reading tells how far it has passed beyond the last mark.

89. Limb Graduations. The limb of the transit is always divided into 360°, and every *tenth* degree is usually numbered, but there are several systems of numbering and several methods of subdividing each degree.

(*a*) *Common systems of numbering limb graduations.* (1) *Full circle,* 0° to 360° (see p. 70); (2) *Half-circle,* 0° to 180°, then backward to 0° (p. 72); (3) *Quadrant,* two zeroes 180° apart, with numbers running up to 90° both ways from each zero (p. 74).

(*b*) *Common methods of subdividing each degree.* (1) Into two parts, or 30′; (2) into three parts, or 20′; (3) into four parts, or 15′; (4) into six parts, or 10′.

(*c*) *Remark.* On most transits there are two rows of numbers, the outer row often corresponding to one system, the inner row to another. Each system has its advantages and is adapted to some particular kind of work. On an ordinary transit each degree is divided into two or, at the most, three parts; it is only on instruments intended for more precise work that the limb is divided to 15′ or 10′.

90. General Method of Reading Angles. There are three distinct steps: (1) Reading the limb; (2) Reading the vernier; (3) Adding the two readings.

Remark. The majority of mistakes made in reading angles are made in reading the limb, and not in reading the vernier. Moreover, a mistake in reading the limb usually amounts to at least 20′ or 30′ and often to several *degrees,* while a mistake in reading the vernier is seldom over a minute or two. Hence the working rule "*Make sure of the limb reading before looking at the vernier.*" In learning to read angles the same rule holds good.

91. Reading the Limb. (*a*) No matter in which direction the plate carrying the indicator revolves there are always two readings for an angle: one is the value of the angle measured clockwise from the backsight, and the other is the value measured counter-clockwise *from the same backsight.* Each value is the explement of the other, i.e., their sum is 360°. See p. 94.

If angles were always measured to the right (clockwise), the numbers of the graduations on the limb would always increase *clockwise,* and a common source of mistakes would be removed. In some kinds of work, however, as in deflection angles, p. 117, readings are taken right and left and this calls for numbers which increase counter-clockwise as well as clockwise. These two rows of numbers must not be used indiscriminately—a constant source of mistakes with beginners.

(*b*) *Limb readings for angles to the right.* Consider the indicator to have moved *clockwise.* (It is immaterial whether it did or not.) Observe which row of graduations increases *clockwise,* note the last number on that row passed by the indicator, and from this number determine the value of the graduation-mark opposite the indicator or just behind it. (See p. 70.)

(*c*) *Limb readings for angles to the left.* Proceed in a similar *manner,* but consider the indicator to have moved counter-clockwise, and use the row in which the numbers increase counter-clockwise. (See p. 70.)

(*d*) *Remark.* For angles clockwise numbers increase clockwise, and for angles counter-clockwise numbers increase counter-clockwise. If one could stand *inside* the graduated circle, numbers for angles to the right would increase towards his right hand, and for angles to the left they would increase towards his left. Since, however, the observer stands *outside* of the circle, the numbers for angles to the right increase towards his *left* hand, and *vice versa.* This leads to confusion which would be avoided if the terms clockwise and counter-clockwise were always used in place of right and left respectively.

92. Common Mistakes in Reading the Limb. (1) Reading the number *ahead* of the indicator instead of behind it. (2) Reading the inner circle of numbers instead of the outer circle, or *vice versa.* (3) Reading *even* degrees instead of degrees and fractions of a degree, as, for example, 43° 0′ instead of 43° 30′, or 11° 0′ instead of 11° 20′.

93. Method of Reading the Limb Illustrated. On the succeeding pages are shown three different systems of numbering the graduations on the limb. The full circle as shown is about half the true size. Below the circle is a portion of the limb greatly enlarged. The vernier is purposely omitted, but the indicators represent four different positions of the zero of the vernier. The readings noted *below* the arc are for angles taken to the right as indicated by the arrow, while those *above* the arc are for angles to the left. The first two common mistakes noted in the preceding article are illustrated by the incorrect readings in parenthesis. The third mistake will be illustrated in the explanation of vernier readings (pp. 76–82), since it is most often made when adding the limb reading and the vernier reading together.

Suggestions. In studying the illustrations keep in mind the fact that the limb of a transit remains stationary during the interval between the backsight and foresight while the indicator moves over an arc equal to the angle measured. Understand clearly the difference between measuring an angle to the right (clockwise) and to the left (counter-clockwise). The following method of studying the illustrations is suggested:

(1) Choose one of the four indicators, assume the angle to have been measured clockwise, and cover with a card the readings below the arc.

(2) Notice which circle of numbers is increasing clockwise, find the number just behind the indicator, count up the intervening spaces, and read the angle.

(3) Remove the card and see if the reading agrees with that given on the figure. Observe also the mistaken reading in parenthesis, and consider from what cause the mistake might be made.

(4) Repeat the three steps given above for each of the other three indicators; also, for all four indicators assuming that angles were measured counter-clockwise.

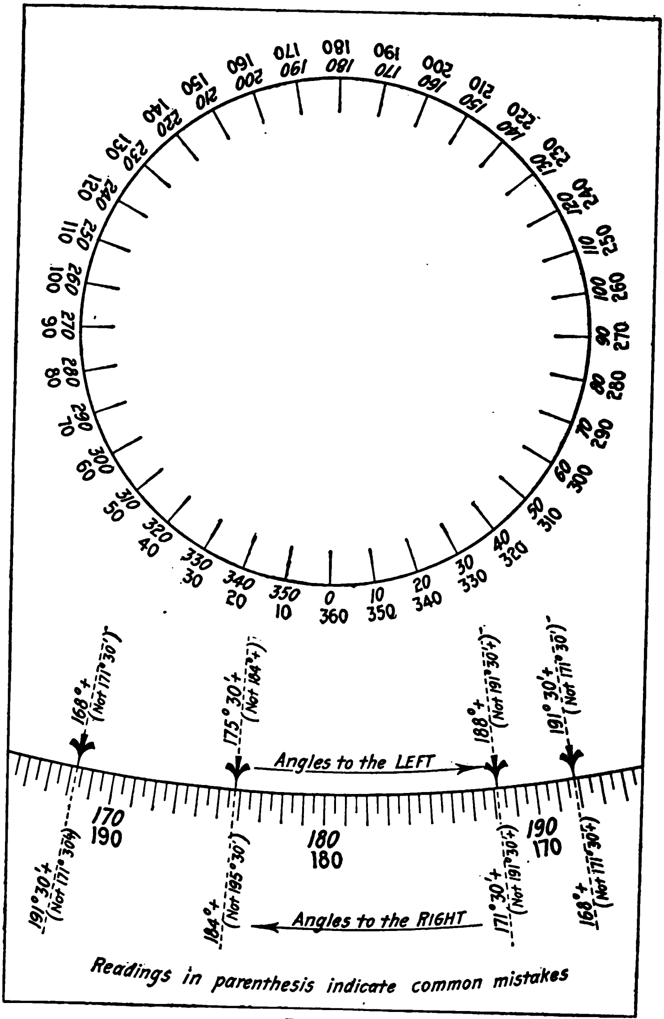
180 180
190 170
200 160
210 150
220 140
230 130
240 120
250 110
260 100
270 90
280 80
290 70
300 60
310 50
320 40
330 30
340 20
350 10
0 360
10 350
20 340
30 330
40 320
50 310
60 300
70 290
80 280
90 270
100 260
110 250
120 240
130 230
140 220
150 210
160 200
170 190
168°+ (Not 171° 30')
175° 30'+ (Not 184°+)
188°+ (Not 191° 30'+)
191° 30'+ (Not 171° 30')
Angles to the LEFT
170 190
180 180
190 170
191° 30'+ (Not 171° 30'+)
184°+ (Not 195° 30')
171° 30'+ (Not 191° 30'+)
168°+ (Not 171° 30'+)
Angles to the RIGHT
Readings in parenthesis indicate common mistakes

FIG. 94.

94. Full-circle System from 0° to 360° in Both Directions. (*a*) This is one of the best systems of numbers for all-around work. Angles may be measured in either direction, to the right or to the left, without confusion, provided simple precautions are taken. Thus for angles to the right always read the outside row if that is the row in which numbers increase *clockwise*, while for angles to the left read the inside row. *When, as is often the case, all angles are taken to the right, ignore the inside row: make it a point not to use it at all.* In some makes of instruments the numbers slant in the direction in which they should be read. (See full circle, opposite page.)

(*b*) The portion of the limb shown enlarged is where the two circles have the number 180° in common, and this is where the mistake of reading the wrong circle of numbers is most likely to occur. Where the numbers are widely different it is usually sheer carelessness to read one for the other. If, for example, an angle of 100° is taken to the right, the indicator will stop opposite 100° and 260°, but an alert transitman would see that the angle could not be over 180° (the backsight produced), and hence would not read 260° in place of 100°. This illustrates, incidentally, the value of *habitually checking readings by estimating angles with the eye.*

(*c*) When the indicator stops just before it reaches a number it is a natural mistake to read as if it had gone by the number. This mistake is illustrated by the two incorrect readings of 171° 30′ and 195° 30′ for angles to the right, and by all of the incorrect readings to the left except 171° 30′ in place of 191° 30′. (What mistake does this illustrate?)

(*d*) Notice that the sum of the outer number and the inner number at *any* point is always 360°. Hence when there is only one circle of numbers (0° to 360° clockwise) and it is desired to read an angle to the left, consider the indicator to have moved to the left (*counter*-clockwise), but subtract the nearest number behind the indicator from 360°, and proceed as if the number were actually engraved upon the limb, as in fact it is when there are two circles. As the subtraction involves only *even degrees*, this method is better than to read to the right and subtract from 360°, for the latter subtraction involves *minutes* as well as degrees. Mountain transits and other transits intended for measuring angles to the right only and hence having but one circle of numbers, may be used for measuring angles to the left by simply imagining the explementary number *engraved on the limb* as just suggested. (See Art. 95 (d).)

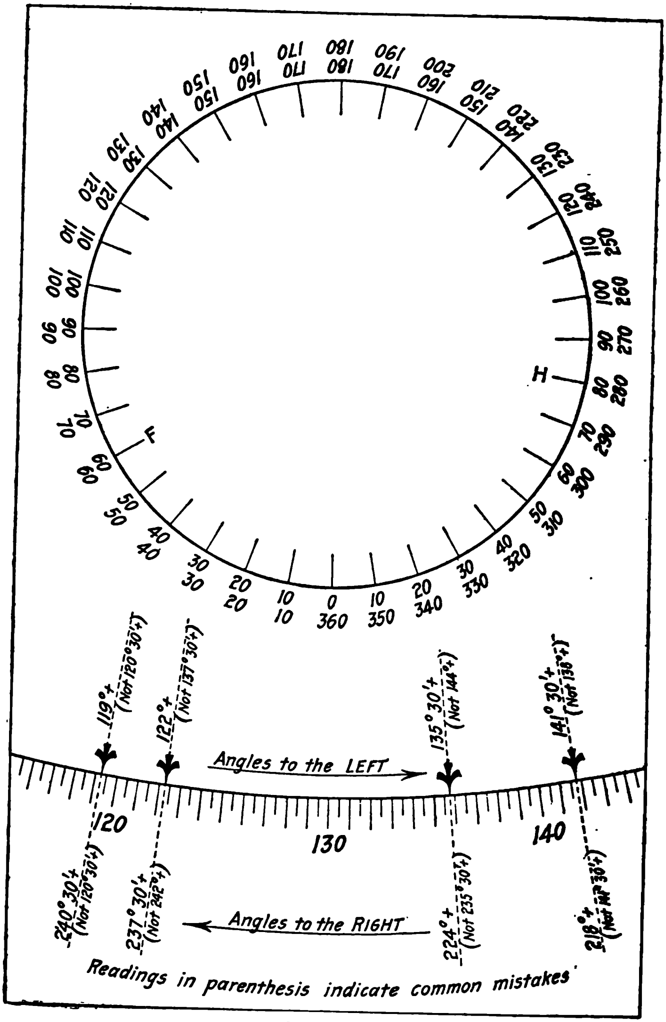
F
H
119°+ (Not 120° 30'+)
122°+ (Not 137° 30'+)
Angles to the LEFT
135° 30'+ (Not 144°+)
141° 30'+ (Not 138°+)
120
130
140
240° 30'+ (Not 120° 30'+)
237° 30'+ (Not 242°+)
Angles to the RIGHT
224°+ (Not 235° 30'+)
218°+ (Not 141° 30'+)
Readings in parenthesis indicate common mistakes

FIG. 95.

95. Combination of Full-circle and Half-circle Systems. (*a*) In the figure on the opposite page the outside circle of numbers is a *full*-circle system, increasing clockwise exactly like the outside circle in the preceding illustration. No further explanation of its use, therefore, is needed.

The inner circle is a *half*-circle system and is used for measuring angles which do not exceed 180°, as, for example, deflection angles (p. 117). Thus, if the indicator is at *F* in the inner circle, the deflection angle is 60° to the right, but if it is at *H*, the deflection angle is 80° to the left.

(*b*) The portion of the limb shown enlarged is near the 230° mark of the outer circle, but the outer numbers have purposely been omitted. The angles to the left are read in the usual manner. The mistaken readings for angles to the left all come from reading the number in front of the indicator instead of the number behind it. There is, of course, the same danger of reading the wrong row of numbers as in the preceding illustration.

(*c*) Notice that the sum of the corresponding numbers in the outer and inner rows is always 360° *after passing the 180° mark of the outer circle.* On some instruments, intended only for measuring deflection angles, the full-circle system is omitted. Suppose that it is desired to measure angles to the right *over* 180° on such an instrument. After the indicator passes 180° the numbers run backward or in the wrong direction for angles to the right. Subtract the number behind the indicator from 360°, and proceed as if the result were engraved on the limb as explained in the preceding article.

(*d*) ***Illustration.*** On the enlarged arc it is observed that the numbers are increasing in the wrong direction for angles to the right. If the indicator is between 120° and 130°, the number behind it, for angles to the right, is 130°. Hence $360 - 130 = 230°$, and proceeding as if 230° were engraved opposite 130, as it really is when there are two rows of numbers (see complete circle), the final reading is 237° 30′+. Likewise, if the indicator is between 140° and 150°, the number behind it (outside the limits of the drawing) is 150, and hence $360 - 150 = 210°$, from which 8° more is counted clockwise, making the final reading 218°+. The other readings, 224°+ and 240° 30′, are obtained in a similar manner.

(*e*) ***Remark.*** In taking deflection angles it is desirable to keep the zero of the limb nearly under the eyepiece of the telescope to save walking around the instrument when reading angles. If the half-circle system is used, it will be necessary, after setting the indicator at zero, *to plunge the telescope before backsighting.* In order to avoid this the *quadrant* system of numbering is often preferable, especially in railway work, where deflection angles are seldom more than 90°. This system is illustrated on the next page.

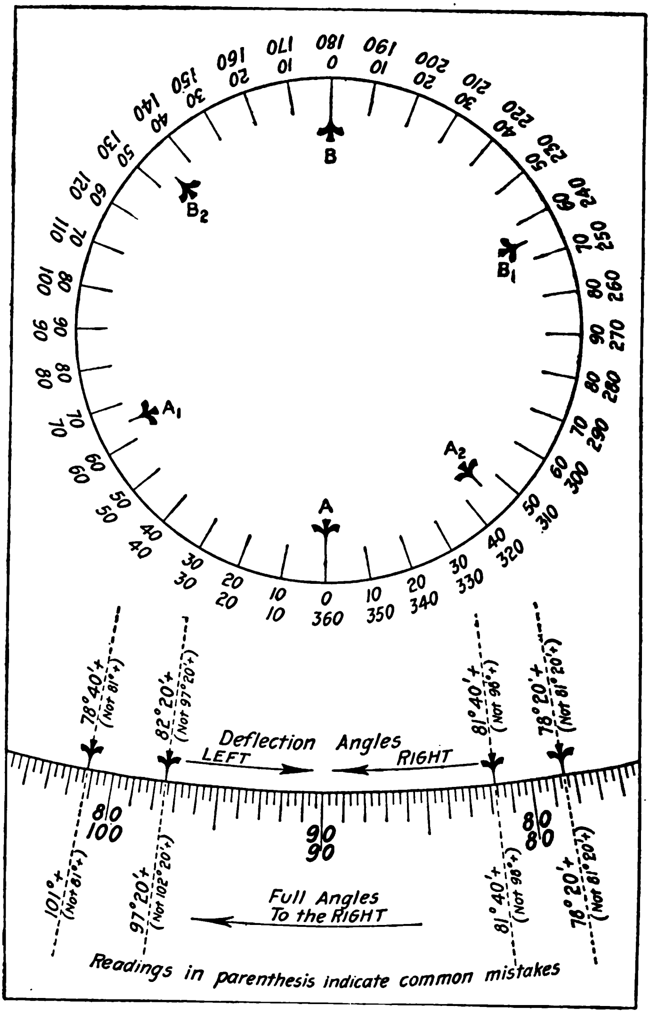
180 0
170 10
190 10
160 20
200 20
150 30
210 30
140 40
220 40
130 50
230 50
120 60
240 60
110 70
250 70
100 80
260 80
90 90
270 90
80 80
280 80
70 70
290 70
60 60
300 60
50 50
310 50
40 40
320 40
30 30
330 30
20 20
340 20
10 10
350 10
0 360
B
B₁
B₂
A
A₁
A₂
78°40'+ (Not 81°+)
82°20'+ (Not 97°20'+)
Deflection Angles
LEFT
RIGHT
81°40'+ (Not 98°+)
78°20'+ (Not 81°20'+)
80
100
90
90
80
80
101°+ (Not 81°+)
97°20'+ (Not 102°20'+)
Full Angles
To the RIGHT
81°40'+ (Not 98°+)
78°20'+ (Not 81°20'+)
Readings in parenthesis indicate common mistakes

Fig. 96.

96. Combination of Full-circle and Quadrant Systems. (*a*) There are two zero points in the quadrant system (see inner circle), and there should be two verniers to correspond, the *A* vernier, nearly under the eye-end of the telescope, and the *B* vernier, across the circle 180° from *A*. Observe the following rules: 1. *Always set the A vernier at the 360° mark or zero of the full-circle system;* (2) *For all angles except deflection angles ignore the quadrant numbering and invariably use the full circle exactly as explained on p. 71;* (3) *For deflection angles use the inner row just as it stands for angles, right or left, up to 90°.* For angles over 90° subtract the number on the inner row from 180°, and proceed as if the result were engraved on the limb in place of the number subtracted. (See similar case, preceding article.)

(*b*) *Caution.* A common mistake is to set the *A* vernier at one of the zeroes of the quadrant system but opposite the 180° mark of the full circle, and then to read from the full circle as if the *A* vernier had been set at the 360° mark. This mistake of 180° may be avoided by invariably using the full circle in setting the *A* vernier, even in deflection angles.

(*c*) *Deflection angles.* When using the quadrant system it is not necessary to plunge the telescope before backsighting. This is its chief advantage over the half-circle system. In setting the *A* vernier at 360° the *B* vernier is necessarily brought opposite one of the zeroes of the quadrant system, and after the telescope is plunged for *foresighting* the *B* vernier will be the one under the eyepiece. Hence the angle is read from the *B* vernier to save walking around the instrument.

(*d*) *Illustration.* If the indicator of the *A* vernier is set at *A* (see opposite page), the *B* vernier will be at *B*. After plunging for the foresight the *B* vernier will be under the eye-end of the telescope, and if it goes to B_1, the deflection angle is between 60° and 70° to the *right;* if it goes to B_2, the deflection angle is between 40° and 50° to the *left.* The same results could be obtained by reading the *A* vernier in the positions A_1 and A_2 respectively, but it would be necessary to walk around the instrument.

The portion of the limb shown enlarged is one of the two places where mistakes are most likely to occur; the other place is across the circle near the 270° mark. Notice that the smallest division on the limb is 20′—a division which may be used as well with any of the other systems of numbers. It is perhaps more usual to use a half-degree division with the quadrant system, but the one-third degree division is shown here for practice in reading. The readings given below the arc are for direct angles taken clockwise, and were obtained from the outer row of numbers; readings above the arc are for deflection angles and were obtained from the inner row.

97. Verniers. (*a*) It is easier to read a vernier correctly, once the least count has been ascertained, than it is to read the limb, and mistakes are less costly (p. 68). Generally if a vernier is divided into ten equal spaces between numbers as shown on p. 78, it reads to minutes, i.e., each space is *called* a minute though it really covers nearly a degree. If the equal spaces are again subdivided into two parts, each part is *called* 30″ (p. 80); if subdivided into three parts, each of the smallest parts is *called* 20″ (p. 81); if into six parts, each part is *called* 10″ (p. 82).

(*b*) *Remark.* It helps, in reading the vernier, to think of the larger divisions as minutes, and the smaller subdivisions, if there are any, as 30″ or 20″ or 10″, as the case may be. It is well to remember, however, that each space on the vernier really covers a much greater arc than is indicated by what we call it.

(*c*) *The least count* as determined by inspection should be verified when using any vernier for the first time. From the principle of the vernier the least count, i.e., the difference between l, the smallest division on the limb, and v, the smallest division on the vernier, is $x = l - v = \frac{l}{n+1}$, where n represents the number of spaces on the limb corresponding to $n+1$ spaces on the vernier, or $nl = (n+1)v$, from which equation the value of x is derived. For verniers as usually constructed the following rule will also apply:

Divide the value of the smallest division on the limb by the number of parts on the vernier.

98. To Read a Vernier. Keeping in mind the value of the smallest division on the vernier it is an easy matter to get the vernier reading. If the zero or indicator exactly coincides with a mark on the limb, the angle is read without using the vernier (p. 67); otherwise find some mark on the vernier which is exactly opposite some mark on the limb. It is immaterial what the mark on the limb is,—the mark on the vernier determines the *vernier* reading. The limb reading to which the vernier reading must be added is determined separately, as explained in the first part of the chapter.

99. Suggestions for Reading Verniers. (1) *Double verniers* like those shown on succeeding pages would be unnecessary if angles were always taken clockwise. *Always read that side of a double vernier which is ahead of the zero or indicator,* the *direction in which the* latter moved having been assumed in reading the limb (p. 68). (2) When reading the limb estimate at a glance the approximate vernier reading. This not only saves time by indicating where to look on the vernier for the line of coincidence, but it often saves blunders such as reading the wrong side of a double vernier. (3) Form the habit of looking at one or two marks each side of the vernier mark which you think is in coincidence. If three marks coincide, take the middle one; if two marks coincide, take the average reading. In the latter case the reading will be closer than the least count of the vernier. For example, the reading will be to 30″ on a vernier which only reads to minutes. (See p. 78.) This means that *the error involved in reading any vernier is not greater than half the least count of that vernier.* (4) In setting the vernier at zero or at any angle, examine in the same way one or two marks each side of the indicator. (5) When there are two verniers 180° apart use the one under the eye-end of the telescope. Avoid setting one at zero and reading the other, except in special cases like that explained on p. 75. (6) Use a reading-glass (p. 98). (7) Practical suggestions for measuring angles are given in Chapter IX.

100. Common Mistakes in Reading Verniers. (1) Mistakes due to reading the wrong half of a double vernier. (2) Arithmetical mistakes in adding the vernier reading to the limb reading, or, what is still more common, dropping the *minutes* from the original limb reading, as 68° 16′ instead of 68° 46′ (p. 78). Do not be so intent on reading the vernier as to lose track of the correct limb reading. Remember that the latter is by far the more important. (3) In verniers reading to seconds the mistake is sometimes made of calling seconds minutes, as 151° 40′ instead of 151° 0′ 40″ (p. 81). This is most likely to happen when the vernier reading is less than a minute.

101. Illustrations of Verniers. In studying the following illustrations of different verniers keep in mind: (1) That the vernier and limb are enlarged to a little more than twice the usual size; (2) The lowest line of each illustration shows how the least count is obtained from the formula on p. 76, Art. 97 (*c*); (3) The whole process of reading the angle is indicated: first the limb reading, then the plus sign, then the vernier reading, and finally the total reading. The mistaken readings in parentheses are wholly mistakes in vernier readings or in *dropping minutes from the* original limb readings.

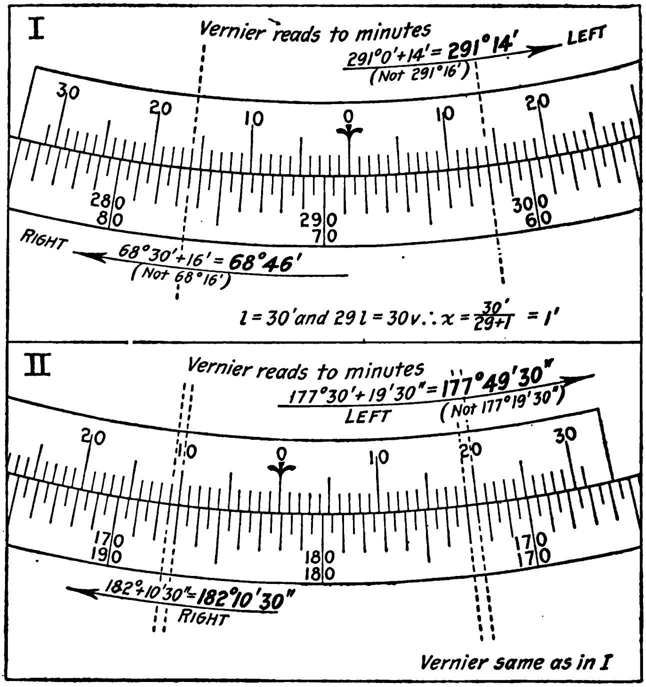

FIG. 102.

102. I. and II. Verniers Reading to Minutes. (*a*) This is one of the forms of verniers most used in ordinary work. The mistake of 291° 16′ instead of 291° 14′ came from reading the wrong half of the double vernier. In the mistake of 68° 16′ instead of 68° 46′ the 30′ of the original limb reading was forgotten when it came to adding the readings. The mistake of 177° 19′ 30″ instead of 177° 49′ 30″ came in a similar manner from dropping the minutes from the original limb reading.

(*b*) In II two marks on the vernier apparently coincide with two on the limb, and hence the mean of the two readings is correct. This illustrates how 30″ may appear in a reading though the smallest space on the vernier is *called* a minute. In a great deal of work it is not worth while to take account of seconds in a case like this,—simply read to either whole minute.

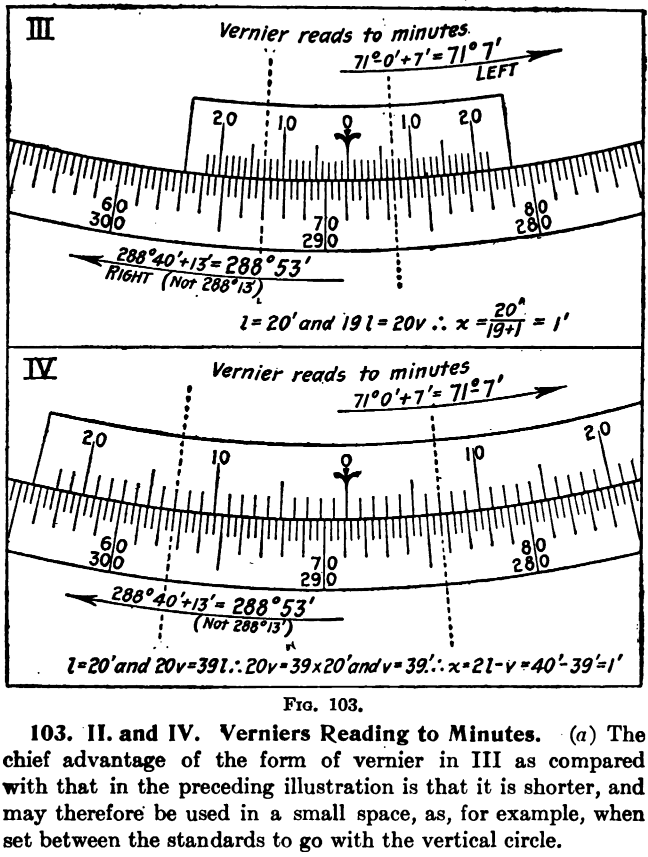

FIG. 103.

103. II. and IV. Verniers Reading to Minutes. (*a*) The chief advantage of the form of vernier in III as compared with that in the preceding illustration is that it is shorter, and may therefore be used in a small space, as, for example, when set between the standards to go with the vertical circle.

(*b*) In Illustration IV the vernier is constructed like that on p. 80 except that every alternate mark on the vernier is omitted. Each division on the vernier is called *one* minute, but it is really 39′ long or 1′ short of two spaces on the limb. The advantage of this construction is that the division-marks on the vernier itself are farther apart than those in I, II, and III, and hence are easier to read. Though this is an excellent form of vernier it is not as common a one as that on p. 78.

The *mistaken readings* in both III and IV are due to *dropping the minutes from the* original limb reading.

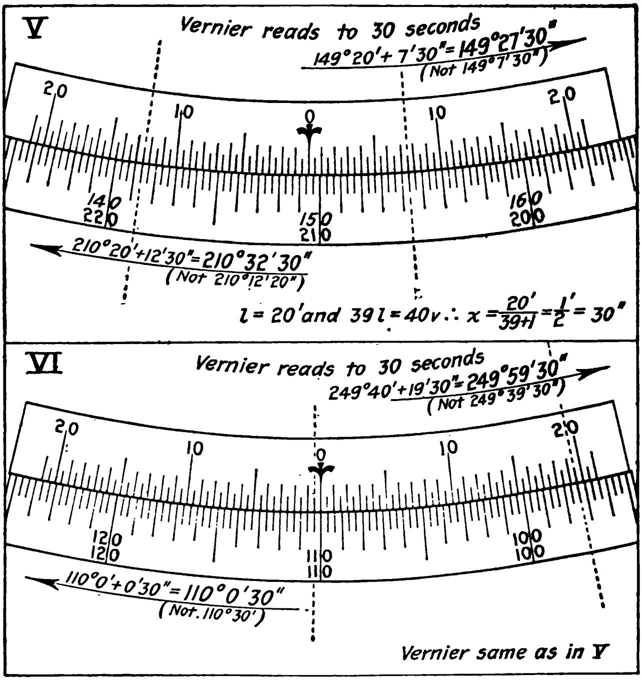

FIG. 104.

104. V. and VI. Verniers Reading to 30 Seconds. The limb is graduated exactly like the limb in III and IV, but the vernier is more finely graduated, each of the small divisions being called 30″.

Both of the mistakes in V are due to dropping 20′ from the original limb reading. Notice also in the reading 210° 12′ 20″ the mistake of calling the smallest division on the vernier 20″ instead of 30″. This is a natural mistake, since the smallest division on the limb is 20′. The mistake of 249° 39′ 30″ was also due to dropping 20′ because of the impression that the limb reading was 249° 20′ instead of 249° 40′.

The mistake of 110° 30′ instead of 110° 0′ 30″ came from adding the vernier reading as if it were 30′ instead of 30″.

The fact that 30″ and 20′, the smallest divisions on the vernier and the *limb respectively, are unequal numerically* leads to confusion as indicated *above, and makes this form* much less desirable than that on p. 81.

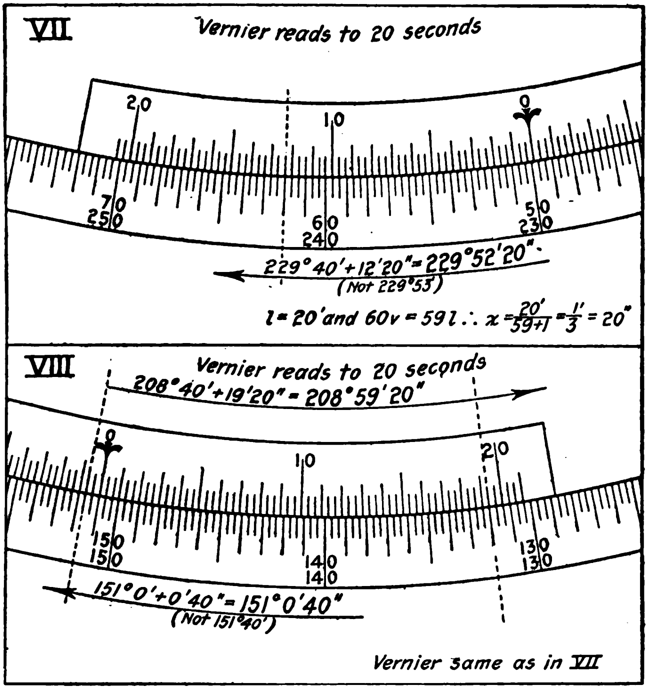

FIG. 105.

105. VII. and VIII. Verniers Reading to 20 Seconds. This form of vernier, in which the smallest space is called 20″, is used for city work, simple triangulation, and other work of high accuracy.

The mistake of 229° 53′ instead of 229° 52′ 20″ came from observing the 12′ mark on the vernier and then counting the next space as if it were a whole minute instead of 20″, thus getting 13′ to add to the limb reading instead of 12′ 20″.

The mistake of 151° 40′ instead of 151° 0′ 40″ came from calling the vernier reading 40′ instead of 40″. This would probably not have occurred had the vernier reading been a full minute or more.

Question. Keeping the graduations on the limb the same, how *could the vernier* be changed so that its least count would be 10″?

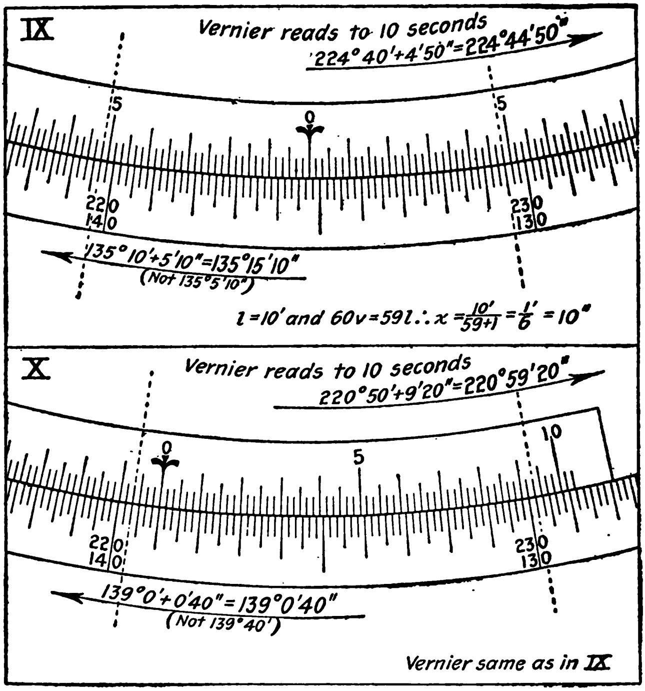

FIG. 106.

106. IX. and X. Verniers Reading to 10 Seconds. This form of vernier, in which the smallest space is called 10″, is for extremely accurate work, and is seldom found except on sextants and on high-grade instruments. On a vernier like this, several marks may appear to coincide with marks on the limb. Take the middle one.

The mistake of 135° 5′ 10″ instead of 135° 15′ 10″ came from dropping the 10′ from the original limb reading.

The mistake of 139° 40′ instead of 139° 0′ 40″ came from calling the vernier reading 40′ instead of 40″—a mistake most likely to occur when the vernier reading is less than one minute.

Question. What is the maximum error involved in a reading when a vernier like that in IX and X is used, assuming that no *actual mistake* is made? (See p. 77.)

107. Special Forms of Verniers. Special forms of verniers may be devised at will, as, for example, verniers reading to a *decimal* of a degree (convenient for railway curves).

(*a*) A vernier sometimes used on the sextant has 60 spaces corresponding to 119 on the graduated arc. This makes each space on the vernier a little less than two spaces on the arc. It is used as a direct vernier. (If the smallest division on the graduated arc is 20′, prove that the vernier reads to 20″.)

(*b*) *Retrograde* verniers are seldom used on transits.* A special form of retrograde vernier is used when it is desired to economize space, as, for example, in the compass-box for setting off the declination.† Thirty spaces on the vernier correspond to 31 on the limb, the vernier reading to minutes. There are two rows of numerals on the vernier arranged as follows:

	20	25	30	25	20	
15	10	5	0	5	10	15

The usual rules for reading do not hold good. Thus, if the indicator is to the *right* of the zero, read the lower figures on the *left*-hand vernier, unless the reading is over 15′, and then the upper figures on the right-hand vernier are used. If the indicator is to the left of the zero, read lower right-hand figures for less than 15′, upper left-hand for more than 15′. (See from the figure that this must be so.) Note that this vernier is only half as long as the ordinary double vernier, and that the lower left-hand and the upper right-hand verniers form one vernier and *vice versa*.†

108. Summary. For a summary of this chapter see Arts. 131, 132, pp. 96–97.

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* Verniers in which the smallest space is larger than the corresponding smallest space on the limb.

† A similar "folded" vernier is often used for setting declinations which is direct instead of retrograde, 30 spaces on the vernier corresponding to 29 half-degrees on the limb.

CHAPTER IX

USE OF THE TRANSIT

THE only way to learn to use the transit is to practice with it in the field, but many suggestions and directions may be given which will help to make that practice intelligent and effective. After a preliminary reading, this chapter will best serve its purpose if the different articles are consulted *at the time that corresponding work with the transit is being done* in the field. Study, first, *methods* of work, and then consult the chapter frequently, thus gradually acquiring *habits* of work. It may be added that directions often seem trivial to the beginner which are nevertheless well worth following when learning to use such an important instrument as the transit.

The chapter on *The Transit*, Part III, shows how the instrument is constructed, and explains the workings of its most important parts. The care and adjustment of the transit is also treated in Part III.

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109. Caution. First of all, treat the transit kindly. Remember, many of the parts are of soft metal. Never force a screw—all screws should work easily; if they do not, there is some trouble which brute force will not help. The diameter of the head of a thumb-screw often gives a leverage out of all proportion to the strength of the screw, and it is comparatively easy to strip the thread. Beginners too often feel that clamps should be set as tight as possible, with the result that each time they have to be set a little tighter and soon become useless. The condition of the clamps on comparatively new school instruments should be a lasting object lesson. If the object-glass slide in the telescope moves hard, probably dust or grit is inside, and it will not take a very strong man either to destroy the rack and pinion, or to cut deep scratches and grooves along the surface of the slide. *Rubbing* dust from the object-glass with the handkerchief may get the dust off, but it often leaves scratches on. Blow it off or brush gently.

Do not let a transit stand with the tripod legs so close together that it is easily upset; on the other hand, if the legs are too far apart they may open out and let the transit down; this is especially likely to happen on a smooth floor. Be sure that the head of the transit is firmly screwed on its tripod.

Before taking the transit up to carry it from one place to another do three things: (1) Loosen the lower clamp so that both plates will turn freely if accidentally hit; (2) Point the telescope straight up, leaving it free to turn on its supporting axis at the slightest knock; (3) See that the needle is lifted so that the delicate point of its pivot will not be injured. As a further precaution do not carry the transit on the shoulder when going through a doorway or through any other place where it is likely to get bumped. Carry it under the arm.

110. Plumb-bob. Do not begin the day's work by leaving the plumb-bob behind. The string to the bob is looped over a hook under the center of the transit, and has a sliding knot by which to adjust its length. The figure shows the sliding knot before it is drawn tight. But instead of putting the end of the string through the loop as shown in the figure, double it back upon itself so that the knot may be untied at any time by simply pulling the free end, as in the case of a bow-knot. Avoid any other knot in the string, even one at the end.

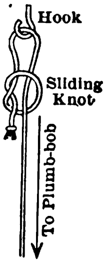

FIG. 110.

Another device is to fasten one end of the string to a piece of wood containing two holes, while the other end of the loop passes through the holes and down to the plumb-bob. Friction and the crook in the string hold the device in any position as it slides up or down the plumb-line. Bone or leather may be used in place of the wood.* If the wind is blowing, shield the bob with something (a hat will do) to keep it from swinging.

111. Using the Telescope. (See also the chapter in Part III on *The Telescope*). Point the telescope toward the sky and move the eyepiece in or out until the cross-hairs are clearly visible. On some instruments the eyepiece is moved by screwing or twisting it, while in others a focusing-screw with rack and pinion is used. Once the eyepiece is adjusted to the eye of the transitman, it is not necessary to disturb it again. (See *parallax*, Part III.)

2. To get the line of sight on an object quickly, sight along the top of the telescope before sighting through it.

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* *See Eng. News.*, Aug. 16, 1900, or Baker's Surveying Instruments, p. 35.

3. In focusing, remember that the nearer the object the farther out the object-glass must be run. "Near-far and far-near."

4. Do not use the upper or lower part of the vertical cross-hair for sighting when its intersection with the horizontal hair can be used just as well. The vertical hair may not be truly vertical.

5. In sighting on a pole, sight as far down as possible (at the very bottom if that can be seen), for the pole may not be held exactly plumb.

6. If the telescope has stadia-wires, do not mistake either of them for the regular horizontal hair.

7. Do not take hold of the telescope itself to move it sidewise; form the habit of turning it by taking hold of the plate on which the standards rest.

8. If the telescope is used in the dark, the cross-hairs must be illumined by reflecting light down the barrel of the telescope. Mining transits are furnished with a device for this purpose. (See also observation on Polaris.)

112. Transiting, Plunging, or Reversing the Telescope. Synonymous terms for turning the telescope completely over on its supporting axis, thus bringing it upside down, and making it point in exactly the opposite direction from what it did before. The upper and lower plate clamps are set so that the telescope cannot move sideways while it is being reversed.

113. Setting up the Transit. The transitman aims at two things in setting up a transit. viz.: (1) He must center it over some given point; (2) He must "level up," i.e., make the graduated plate horizontal. The given point is usually a stake driven into the ground, the head of a tack or nail marking the exact point on the stake. The tripod, plumb-bob, and shifting-head are used to "center" the transit, the thumb-screws and plate-levels to "level up." The process may be outlined as follows:

1. Set the tripod, with shoes pressed firmly into the ground, in such a position that, when its head is *approximately level* (judged by the eye), the plumb-bob will come to rest *nearly* over the point (tack). (See suggestion 5, Art. 114.)

2. Loosen two *adjacent* thumb-screws, and, using the shifting-head, bring the point of the plumb-bob over the required *point (center of* tack-head). Use the sliding device on the

plumb-bob string, if necessary, so that the point of the bob just clears the stake.

3. Tighten the two thumb-screws (previously loosened for shifting), and by means of all four screws level up.

4. See that the plumb-bob still hangs over the center of the tack when the plates are level. If in "leveling up" it has been disturbed, bring it back to the center by using the shifting-head and level up again.

Speed in setting up a transit is worth a great deal, for a slow transitman can delay the work of a whole party. Moreover, setting up over a railway track "between trains" often means quick work. It is true that the knack of setting up a transit comes from practice, but make it *intelligent* practice from the start. Consult the next article frequently and practice until the methods suggested become second nature.

114. Suggestions for Setting Up a Transit.

(1) The clamps in the upper part of the tripod should be adjusted so that they are just tight enough for friction to hold the legs in any position.

(2) Spread the tripod legs just far enough to bring the telescope at a convenient height: it saves stretching up or stooping down.

(3) Manipulate the tripod so that its head is approximately level to begin with. The less the thumb-screws are used the better: saves wear, saves time. On level ground set the shoes about equal distances from the tack. On a slope place *two* legs on the down-hill side.

(4) If the tripod head is too much out of level, the thumb-screws in "leveling up" will bind; do not force them; start over again by moving the legs of the tripod so as to make the head level.

(5) *A most important suggestion.* Set up the transit at first within a foot or two of the station without any attempt to bring the bob over the tack, the main object being to get the plates *approximately* level, as judged by the eye, and the telescope at a convenient height, with the tripod shoes merely resting on the ground. This having been accomplished in two or three quick movements, lift all three shoes off the ground and move the transit bodily until the bob is over the tack, push the legs into the ground and proceed to use the shifting-head and leveling-screws as already explained.

Remark. It seems difficult to convince students that the preliminary setting up to one side of the station is not a waste of time. If done correctly, however, it takes but a few seconds, and a fair trial will show that *it actually saves time,* especially on a side hill. As the transit is taken *off the shoulder, set it* down with one leg up hill, take the other two legs, *one in either hand, and* bring it to position, then move it bodily over the *station.*

(6) *Three important suggestions for manipulating the tripod.* When the bob is within a few inches of the tack, but too far away to be brought over it with the shifting-head, the legs of the tripod must be moved slightly. Keep in mind these three suggestions:

(*a*) If the plumb-bob is off the tack, but the plates are nearly level, move a tripod leg on its own pivot towards or away from the tack. (Changes the position of the bob, but does not throw the plates greatly out of level. This is the condition most often met with.)

(*b*) If the bob is over the tack, but the plates are out of level, swing a tripod leg right or left without moving it on its own pivot. (Tends to level the plates without changing the position of the bob.)

(*c*) If the bob is only a little off the tack, first get it "on line" with one leg and the tack, and then bring it over the tack by moving that leg thus: If bob is at *a*, Fig. 114 (*c*), bring it to *b*, or to *c* by moving legs *d* and *e*, and then bring it over the tack by moving leg *f*, or by moving both legs *d* and *e*.

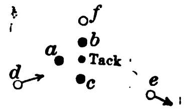

Fig. 114 (*c*).

(7) The tripod must stand firmly. On soft ground push the legs well in, but not as if they were to stay for all time. Pushing against a leg crosswise is a waste of strength and may break it by bending; push lengthwise. On hard, smooth surfaces, such as floors and pavements, put the shoe-points, if possible, in cracks or small holes, to prevent slipping.

(8) Avoid setting a tripod leg "on line" where it might interfere with stretching the tape in chaining between stations.

(9) If the wind is blowing, kneel on the windward side of the plumb-bob and shield it with the body or with a hat.

Suggestions for using the thumb-screws in leveling up.

(10) Two *diagonally opposite* screws are worked at the same time, first one pair, then the other, alternating until the plates are level. The thumbs always move toward or away from each other. The knowledge of which way to turn the screws should soon become instinctive. It may be helpful, however, to remember *that the bubble will move in the direction in which the left thumb is turned.*

(11) If there is only one plate-level, it is placed parallel to *the first pair of* thumb-screws used, and then parallel to the

other pair. If there are two plate-levels, each is placed parallel to a pair of diagonally opposite screws.

(12) Keep all four thumb-screws bearing evenly and firmly. If one of a pair is turned faster than the other, it will either bind or be raised off the bearing plate; stop turning this screw until the other catches up. If all are turning hard, the tripod head is too much out of level, or one pair has been forced, or possibly the screw-threads need cleaning. Consult an instructor.

Remark. The leveling-screws should turn so easily that they may be *rolled* between the palms of the hands. This is quicker than turning them with the fingers.

(13) After leveling up, take hold of the graduated plate and see if it is firm; if it rocks, one or more of the thumb-screws is not bearing properly.

(14) In setting up to prolong a straight line, it is well to have one pair of diagonally opposite thumb-screws in the line, the other pair at right angles to it.

(15) In precise work, especially where the transit is to be used for leveling, it is better to be guided by the level attached to the telescope, as this is more sensitive than the plate-levels. The latter may be used until the plates are approximately level. The telescope is then clamped in a horizontal position, and brought first over one pair of leveling-screws and then over the other, just as in setting up a level. It must be carefully noted, however, that in the case of the transit the bubble is brought halfway back to the center of its tube with the leveling-screws, and the rest of the way by moving the telescope up or down with the slow-motion screw which controls the revolution of the supporting axis.

115. Accuracy Required in Setting Up a Transit. It is not always necessary to have the bob exactly over the tack or the plates exactly level. Like all surveying operations, "setting up" demands care in proportion to the results required.

If angles are being read to the nearest minute, and points sighted at are from 50 to 100 feet away, the point of the bob can be from ¼ to ½ in. from the center of the tack without seriously affecting results. (See the next chapter.) An experienced transitman, however, should be able to bring the bob to within less than a quarter of an inch of the point without special effort or waste of time.

In leveling up, the following points should be remembered:

1. In measuring the horizontal angle between two points about on the same level, the plates need be only approximately level. (They can often be leveled by the eye alone.)

2. In measuring the horizontal angle between a high and a low point, more care should be taken in leveling up—especially should the bubble be in the center of that tube which is at right angles to the telescope.

3. In measuring the vertical angle between points in the same vertical plane, it is only that plate-level parallel to the telescope which requires special attention.

The student will be well paid if he will experiment for himself along the lines suggested by this article.

Errors caused by poor setting up are: (1) From not getting the center over the tack; (2) From not getting the plates level. Errors from these sources are as a rule comparatively small.

116. Backsight—Foresight. A transit stands at B. It is desired to measure the angles ABC and ABD, and to produce the line AB to E. In measuring either angle or in producing the line, if the point A is the first point sighted at, then AB is called the backsight, while the sights on C, E, and D are called foresights. An indefinite number of sights to other points (all foresights) could be taken, the angles to which could all be measured from AB as a backsight. A backsight is a point of reference (or more strictly a line of reference from the transit to some other point); from it angles are measured to lines (foresights) to other points. If the angle is 180°, the line of backsight is simply produced. A backsight is usually the first sight taken after the transit is set up, and ordinarily only one backsight is used no matter how many foresights may be needed.

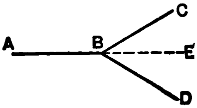

FIG. 116.

Note. The terms backsight and foresight are unfortunate in that they imply direction: they have nothing to do with direction. Thus, if only the angle ABC were to be measured, one might sight on C first, in which case CB would be the backsight.

(1) *In this book when three letters or numbers are used to denote an angle the* MIDDLE *letter or number represents the* VERTEX *of the angle (i e., where the transit stood when the angle was measured). Unless otherwise stated it may be assumed also that the* FIRST *letter or number represents the* BACKSIGHT *and the* THIRD *letter or number the* FORESIGHT.

Illustration. If $XYZ=40°$, Y is the vertex, X the backsight, and Z the foresight.

117. Permanent Backsight. A backsight is usually taken on a pole or pencil held by one of the party. It is often desirable to establish a *permanent backsight* so that the transitman can sight to it whenever he desires without keeping a man there to give the sight. When the line of sight is set on the back*sight for the first time,* before unclamping, move the telescope up or down. Usually the line of sight will strike some object definite and prominent enough to use as a backsight in place of the other, thus allowing the flagman to be called in. A crack in the fence, a mark on a tree, some definite corner of a house, or something similar, can be used, but if none is readily found, it often pays to have a flagman *make* a "permanent backsight" by sticking in a flagpole or a chain-pin, or by making some mark that can be easily distinguished through the telescope.

118. Permanent Foresight. When, as is often the case, a number of points are to be "lined in" with some point ahead, it is an excellent plan to establish a *permanent foresight* as far ahead as practicable, so that if the line of sight gets off the point the transitman will notice it and bring the line in again.

119. Use of the Upper and Lower Clamp.

The *lower clamp* and corresponding slow-motion screw are used in getting a *backsight; these must never be disturbed while taking a foresight.*

The *upper clamp* and tangent-screw are used in setting the indicator at zero and in getting a foresight.

(*a*) *Note.* The importance of keeping the above in mind is readily seen. If, *after* taking a backsight, the transitman *will keep his hands off the lower part of the transit,* he will avoid the danger of disturbing his backsight. When in doubt, turn the upper screw—that can do little harm. Disturbing the backsight may mean beginning all over again. *Note that a tangent-screw will not work until the corresponding clamp is set.* Errors often arise from *thinking* clamps are set when they are not, or from clamps slipping. Clamps should not be forced, but they must be set tight enough to hold. After setting on the backsight, it is well to try turning the upper plate *gently,* as a test before beginning to take angles. If the cross-hairs move off the point and do not return, the clamp must be tightened.

120. Lining in, or Sighting in Line. Be sure the line of sight is on the desired point and all clamps set. Look along the top of the telescope and motion the flagman approximately into line, then look through the telescope and, sighting at the point of the rod if possible, signal into line. (Make the signal correspond to the distance the rod must be moved: a vigorous motion means to move it quite a distance, a slight motion means a short distance). For sights three hundred feet or less, a lead-pencil is often used instead of a rod. Do not split hairs in lining in the chainman. (See Art. 63, p. 41.)

In setting the stake for a transit station, first line the stake in and, if possible, watch it as it is being driven; make the axeman drive it so that its middle is about on line, and then set the tack with exactness, using a pencil to sight on when possible.

121. Reversing in Azimuth and Altitude, or Double Reverse. A technical term for the following process: A transit stands at *T*. Clamp the upper and lower plates together at the start and do not unclamp during the process. This means that *no angles are read*, and no use whatever is made of the vernier,—it is not necessary even to set it at zero.

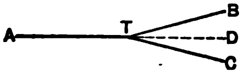

FIG. 121.

(1) Backsight on *A*. (Using the lower clamp and tangent screw.)

(2) Plunge the telescope. If the transit is in adjustment, the line of sight will strike some point in the line *AT* produced, as, for example, the point *D*; if, however, it is out of adjustment, it will strike some other point, as *B*.

(3) Let the telescope remain bottom side up, unclamp the lower screw, turn the two plates together until the telescope (still bottom side up) again points at the backsight on *A*. Clamp the *lower* clamp and bring the cross-hairs exactly on *A* with *lower* tangent-screw.

(4) Plunge the telescope. (The telescope now becomes right side up.) If the transit is in adjustment, the line of sight will again strike the point *D*, but if it is out of adjustment, the line of sight will strike a point *C* which is as far on one side of *TD* as *B* is on the other side. In practice, unless the instrument is badly out of adjustment, the points *B* and *C* are usually within an inch or so of each other, and the true point *D*, halfway between *B* and *C*, is easily established by measurement.

Note. No use is made of the graduations on the lower plate, thus eliminating error from incorrect graduation or from not setting the vernier at exactly 180°. Notice also that either the upper or the lower clamp can be used, provided both are tight when the telescope is plunged. It is better, however, *to set the upper clamp once for all before taking the first backsight and then leave it alone.* If, in setting up the transit, hint (14), page 89, is followed, the error due to the plate-level being out of adjustment may be eliminated by leveling for the bubble in that tube at right angles to the line, just before each backsight is taken. Do not take hold of the telescope to turn it around horizontally.

122. Prolonging a Straight Line. (Four methods.)

1. Set up at one end, sight at the other end and line in.

2. Set up at one end, backsight on the other end and, plunging the telescope, line in.

3. If not sure that the transit is in adjustment, use the d of double reverse. In important work this method be used as a precaution whether the transit is in ad- nt or not.

alance in the transit on any point in the line produced rt. 124), and set other points as desired.

Double Centering. A term sometimes used to denote rd method in the preceding article of finding a point on prolonged.

Balancing In. Getting the transit on line with two when the point at which the transit is to stand has en previously established. This is done by trial. When of the given points can be seen from the other and ansit stands between them, the work is more difficult. ethod pursued is given on p. 201.

Measuring Horizontal Angles. In measuring an keep in mind that the lower part of the instrument is a protractor, and the upper part merely er with the zero mark of the vernier for an or. The outline of the actual work done in ing an angle, as ACB, is:

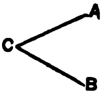

FIG. 125.

Set up at C.

With upper and lower clamps loose, revolve both plates e zero on the vernier coincides as nearly as may be with o on the limb. Set the upper clamp and with the corling tangent-screw make the zeroes exactly coincide. s called setting the vernier at zero.) Observe the marks de of the zero.

Backsight at A. When the line of sight is nearly on A lower clamp and make it exactly strike A with the corling tangent-screw (lower).

Loosen the upper clamp, turn the upper plate until e of sight is nearly on B, set the upper clamp and he line of sight exactly strike B with the upper tangent-

Read the angle.

g the wrong clamp or tangent-screw is a frequent source r. See Art. 119 (a), p. 91. Notice which is used for eration, viz.:

ng the vernier (2). Upper clamp and tangent-screw.
sighting on A (3). Lower clamp and tangent-screw.
ighting on B (4). Upper clamp and tangent-screw.

It will help matters to remember that in measuring angles the only use made of the lower clamp and lower tangent-screw is in *backsighting*, and that the upper clamp and the upper tangent-screw are used for everything else, including *setting the vernier and foresighting. The upper clamp or tangent-screw must never be disturbed when backsighting, and the lower clamp or tangent-screw must never be disturbed when foresighting.*

For a more accurate method of measuring angles (by repetition) see Art. 138, p. 99.

126. Angles Taken to the Right or Left. Any angle as *BCA* has two values, one measured clockwise, the other counterclockwise. If measured clockwise, it is said to be taken to the right; if measured counterclockwise, it is taken to the left. The difference lies wholly in *the direction in which* the angle is read; it makes no difference whatever in which direction the telescope is turned when moved from the backsight to the foresight. This will be clear if the methods of reading right and left angles as explained in the preceding chapter are understood. In ordinary surveying it is better on the whole to take all angles to the right, but in some lines of work it is more convenient to take them right or left. Keep in mind that, except for deflection angles, 360° minus an angle to the right equals the corresponding angle to the left and *vice versa.*

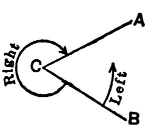

FIG. 126.

127. Taking a Number of Angles from the same Backsight.

1. Set the vernier at zero. (Upper clamp and tangent-screw.)
2. Backsight on *A*. (Lower clamp and tangent-screw.)
3. Foresight on *B*. (Upper clamp and tangent-screw.) Read the angle.
4. Foresight on *C*. (Upper clamp and tangent-screw.) Read the angle.
5. Foresight on *D*. (Upper clamp and tangent-screw.) Read the angle.

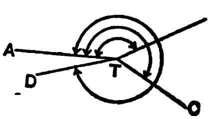

FIG. 127.

Thus any number of angles may be taken from *A* as a backsight. Note again that the lower clamp and corresponding tangent-screw are *not to be disturbed* after the backsight is taken.

128. Setting the Vernier. Setting the vernier at zero is merely a matter of convenience. In some cases it is better to set it at random, read it before and after measuring the angle, and take the difference of the two readings.

Illustration. The vernier is set at 30° 10′ before backsighting. The reading from the foresight is 70° 21′. The value of the angle is: 70° 21′ − 30° 10′ = 40° 11′.

129. Suggestions for Measuring Horizontal Angles.

1. Remember that the telescope cannot be turned right or left without turning the upper plate with it.

2. If there are two verniers, it is sufficient in ordinary work to use only one, and that should be the one which is more nearly under the eyepiece of the telescope. (Saves walking around the transit.) The other vernier across the circle should not be used at all except in special cases. Avoid the mistake of using first one and then the other indiscriminately, or getting the two confused.

3. Form the habit of estimating by eye whether an angle is between 90° and 180°, or between 180° and 270°. This is one of the best hints that can be given, and if the beginner will check his readings in this way he will save himself absurd mistakes of an even 90° or an even 180° due to reading the wrong vernier or the wrong number on the limb. (See p. 71, Art. 94 (*b*).)

4. After the backsight is taken see that all clamps are tight and that the vernier is set exactly at zero before loosening the upper clamp to take angles, but *don't force the clamp*. To make sure, test the lower clamp by trying *gently* to turn the plate. If the cross-hairs move slightly off of the backsight but come back again when the hands are removed from the plate, the clamps are tight.

5. Do not rest the hands on the tripod or upon any other part of the instrument.

6. In taking a large number of angles from the same backsight a good transitman will find time, every few angles, to sight on his "permanent backsight" to make sure that the lower part of the transit has not been disturbed. After taking a set of angles always do this the last thing before leaving the station.

7. In taking a number of angles from the same backsight, it is not necessary to set the upper clamp each time if the tangent-screw is not used. Where angles have to be read rapidly and to the nearest minute or two, it is better not to use the upper clamp at all.

8. If a number of angles are taken from the same backsight, and if, as is often the case, one is more important than the others, aim to take the important one first as soon as possible after backsighting, to avoid the error from slight disturbances of the transit.

9. In taking angles to the right it is not necessary to turn the telescope to the right. Turn it through the shorter arc.

10. Follow the suggestions for using the vernier and reading angles given in Art. 131.

130. Sources of Error in Measuring Horizontal Angles.

If the transit is in good working order, well adjusted and no parts loose, the most common sources of error are:

1. Careless setting up,—not over the tack, or plates not level, or legs not firm.

2. The slipping of a clamp when it is supposed to be tight.

3. Rough or careless handling of the instrument.

4. Careless sighting,—intersection of cross-hairs not used, or sighting at the top instead of the bottom of the rod.

5. Instrument disturbed or jarred out of position by walking around it, by passing teams or cars; vibration of telescope in strong wind.

6. Mistakes in setting the vernier or in reading the angle. A discussion of these errors will be found in the next chapter.

131. Suggestions for Using the Vernier and Reading Angles.

(A summary of Chapter VIII.)

1. Use the upper clamp and tangent-screw for setting the vernier. Loosen *both* clamps, upper and lower, until the setting is near enough to use the tangent-screw, then tighten the upper clamp; after that remember that the upper tangent-screw cannot be turned *without disturbing the setting of the vernier.*

2. If there are two verniers, 180° apart, use the one that is nearly under the eyepiece of the telescope. Do not use the other vernier at all except in special cases.

3. Ascertain from inspection whether the center (or inner) row of numbers on the limb is to be read for angles to the right. Use this row exclusively except in work where angles are taken to the left.

4. Do not make the mistake of setting the vernier at the wrong zero. If, for example, the outer row is graduated from 0° to 360°, and the inner row is graduated by the half-circle method or by the quadrant method (see pages 73–75), one of the

zeroes on the inner row will come opposite to the 180° mark on the outer row. A common mistake is to set the vernier at this zero and then read the angle as if it had been set at the 360° mark (or corresponding zero) of the outer row. This will make an error of 180° in the angle.

5. In setting the vernier as well as in reading angles, look at two or three vernier marks rather than at a single mark.

6. Before looking at the vernier make sure that you have the limb reading correct, especially the even number of degrees. Always keep in mind the value of the smallest division on the limb and do not neglect the fraction of a degree. Estimate roughly from inspection of the limb what the vernier reading should be before looking at the vernier.

7. In a double vernier read the vernier which is in advance of the indicator.

8. In reading the vernier keep in mind the value of the smallest division, and if it is in *seconds* do not call it *minutes*. Notice if the final reading agrees with that previously estimated from the limb.

9. Finally. The order in reading an angle is: (1) Note the limb reading *carefully;* (2) Estimate the vernier reading; (3) Read the vernier; (4) Add the vernier reading to the limb reading.

132. Common Mistakes in Reading Angles. (See Chapter VIII.)

1. Setting the vernier at a zero of the inner circle, but not at the zero of the outer circle, and reading the angles thereafter from the outside circle, or *vice versa.*
2. Omitting the *fraction* of a degree in reading a limb.
3. Reading the inside circle of numbers on the limb instead of the outside circle, or *vice versa.*
4. Getting the number of degrees wrong when the limb is not numbered continuously from 0 to 360.
5. Reading the *B* vernier instead of the *A* vernier, or *vice versa.*
6. Reading the wrong side of a double vernier.
7. Calling the smallest division on a 30″, 20″, or 10″ vernier minutes instead of seconds.
8. Looking at a single vernier mark instead of at two or three.
9. Arithmetical mistakes in adding the vernier reading to the limb *reading.*

10. Finally: Mistakes are more likely to occur in reading the limb than in reading the vernier, and mistakes in reading the limb are the more serious. Many such mistakes may be avoided by habitually estimating angles by eye as a rough check on the reading. (See suggestion 3, p. 95.)

133. The Reading-glass. A small magnifying-glass is used for reading the vernier; one about an inch in diameter is a good size. Steady the glass by resting the hand on the plate near the vernier, and bring the glass slowly toward the eye until the best results in clearness and magnification are obtained. In bright sunlight a good way is to use both hands, one resting on either side of the vernier, with the reading-glass supported between them. This effectively shades the vernier.

134. Taking Angles from the Backsight Prolonged. In some classes of work, especially in railroad work and in surveys which continue more or less in a straight line, it is more convenient after backsighting to plunge the telescope and take angles right or left. Thus in Fig. 134, backsight on A, plunge the telescope and measure an angle CTD to the right or CTB to the left. Such angles are called *deflection angles*. (See p. 120.) *Deflection angles are never greater than 180°.*

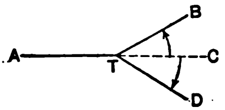

FIG. 134.

135. Vertical Angles. (*a*) *With the horizontal.* If necessary, first adjust the vertical arc so that it is at zero when the line of sight is level (as shown by the level-bulb attached to the telescope). Sight approximately at the point, set the clamp that controls the movement of the telescope in the vertical plane, and use the corresponding tangent-screw to bring the line of sight exactly on the point. Read the vertical arc.

Caution. On some transits the vertical arc is fastened to the supporting axis by a clamp, and when this clamp-screw is loose the axis may turn without taking the vertical arc with it. Be constantly on guard when measuring vertical angles with such an instrument, to make sure that the reading is zero when the bubble is at the center of its tube.

If neither the vertical arc nor the corresponding vernier is adjustable, note the reading when the line of sight is horizontal. This reading, sometimes called an *index error*, must then be applied *with its proper algebraic sign* to all readings, just as if the vertical angle were taken between any *two* points, as explained in the next paragraph.

(*b*) *Vertical angle between two points.* Sight first at one point, note the reading on the vertical arc, then sight at the other and note that reading. Use the clamp and tangent-screw attached *to the supporting* axis. Inspection of the vertical circle will make it obvious whether to add or to subtract the readings. If both sights are above or below the horizontal, subtract; if one is above and the other below, add. It is not usually necessary in surveying to measure vertical angles with the same precision required for horizontal angles.

136. To Turn off a Horizontal Angle. The reverse of measuring an angle. Set the vernier at zero and clamp the plates together (upper clamp). Backsight and set the lower clamp. Loosen the upper clamp and set the vernier at the angle to be laid off. Set the upper clamp, "line in" the pole, stake, tack, or whatever is used to mark the desired point.

Note. Notice that, as in measuring angles, the lower clamp is only used for backsighting. The most common angle to lay off is 90° for foundations of structures, for rectangular pieces of land, for triangulation, and for many other purposes. For a more accurate method (by repetition) see Art 142.

137. To Measure Horizontal Angles with Great Accuracy. Errors in adjustment must be eliminated and the angle read to as small a fraction of arc as the instrument will allow. As most transits are more or less out of adjustment, and as the ordinary transit reads to minutes or possibly half-minutes, the method of Art. 125, depending on a single reading, will not give the best results. One of the two methods—by repetition or by series—may be used where greater accuracy is required. Both methods depend upon multiplying the number of the observations.

138. Measuring Angles by Repetition. To measure an angle by repetition means to measure it two or more times, allowing the vernier to remain clamped each time at the previous reading instead of setting it back at zero when sighting at the backsight.

(*a*) Measure the angle once in the usual manner, note the reading (merely as a check), and *let the two plates remain clamped at the reading.* Backsight again and measure the angle a second time in the same direction as before. The reading should now be double the real value of the angle.

(*b*) *Illustration.* Transit at *C*, backsight at *A*. Measure *ACB*; suppose vernier reads 30° 1′. Allow vernier to remain clamped at 30° 1′, backsight at *A*, and measure the angle again. The vernier will now read 60° 2′, which divided by 2 gives the value as before, 30° 1′. But suppose instead of 60° 2′ it reads 60° 3′. This would indicate that the angle is 30° 1′ 30″. But if the transit reads to minutes, this result could not have been obtained from a single observation without estimating the 30″. By letting the vernier remain at 60° 3′ and measuring for a third time, if the reading should be 90° 4′ the value would be 30° 1′ 20″. Thus on a transit reading to minutes results may be obtained to 30″ by doubling an angle, and to 20″ by *tripling it.*

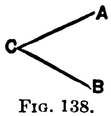

FIG. 138.

(*c*) *Doubling* an angle is measuring an angle by the above method twice before reading, *tripling* three times, *quadrupling* four times, and so on. More than six repetitions are seldom used. Of course, a vernier may be set at random to start with instead of at zero. In running transit lines, doubling angles may be used merely as a check.

Caution. Keep in mind the rule, Art. 119: Backsight—lower: foresight—upper. In repeating an angle, turning the lower tangent-screw when foresighting, or the upper tangent-screw when backsighting, will make it necessary to start all over again.

(*d*) *Adding multiples of 360°.* In repeating angles it is sometimes necessary to add 360° or 720° to a reading. *First illustration.* In doubling an angle of 200° the first reading is 200°, the second reading is 40°, or 40+360=400 and 400÷2=200°. *Second illustration.* An angle of 130° is sextupled, and the final reading is 60°, or 60+360+360=780, and 780÷6=130°. The indicator on the vernier has moved twice around the graduated circle and 60° over.

(*e*) *Measuring deflection angles by repetition.* This involves the method of double reverse (p. 92); otherwise it does not differ from the method just explained.

(1) Measure the deflection angle in the usual way.

(2) Keep the plates clamped at the reading obtained in (1), loosen the lower clamp, and, with the telescope still bottom side up, backsight a second time as in the method of double reverse.

(3) Plunge (the telescope now becomes right side up), and again measure the deflection angle, dividing the final reading by two.

In addition to the usual advantages of doubling an angle this method eliminates errors in adjustment of the line of collimation.

139. Measuring Angles by Series. This method may be used where several angles have a common vertex. Angles are measured in succession around the horizon, and the set "closed" by sighting on the first backsight. If the vernier does not read the same as it did to start with, the error is distributed equally between the angles irrespective of size.

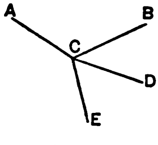

FIG. 139.

Illustration. Transit at *C*. Set vernier at random, say 20°. Backsight on *A*. Vernier reading 20°.

Fore-sight	Vernier Reading	Angle	Subtraction	Value
B	140° 21′	*ACB*	(140° 21′) − 20° 0′)	120° 21′
D	160° 32′	*BCD*	(160° 32′) − (140° 21′)	20° 11′
E	200° 40′	*DCE*	(200° 40′) − (160° 32′)	40° 8′
A	20° 1′	*ECA*	(380° 1′) − (200° 40′)	179° 21′
				360° 01′

Error = 20° 1′ − 20° = 1′ or 15″ to be subtracted from each angle, thus: 120° 20′ 45″ + 20° 10′ 45″ + 40° 7′ 45″ + 179° 20′ 45″ = 360° 0′ 0″. By taking angles in combination, as, for example, *ACD*, *BCE*, and *ACE*, other *equations of conditions* may be obtained.

140. Additional Precautions in Measuring Angles. In very accurate work, when measuring angles either by repetition or by series, one or more of the following precautions may be taken to eliminate error due to lack of adjustment, eccentricity, faulty graduation, and other instrumental errors:

1. Read both (opposite) verniers each time.
2. Take angles to the right, then to the left.
3. Take angles with the telescope normal, then with it inverted.
4. Shift the zero of the vernier along the limb so that different portions of the graduated arc may be used.

Taking angles in sets. When the above precautions are used it is necessary to take the angles in "sets." Each set, complete in itself, gives a value of the angle, and the final value is found either by taking the arithmetical mean, or by adjusting the values from equations of conditions. The latter properly belongs to higher surveying. In Chapter XVI, on *Triangulation*, more explicit directions are given for taking angles in sets.

141. Comparison of the Two Methods. The method of repetition is to be preferred for the measurement of a single angle, and for a transit with coarsely graduated circle. The method by series is used in triangulation when precise instruments with finely graduated arcs are employed. In either method the precautions of Art. 140 will theoretically eliminate instrumental error. In repeating angles the number of sights is doubled, hence the chances for errors in sighting and clamping are doubled, and the time nearly doubled. Thus this method, perfect in theory, is not so superior in practice as at first appears, and the limit of precision is ordinarily reached after a few repetitions.

142. To Lay Off an Angle by Repetition. The reverse of measuring an angle by repetition.

Illustration. Transit at *C*. Backsight at *A*. Required to lay off 30° 1′ 30″, transit reading to minutes.

Backsight at *A* with vernier set at 0°.
Foresight by setting vernier at 30° 1′, point falls at *B*.
Backsight at *A*, vernier still at 30° 1′.
Foresight by setting vernier at 60° 3′, point falls at *D*.
Required point is at *E*, half-way between *B* and *D*.

Note. The first and second points, *B* and *D*, are usually so close together that no difficulty is experienced in getting the half-way point. It may be well, however, having set the final point, to actually measure the angle by repetition in the usual way as a check. Precautions corresponding to those of Art. 140 can be taken if necessary.

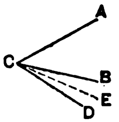

FIG. 42.

Deflection angles may be laid off in a similar manner by reversing the method of 138 (*e*).

143. Reading Bearings. (*a*) The compass-box on a transit is attached to the *upper plate*, and when the telescope is turned sidewise the compass-box must turn with it. When the telescope is right side up the letter N. is directly beneath the front end of the telescope. A line between N. and S. is always directly beneath the axis of the telescope (or line of sight) *no matter in what direction the telescope may be pointing.* When the magnetic needle is lowered to its pivot it will come to rest pointing north, and if the telescope is also pointed to the magnetic north, the north end of the needle will point to the letter N. (Fig. 143 (*a*)). If, however, the telescope is turned in some other direction, the letters N., E., S., and W. turn with it, but the needle will still point to the magnetic north, and each of its ends will usually rest *between* two of these letters. To read the bearing of a line, pay no attention to the telescope except to see whether it is normal or inverted, and do not try to think beforehand whether the bearing is going to be N.E. or S.E., S.W. or N.W. If the telescope is right side up, simply observe between which two letters the north end of the needle rests, and how many degrees it is from the letter N. or the letter S. as the case may be. The south end of the needle is observed instead of the north end when the telescope is inverted. *The north end of the needle is the end without the weight.*

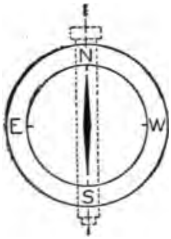

FIG. 143 (*a*).

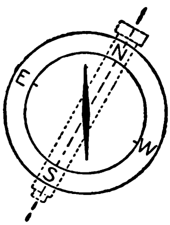

FIG. 143 (*b*).

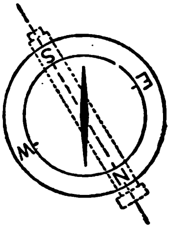

FIG. 143 (*c*).

On most transits the letters E. and W. are interchanged, the E. being on the left and the W. on the right. The reason for this will appear from the following illustrations:

(*b*) *Illustration.* In Fig. 143 (*b*) suppose the telescope to be pointing N. 30° 30′ E. In reading the bearing the *north* end of the needle will be found between the letters N. and E. or 30° 30′ from N. towards E. If W. had been on the left in place of E., as one might naturally expect it to be, the north end of the needle would fall between N. and W., which might lead to the mistake of calling the bearing N.W. instead of N.E.

(*c*) *Illustration.* In Fig. 143 (*c*) suppose the telescope to be pointing S. 30° 15′ E. The *north* end of the needle is observed to be 30° 15′ from S. *toward* E., as it should be. If E. and W. had not been interchanged, *it would have fallen* between S. and W.

transits where the E. and W. have not been interchanged one must guard against the mistake illustrated above of reading east for west *versa.*

Precautions. (1) Never read a bearing from E. or from W. The etter is always N. or S., and the last letter is always E. or W.

In recording a bearing do not omit either the first or the last letter. at value is a bearing recorded N. 40°, or one recorded 35° E.?

In some transits a vernier will be found within the compass circle, used, when desired, for setting off the magnetic declination, so that the needle points to N. the telescope will point to the *true* north in- of the magnetic north. This makes it possible to read true bearings d of magnetic bearings. (See Art. 154.) *When reading magnetic gs be sure that this vernier is set at zero.*

CHAPTER X

ERRORS IN ANGULAR MEASUREMENTS

THE aim in this chapter is to discuss the relative importance of the different sources of error in angular measurement, to suggest limits of error and to point out how the limits of error adopted for angular measurement should be consistent with those adopted for linear measurement.

144. Sources of Error in Measuring Angles.

1. *Instrumental errors.* (1) Errors of adjustment, and (2) errors from defects in construction which cannot be eliminated by adjustment, such, for example, as eccentricity in the graduation on the limb. (See Part III.) Methods of eliminating instrumental errors are given in Art. 140, p. 101.

2. *Mistakes in manipulating the transit.* Summarized in Art. 130, p. 96.

3. *Mistakes in reading angles.* Summarized in Art. 132, p. 97.

4. *Errors in setting up the transit.* (1) Transit not centered over the station, and (2) plates out of level.

5. *Errors in sighting.* Intersection of cross-hairs not exactly on the point sighted at.

6. *Natural sources of error.* Wind, refraction, temperature, obstacles.

145. Relative Importance of Error. In linear measurement the magnitudes of the errors from different sources are pretty well known, but in angular measurement the errors are so affected by varying conditions that they cannot be estimated in advance. Instrumental errors, mistakes in manipulating the transit, and mistakes in reading angles, for example, do not admit of much theoretical discussion. In ordinary surveying such errors can be practically eliminated, and then their relative importance does not matter. The last three sources of error mentioned in the preceding article may be briefly treated.

146. Errors in Setting up the Transit. Contrary to the

neral impression the error due to the plumb-bob being off
e station is not so important as the error due to the plates
ing out of level. This is especially true in hilly ground where
e telescope may be pointed up for one sight and down for
ıother. It is to be remembered also that in measuring an
ıgle the error from either source is greater for short sights
an for long sights, while in laying off an angle the reverse is
ue. It does not take much if any longer for a good transit-
an to so set up his instrument that in ordinary work the erroı
ɔm either source is inappreciable. Although the plumb-bob
ay be as much as ⅓ inch off the station, the resulting error
ll probably not exceed one minute for 100-ft. sights, while
r 1000-ft. sights it will be a small fraction of a minute. As
e plumb-bob can be brought to within ⅛″ of the point with-
ıt special effort and still closer if desired, it is seen that the
ıportance of this source of error may be easily overestimated.
the limb is 6″ in diameter and one edge is 3/16″ lower than the
her, a maximum error of one minute of arc might result. A
fference of ½″ in level might cause a maximum error of from
x to seven minutes, but such a difference in level would be
parent to the eye.

147. Errors in Sighting. (*a*) The practical question here
How much error is caused by not sighting exactly on a
int? Suppose that in *measuring* an angle a line of sight which
ould strike a given point passes a short distance, say one
ch, to one side of that point. The nearer the given point
to the transit the greater the angular error caused by this
parture of one inch. On the other hand, suppose that a
ıall error, say of one minute, is made in *laying off* an angle.
ıe line of sight will depart farther and farther from the true
ıe as the distance from the transit becomes greater and greater.
me idea of the magnitude of errors from these two sources
ay be gained from the following *approximate* values:

ngth of sight in feet	Angular error for one inch or 0.08 ft.	Deviation for error of one minute
100	3 min.	0.03 ft. or ⅜ in.
300	1 min.	0.09 ft. or 1 in.
500	½ min. or 30 sec.	0.15 ft. or 1⅞ in.
1000	¼ min. or 15 sec.	0.30 ft. or 3⅝ in.
1500	⅙ min. or 10 sec.	0.45 ft. or 5⅜ in.
5000	1/20 min. or 3 sec.	1.50 ft. or 18 in.

(*b*) *Note.* It is well to fix two or three of these values in mind. For example, a point an inch out of line at 100 ft. from the transit causes an error of about 3′. This error varies very nearly *inversely* as the distance of the point from the transit, so that one minute corresponds approximately to one inch 300 ft. from the transit. Again, an error of 1′ of arc brings a point 0.3 ft. out of line 1000 ft. from the transit. This error varies *directly* as the distance of the point from the transit, so that at 500 ft. the error is half as much, or 0.15 ft.

(*c*) *Illustration.* A rod is held so that the top is one inch out of plumb. It is 100 ft. from the transit. Sighting at the top results in an angular error of 3′. Hence the importance of sighting at the bottom of the rod or, when this is impossible, of holding the rod plumb. The nearer the rod is to the transit the more need there is for accurate sighting.

(*d*) *Illustration.* The following illustration shows how much more important it is to hold the rod plumb than it is to split it exactly with the vertical hair; incidentally it emphasizes also the importance of sighting at the bottom of a rod when it is possible to do so.

If a rod one inch in diameter is held on a point 300 ft. from the transit, the maximum error due to the vertical hair not splitting the rod will not exceed half a minute of arc, provided the hair covers the rod at all. On the other hand, in sighting at the top of a rod which is, say, two inches out of plumb, the error will be four times as great, or two minutes. It is evident that it is easier to bring the vertical hair on the rod than it is to hold the latter plumb. Moreover, when all but the upper foot or two of a rod is invisible from the transit, the top may be considerably more than two inches out of plumb with a corresponding increase in error.

148. Natural Sources of Error. A high wind, a hazy atmosphere, the sun shining on the instrument, and other natural sources of error often render accurate work with the transit very difficult. For work of great precision atmospheric conditions are usually most favorable directly after sunrise.

149. Elimination of Errors. *Mistakes.* Keep in mind the "danger-points" where mistakes are likely to be made in reading angles. (Art. 132, p. 97). Form the habit of estimating angles by eye as a rough check (p. 95). Make use of other checks whenever possible, such as the check by the magnetic needle or by doubling the angle. Be sure that the line of sight is directed at the point which is supposed to be taken; do not get stations confused, and when it becomes necessary to sight at the top of a rod make sure that it is plumb.

Constant errors. One of the reasons why the measurement of angles is so satisfactory is that constant errors may be largely eliminated by taking observations in pairs. Thus, for example, plunging the telescope between two observations eliminates instrumental errors, and reading both verniers eliminates eccentricity of graduation. Other precautions are given in Art. 140, p. 101, and an illustration of the method of procedure when it is desired to measure angles with high precision is given in Chapter XVI, *Triangulation.*

Accidental errors are eliminated by adjusting values of angles, aking the mean of several observations and by distri buting total error when several angles are related by some fixed ition (Art. 24 (*b*), p. 14).

50. The Precision of Angular Measurements may be ed in one of two ways: (1) By comparing the results of ral independent measurements of the same angle, and (2) bserving how closely the values of different but *related* angles l known conditions.

) *First method.* In ordinary work if an angle is important gh to check at all, it is usually doubled. (Method of ition, p. 99.) In more important work an angle may be ured a number of times and the precision will then be ated by the probable error (pp. 15–19). The more com- ly the assumptions of least squares (p. 15) are fulfilled nore nearly will the probable error represent the true error 7, Art. 27 (*a*)).

) *Second method.* When the angles of a polygon are meas- the *true* error of their *sum* is known (p. 119), but not the of any one angle. The same is true of several angles ng vertices at a common point, when, as in Fig. 139 (p. 100), sum should equal 360°. When, however, the sum of two ore angles should equal a *measured* angle the *discrepancy* een the sum and the measured angle is not an error in the sense of the word, but simply the difference between two tities both of which contain an error (see p. 14). In hree cases errors in one angle may be compensated by s in another angle, so that the error of the sum does not sent the actual precision of the work.

Illustration. The three angles of a triangle often add up exactly to when measured with a transit reading to minutes. This does not that no error has been made, but that the errors are exactly com- ting. It is not likely that any one of the three angles can be ex- d in even minutes. If, therefore, they are measured with a finely ated transit reading to 10″, their sum is less likely to add up to 180° if a transit reading to minutes is used, although the precision with each angle is measured may be much greater. Thus, for example, angles were measured with the following results:

	Transit reading to 1 minute	Transit reading to 10 seconds
1...........	68° 10′	68° 9′ 40″
2...........	79° 40′	79° 40′ 25″
3...........	32° 10′	32° 9′ 40″
	180° 0′	179° 59′ 45″

error of the sum in the second set is 15″, yet the precision for each is probably *much greater* than in the first set where the error of the *zero. In a similar* manner the angles of a triangle as determined by reading a compass to the nearest 15′ will often add up 180°, while the same angles measured with a transit fail to do so, yet the precision of the work done with the transit is probably far greater.

151. Conditions which Affect the Precision of Angular Measurements in addition to the sources of error already treated are:

(1) *Instrument used.* How well made; how finely graduated; power of telescope, sensitiveness of levels, and rigidity of tripod.

(2) *Local conditions.* Conditions affecting the setting up and the stability of the transit. Time allowed for completion of the work.

(3) *Atmospheric conditions.* Wind, haziness, temperature, and refraction.

(4) *Method* used in measuring the angle.

152. Customary Limits of Error. (*a*) Varying conditions render it difficult to state what constitutes good work with the transit. Moreover, as already pointed out on p. 44, the limits of error are largely governed by the purpose of a survey, and even in the same survey angles are not all measured with the same precision. Suppose, however, a limit of error has been fixed for chaining and it is desired that the precision of angular measurement shall correspond. The question at once arises: What is the largest permissible error for each angle? In the following table the errors in feet caused by different angular errors are given for five lengths of sights, viz., 100 ft., 500 ft., 1000 ft., 1500 ft., and 5000 ft.:

Error	**100**	**500**	**1000**	**1500**	**5000**	**Ratio**
1′	0.0291	0.1454	0.2909	0.4363	1.4544	$\frac{1}{3440}$
30″	0.0145	0.0727	0.1454	0.2182	0.7272	$\frac{1}{6880}$
20″	0.0097	0.0485	0.0970	0.1454	0.4848	$\frac{1}{10300}$
10″	0.0048	0.0242	0.0485	0.0727	0.2424	$\frac{1}{20600}$
6″	0.0029	0.0145	0.0291	0.0436	0.1454	$\frac{1}{34400}$
4″	0.0019	0.0097	0.0194	0.0291	0.0970	$\frac{1}{51600}$
2″	0.0010	0.0048	0.0097	0.0145	0.0485	$\frac{1}{103100}$

0.0104 ft = ⅛ *in.* 0.0208 ft. = ¼ in. 0.0417 ft. = ½ in.
0.0833 ft. = *1 in.* 0.2500 ft. = 3 in. 0.4167 ft. = 5 in.

This table has two uses: (1) It gives the permissible angular error for a given ratio of precision; (2) it gives an idea of the error *in feet* at different distances from the transit caused by a given error in *minutes or seconds.*

(*b*) *Permissible angular error.* The ordinary transit reads to minutes. If an error in an angle does not exceed one minute, the ratio of precision is about $\frac{1}{3400}$. It is not difficult to keep within this limit in chaining, and in a large part of the surveyor's work greater precision is not required. A ratio of $\frac{1}{5000}$ is classed as fair work for measurements with a steel tape (p. 49). To come within this limit angles should be read to 30″ (see table), and this can easily be done with a transit reading to minutes (p. 78). For a precision of $\frac{1}{10000}$ the angles should be read to 20″, therefore a transit graduated to 20″ is preferable. For the higher limits of $\frac{1}{35000}$, $\frac{1}{50000}$, and $\frac{1}{100000}$ it is seen that angles must be read to 6″, 4″, and 2″ respectively. This requires the use of accurate instruments, finely graduated, with corresponding precautions in sighting and manipulation. Moreover, a number of independent measurements of each angle must be taken (either by repetition or by series, see pp. 99, 100), from which the probable value of the angle may be deduced.

It is evident that in any given survey the limit of error for angles and the limit of error for linear measurements should be consistent. This is too frequently overlooked, one limit being set for chaining, another for transit work. An exception to be noted is in rough chaining when corresponding angular measurements can be made with greater precision than the linear measurements without special effort or loss of time.

(*c*) *The permissible error in the position of a point,* if kept in mind, will often save unnecessary refinement in measuring angles in certain classes of work, while in other work it will emphasize the necessity for accurate measurements. This is shown in the following illustrations.

(*d*) *Illustration.* In a topographic survey it is sufficient to locate contour points within, say, a foot or two. The longest sight is, say, 400 ft. From the table the error for 1′ for a 100-ft. sight is 0.029 ft., hence for 400 ft. it is about 0.12 ft. To come within 1.5 ft. the angular error must not exceed $1.5 \div 0.12 = 12$ minutes. For sights less than 400 ft. the permissible error is still greater. As limb readings can easily be estimated to within 10′, this means that for 400-ft. sights, or less, it is not necessary to consult the vernier—an obvious saving of time.

(*e*) *Illustration. A certain point about* 500 ft. from the transit must *be located within* $\frac{1}{4}$″. From the table, $\frac{1}{4}$″ (or 0.0208 ft.) for 500 ft. corresponds to an error of a little less than 10″. This is the maximum permissible angular error.

(*f*) *Error in the sum of angles.* Since errors in angular measurements are largely compensating, when several angles are added together the total error of their sum may be expected to vary about as the square root of the number of angles. Limits of error may be indicated as in chaining (p. 45); thus $R_s = r\sqrt{N}$, where R_s is the total error of the sum, r the permissible error for one angle, and N the number of angles either in a polygon or about a point. From the following table the value of R_t in seconds can be obtained for different values of r and N:

Permissible errors in the sums of angles for different values of r

N	$r=60''$	$r=30''$	$r=20''$	$r=10''$	$r=6''$	$r=4''$	$r=2''$
3	104″	52″	35″	17″	10″	7″	3.5″
4	120″	60″	40″	20″	12″	8″	4″
6	147″	73″	49″	24″	15″	10″	5″
9	180″	90″	60″	30″	18″	12″	6″
12	208″	104″	69″	35″	21″	14″	7″
16	240″	120″	80″	40″	24″	16″	8″

In ordinary work the value of r may be taken as 30″ (see p. 109). The error which may be expected in the sum of the angles of a triangle is 52″, or about a minute. For a four-sided polygon the table gives exactly a minute, while for one of sixteen sides it is two minutes. Of course these values are not absolute,—they are for guidance only. In selecting the value of r the correlation of angular and linear measurements already discussed should be kept in mind. The tendency is to measure angles with unnecessary accuracy, and to neglect important sources of error in linear measurements.

CHAPTER XI

BEARINGS AND AZIMUTHS

Bearing and *azimuth* mean so nearly the same thing in surveying that the two terms are often confused. Each is used to designate the angle which a line makes with a north and south meridian, but there is an important distinction as to how this angle is measured. The distinction is made clear in this chapter, as is also the distinction between *forward* bearing and *back* bearing, and between *forward* azimuth and *back* azimuth.

For the method of calculating the bearing of one line from the bearing of another line when the angle between the two is known see chapter on *Calculation of Bearings*, Part II. For properties of the magnetic needle consult Part III. For the method of surveying with the compass see the chapter on *Compass Surveying.*

---

153. Magnetic Declination. In Part III it is explained that—

1. Except in a few localities the magnetic needle does not point to the *true* north, but to a *magnetic* north. The angle between a true north-and-south line and a magnetic north-and-south line as observed at any place is the *magnetic declination* for that place.

2. The *magnetic declination* differs for different places, and varies slightly from year to year in the same place.

3. The *agonic* line is a line passing through places where the declination is zero. In the United States it passes through the central part of South Carolina, Ohio, and Michigan.

4. For places *east* of this line the declination is *west*, and for places *west* of it the declination is *east*. This fact helps the memory in deciding whether the declination for any place is east or west.

5. Local attraction is a term used to denote any influence which prevents the needle from pointing to the magnetic north in a given locality.

154. The Bearing of a Line. If a north-and-south line passes through one end of a given line, two supplementary angles are formed; the *smaller* of these two angles is the *bearing* of the given line. Bearings are always measured from the north point or the south point so many degrees east or west, *never* from the east point or the west point so many degrees north or south.

Illustration. In Fig. 154 the four possible directions in which bearings are measured are indicated by the arrows. Thus the bearing of *CA* is N. 30° E., *not* E. 60° N.; of *CB*, S. 60° E., *not* E. 30° S.; of *CD*, S.° 70° W., *not* W. 20° S.; of *CF*, N. 40° W., *not* W. 50° N.

By definition a bearing can never exceed 90°. For example, a line 120° from north measured clockwise is *not* N. 120° E., but S. 60° E.

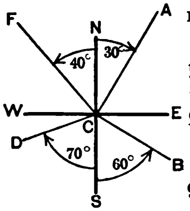

Fig. 154.

True bearing. The bearing of a line with respect to a *true* north and south meridian.

Magnetic bearing. The bearing of a line with respect to a *magnetic* north-and-south meridian. This is the bearing that is read from a compass (unless the declination arc has been turned off), and differs from the true bearing at any place by the amount of the magnetic declination at that place.

Question. Why will the true bearing of a line change if the line is prolonged far enough with the transit?

155. Forward Bearing and Back Bearing. Every line of definite length has two bearings, one for each end of the line. If the bearing of any line from one end is N.E., from the other end it will be S.W. and *vice versa*; likewise if from one end of a line the bearing is N.W., from the other end it must be S.E. This is an important distinction because it indicates direction northerly or southerly. Thus, for example, if the bearing of a line *AB* (Fig. 155) is given indiscriminately as either S. 30° E. or N. 30° W., one cannot tell whether *B* is southeast or northwest of *A*, but if from *A* to *B* (*AB*) it is S. 30° E. and from *B* to *A* (*BA*) it is N. 30° W., all doubt is removed. The *forward bearing* of a line is the bearing in the direction in which the survey is being run, while the *back bearing* is that

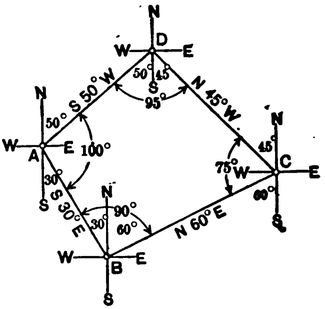

Fig. 155.

in the opposite direction. The two bearings of a line are numerically equal but with opposite letters.

Illustration. In Fig. 155, if the survey is made from *A* to *B* to *C* to *D* to *A*, the bearings as given on the figure are all *forward* bearings, but if the survey is made from *A* to *D* to *C* to *B* to *A*, the bearings as given are *back* bearings. Assuming the stations to have been occupied in the order *ABCD*, the bearing of *AB* is S. 30° E., or back bearing is N. 30° W.; *BC* is N. 60° E., or back bearing is S. 60° W.; *CD* is N. 45° W., or back bearing is S. 45° E.; *DA* is S. 50° W., or back bearing is N. 50° E.

Note. In this book when referring to the bearing of any line, that end of the line at which the bearing was read will be given first. Thus if the bearing of a line *AB* read at *A* is S. 30° E., then *AB*=S. 30° E. and *BA*=N. 30° W., i.e., the bearing of *BA* is the back bearing of *AB*.

156. Reading Bearings. The method of reading the needle in a compass is explained on page 102.

157. The Bearings Kept in a Survey are usually *all forward* bearings. It will not do to record forward bearings and back bearings indiscriminately. While it is true that one can always determine from the needle whether the forward bearing is N. or S., nevertheless one can usually do this without consulting the needle, since after working in any locality a short time the compass-points are well fixed in mind.

Note. When puzzled in the field as to whether the first letter or bearing is N. or S., imagine yourself walking along the line *in the direction in which the survey is being made*, and note whether you are going in a northerly or southerly direction; if the line should happen to be so nearly east and west that you cannot tell, then ask yourself if you are going easterly or westerly. If, for example, you know that the bearing of a line must be either N.E. or S.W., you can usually determine in this way which of the two is the *forward* bearing.

158. Calculated Bearing is the bearing of one line found by calculation from the bearing of another line, taking into account the horizontal angle formed by the two lines.

Note. The way in which calculated bearings are used in transit surveying is explained in the next chapter. The method of calculating bearings is given in the chapter on the *Calculation of Bearings*, Part II.

159. Things to Remember about Bearings—Summary.

1. The magnetic bearing differs from the true bearing by the amount of local magnetic declination.

2. Bearings are reckoned from either the north or the south points, never from the east or west.

3. Every bearing must be given by two letters. The first must be either N. or S., the second either E. or W. The omission of either letter renders the bearing of no value.

4. No bearing is over 90°.

5. Every line has two bearings, forward and back, equal numerically but with opposite letters.

6. The order in which the two ends of a line are given should indicate from which end a corresponding bearing is taken.

160. The Azimuth of a Line is the *angle* which that line makes with a *north-and-south* line as a reference-line. Azimuths are measured *clockwise* from the *north* as a starting-point. *Azimuth* differs from *bearing* in that it is measured in one direction only (clockwise) and from but one point of the compass (N.). A line the azimuth of which is anywhere between 0° and 90° is in the N.E. quadrant; between 90° and 180° in the S.E. quadrant; between 180° and 270° in the S.W. quadrant; between 270° and 360° in the N.W. quadrant. Thus in speaking of, or recording, azimuths, the compass-points N., E., S., and W., are not used, the *magnitude* of the azimuth alone indicating the direction of the line—a decided advantage.

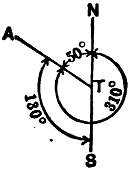

Fig. 160 (*a*).

(*a*) *Illustration.* A transit standing at *T*, Fig. 160 (*a*), is pointed at *A*. The angle *NTA* measured *clockwise* is 310°, and if *NT* is a north-and-south line, the azimuth of *TA* is 310°. If the bearing of *TA* is required, it must be from the azimuth, 360 − 310 = N. 50° W.

(*b*) The meaning of the term *azimuth* as given above is a restricted one, but it is the one to be everywhere understood in this book unless otherwise specified. Many surveyors measure azimuths from the south as a starting-point, in which case the azimuth of *TA*, Fig. 160 (*a*), would be 130° instead of 310°.

Note. The chief advantage in measuring azimuths from the north instead of from the south lies in the fact that by so doing the algebraic signs will accord with the directions to which the student is accustomed in trigonometrical computations. (See chapter on *Latitudes and Departures.*)

(*c*) *A general definition of azimuth.* The angle (measured *clockwise*) which a line makes with *any* reference-line. For example, in Fig. 160 (*c*), let *BC* be any line of reference *not* a N. and S. line. The azimuth of *TA with respect to BC* will be *BTA* if *B* is the end chosen as a starting-point, or *CTA* if *C* is the end from which azimuths are reckoned. For example, in any survey the first transit line may be used as a reference-line for all the other transit lines if so desired. The usual method, however, is to assume the *magnetic* north-and-south line as a reference-line, so that the *azimuth of any line* can be checked by the compass-needle.

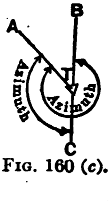

Fig. 160 (*c*).

Back azimuth. Every line of definite length has two azi-
, one for each end of the line.
in Fig. 160 (*d*), if the azimuth of
taken at *A* it is 40°, but if taken at
220°. If the survey is being made
general direction from *A* to *B*,
0° is the azimuth and 220° the back
th.

back azimuth of a line is the azi-
of that line measured at its *forward*
Since azimuths are measured always
direction (clockwise), the back azimuth is always equal
azimuth plus 180°. If this sum is more than 360°, sub-
60°.

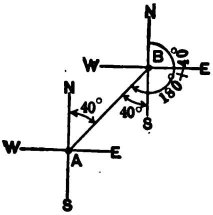

FIG. 160 (*d*).

. Facts to Remember about Azimuths.

Bearings are measured from either N. or S., but *azimuths*
m N. only (or from S. only).

Bearings are measured either right or left, while *azi-*
are always measured to the right (clockwise).

Bearings are never over 90°, but *azimuths* are from 0°
°.

In recording a *bearing*, *two* letters (compass-points) are
ed, but in recording *azimuths no* letters are used.

The back azimuth is equal to the forward azimuth plus
nd *vice versa*.

If desired, *any* line of reference may be used in place of
nd S. line, but the magnetic N. and S. line is usually the
onvenient because of the check afforded by the needle.

The term azimuth does away with all ambiguity as to
on, but since custom differs it should be plainly stated
notes of a survey, once for all, whether azimuths are
red from N. or from S.

CHAPTER XII

METHODS OF RUNNING TRANSIT LINES AND TRAVERSES

In this chapter four methods of running transit lines are explained. The *theory* of each method is given without entering upon the practical details of the field work. (See Chapter XIV for *field work.*) In order that the different methods may be compared, the advantages and disadvantages of each method are stated.

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162. Transit Station. The point over which the transit stands when in use. (Usually a stake with a tack in it to mark the exact point.)

163. Transit Line. An imaginary straight line between two transit stations.

164. Transit Angle. Any angle measured with a transit. It must, from the way in which the transit is constructed, be either a *horizontal* angle or a *vertical* angle. (See chapter on *The Transit,* Part III.) Unless otherwise specified it is usually understood to be a horizontal angle.

165. The Transit Line a Line of Reference. Transit stations are so chosen by the surveyor that the lines between them may be used as reference-lines, by which and from which boundaries, buildings, streets, or whatever he may wish to survey are located. Methods of locating details of a survey such as these are given in the next chapter.

Transit lines, as a rule, will not be inked in on the finished map, and should be regarded merely as a means to an end.

166. Running Transit-lines. A term for establishing transit stations and locating these stations with respect to each other. The work usually consists in chaining distances between successive transit stations and in measuring the angle which each transit line makes with some other transit line, or with some reference meridian.

Note. The field work of a survey may be divided into two parts: (1) *running transit lines,* and (2) locating details from those transit lines. *This chapter has to do with* the first part only.

167. Transit-line Angle. The horizontal angle between any two transit lines that meet at a transit station. It may be either the larger or smaller angle and each of these angles may be measured clockwise or counter-clockwise; in practice, therefore, *it is not enough to note the value of an angle, but the backsight and direction in which the angle is measured must be noted also.*

Illustration. In Fig. 167 let *A*, *T*, and *B* represent the *correct* location of three transit stations, the transit standing at *T*. Suppose the notes give the angle *ATB* as 45° from a backsight on *A*, but it is not stated whether the angle is measured to the right or to the left. If the draftsman assumes that it was taken to the left, plotting 45° he will get a line *TC* instead of the line *TB*.

Again, suppose that the notes say that the angle of 45° was taken to the right, but it is not stated whether *A* or *B* is the backsight. Should the draftsman assume that *B* was the backsight, plotting 45° to the right he would get a line *TD* instead of the line *TA*.

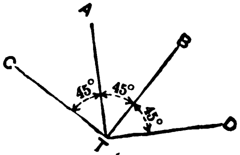

Fig. 167.

The notes, therefore, must not admit of ambiguity in respect to the backsight, or to the direction (right or left) in which an angle is taken.

168. Deflection Angle. The angle which a line makes with a transit line produced from the preceding station. Deflection angles are measured to the right or to the left, hence the *direction* should always be entered in the notes.

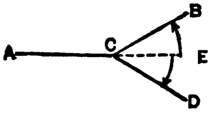

Fig. 168.

Illustration. A transit stands at *C*, Fig. 168; the backsight is at *A*; the telescope is plunged after backsighting, and then points at *E*. If turned to the right, any deflection angle, as *ECD*, may be measured to the right; if turned the other way, any deflection angle to the left may be measured, as *ECB*. When deflection angles are used at all, it is customary to measure each deflection angle to the right or to the left, according to which side of the line the point sighted at lies.

169. A System of Transit Lines. Two or more transit lines so chosen and connected as to form the basis of a survey. The transit lines are connected by the angles which they make with each other at transit stations. The distances between the stations must also be known.

Illustration. The system may form a closed polygon, as in Figs. *A* and *B*, or a series of lines which do not close, as in Figs. *C* and *D*, or a network of lines, as in Fig. *E*, p. 118.

170. Traverse. A term almost synonymous with a system of transit lines, but somewhat broader and more comprehensive in its meaning. For example, "running a traverse" with a transit, a compass, or a plane table means to run out a series of connected lines, locating the stations with respect to each other by a combination of linear and angular measurements. The term may have been borrowed from navigation, where its nautical meaning is the zigzag line described by a ship when sailing on different courses. When a traverse comes back to the starting-point it is a *closed traverse.*

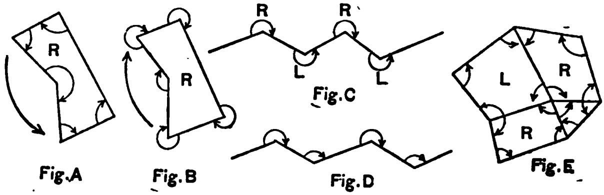

FIG. 169.

171. Triangulation. A network of connected triangles, in which the angles are measured, but not the sides. The latter are found by trigonometrical calculation, starting from some measured base-line. (See Chapter XVI.) The distinction between traverse and triangulation lies in the fact that the latter term implies the measurement of angles only.

172. The Transit-line Angles—Right or Left. Angles may be measured all to the right from a backsight on the preceding station, as in Figs. *A, B,* and *D,* or all to the left, as in two of the polygons in Fig. *E,* or right and left indiscriminately, as in Fig. *C.* (In Fig. 169 the direction in which the angles were measured is indicated by arrows; *R* in a polygon means that all angles in that polygon were measured to the right; *L* means that all angles were measured to the left.) For beginners, at least, it is better to measure all angles to the right.

173. Transit-line Angles—Interior and Exterior. It is usually more convenient, but by no means necessary, to measure the *interior* angles of a polygon instead of the exterior. To do this when all angles are measured to the right, go around the polygon *counter-clockwise* (leave the polygon on the left), as in Fig. *A.* The other direction (Fig *B*) gives the *exterior* angles.

174. A Survey Closes when the transit lines form a closed *polygon, as in Figs. A, B,* and *E.* In farm and city surveys transit lines usually close, but in railway or highway work they seldom do.

175. Closing Check on Transit-line Angles. From twice the number of sides of a polygon subtract four, and multiply the result by 90°. The final result is the sum of the interior angles.

Illustration. Triangle: $(2\times3-4)\times90°=180°$. Four-sided: $(2\times4-4)\times90°=360°$. Five-sided: $(2\times5-4)\times90°=540°$. Six-sided: $(2\times6-4)\times90°=720°$. Algebraically: If N = number of sides, S = sum of the interior angles, then $S=(2N-4)\times90°$.

176. The Error of Closure—Transit Lines. In any polygon of transit lines the location of each station with respect to any other station can be *calculated* from the angles and distances measured in the field. If the position of any station is assumed as a starting-point, and then following around the polygon, the position of this same station is found by calculation; the difference between the assumed position and the calculated position is the *error of closure.* This error may be due to: (1) errors in measuring transit-line angles; (2) errors in measuring the *lengths* of transit-lines. In practice a survey seldom closes exactly, due to error from both sources. The permissible error of closure is treated in Chapter XIV on *Field Work.*

177. The Methods of Running Transit Lines most used differ only in the method of measuring and recording the transit-line *angles;* the methods for *linear* measurements are the same, i.e., chaining or measuring with the stadia. The four methods given in the succeeding articles of this chapter may be divided into two groups:

$$I\begin{cases}\textit{Running transit lines by } \text{DIRECT ANGLES.}\\ \textit{Running transit lines by } \text{DEFLECTION ANGLES.}\end{cases}$$

$$II\begin{cases}\textit{Running transit lines by } \text{AZIMUTHS.}\\ \textit{Running transit lines by } \text{BEARINGS.}\end{cases}$$

In the first group the angles measured are those which the transit lines make with each other.

In the second group some line of reference, for example, a north-and-south line) is established for the entire survey, and the angles measured are those which the different lines make with lines *parallel to this* line of reference.

FIRST METHOD.

178. Running Transit Lines by Angles. This method consists simply in measuring each angle directly from a backsight on a preceding station. If desired, the angle can be doubled, tripled, or repeated any number of times. The lengths of the sides may be measured with a chain, a tape, or the stadia. The method may be used whether the transit lines form a closed polygon or not. On the whole it is better in this method to take all angles clockwise.

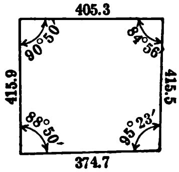

Fig. 178.

Illustration. In Fig. 178 each interior angle was measured to the right (clockwise) from a backsight on the preceding station. If more convenient, the exterior angles could have been measured. (Art. 173.)

Field Notes. Some of the forms of field notes used for this method are given in Chapter XV.

Remark. Triangulation is a special case of the angle method of running transit lines.

SECOND METHOD.

179. Running Transit Lines by Deflection Angles. A modification of the angle method especially applicable to transit lines which do not close, but progress more or less in one general direction. Instead of measuring the angle directly from the backsight, the telescope is plunged and the deflection angle (see Art. 168) is then measured right or left. (Fig. 179.)

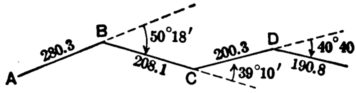

Fig. 179.

Field Notes. Some of the forms of field notes used for recording deflection angles are given in Chapter XV.

THIRD METHOD.

180. Running Transit Lines by Azimuths.* It will be assumed in this explanation that the azimuth of each line is measured from the *north* end of a *magnetic* north-and-south line. The method holds good, however, no matter what line is chosen as a reference-line. Keep clearly in mind what is meant by *azimuth* and *back* azimuth. (See Arts. 160–161, pp. 114–115.)

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* In most books on surveying this method is called *traversing*. As usually *defined this term includes* not only the azimuth method, but the quadrant *method also. Moreover, the* term traversing (or traverse surveying, or

(*a*) In the azimuth method the main thing to remember is simply this: At whatever station the transit may stand, *when the telescope is pointed north the vernier reading must be zero.* This is *not* accomplished by the use of the compass-needle except at the first set-up; on the contrary, the needle is consulted only as a rough check at all stations except the first.

There are two azimuth methods. In one the telescope is *inverted* when taking a backsight, and is *normal* when taking a foresight; in the other method the telescope is always *normal.* The essential difference between the two is that in the first method no setting of the vernier is required after the first set-up, while in the second method the vernier is set anew at each station.

(*b*) *First Method.* Let the first set-up of the transit be at station *A* (Fig. 180 (*b*)). With plates clamped at zero, point the telescope to the magnetic north as indicated by the needle. This is equivalent to *backsighting on the magnetic north, but this first station is the only one at which the needle is consulted except as a check.* Set the lower clamp as usual for a backsight, and loosening the upper clamp, sight on *B*. Suppose the vernier now reads 50°, then this angle, taken clockwise from the magnetic north, *must be the azimuth of AB. Allow the vernier to remain clamped at* 50°. Setting up at *B, plunge* the telescope and backsight on *A*. Having backsighted, plunge the telescope again; it now becomes *normal* and points in the direction of *AB produced* along the line *BP*. If the azimuth of *AB* is 50°, the azimuth of *BP* must also be 50°, and since the ver-

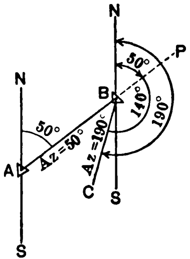

FIG. 180 (*b*).

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running a traverse) is used by many surveyors to indicate any one of the four methods of running comparatively short lines where angles and distances are both measured, contrasted, for example, with triangulation. This accords with the definition of *traverse* as given on page 117. To avoid all ambiguity, however, the method explained in Art. 180 will be called *the azimuth method.*

nier has not been disturbed, it reads 50° just as it did when the transit stood at *A* and the telescope pointed at *B*. If now the telescope is pointed north, the vernier reading will be zero, and if it is pointed to any other station, the vernier reading will be the *azimuth* of the line from station *B* to that station.

(*c*) *Illustration.* (Fig. 180 (*b*).) Set up the transit at *A*, and backsight on the magnetic north with the vernier at zero. Foresight on *B*, the vernier reading will be 50°, or the *azimuth* of *AB*. Without disturbing the vernier set up at *B*, backsight on *A* with *inverted* telescope, *plunge* the *telescope* (it now becomes normal again, the vernier still reading 50°), and foresight on *C*. The vernier will read 190°, or the *azimuth* of *BC*. Allow the vernier to remain clamped at 190, set-up at *C*, *invert* the telescope, backsight on *B*, *plunge* the telescope and sight on the next station. Thus the process can be repeated for any number of stations.

Note. Usually there are azimuths to *other points* beside transit stations which must be taken at each set-up; if so, then the *last* sight before leaving a station must be on the next station, thus automatically setting the vernier for the next set-up.

(*d*) *Second Method.* In this method the telescope is *not* inverted for the backsight, but instead the vernier is set anew at each station so that its reading will be the *azimuth of the backsight*. The *azimuth* of the backsight is equal to the *back azimuth* of the foresight as taken at the preceding station, i.e., add 180° to the azimuth already recorded. For example, if the transit stands at *B* (Fig. 180 (*d*), before backsighting on *A* the vernier is set so that the reading is the *back* azimuth of *AB*, or 230° (obtained by adding 180° to 50°). If, therefore, the vernier is set at 230° and a backsight taken to *A*, upon turning back to zero the telescope will point north. Moreover, if a foresight is taken to any other point, as *C*, the reading of the vernier will be the azimuth of the line from the transit to that point, or the azimuth of *BC*.

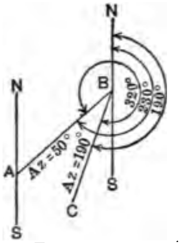

FIG. 180 (*d*).

Illustration. In Fig. 180 (*d*) the transit is set up at *A* and the azimuth of *AB* obtained exactly as in the first method. Set up at *B*, set the vernier at 230° (50°+180°), backsight on *A*, foresight on *C*: the new reading will be 190°, or the azimuth of *BC*. Set up at *C*, set the vernier at 190°+180° (i.e., 10°), backsight on *B* and foresight on the next station. Thus the process is repeated at each succeeding station. The telescope is always normal, never inverted for backsight or foresight.

181. Outline of the Two Azimuth Methods.

In both methods the azimuth of the first transit line is established in exactly the same manner by the use of the magnetic *needle*. *There are* two other ways of getting this first azimuth (explained in Arts. 185 and 186), which can be used with either method. The difference between the two methods of traversing is simply one in *orienting the transit.* (Art. 187.)

First Method	*Second Method*
1. Vernier is left clamped at the forward azimuth of the preceding line.	1. Vernier is reset at the forward azimuth of the preceding line *plus 180°*.
2. Telescope is inverted before backsighting.	2. Telescope remains normal when backsighting.
3. Telescope is plunged after backsighting.	3. Foresight is taken without plunging.

In both methods the azimuth can be read directly from the vernier, *whether in turning off the angle the telescope is moved to the right or to the left.* Transits should be graduated from 0° to 360°, preferably in one direction only, to correspond to angles taken to the *right,* or clockwise.

Field Notes. Some of the forms of field notes used for the azimuth method are given in Chapter XV.

182. When Azimuths and Magnetic Bearings Disagree. The back bearing of the *first* line should be read at the second station. If it agrees with the forward bearing as taken at the first station, it may be assumed that there is no local attraction at either place. If the two bearings disagree, it is necessary, before proceeding with the survey, to establish some transit line the back bearing of which agrees with its forward bearing. Having done this, a new start may be made from either end of the line, since in all probability the azimuth of the line taken at either end will be correct. Thereafter should the azimuth of any transit line disagree with the magnetic bearing and no mistake be found in the azimuth, local attraction may be assumed as the cause. It often happens that such a disagreement will occur for several successive transit lines, but should bearings and azimuths continue to differ by about the same amount the original azimuth upon which all others depend is probably incorrect. If the magnetic needle is to be used as a check throughout the survey, it is obviously important that the original azimuth should be as nearly correct as possible. Herein lies the advantage of starting from a true north-and-south meridian established by observation, and in extended surveys this is often the method used. (See Art. 185.)

183. Comparison of the Two Azimuth Methods.

The first method has no disadvantages as compared with the second method except such as arise from plunging the telescope. If, for example, the line of collimation is badly out of adjustment, an error is introduced each time the telescope is reversed. The advantage of this method over every other method of running transit lines is that after the first set-up it is not necessary (or desirable) to set the vernier, since the *foresight* on each station automatically sets the vernier for that station. Thus the error which is so apt to occur in setting the vernier is eliminated. (For disadvantages see Art. 190, III.)

The second method is used when for any reason it is not best to plunge the telescope, or when, as on the theodolite, the telescope cannot be plunged. A disadvantage as compared with the first method is that the vernier must be set anew at each station.

184. Checking Azimuths by the Magnetic Needle. Any azimuth may be easily reduced to a bearing. (See chapter on *Calculation of Bearings*, Part II.) If, therefore, the *magnetic* north-and-south line is taken as the reference-line for a survey, the azimuth of *any* line may be roughly checked by glancing at the needle while the telescope is still pointing along the line and mentally noting whether or not the azimuth and bearing agree.

185. A True North-and-south Line as a Reference Meridian is often used in place of a magnetic north-and-south line. In that case the azimuths are more readily checked by the needle if the compass-box is adjusted so that when the telescope is pointed to the *true* north the needle will point to the N. of the compass-box. If the declination is known, this may be accomplished by turning off the declination arc, using the vernier inside the compass-box; otherwise a true meridian must be established by an observation on the North Star or by some other method.

Notice that after the declination arc is turned off the N. of the compass-box is no longer exactly underneath the axis of the telescope. In many transits no provision is made for turning off the declination arc.

The advantage of starting from a true meridian established by observation is explained in Art. 182. In ordinary surveys of small extent, however, there is usually no time for establishing such a meridian.

186. Any Reference Meridian may be Used instead of a north-and-south line, provided no use is to be made of *bearings*. *For example, the line* between the first two stations may be assumed as a reference meridian (zero azimuth); the azimuth of each transit line is then reckoned from a line parallel to the first transit line. (In that case the meaning of *azimuth* is that given in Art. 160 (*c*), p. 114.)

In this method the transit lines are all located *with respect to each other*, which is often all that is necessary, but the needle cannot easily be used for checking azimuths, and this is a disadvantage.

187. Orienting the Transit. In running lines by azimuths, the first step after setting up the transit at any station is to get the graduated plate into such a position that *when the vernier reading is zero the line of sight will be parallel to the reference meridian.* This is called *orienting the transit.*

Illustration. If the reference meridian is a north-and-south line, *orienting the transit* results in the telescope pointing north (when the vernier reading is zero and the telescope is normal). In the first method this is accomplished by backsighting with inverted telescope and then plunging; in the second method by setting the vernier at the *back* azimuth and backsighting.

FOURTH METHOD.

188. Running Transit Lines by Bearings. This method is essentially the same as the azimuth method except that *bearings* instead of azimuths are read directly from the transit. For this purpose the graduations on the limb should be numbered from 0° to 90° to correspond to quadrants of bearing in the compass. (See page 75.)

When the plates are left clamped at the *forward* bearing of a transit line, the transit is oriented at the next station by backsighting with inverted telescope and then plunging; otherwise it is oriented by setting the vernier at the *back* bearing and then backsighting. (The two methods of orienting correspond to those used for azimuths.)

Either a *true* north-and-south line or a *magnetic* north-and-south line may be taken as a reference meridian. If the former is used, it is convenient to turn off the declination arc (see Art. 185) so that bearings read from the limb may be checked by the needle without thinking of the magnetic declination.

Note. The two zeroes on the limb (180° apart) may be called the *north* zero and the *south* zero; *the one with which the zero of the vernier coincides when the telescope points north* (*the transit having been oriented*) *is the* NORTH *zero.* (Why, in most cases, will this zero really be nearer the *south* end of a telescope pointed north?)

Illustration. In Fig. 180 (*d*), if the transit is oriented at Sta. *B* and is then pointed to Sta. *C*, the vernier reading will be 10°, and since the vernier zero is to the *left* of the S. zero on the limb, the bearing of *BC* is S. 10° W. (Draw a sketch showing the relative position of the telescope, the N. and S. zeroes on the limb, and the vernier zero. (See page 102.)

Caution. In reading bearings, it is evident that an error of 180° will be introduced by mistaking the north zero for the south zero or *vice versa*. Thus in the illustration a common mistake would be to read N. 10° E. instead of S. 10° W.

Note. Precautions should be taken to eliminate error due to local attraction in the bearing of the first line. (See Art. 182.)

A true north-and-south line may be established by observation if desired, and used as a reference meridian for bearings.

Field Notes for this method are similar to those used with azimuths, except that care must be taken to designate in which quadrant of bearing each line lies, i.e., to give its *forward* bearing.

189. Running Transit Lines by Angles and Calculated Bearings. This can hardly be called a separate method. Transit lines are run exactly as in Method I or Method II, but in addition the bearing of each line is *calculated* instead of being read directly from the transit as in Method IV. The bearing of the *first* transit line is obtained from the needle, the angle which that line makes with the *second* transit line is measured, and the *calculated* bearing of the second line found by the method explained in the chapter on *Calculation of Bearings*, Part II. Thereafter the bearing of each transit line is *computed* from the *calculated* (NOT the magnetic) bearing of the preceding transit line, and *is entirely independent* of the magnetic bearing. Since, however, the *first* bearing calculated (i.e., the calculated bearing of the *second* transit line) was obtained from the *magnetic* bearing of the first transit line, all other calculated bearings should agree *approximately* with the corresponding magnetic bearings, and hence any *large* error in a transit-line angle may be detected. The check thus afforded is a valuable one, especially in running the transit lines of a survey which does not close, provided, of course, that the needle is not continually affected by local attraction.

Remark. If the cause of local attraction is not about the person of the observer, but remains stationary, an angle can still be checked by the needle provided two readings are taken at the station where the transit stands, one the back bearing of the preceding line, the other the forward bearing of the line ahead.

Note. Precautions to eliminate errors due to local attraction should be taken in assuming the bearing of the first line in order to use the needle throughout the survey as a rough check.

(*a*) *Illustration.* The bearing of *AB*, Fig. 189 (*a*), as read at *A* is N. 20° E. *The back bearing*, read at *B*, is S. 20° W.; hence N. 20° E. is assumed as correct. The angles at *B*, *C*, and *D* are indicated on the figure. The calculated bearings of the different lines are also indicated.

Using the method explained in Part II the calculation is as follows:

$$
\begin{array}{rl}
AB = & \text{N. } 20^\circ \text{ E.} + \text{ (S.W.)} \\
ABC = & \quad 60^\circ \quad - \\
\hline
BC = & \text{S. } 40^\circ \text{ E.} - \\
BCD = & \quad 30^\circ \quad - \\
\hline
CD = & \text{S. } 70^\circ \text{ E.} - \text{ (N.W.)} \\
CDE = & \quad 100 \quad + \\
\hline
DE = & \text{N. } 30 \text{ E.} +
\end{array}
$$

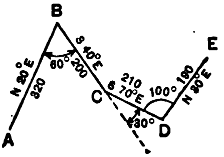

FIG. 189 (*a*).

Notice that since the angle $BCD = 30^\circ$ is a *deflection* angle, the *forward* bearing of *BC* is used instead of the back bearing, i.e., the letters S.E. are not changed to N.W. before adding 30° algebraically.

For other examples of the calculation of bearings see the chapter on *Calculation of Bearings*, Part II.

Note. In extended surveys a true north-and-south line is often established by observation, and the bearing of the first transit line is then *calculated* from the angle which it makes with the line thus established. See, for example, line *YV* in the sketch of transit lines (p. 390).

(*b*) *Check for a closed polygon.* The bearing of the first line assumed from the needle reading is sometimes called a calculated bearing, and hence it may be said that the calculated and magnetic bearings are *exactly* the same for the first line, although this is not likely to be true of other lines. If the transit lines form a closed polygon, a second calculated bearing may be obtained for the first line from the calculated bearing of the last side and the closing angle. The discrepancy between the assumed bearing of the first line and the final calculated bearing as thus obtained is equal to the total error in the sum of the interior angles, as found by the check of Art. 175, p. 119.

Illustration. The assumed bearing of the first side of a triangle is N. 30° E. The calculated bearing of the same side found by following around the triangle is N. 30° 1′ E. This indicates that there is an error of 1′ in the sum of the interior angles. In a similar way the sum of the interior angles of any polygon may be checked no matter how many sides it has.

Field Notes. Some of the forms of field notes used with this method will be found in Chapter XV.

190. Methods of Running Transit Lines Compared. Use the method best adapted to the work in hand. There is not much difference between the methods in the time required for field work.

I. METHOD BY DIRECT ANGLES. Used in all kinds of work, but especially in small surveys where there are but few transit lines and where the survey closes.

Advantages. (1) Angles at any station may be measured by repetition or by series, a decided advantage in accurate work. (2) Stations may be occupied in any order. (3) The angle at one station is not affected by the angle measured at a preceding station.

Disadvantages. (1) Angles in order to be checked by the needle must be reduced to calculated bearings. (2) A distinction must be made in the field notes between right and left angles unless *all* angles are taken in one direction (clockwise). (3) It must be made plain in the field notes which station was used as a backsight for each angle. (4) The vernier must be set anew at each station.

II. METHOD BY DEFLECTION ANGLES. Used chiefly in railway and highway work,—adapted to surveys in which each transit line makes a small angle (less than 90°) with the preceding transit line produced. Used in running in curves.

Advantages and disadvantages are practically the same as for the preceding method. Railroad curves are usually laid out by means of deflection angles and hence the method is used in nearly all railroad surveying. For other kinds of work, however, the advantages of the method are often overestimated. (1) Unless each angle is doubled it involves practically every error in the adjustment of the transit, especially the error caused by plunging the telescope when the line of sight is not perpendicular to the supporting axis; in the preceding method this error is not involved. (2) It is necessary to take angles right or left, a common source of mistakes in note-keeping, while in the preceding method this can be avoided by taking all angles clockwise. Deflection angles never exceed 180°, and this is of some advantage in plotting, but even this advantage is very slight, and it is always doubtful policy to save office work at the expense of field work.

III. RUNNING TRANSIT LINES BY AZIMUTHS. Almost always used in stadia surveying and for certain forms of topographic work. It can often be used to advantage in railway and highway work. The method is a valuable one and should be used, perhaps, even more than it is.

Advantages. (1) The needle is a constant check on the azimuth of each line. (2) The method avoids nearly all the disadvantages given for the two preceding methods, i.e., it is not necessary to enter the backsight in the field notes, or to make a distinction between right and left angles or to set the vernier anew at each station. (3) The azimuth gives the bearing without using the letters that correspond to the compass-points. (4) Field notes are usually simple, definite, *and easily plotted.*

Disadvantages. (1) The method does not lend itself readily to measuring angles by repetition or by series. (2) Each station must be occupied in order without skipping any. This may be a serious disadvantage if for any reason it is impossible or undesirable to set up over a transit station at any given time. (3) A mistake in the azimuth of one line affects the azimuths of all succeeding lines. This does not mean, however, that all the work is lost, for the azimuth of each succeeding line is affected by the *same* amount, and though these succeeding lines are all wrong with respect to the reference meridian of the survey, they may still be correctly located with respect to each other. If the mistake is discovered, the notes are easily changed by applying the correction to each succeeding line, though if there are many side shots this may cause considerable extra work. (4) If local attraction exists for any considerable number of stations, the advantages of checking azimuths by the needle is lost. (5) In the first method of running azimuths, plunging the telescope involves the same error in the adjustment of the line of sight that is involved in the deflection-angle method.

IV. RUNNING TRANSIT LINES BY BEARINGS. This method, probably a survival of the compass-survey method, is little used at present.

Advantages. It has no advantage over the preceding method except, possibly, a slight advantage in checking lines by the needle, while it has all its *disadvantages* and in addition: (1) A special numbering of the graduations on the limb (0° to 90°) is called for; (2) Likelihood of mistaking the north zero for the south zero, or *vice versa;* (3) Letters corresponding to compass-points must be used.

V. CALCULATED BEARINGS when used in connection with either of the first two methods serve as an excellent check on the transit-line angles, provided local attraction does not exist. While mistakes of less than 15 minutes are not detected by the use of the needle, nevertheless in a survey which does not close there is no other check unless angles are measured by repetition.

VI. Measuring Angles by Repetition. Many surveyors, when running transit lines by either of the first two methods, believe in doubling all transit-line angles, if for no other reason than to guard against error. It requires more time in the field, but often saves time in the end. Angles can be measured with greater accuracy by repetition than by the ordinary method. (See page 99.)

191. Transit Lines—Summary.

1. Transit lines are imaginary lines of reference by which and from which the details of a survey are located. They are the skeleton lines of the survey.

2. A system of transit lines may form a closed polygon, or a network of polygons, or it may consist of a series of lines which do not close.

3. In any polygon of transit lines either exterior or interior angles may be measured. A check is afforded by adding interior angles. (Art. 175, p. 119.)

4. Error of closure is usually due both to error in lengths of transit lines and error in angles.

5. There are four common methods of running transit lines, two of which depend upon measuring the angle which one transit line makes with another transit line; the other two depend upon measuring the angle which each transit line makes with a line parallel to a reference meridian (as, for example, a north-and-south line).

6. The four methods are: (1) Angles; (2) Deflection Angles; (3) Azimuths; (4) Bearings.

7. In the *angle* method all angles are measured directly from the backsight without plunging the telescope, while in the *deflection*-angle method all angles are measured from the backsight *produced* by plunging the telescope. This is the principal difference between the two methods.

8. In the azimuth method the azimuth of the first line may be determined in one of three ways.

(1) By the magnetic needle; all azimuths will then be *magnetic* azimuths.

(2) By establishing a true north-and-south meridian by observation, and measuring the azimuth of the first line from this meridian. All azimuths will then be *true* azimuths. In this case the declination arc is often turned off.

(3) If bearings are of no importance, the azimuth of the first

line may be assumed as zero or any other convenient quantity. All azimuths will then be *relative,* i.e., azimuths with respect to the first transit line as a reference meridian.

9. There are two methods of orienting the transit in running by azimuths. In one the vernier must be set anew at each station (at the back azimuth) and the telescope is never inverted. In the other the vernier is set automatically for each station by a foresight taken while at the preceding station, but the telescope must be inverted before backsighting and plunged before foresighting.

10. If the azimuth of the first line is correct, a discrepancy between the azimuth and the bearing of any other line indicates a mistake in measuring the angle; if no mistake is found in the angle, local attraction may exist.

11. Running transit lines by bearings is similar to the azimuth method, but calls for the quadrant system of numbering graduations on the limb. Errors are much more likely to occur and the method is but little used.

12. Calculated bearings may be used as a rough check in running transit lines by angles or by deflection angles.

13. Any method of checking transit lines by the magnetic needle is useless in locations where local attraction exists.

14. Doubling angles, or measuring angles by repetition, is a more certain check than that afforded by the magnetic needle, and it is also a much more accurate one. By this method angles can also be measured with greater precision than by the ordinary method.

CHAPTER XIII

METHODS OF LOCATING DETAILS

In this chapter are given the geometrical methods in common use for locating boundary lines, street lines, buildings, fences, rivers, bridges, and other details of a survey. The different methods are outlined without entering upon the routine of the field work, or discussing the practical questions which arise in connection with that work.. (See the next chapter for field work.)

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192. Transit Survey—General Method. A transit survey is one in which the transit is the principal instrument used: this implies the measurement of angles as the distinguishing characteristic. Part of the work in such a survey consists in running the transit lines according to one of the methods explained in the preceding chapter, and the rest of the work consists in locating details of the survey using these transit lines as reference-lines. It is desirable to have either a transit station or a transit line conveniently near each detail that is to be located, such, for example, as a house or a fence-corner; in establishing transit stations this should always be kept in mind. (See Art. 216, p. 149.) The different geometrical methods used are exactly the same as those employed by draftsmen for locating points on paper. Some involve linear measurements only, others are based entirely upon the measurement of angles, while still others combine linear and angular measurements.

193. Two Methods of Carrying on the Work. Having chosen the method of running the transit lines (Art. 217, p. 151) there are two ways to proceed: (1) Lengths and angles of the transit lines may be measured and the entire system established before work is begun on the location of details. This means going over the ground twice, once for transit lines and once for details. (2) A more common method is to complete all the measurements at a station at one set-up, carrying on the work of running the transit lines and of locating the details simultaneously, and establishing new stations as fast as they are needed.

The first method is good for a survey of considerable extent, in which a network of transit lines must be established with care. The second method is better for a survey which does not close, or for one in which it makes little difference where the transit lines are run, or for small surveys with comparatively few stations where the work can be done in a short time.

As a compromise between the two methods it is often best before beginning a survey to decide upon the approximate positions for the different stations, and then, working by the second method, set the stations more exactly as fast as they are needed.

194. Locating Objects from Transit Lines. Seven methods of locating a point have already been given on page 4. Locating details consists simply in locating two or more points of each object by one or more of these seven methods. Transit lines form the skeleton upon which the survey is built. Objects are usually located from the nearest or most convenient transit line. Enough points are taken to completely locate the object, with perhaps an additional point or two for a check. Thus, for example, it is enough to determine the position of two corners of a rectangular building directly from a transit line, the other two corners being located indirectly by measuring the building itself; it is well, however, in accurate work to locate a third corner from a transit line as a check.

Sometimes it is necessary to locate different parts of the same object from different stations or transit lines. For example, part of a large building may be located from a station near one end, while other portions of the building are located from a station near the other end. Fence-lines and street-lines are nearly always located from at least two stations. For example, in a survey of a rectangular field each corner of the field may be located from a station near it, and each boundary line will thus be fixed by two points located from different stations, and possibly from different transit lines also. See page 141.

In locating the different points of any detail of the survey the surveyor must be quick to see which of the seven methods given on page 4 is best adapted to his purpose. (See Art. 223. p. 156.) Sometimes, however, he has no choice but must use the one method possible, as, for example, the method of triangulation in locating an inaccessible point. Of the seven methods referred to, one depends upon the intersection of two straight lines, two depend upon linear measurements only, while the other four involve measurements of angles. Since the transit is an instrument for measuring angles, these last four methods are most characteristic of transit surveying. In the succeeding pages, however, there will be found illustrations which are intended to give a general idea of the way in which all seven methods are used in locating objects from transit lines.

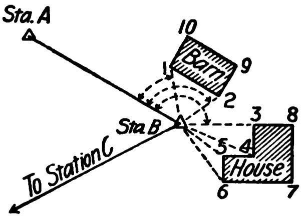
First those methods which combine linear and angular measurements will be given, and then those methods which involve linear measurements only.

More complete illustrations in which different methods are combined will be found in the various forms of field notes given in Chapter XV.

METHODS INVOLVING ANGULAR MEASUREMENTS

195. Locating Objects by Angles and Distances.

(*a*) *Illustration.* Any number of points may be located from a station by taking an angle to each point and measuring the corresponding distance from the station to the point. (Method 3, p. 4.) The angles are usually all taken from the *same* backsight. (Art. 127, p. 94.) In Fig. 195 (*a*) the transit is at station *B*, backsight on station *A*. Angles are taken to *1* and *2* (corners of the barn) and *3*, *4*, *5*, and *6* (corners of the house), and the corresponding distances from station *B* to these corners are measured. "Measuring up," i.e., measuring the lengths of the sides of the barn and the house, completes the field work of location. In many cases it would be sufficient to locate two corners of the house, as, for example, *3* and *4* or *5* and *6*.



195 (*a*).

Locating objects by azimuths and distances. In the preceding illustration, and in many that follow, it will be observed that the method of locating objects holds good when azimuths are used in place of the usual angle taken from some backsight. *Indeed, azimuth* means an angle taken clockwise from a north-nd-south line or from some other meridian as a backsight.

(*b*) *Illustration. Locating an object by an angle and distance to one point only.* In Fig. 195 (*b*) a method is shown which is often used for locating a rectangular object. A point *c* is "lined in" with the transit between stations *F* and *G*, and at the same time brought into line *by eye* with one side of the house, as *ab*. The point *c* being thus established the transit is set up at *c*, the angle *Fcb* is taken, and the distances *Fc* and *cb* are measured. One corner of the house *b* having been located in this way, the other corners are found by "measuring up" the house itself. This method is most used when the object is too far away to be located from a regular transit station, and yet is comparatively near one of the transit lines; although it is not the most accurate method, it will often answer the purpose. A method similar to this but one which does away with the setting up of a transit at *c* is given on page 139, Fig. 199 (*a*).

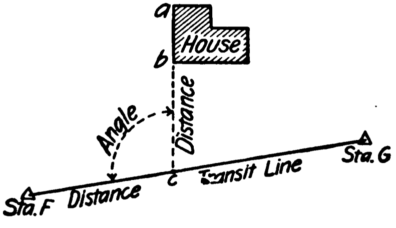


FIG. 195 (*b*).

(*c*) *Illustration. Locating curves by angles and distances.* The simplest way of locating a curve made up of *circular* arcs of small radii is to find by trial the radius and center of each arc, locate each center from the nearest transit station, and in addition, if necessary, locate the points where each curve ends or where it changes to a different curve. When, however, the radii are large or when the curve is irregular, i.e., not composed of *true* circular arcs, some other method must be used. One method, especially adapted to the location of small, sharp, irregular curves such as driveways, park walks, and other similar curves, is given in Fig. 195 (*c*). Notice that for each angle measured two points are located. Thus, with transit at *E*, backsight at *D*, an angle to 7 is taken, 7′ is lined in, and the two distances to 7 and 7′ are measured, the object being to save work in plotting. The second point can usually be set in line with the first by eye. Chain-pins or stakes can be stuck into the ground at intervals, close together where the curve changes rapidly, farther apart where the curve is less abrupt, and the angles and distances measured to these points, or, on the other hand, angles can be turned off, say every 15°, the corresponding points on the curve marked, and the distances to these points then measured. It often happens that, by making the same angle serve for the location of two or more points, work in plotting is saved not only in the location of curves, but also in the location of all kinds of objects.

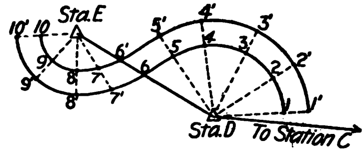


FIG. 195 (*c*).

(*d*) *Illustration. Locating a point by an angle and distance when the distance is measured from some fixed point other than the transit station.* (Method 4, page 4.)

In the three preceding illustrations the distance to any point located was measured from the station where the transit stood when the angle to that point was taken. (Method 3, p. 4.) Instead of measuring from the transit station it is sometimes more convenient to measure from some other point already located as shown in Fig. 195 (*d*) and Fig. 195 (*e*).

In Fig. 195 (*d*) corner *b* is located in the usual way. Corner *a* is located also by an angle from the backsight on station *G*, *but the distance is measured from b* instead of from station *H*.

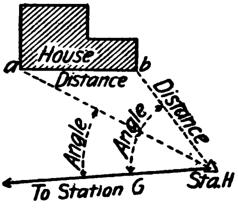


FIG. 195 (*d*).

(*e*) This method may also be used for determining the direction of a street which branches off from the main survey. As only enough of the street will be shown on the map to indicate its direction, it will usually suffice to locate a single point, as, for example, *b* in Fig. 195 (*e*). The corner *a* having been located by an angle and distance in the usual manner, an angle is taken to *b*, *but the distance from a to b is measured instead of* the distance from *Sta. B* to *b*. This method might be used, for example, if on account of fences it is much easier to measure *from a to b than it is to* measure from *Sta. B* to *b*. If more con-

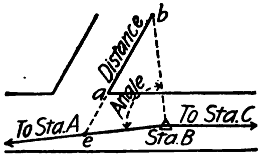


FIG. 195 (*e*).

venient, produce *ba* to *e* and measure from *Sta. B* to *e*. (See p. 140, Fig. 199 (*c*), for another method of getting the direction of a street.)

196. Locating Points by Two Intersecting Angles.

Illustration. (Method 5, p. 4.) In Fig. 196 a transit at *B*, backsight at *C*, measures the angle *CB*2. At the same time another transit at *C*, backsight at *B*, measures the angle *BC*2. The distance *BC* being known, the point 2 is thus located. Any number of points along the bank may be determined in a similar manner. When this method is followed two transits are generally used.

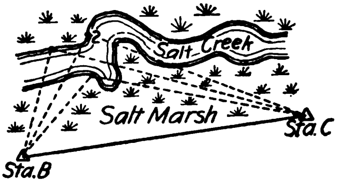


FIG. 196.

This method may be used for locating any point which can be seen from two transit stations, as, for example, a fence-corner or the corner of a building. The method is not as accurate as the preceding methods unless the base-line and angles are measured with great care. If the angle at the intersection of the two lines is too acute, it makes it difficult in plotting to determine the exact point of intersection.

This method is especially adapted to locating inaccessible points and for taking soundings. It is much used in plane-table work.

197. Locating Points by the Three-point Method.

(Method 7, p. 4.) Let *H*, *I*, and *K* in Fig. 197 be stations on shore the positions of which are known. Let 1, 2, and 3 be points on the water at which buoys are placed or soundings taken. At each of these points two angles between lines to the three stations on shore are taken, as indicated on the figure, thus locating each point. The angles are usually measured with a sextant. The peculiarity of this method is that *the angles are all taken at the point to be located*. The points could of course *be located by the* method of the preceding article, the two

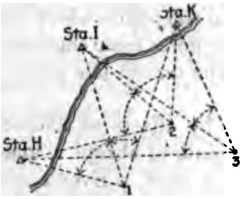


FIG. 197.

transits on shore occupying two of the three stations, as Sta. H and Sta. I, or Sta. I and Sta. K.

Methods Involving Linear Measurements only

All the methods on the preceding pages depend upon the measurement of one or more angles. Much of the work of locating objects, however, can be done without measuring angles. Of the two methods involving only linear measurements (page 4) that of locating a point by offsets is the one more used.

198. Locating Objects by Offsets.

(*a*) This method corresponds to method 2, p. 4. The *perpendicular* distances to the transit line from different points of an object are called *offsets.* (For methods of erecting perpendiculars see p. 61.) Enough offsets are taken to completely locate the object, the distances *between* the offsets being measured *along the transit line.* In Fig. 198 (*a*) only two offsets are needed to locate the building.

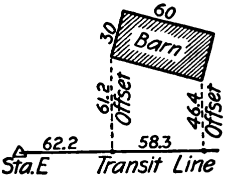


Fig. 198 (*a*).

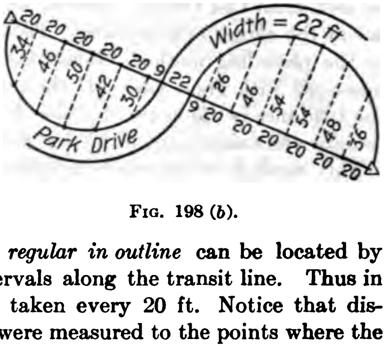


Fig. 198 (*b*).

(*b*) Anything which is *regular in outline* can be located by offsets taken at *equal* intervals along the transit line. Thus in Fig. 198 (*b*) offsets were taken every 20 ft. Notice that distances (9 ft. and 22 ft.) were measured to the points where the transit line crosses the curves. In keeping notes some engineers prefer to consider offsets on one side of the transit line *plus,*—on the other side minus.

(*c*) *Irregular outlines* are best located by offsets at *unequal* intervals, for in this way only can the points of abrupt change in the line be determined. The edge of a bluff, a river bank, or any other irregular line can be located by running a transit line near it and taking offsets as illustrated in Fig. 198 (*c*).

(*d*) *In locating* objects by offsets remember three things:

(*1*) *The distance* from one of the offsets to a transit station (or some other known point on the transit line) must be measured. If, for example in Fig. 198 (*c*), the distance 30 ft. from *Sta. A* to the first offset were not measured, the rest of the work would be useless.

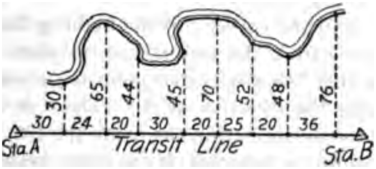


FIG. 198 (*c*).

(2) An offset should be taken at every point where there is an abrupt change in the outline to be located.

(3) Offsets should be taken to all points of the outline which are *farthest from* or *nearest to* the transit line.

Note. It will facilitate plotting (and the field work too) if, instead of measuring distances between offsets as shown in the figure, the total distance of each offset from the transit station is measured. Thus, starting from *Sta. A*, the successive points would be numbered according to the system explained on page 147, +30, +54, +74, 1+04, 1+24, 1+49, 1+69, 2+05. This shows that the last offset is 205 ft. from *Sta. A*, and, if the remaining distance to *Sta. B* is now measured in the same way, the result should equal the total distance previously found by chaining between *A* and *B*. This is a valuable check on all the intermediate measurements along the transit line as well as upon the total measurement itself. In getting the plus stations *no arithmetical addition is necessary;* simply stretch the tape its full length each time and read the plus for each point directly from the tape.

199. Locating Objects by Measurements Oblique to the Transit Line.

Illustration. In Fig. 199 (*a*) and Fig. 199 (*b*) are shown two different methods of locating a building, each of which, however, depends upon the same general principle. (Method I, p. 4.) In Fig. 199 (*a*) points *d* and *e* are two points *on the transit line*, and the distance of one of these points from the nearest station is measured. Point *e* was taken on a line with one side of the building, and point *d* is any convenient distance from *e*. The corner of the building is then located by the distances from *d* and *e*. In Fig.

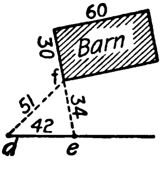


FIG. 199 (*a*).

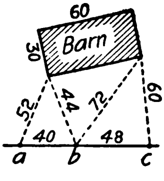


FIG. 199 (*b*).

199 (*b*) each of the two corners of the building nearest the transit line are located by two distances from points (*a* and *b* or *b* and *c*) on the transit line.

In Fig. 199 (*a*) if the corner *f* of the building had been a considerable distance from the transit line, in place of the 51 ft. line to *f* a "tie-line" to some other point on *fe* could have been used, thus fixing the *direction* of *fe*. The "tie-line" answers the same purpose as measuring the angle (p. 135, Fig. 195 (*b*)), and while not quite as accurate, it can often be used to advantage. In Fig. 199 (*c*) another illustration of the use of "tie-lines" is given.

(*c*) *Use of tie-lines.* In Fig. 199 (*c*) the intersection *c* of two street-lines is located from *Sta. B* by an angle and distance in the usual way. No other point is located from *Sta. B*, *the street-line cef being located by a second point taken from some other station not shown in the figure.* The point *d* determines the direction of a side street. Measure from *c* to *d* and from *c* to any other point *e* on the line *cf* The length of the "tie-line" *de* determines the angle *dce*. (Another method: produce *dc* to *a*, see p. 136.) In a similar way if a point *n* has been lined in by eye with a division-fence, a tie-line from any point *m* to any point *o* will determine the direction of the fence and thus avoid a "set-up" at *n*.

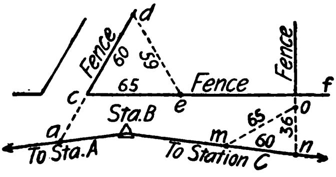


FIG. 199 (*c*).

(*d*) *Offsets oblique to the transit line.* In Fig. 199 (*d*) the points *d* and *c* on the transit line were lined in by eye with two sides of a building, and the distances *ad* and *bc* were measured. The point *d* was located from *Sta. A*, and the building was measured up. Notice that *dc* was *not* measured. This method is not geometrically correct, for, in plotting, the distance *ab* and *dc* are assumed equal. They are equal for all practical purposes, since this method *would not be used* except in cases where the transit line runs

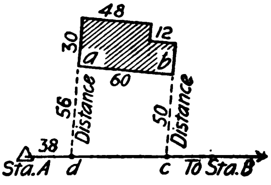


FIG. 199 (*d*).

approximately parallel to the side of the house, and where the map will be plotted to such a small scale that the discrepancy between the lengths of *ab* and *dc* will not show.

200. Locating Boundaries. In surveying boundary-lines it is not often practicable to make the transit lines coincide with the boundary- or fence-lines; they are therefore run around the field outside or inside as may be more convenient. It is not necessary to establish the transit lines parallel to the boundary-lines, although when it is desired to find the angle at the intersection of the boundaries it may be well to do so; even then, however, it is not really necessary, since the required angles can be calculated.

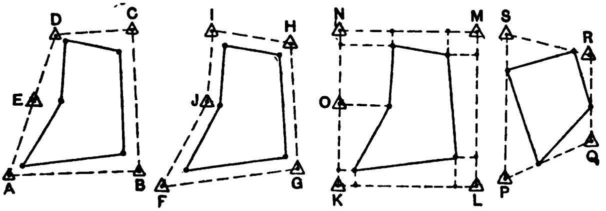


FIG. 200.

Illustration. In Fig. 200 three methods are shown for locating the boundary-lines of the same plot of land. It is assumed that buildings, fences, or other obstacles prevent the transit lines (represented in the figure by broken lines) from being established on the boundary-lines.

(1) Transit lines *ABCDE* are run *at random*, and each corner of the plot is located from the nearest station by one of the usual methods,—in most cases by angle and distance. (See p. 134.)

(2) Transit lines *FGHIJ* are parallel respectively to corresponding boundary-lines, and the corners of the plot are located as in the first method; or each boundary-line is located by offsets from the corresponding transit line. (See p. 138.)

(3) Any convenient rectangle of transit lines, as *KLMNO*, is established, and each corner of the plot is located by perpendicular distances from the nearest transit lines.

In the figure the transit lines in each case are shown wholly outside of the tract surveyed. If more convenient, they may be wholly inside or partly inside and partly outside.

The *first method* is the one most used because it involves the least field work. The *second* method calls for extra work in establishing the transit lines parallel to the boundaries, but it gives the angle at each corner of the plot directly without computation. In the third method the difficulty lies in getting the points where the perpendiculars from the corners of the plot intersect the transit lines, especially if this must be done with considerable precision, but when it is desired to save office work at the expense of field work this method is a valuable one. It is evident that much time is saved in plotting and in office computations.

In the second and third methods, buildings, fences, and other obstacles may render it difficult to establish the transit lines, while in the first method, since the transit lines are run at random, obstacles are more easily avoided.

This illustration shows how it is often necessary to locate different points of an object from different transit stations. Thus, for example, any one of the three methods could be employed for locating a large building. For staking out a building the third method is generally used. (See p. 210.)

(4) The lines *PQRS* illustrate a fourth method, which is not a common one, but which can be used sometimes to advantage where conditions are favorable. It consists simply in running a transit line through each corner of the lot, and measuring along these lines to the respective corners. Not much work is saved in the field by this method, but the office work of plotting and of calculating the area is simplified. It has the further advantage of definitely locating each corner with the fewest possible measurements.

In the Field Notes of Chapter XV will be found more complete illustrations of the use of transit lines for locating boundaries.

201. Practical Application of Methods. The field work of locating details, involving such questions as which methods to use and how to apply them, is discussed in the next chapter. A careful study of the illustration in the second part of Chapter XV on Field Notes, especially the survey for an electric road (Illustrations V and VI, pp. 184–186), and the survey of a city street (Illustration VII, p. 188), will suggest how several methods may be combined in practice. It is well worth while to plot the notes referred to in order to gain an idea of the actual field work of locating details.

202. Summary. Transit lines form a skeleton upon which the survey is built. Details are located from the nearest or most convenient transit line. There are seven methods of locating any point, four of which involve the measurement of one or more angles, two depend upon linear measurement only, and one upon the intersection of two straight lines. No one method should be used to the exclusion of all others, and the surveyor must be quick to see which is best adapted to his purpose.

1. The most common method for locating a point is that *by angle and distance.* It is quick and accurate, and should ordinarily be used for points near a transit station, when measurements from the station to the points can be made without difficulty. For a number of such points angles should be taken from the same backsight. One angle will often serve to locate several points.

2. A *second method* differs from the preceding in that the *distance* to the point located is measured from some other point instead of from the transit station. Used when it is more convenient to measure from a near-by point already located than to measure from the transit station itself.

3. A *third method* for locating a point depends upon two angles, one taken at one transit station, the other at another transit station, with the line between the two stations as a base-line. Two transits are ordinarily used, although the work can often be done with one. Used for locating inaccessible points or points too far from transit stations for linear measurements to be taken. Especially adapted to locating points on the water. The method is not accurate unless the base-line and angles are measured with great care. With ordinary precautions, however, it answers the purpose in locating unimportant details.

4. A *fourth method* involves the measuring of two angles *at the point to be located.* The angles measured are formed by three lines radiating from the unknown point to three known points. Used for hydrographic surveying and similar work. This method is the so called "three-point" method.

5. A *fifth method,* locating points by offsets, involves linear measurements only. Used for locating objects which are near a transit line but not near a transit station. Especially adapted to locating curves, and irregular lines such as river banks, and edges of bluffs. Also used for locating buildings.

6. A *sixth method* is that in which each point is located by two *distances, one from* each of two known points. This method

is quite accurate, and is often more convenient to use than one involving the measurement of angles. The "tie-line," one form of this method, will often save setting up the transit, as it virtually determines the angle required.

7. A *seventh method* is that of locating a point by two intersecting straight lines from four known points. Used principally for referencing points, but it can also be used to some extent in locating points.

8. No matter how many points of an object may be located, the dimensions of the object itself should be ascertained. This means that lengths of boundary-lines, lengths of street-lines, and lengths of all other important lines, especially straight lines, should be measured along the lines themselves in addition to locating the ends of the lines from transit stations.

9. *Rectangular objects,* such as buildings, are commonly located in one of the following ways: (It is assumed that the dimensions of the object are also ascertained.) (1) By angles and distances to two or more corners; (2) By the angle and the distance to *one* corner from a point on the transit line which is also in line with two corners of the object, one of which is the corner located; (3) By an angle and distance to one corner and the angle (but not the corresponding distance) to another corner; (4) By offsets; (5) By producing one side of the object to an intersection with the transit line and measuring a "tie-line"; (6) By locating two corners, each by two distances from known points.

10. The *direction* of a street-line or a division-fence line or any similar line branching from the main survey may be determined by producing the line until it intersects the transit line (or some other known line), and then locating one additional point on the line by angle and distance, or by a "tie-line."

11. Unbroken curves may be located either by offsets, or by locating controlling points by any of the usual methods.

12. *Finally.* If an object covers considerable ground, it may be necessary to locate different parts of it from different stations or different transit lines; in locating long lines, such as street-lines and boundary-lines, this is especially true. In locating any object it may be well to use one method for locating some of the points, and another method for locating other points; thus a combination of two or more of the seven methods may be used often to good advantage.

CHAPER XIV

TRANSIT SURVEYING. FIELD WORK

It is comparatively easy to learn the theoretical methods of running transit lines and of locating details given in the two preceding chapters. On the other hand the judgment required for choosing methods best adapted to the work in hand, and the skill necessary for carrying out the field work with accuracy and despatch, can only be acquired from a long and varied experience in the field. It is not desirable, therefore, to lay down hard and fast rules for field work. The most that can be done is to outline the routine usually followed, and to discuss in a general way practical questions which arise in the field.

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203. Transit Party. Transitman, chainmen, flagman, axeman, and others who work together as one party in making a transit survey. The size of the party depends upon the nature of the work. Two men may constitute a transit party, but usually three can work to better advantage. When much chaining has to be done, and many angles taken, a party of four or five men is still better. Sometimes several transit parties are at work on the same survey. The duties of each member of the party are outlined at the end of this chapter.

204. The Instruments and Equipment for a surveying party depend upon the work to be done. For ordinary surveys the different members of a transit party should be equipped as follows:

1. *Transitman.* Transit, plumb-bob, reading-glass.

2. *Note-keeper.* Note-book, pencil, eraser, straight-edge, scribbling-pad, and trigonometric tables.

3 and 4. *Chainmen.* Steel tape, chaining-pins, plumb-bob.

5. *Axeman and Flagman.* Axe, stakes, transit-pole, crayon, tacks.

In addition to the above list it is well habitually to carry in the pocket a short steel tape, reading-glass, sharp knife with screw-driver blade, pad of scribbling-paper, string or spool of fine wire, pencil, and eraser.

For a more complete list of instruments and equipment consult Part III.

205. Signals in common use in surveying are as follows:

1. In "lining in" a motion is made with one hand or the other in the direction in which the pole, stake, or pencil is to be moved: a vigorous motion, move quite a distance; a slight motion, a small distance.

2. Waving the arms so as to bring the hands together over the head means "all right."

3. When the transitman wishes a rod held for a sight he raises an arm to a vertical position.

4. The left arm held *vertically* means to plumb the rod in that direction; the right arm, plumb in the opposite direction.

5. When a transitman is through with a flagman for the time being, but is likely to need another sight on the same point, he simply signals all right. When, however, he is entirely through and wishes the flagman to "come in" or to move to the next station he waves a handkerchief. Unless there is some clear understanding on this point a flagman is apt to come in at a signal of "all right," and thus cause needless delay.

5. When the flagman holds his transit-pole in a horizontal position over his head he means that he is going to give a sight on a transit station. The same signal is used when establishing a transit station to indicate that the tack is set and the station completed.

Additional signals are often used, such, for example, as "give a point on line" and "another sight wanted."

Any system of signals may be used in place of those given. It is a matter of small consequence what the signals are, provided they are clearly understood by every member of the transit party. Avoid shouting: it is an evidence of "greenness."

206. Transit Survey—General Method. Transit lines, run according to one of the methods of Chapter XII, are used as reference-lines. The field work consists: (1) In running the transit lines, and (2) in locating points of the survey with reference to those transit lines. Keeping notes is also an important part of the work.

The work of running the transit lines and of locating details may be carried on simultaneously, or the entire system of transit lines may be established before the work of locating details is begun. The question of when to use one method and when to use the other has already been discussed on pp. 132–133.

207. A Transit Station may be any well-defined point, as a tack in a stake, a nail in the crack of a curb, or an X scratched on a pavement. If a stake is used, it should be driven flush in the ground and a guard-stake put near it. The former stake is called a "hub." A transit station may be established in two ways:

(1) Its position may be chosen, and the exact point marked before any sight is taken on it, or (2) it may be "sighted in" from the transit, as, for example, when a station is to be established "on line" with two other stations. When the transit station is to be lined in the flagman first holds the transit-pole or, better still, the stake itself, and moves it into line in accordance with the signals from the transitman. (See Art. 205.) Having determined the required point, the stake is driven, and the pole (or a pencil if it can be seen) is again held to find the exact point on the stake for driving the tack. In rough work the tack is unnecessary, the stake itself answering every purpose.

208. Guard-stake. A stake driven partly into the ground, a foot or so from the station, so that the station itself may be easily found and identified. The station number should be marked on it *from the top towards the bottom,* and the stake should be driven on the left of the transit line, so that the number may be easily read as one walks along the line. If it is driven at a *slant* (with top inclined away from the station), it will not be mistaken for the hub itself, and the lettering is more easily read than if the stake is vertical. If for any reason a guard-stake near a station is likely to be knocked over, it may be driven a few feet away and marked accordingly, as, for example, Sta. 15 (12 ft. North). Flat stakes are often used for guard-stakes, and square stakes for hubs.

209. Numbering or Lettering Stations. Stations are usually numbered or lettered in the order in which they are established, but this is not essential. Numbering is better than lettering where there are many stations. In railway work and in some other kinds of work the first station is often numbered zero, and the other stations are numbered to indicate how many feet they are from zero *measured along the transit line*—one number for every one hundred feet. Thus: 15=1500, 15+50=1550, 15+83.5=1583.5, 16=1600 feet from zero, and so on.

Remark. The surveyor's crayon or "keel," used for marking stakes, differs very much in its water-proof qualities. Test by marking a stake and then trying to wash it off. Make all numerals bold and distinct, pressing the crayon into the surface of the wood.

210. Choosing a Place for a Transit Station. Do not choose a place for a station where the transit is easily jarred or disturbed. Avoid stations close to car-tracks, on loose planking, soft or marshy ground, smooth pavements, or other places where the legs of the tripod are easily moved.

Do not choose a station where the line of sight will be continually obstructed by passing wagons, cars, or people, if it can be avoided. In a crowded city street the surveying may have to be done very early in the morning.

Illustration. Near the curb is a better place for a station than in the middle of the sidewalk, and, where there is much traffic, stations on the same side of a street are better than on opposite sides. Avoid choosing a station where it will be destroyed when the work of construction begins.

211. Referencing a Station. When a station is likely to be used more than once it should be referenced, so that if lost or destroyed it can be restored. This precaution is especially necessary when stations are where carting, excavating, grading, and other construction work is going on. Three methods for referencing a point with a tape are illustrated on p. 54. The third method, perhaps, is the one most used in transit surveying, since the four points of reference can be easily established from the station by plunging the telescope. These points of reference are usually stakes driven flush, with a tack to mark each point, located at random, but in such a way that the intersection of the two lines will be sharp and definite. The stakes should, of course, be set where they are not likely to be disturbed, and each one should have a guard-stake. A scratch on a pavement or any other well-defined point that would answer for a transit station may be used for a reference-point in place of a stake. Marks on buildings, nails in fences, and similar points are often used where only a point of sight is needed.

212. Questions of Judgment. (1) What *limits of error* to adopt, i.e., how close to measure angles and distances. (2) *Where* to run transit lines, i.e., the general plan of transit lines. (3) *Method* of running transit lines: which one of the four to use. (4) *Method* of locating details: which is the best for any given case. (5) Method of keeping notes: which is the best adapted to the work in hand. These closely related questions are discussed in the succeeding articles.

213. Obstacles to Surveying often make it impossible to *do the field work in the* usual and most direct way, and render it necessary to adopt extreme precautions in order to obtain even an ordinary degree of accuracy. A line runs through a tree; a distance is wanted between two points, but a house-corner intervenes; points to be located are inaccessible. Obstacles such as these give rise to special problems in the use of the tape or the transit or both. They may also influence the surveyor in deciding some of the questions suggested in the preceding article. Problems of this kind which occur in chaining are explained in Chapter VII. Similar problems which arise in transit work are explained in Chapter XVIII.

214. The Purpose for which a Survey is Made is the first consideration governing the field work. It largely determines the methods used, the data required, the limits of error adopted, and the general plan of the field work.

Illustration. If a map is to be made, ask yourself what will it be used for, and to what scale will it be drawn? (See Art. 11, p. 6.) Are there any measurements which as a matter of record should be more accurate than the mere plotting of the map requires? (Art. 70, p. 44.) If work is being staked out, how will the stakes be used, and how close will the men who use them work?

215. The Relation between Field Work and Office Work is so close that questions which arise in the field can often be settled by the bearing which they have upon work in the office. When, for example, two or more methods of field work seem equally good, consider which will require the least office computation, or which is the best for plotting. To facilitate office work in this way, by forethought in planning field work, does not mean, however, that it always pays to do extra field work in order to save office work. Since the work of an entire transit party can often be plotted by one man an hour in the field for, say, three men may cost as much or more than three hours in the office for one man. Moreover, field work must often be done in a limited time and under favorable weather conditions, while the corresponding office work can be done more leisurely and regardless of the weather.

216. Where to Run Transit Lines Much depends upon the general plan of the transit lines, for they form the skeleton of a survey. While in general it is more important to plan carefully for a closed system of transit lines, especially if it is a complicated network, still even in a random survey which does not close unnecessary work may be saved, both in the field and in the office, by judiciously choosing the location of transit stations.

When possible, transit stations should be so chosen that the work of running transit lines, locating points of the survey, and plotting the map is reduced to a minimum. Before beginning a survey, go over the ground and note the most important features to be located. Place transit stations where they will be of the most use in locating those features. If the main purpose is to survey boundary-lines, then a transit line should run near each boundary (but not necessarily parallel to it). If buildings are to be located, plan to have stations near the buildings. Obstacles often prevent a surveyor from running his transit lines just where he would like to run them, but he can usually find some good way out of the difficulty.

See if the transit lines can be run readily. Changing a station a few feet one way or the other may save unnecessary field work or unnecessary work in plotting. Make a rough sketch of the transit lines and see if the system as a whole is well balanced. Are there any unnecessary stations? Are distances between stations too long or too short for accurate sighting? Are there any unnecessary obstacles to sighting or chaining? Half an hour spent in choosing transit stations may save hours of field work.

(a) *Illustration—Boundary Survey.* Fig. 216 (a) illustrates how field work and office work are affected by the location of transit lines. The lines 1-2-3-4-5-6-7-1 are boundary-lines, and the question is *where* to run the transit lines. They can be located in four different ways.

1. To *coincide* with the boundary-lines.
2. To form a reference *rectangle*, as *WXYZ*.
3. To form a polygon each side of which is *parallel* to and a short distance outside or inside a corresponding boundary-line. (This polygon is not shown in the figure.)
4. To form *any* convenient polygon, as *ABCDEFA*.

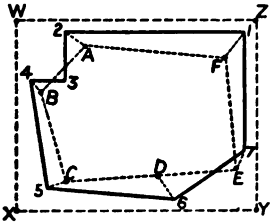

FIG. 216 (a).

(b) *Comparison of methods.* (1) The first method, theoretically ideal, is often impracticable because of existing fences and other obstacles. (2) The second method facilitates office work, but is often unsatisfactory or impracticable in the field. (3) The third method involves considerable extra field work in establishing each transit line parallel to the corresponding boundary-line. (4) The fourth method is the one most used. Its chief disadvantage is that neither the length of boundary-lines nor the angles at the corners are measured directly. They can be easily found, however, by computation, and if desired the lengths of the boundary-lines may be measured in addition.

Note. A fifth method sometimes used is discussed on p. 141.

(c) The choice of stations and its effect on field work and office work are

further illustrated in the polygon *ABCDEFA*. Notice that station *B* was so chosen that *two* corners, 3 and 4, could be located from it, and that when the transit was at *C* station *D* and *E* were established "in line." In this way a five-sided polygon is made to answer in place of a seven-sided polygon, and a corresponding gain in both field work and office work is the result.

(*d*) *The relation between field work and office work* discussed on page 149 is illustrated by the third and fourth methods. Suppose that the lengths of the boundary-lines and the angles at the corners are required. If the fourth method is used, they must be computed. Is it worth while to save this extra office work by running parallel transit lines according to the third method? The computation involves finding one unknown side in each of seven polygons. For example, if the lengths and directions of the lines 2*A*, *AF*, and *F*1 are known, the length and direction of the missing side 1-2 can be computed. (See Part II.) In the polygon 1-*F*-*E*-7 the missing side 1-7 can be found in like manner. The direction of 1-2 and 1-7 having been determined, the angle 2-1-7 is easily found. The extra office work involved is not so much as at first appears, especially if the latitude and departure method is used for plotting. It should not exceed in this particular case more than two hours *for one man*. If the third or parallel-line method is used, the extra field work involved might easily amount to more than two hours *for an entire party*. In city work it often pays to run transit lines parallel to corresponding street lines, but with this exception the third method is not generally favored.

217. Methods of Running Transit Lines have already been compared in Chapter XII. No definite rule can be given for choosing the method best adapted to the work in hand, but if the advantages and disadvantages of each method as given on pp. 127 to 130 are understood, it is not a difficult matter to decide which to use. In general it may be said that for work in the city where accuracy is required and where the magnetic needle is often useless, one of the first two methods (angles or deflection angles) is used. Angles can then be measured by repetition or in series. In country surveying where local attraction does not exist to so great an extent, and where less accuracy is required, the azimuth method is often used, the magnetic needle furnishing an approximate check on the azimuths of the different lines. This method is also especially adapted to stadia and topographic surveying. On the other hand, when the transit-line angles of a survey must be measured by repetition or when the stations cannot be occupied in order, the azimuth method should not be used. The method of deflection angles is used in railway work and is a favorite with many engineers for other kinds of work. Its advantages are sometimes overestimated, especially as it is subject to about all the instrumental errors. The quadrant method has little to commend it. In extended surveys two methods are often combined. For example, in topographic surveys primary stations are often established by triangulation (a special case of the angle method), and from these controlling stations secondary transit lines are run by the azimuth method. (See chapter on *Topographic Surveying.*)

218. Suggestions for Running Transit Lines. Whichever method of running transit lines is adopted, by taking a few simple precautions the transitman can be reasonably sure of avoiding serious mistakes. Instead of depending entirely upon his own care and vigilance he should adopt some method of procedure by which a mistake will become self-evident as soon as it is made. This usually involves doing something as a check that otherwise would be unnecessary, but the extra time required is trifling compared to the assurance that his work is free from blunders. Indeed, with his mind thus relieved from unnecessary strain, the transitman will usually work more quickly and accurately than he otherwise would.

Precautions are taken for two reasons: (1) to insure accuracy and (2) to avoid actual mistakes or blunders. Sources of error in manipulating the transit are pointed out on p. 96, and corresponding precautions can be taken. Common mistakes in reading angles are pointed out in Chapter VIII and summarized on p. 97. Two precautions in manipulation to be emphasized are: (1) After backsighting try turning the plates *gently* to make sure that the lower clamp is tight before taking the foresight. (2) After backsighting take the foresight on the next station at once, and leave the side shots until afterwards. The shorter the interval between backsight and foresight the less likely is the instrument to be disturbed.

Checks. A system of checks affords assurance that precautions have not been in vain. There are two checks which may be employed in any method of running transit lines: (1) magnetic bearings and (2) setting the vernier to see if a line strikes a given point that it should strike.

(1) Any angle may be roughly checked by the magnetic needle regardless of local attraction (unless the latter exists in the transit or about the person of the transitman), provided the bearings of both lines are read at the same station, i.e., the back bearing of one line and the forward bearing of the other. Where local attraction does not exist, however, it is better to calculate bearings in continuous form. (See p. 126.)

In this case make sure of some correct magnetic bearing at the beginning of the survey by reading the needle at each end of a line, and assume this bearing as the basis of all calculated bearings (p. 126). In a closed survey the discrepancy between this assumed bearing and the calculated bearing of the same line is the error in the sum of the interior angles, an error which may also be detected by the check on p. 119.

(2) After measuring any important angle the vernier may be set at zero to see if the line of sight still strikes the backsight. In this way if the clamp has slipped or a wrong tangent-screw has been turned or the transit has been disturbed, the movement of the lower plate becomes evident. A slight deviation from the original backsight, due to inaccuracy in setting the vernier, is of course to be expected. The work will be facilitated if a permanent backsight is established.

When, in addition to the usual angle, side shots have been taken, before leaving the station the vernier may be set at the original angle reading to see if the line of sight strikes the *foresight.* This is a double check: (1) any movement of the lower plate while the side shots were being taken becomes evident, and (2) any mistake in the original angle reading is likely to be detected.

219. Methods of Procedure in Running Transit Lines. *Method of procedure* implies systematic precautions and checks, such as:

(1) Precautions in the manipulation of the transit, p. 95.
(2) Precautions in reading angles, p. 96.
(3) Precautions to eliminate instrumental errors of adjustment and of faulty graduation or construction, pp. 99–101.
(4) Precautions to prevent or forestall slight displacement of the transit.
(5) Checks to detect displacement of the instrument or mistakes in manipulation.
(6) Checks to detect mistakes in reading angles.

Any method of procedure may be used which accomplishes the desired result without waste of time, but it is wise to adopt some one method and stick to it, for then in many of the operations the mind will come to work automatically, which, in itself, is an efficient means of avoiding mistakes. It will be found necessary, however, to adopt different methods of procedure corresponding to the four methods of running transit lines.

Those suggested in this article are adapted to ordinary work where great precision is not required, and each method should be *modified to suit conditions.* When producing straight lines the best check is that furnished by the method of double reverse (p. 92).

Remark.—Surveyors differ as to minor details, but there is a general agreement as to the value of having a fixed method of procedure. Those outlined below are likely to seem somewhat intricate to the student, and hardly worth while following in all their details, but this is apt to be the case where any operation in the field is described in print. As a matter of fact the methods are simple, and after a little practice the extra steps taken as precautions or checks become instinctive. This book contains no better "pointer" for inexperienced transitmen than this: *Adopt some method of procedure well adapted to your work and stick to it in all its details;* it will save mistakes, relieve the mind of unnecessary strain, and inspire confidence in results.

Methods of procedure corresponding to different methods of running transit lines are given below. In each case the method of procedure indicates what the transitman actually does at any one station, from the time he has finished setting up the transit until he is ready to move to the next station. The extra steps taken as checks or precautions are indicated in italics to distinguish them from those that are absolutely necessary.

220. Method of Procedure for Direct-angle Method.

Suggestions. (1) Use a transit graduated clockwise from 0° to 360° and take all angles clockwise: saves confusion in reading and recording angles. (2) Invariably read the A vernier, consulting the B vernier, if at all, merely as a check: avoids mistakes of 180° in reading angles.

(*a*) METHOD OF PROCEDURE WHEN ANGLES ARE NOT DOUBLED. (1) Set the A vernier at 360°,—or, if at zero, *make sure that it is the zero corresponding to* 360°. (See p. 75.)

(2) Backsight; *glance to see if A vernier is still at zero; see if the back bearing agrees with the forward bearing.*

(3) *Test to see if clamps are tight;* foresight *immediately* on the next station before the transit is likely to be disturbed; *establish a permanent foresight.*

(4) Read and record the angles.

(5) *Read and record the forward bearing* (*from the needle*).

(6) *Calculate and record the forward bearing and see if it agrees with the magnetic bearing.*

(7) Take all side shots.

(8) *Set the vernier at the original angle reading and see if the line of sight strikes the permanent foresight.* (Double check.)

Comments. Instead of relying upon the needle for a check, the transit-line angle and any other important angle may be doubled: this is the surest check. If the extra time required for doubling angles is not warranted and it is desired to use the needle as a check, the azimuth method is better adapted to the purpose, provided stations can be occupied in regular order.

(*b*) *METHOD OF* PROCEDURE WHEN ANGLES ARE DOUBLED. The *first four steps are* the same as in the preceding method except that no attention is paid to the needle in backsighting, and a permanent *backsight* is established in (2) instead of a permanent *foresight* in (3).

(5) *Double the angle; record the double value, and see if it agrees with the single reading.*

(6) Set the A vernier at 360° again, backsight as before, and take all side shots.

(7) *Set the vernier at 360° to see if the line of sight strikes the backsight.*

221. Method of Procedure for Deflection-angle Method.

Suggestion. Use a transit graduated according to the quadrant method (see p. 75), and make sure that the line of collimation is in adjustment.

(*a*) METHOD OF PROCEDURE. (1) Set the A vernier at 0°; *glance at the B vernier to see that it is opposite* 180° (*and the corresponding* 0°).

(2) Backsight; *glance to see if the A vernier is still at* 0°; *see if back bearing agrees with forward bearing.*

(3) *Test to see if clamps are tight;* plunge; foresight *immediately* on the most important point, usually the next station; make a mental note whether the angle is "right" or "left"; *establish a permanent foresight.*

(4) Read the angle from the B vernier, *verifying the direction* (*right or left*) *previously noted;* record the angle, *making sure it is in the proper column "left" or "right."*

(5) *Read and record the magnetic bearing.*

(6) Calculate and record the forward bearing, and see if it agrees with the magnetic bearing.

(7) Take all side shots.

(8) *Set the B vernier at the original deflection angle and see if the line of sight strikes the permanent foresight.* (Double check.)

(*b*) *Additional suggestions.* (1) It makes no difference whether the telescope is normal or inverted when backsighting if the limb is graduated in quadrants (p. 75). When, however, the limb is graduated according to the half-circle method (p. 73) make it a point to have the telescope *inverted* when backsighting and *normal* when foresighting, so as to bring the *A* vernier under the eyepiece when reading angles (p. 73). In using such a transit a very valuable check is to observe and record both *A* and *B* vernier readings: their sum should always equal 180°.

(2) Some engineers double important angles instead of relying upon the needle. This also eliminates error in the line of collimation. (See p. 100.) In this case establish a permanent backsight in (2) instead of foresight in (3).

(3) When calculated bearings are kept the following check may be employed:

Check on the calculated bearings. Add all angles to the left; add all angles to the right; the difference of these two sums applied to the bearing of the first line should give the calculated bearing of the last line.

Illustration. 1-2 = N. 42° 18′ E. 2-3 = 16° 21′ (R.), 3-4 = 18° 20′ (L.), 4-5 = 28° 30′ (L.), 5-6 = 81° 19′ (R.), 6-7 = 8° 11′ (R.). Hence bearing of 6-7 = (16° 21′ + 81° 19′ + 8° 11′) − (18° 20′ + 28° 30′) = 59° 01′ to be applied to N. 42° 18′ E. Since 59° 01′ is R. (clockwise) it is added to N. 42° 18′ E. and the bearing of 6-7 is S. 78° 41′ E. Verify this bearing by calculating it in the usual way.

222. Method of Procedure for the Azimuth Method. *Suggestions.* Use a transit graduated clockwise from 0° to 360°, and invariably read the A vernier. As one advantage of this method is the ease with which azimuths are checked by the needle, make sure that the latter is in good working order, and that proper precautions are taken in getting the azimuth of the first line. (See p. 123.)

(*a*) METHOD OF PROCEDURE, FIRST AZIMUTH METHOD, p. 121. (1) *Glance at the vernier to see if it is still set at the forward azimuth;* plunge and backsight on the preceding station.

(2) *Test to see if the clamps are tight;* plunge; foresight on the next station; *establish a permanent foresight.*

Note. In these first two steps make it a point always to have the telescope *inverted* when backsighting and *normal* when foresighting.

(3) Read and record the azimuth; *check the azimuth by the needle reading, and record this magnetic bearing.*

(4) Take all side shots, *checking one now and then by glancing at the needle.*

(5) Set the vernier at the forward azimuth and *see if the line of sight strikes the permanent foresight.* If, as is usually the case, the line of sight is *slightly* off, say the width of the pole, bring it on with the UPPER tangent-screw, thus automatically setting the vernier for the next set-up. The vernier reading should still agree with the forward azimuth originally recorded.

(*b*) *Note.* The same system of checks may be used for the second azimuth method (p. 122). In the above method of procedure any movement of the lower plate occurring between the backsight and the first foresight to the next station would not be detected. The check for this would be to set the vernier at the azimuth of the preceding line, plunge and see if the line of sight strikes the backsight. This could be done immediately after foresighting on the next station, or it could be deferred until after the side shots were taken. When doubtful whether or not the transit has been oriented correctly, set the vernier at zero: the north end of the needle should point to N. This is only a rough check, but it will disclose any large error.

(*c*) *Note.* In beginning the survey it is necessary to decide which of the three methods to use in obtaining the azimuth of the first line. (See p. 124.) Unless a north-and-south meridian is chosen, the advantage derived from using the magnetic needle as a check is lost, and since in the ordinary survey it does not pay to establish a *true* north-and-south line by observation, a *magnetic* north-and-south line is the reference meridian most often used.

(*d*) THE METHOD OF PROCEDURE FOR THE QUADRANT METHOD (p. 125) is simply a modification of the method of procedure for the azimuth method, and will suggest itself after studying the preceding article.

223. Methods of Locating Details have been given in the preceding chapter. The two most used are the method *by angle and distance* and the method *of offsets*, but one of the *other five methods* can often be used to advantage. The accuracy required will frequently help to determine which method to employ. When, so far as accuracy is concerned, there seems to be no choice, it is usually a question of time. If, for example, the transitman is busy and the chainmen are not, it may save time to locate a point by some method which does not involve the measurement of angles. For the same reason, methods of locating details which save unnecessary "set-ups" of the transit are often used. (See p. 140.) Sometimes the surveyor is practically limited to one method, as, for example, in locating inaccessible points by angles (p. 137) or in locating irregular lines (such as river banks) by offsets (p. 139). Some obstacle to measurement or to the line of sight often compels the use of a method different from the one which would ordinarily be used, as, for example, in locating a point on the fence-line (p. 136). If the student will acquire a working knowledge of *all* the methods of locating details explained in the preceding chapter, he will soon learn by actual experience which method to chose in any given case.

In locating any detail select points which will best serve the purpose. Thus, for example, in locating a building choose a long side in preference to a short side, so that the corners located may be as far apart as possible.

Do not locate points unnecessarily. For example, do not locate both rails of a track, but take points on the gage-line of one rail, and make a note of the gage.* Sometimes it is better to locate the center-line of track, as is done in staking it out.

In filling in the details of a survey it is often a waste of time to take all measurements with equal accuracy. It is far better to keep in mind what each angle or distance will be used for, and measure it with corresponding accuracy. Measurements required solely for plotting may be taken, as a rule, with less precision than those needed for permanent record, and measurements taken to fix the position of an object are usually less important than measurements of the object itself. This is especially true if the scale of the map is small. Thus, for example, suppose that several buildings stand on a piece of land. The map will be plotted to a scale of 1 inch = 50 ft. An error of ½ ft. in the relative positions of the buildings will hardly show on the map. Measurements made solely for plotting the buildings and the corners of the boundaries could, therefore, be made to the nearest *foot*. On the other hand, if the dimensions of the buildings and the lengths of the boundary-lines are to be recorded on the map, or are to be preserved elsewhere as a matter of record, they should be measured with greater accuracy. Rough measurements will, of course, suffice for *indefinite* details, such as banks of streams or stone walls, and for *temporary* objects, such as walks and driveways.

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* *The gage-line is a line* on the inside of the rail ½" below the top.

Locating street-lines. In the larger cities it is customary to set stone monuments or *merestones* at street-corners, thus marking the intersection of street-lines. In small towns, however, and in country highways the surveyor must often depend upon existing fences or upon whatever information he can gather from old maps, old landmarks, and other sources. On the other hand it cannot be taken for granted that existing fences are on the street-lines, for it is a common experience to find that they are not.

Practical suggestions. (1) When a definite object such as a building is located, remember that the work is not completed until the object is "measured up," i.e., enough dimensions obtained to plot it. (2) If the two ends of a boundary-line are located, it is a good rule to measure the actual length of the line as a check and as a matter of record. (3) Transitman, note-keeper, and chainmen must all cooperate to prevent the too common mistake of measuring to one point of an object and then through carelessness taking the angle to some other point or *vice versa*. (4) A common mistake in note-keeping is to interchange the angles or the distances to two points so that the angle to one point and the distance to another point are recorded as if they were taken to the same point. (5) In measuring from a transit station to a number of near-by points it is well to have the zero end of the tape at the station. If the tape is not graduated to decimals of a foot throughout, but only the end foot is subdivided, the chainman who is not at the station will simply read the *whole* number of feet; the chainman at the station, pulling the tape taut, notes the decimal of a foot, and after subtracting one foot from the other chainman's reading, gives the final reading (p. 34). (6) In taking a large number of side shots, sight occasionally at the permanent backsight to see if the transit has been disturbed. (7) Keep in mind the relation between angular and linear measurement pointed out on p. 108. Thus, for example, one minute is equivalent to only 0.03 ft. at 100 ft. from the transit or 0.3 ft. at 1000 ft.; likewise one inch at a distance of 100 ft. from the transit may cause an angular error of 3 minutes, while at 1000 ft. an inch is equivalent to only 15 seconds (p. 105).

Question. If a corner of a building is, say, 50 ft. from a transit and the building is to be plotted to a scale of $1'' = 100$ ft., can the angle to the corner be read from the limb with sufficient accuracy for plotting, ignoring the vernier reading?

224. Staking Out Work, i.e., setting stakes for construction, is the reverse of locating objects which already exist; hence much that applies to the location of details applies to this kind of work also. The chief difficulty that the young surveyor encounters is in knowing what stakes will be needed by the contractor. This knowledge is best gained from long experience, from conversations with foremen, and from actual observation of what is actually needed in various lines of construction. Each piece of work is a problem in itself, but a few general suggestions may be given which apply to nearly all kinds of work. In Chapter XVII a number of examples of special work in staking out buildings, roads, bridge piers, and other objects show how these suggestions are carried out in practice.

General suggestions. (1) It is better to give a few points which will determine the main lines of a structure than to give so many points that they are likely to be confused; hence do not give lines which the foreman can easily establish for himself.

(2) Where excavation is required it is often best to give lines *roughly* at first for digging,—more accurately afterwards. (Art. 265, p. 208.)

(3) Set stakes where they are least likely to be disturbed during construction, yet near enough to the object so that they are convenient for use.

(4) For buildings, masonry structures, and certain other kinds of work where elevations as well as lines are required, batter-boards are used in place of stakes. (See Art. 265, p. 208.)

(5) Any important point which, if destroyed during construction, could not be re-established readily should be referenced (p. 148). This may often be done by setting nails or marking points where lines prolonged strike neighboring fences or buildings. (See Fig. 266 (*a*), p. 210.)

(6) So far as possible follow a uniform system of giving points, even for different contractors: saves confusion in the office and in the field. Thus, for example, in staking out a long retaining-wall, do not set stakes a certain distance in *front* of a neat line at one place and *back* of it at some other place, but, if possible, set stakes at the same distance on the same side of the same neat line throughout the length of the wall.

(7) So plan the location of stakes and points that no serious delay will follow if one here and there is knocked out or covered up, or rendered useless for purposes of sighting by intervening piles of material. One method of accomplishing this is to plan for at least two different ways of arriving at or locating each important point. In large works of construction such a plan is of great advantage, and saves work which would otherwise be necessary in referencing points.

(8) A sketch in the field notes should show the location of all important stakes and reference points so clearly that they can be found and used by a surveyor who did not see them set.

(9) Have some good system of marking the stakes themselves, one readily understood by the contractor. Suggestions for marking grade-stakes will be found in the chapter on *Leveling,—Field Work.*

225. Field Notes. In the next chapter some of the forms of field notes used in transit surveying are given, and the different methods of keeping notes are discussed. It is not so important *which* method is used as it is essential that the notes be clear and self-explaining, so that any intelligent draftsman can plot them. There is little excuse for notes so obscure and confused that only the man who took them can understand them.

226. Errors and Limits of Error. To come within a given limit of error in transit surveying both linear and angular errors must be under control within fixed limits. The most important points brought out in the discussion of errors (Chapters II, V, and X) may be summarized as follows:

1. The true error of any measurement is never known, but it can be estimated from the discrepancy between duplicate measurements or from *probable errors*. The more completely constant errors are eliminated the more nearly can the precision of the work be estimated.
2. Accidental or compensating errors, as a rule, are less important than constant or cumulative errors.
3. Not only should the methods of eliminating or correcting errors from different sources be understood, but the relative importance of the errors should be known also. (Art. 69, p. 43.)
4. Field-work requirements corresponding to the adopted limit of error should be kept in mind. (Art. 76, p. 51.)
5. Consistent accuracy implies the correlation of linear and angular errors. (Art. 152, p. 109.)
6. Customary limits of error should be adopted so far as the conditions governing a survey will permit. (p. 46 and p. 108.)
7. Not all the work of a survey is done with the same precision. For example, an error in the transit lines affects the whole survey, while an error in locating a detail may only affect a single point; hence the greater need of precision in running transit lines. *Make the accuracy of each step in the field work correspond to its importance.*
8. Limits of error should not be followed blindly. The best surveyor is not he who does all his work with great precision, but he who does it just accurately enough for the purpose without waste of time and money.

227. Permissible Error of Closure. (See Art. 176, p. 119, for definition.) Let l = the difference between the sums of the north and south latitudes, and d the difference between the sums of the east and west departures, found by the methods explained in the chapter on *Latitudes and Departures*, Part II; then the error of closure $E=\sqrt{l^2+d^2}$. This error is usually expressed as a ratio $\frac{E}{P}$, where P is the total length of the perimeter. For ordinary transit work this ratio should seldom be more than 1:1000, and in city work should average not less than 1:5000. In some cities the limit is from 1:10 000 to 1:15 000 for precise traverses with repeated angles. This is probably as high a degree of precision as should be required in traverses run with a transit and steel tape.

A general formula may be derived as follows: Let
N = number of sides to the polygon, also number of interior angles;
c = coefficient of chaining adopted for one tape-length;
E_c = that part of the error of closure due to chaining;
E_a = " " " " " " " " " angular measurement;
t = *length of tape*;
a = *permissible error in one angle.*

From Art. 71, p. 45,

$$E_c = c\sqrt{N} = c\sqrt{\frac{P}{t}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The error for any one side due to angular measurement will be proportional to the length of that side and hence for the whole perimeter it will be the total angular error times the length of the perimeter. But from p. 110 the total error in the sum of the interior angles may be expected to be $a\sqrt{N}$; hence $E_a = a\sqrt{N} \times P$. Since an angular error of one minute = 0.0003 ft.,

$$E_a = 0.0003aP\sqrt{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The error from the combined sources from the theory of errors may be the square root of the sum of the squares of the separate probable errors, p. 18; hence

$$E = \sqrt{(0.0003aP)^2N + c^2\frac{P}{t}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In ordinary chaining the second term under the radical is so small that it may be ignored and formula (3) then reduces back to formula (2), in which form it is usually used.

$$E = 0.0003aP\sqrt{N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Illustration. Suppose that the permissible error for one angle is 15" or ¼ minute; that the total length of the perimeter of a ten-sided polygon is 4000 ft. The error of closure should not exceed

$$0.0003 \times \tfrac{1}{4} \times 4000 \times \sqrt{10} = .95 \text{ ft.}$$

228. The Duties of the Different Members of a Transit Party depend upon the number of men in the party. In class work, if the party is composed of five men the duties may be outlined as follows:

THE CHIEF OF PARTY plans the survey and directs the field work. Oftentimes in addition he acts as transitman or as note-keeper. He should thoroughly understand the advantages and disadvantages of the different methods of running transit lines (Chapter XII and Art. 217), and be able to choose wisely between the different methods of locating details (Chapter XIII and Art. 223). He should be familiar with the customary limits of error, and with the sources and relative importance of errors (Chapters II, V, and X; also Art. 226). In class work the instructor necessarily decides most of the questions which are suggested above, but he should discuss with the students the reasons which influence him in his decisions.

1. THE NOTE-KEEPER often acts as chief of party, and in addition to directing the field work keeps the field notes in the party note-book. During the early part of the course *he should also read each angle as a check on the work of the transitman.* Read Chapter III for general directions and consult Chapter XV *for the different* forms of transit notes. Pay particular

attention to *Special Directions for Class Work,* p. 30. The position of note-keeper is a responsible one, and upon his alertness and his ability to keep good notes will depend to a large extent the value of the work done by the whole party.

2. The Transitman is responsible for all work done with the transit; also responsible for any injury done to the instrument either while in use or while being carried from one place to another. *The transitman must not leave the transit standing unguarded.* The use of the transit is explained in Chapter IX. Particular attention is called to the following articles:

1. *Caution*—to prevent injury to the transit, p. 84.
2. *Use of the Telescope*—focusing eyepiece, getting line of sight quickly, etc., p. 85.
3. *Setting up the Transit*—hints for setting up *quickly,* pp. 86–89.
4. *Use of Clamps*—warning against using wrong clamps, p 91.
5. *Method of Double Reverse*—to prolong a straight line accurately, p. 92.
6. *Lining In*—sighting chainmen or flagman in line, *setting transit* stations, p. 91.
7. *Measuring Horizontal Angles*—different methods, pp. 93–95. Pay particular attention to hint 6, p. 95.
8. *Measuring Angles by Repetition*—especially doubling an angle, p. 99.
9. *Reading Angles*—use of vernier and common mistakes, pp. 96–97.
10. *Reading Bearings,* p. 101.
11. *The Sources and Relative Importance of Errors in Angular Measurements* (Chapter X).
12. *Suggestions for Running Transit Lines*—precautions and checks, p. 152.
13. *Methods of Procedure in Running Transit Lines*—to avoid blunders, p. 153.
14. *Care and Adjustment of the Transit* (Part III).

3 and 4. The Chainmen are responsible for the measurement of all distances. In addition they may be called upon to give sights, drive stakes, assist in setting up the transit, and to help in other ways to carry on the field work. They are also held responsible for any injury to the tape. Directions for chaining are given in Chapter IV. Special attention is called to the following articles:

1. *Care of the Steel Tape*—to prevent bends, breaks, and rust, p. 31.
2. *To Do Up and Undo a Tape*—in a figure 8, p. 31.
3. *General Method of Chaining*—p. 33.
4. Precautions to avoid error in reading odd feet and tenths of feet, note (*c*), p. 34.
5. *Setting Chain-pins*—directions to insure accuracy, p. 35.
6. *Chaining on a Slope*—especially the second method, p. 36.
7. *Sources of Error,* methods of correction and elimination, Chapter V, especially the rule for the correction of errors in the length of the tape, p. 40, and the correction for temperature, p. 40.
8. *Relative Importance of Sources of Error*—p. 43.
9. *Customary Limits of Precision,* and the field-work requirements *corresponding to* different limits, pp. 44–53.
10. *Summary—of things* which the chainmen should do and should not *do, p. 37.*

5. **The Axeman or Flagman** drives the stakes for transit stations, marks the guard-stakes and sets them, holds the transit-pole or pencil for backsights and foresights, and clears the way through underbrush, when necessary, for the line of sight.

1. For the method of establishing a transit station, read Art. 207, p. 146.
2. For the method of marking and setting a guard-stake, read Art. 208, p. 147.
3. In holding a transit pole be sure that it is *plumb*. Stand squarely behind it, and using both hands, suspend it so that it will swing freely until it becomes plumb, then keep it there. Oftentimes the vertical edge of a building or some other fixed vertical line will aid in keeping the pole plumb.
4. There is a knack in driving a stake plumb; acquire it. Use a broad flat-headed hand-axe, and strike squarely to prevent brooming. When a stake begins to get out of plumb do not bring it back by striking one side of it, but "draw" it over by striking it on the head with a peculiar blow, in which the axe-head moves towards the body as it hits the stake; if the stake is not too far down, pull it over with one hand while using the axe with the other.

Remark. Surveyors complain that axemen are either careless in driving stakes or that they do not know how—a constant source of annoyance.

5. If a backsight or foresight is wanted at a station where another transit is set up, hold a handkerchief behind the plumb-bob line, keep the bob from swinging, and see to it that members of the other party are not in the line of sight. Another way of giving a sight in such a case is to hold a pencil on top of the center of the transit telescope.
6. Do not "come in" from giving a sight until signaled to do so. Have a clear understanding in regard to signals, p. 146.

CHAPTER XV

TRANSIT SURVEYING—FIELD NOTES

In the first part of this chapter three general methods of keeping the field notes for a transit survey are discussed, and those things which are essential to good transit notes in any form are pointed out. In the second part of the chapter several methods of keeping notes are illustrated by reproductions of actual field notes. A study of these notes should also result in a clearer idea of how the methods of running transit lines (Chapter XII) and of locating details (Chapter XIII) are applied in actual field work. The comments accompanying the illustrations are intended to lead the student to a comparison of the different methods of field work and of keeping notes, so that ultimately he will be better prepared to choose methods for himself, modify them, combine them, or devise new ones especially adapted to any work he may have in hand.

229. Different Methods of Keeping Field Notes. There are many forms of field notes in common use, some adapted to one kind of field work, others to other kinds. The best method of keeping notes for a given piece of work is a matter of opinion, and an experienced surveyor is likely to use his own method,—one which is "the survival of the fittest." Any system may be used which results in a clear and accurate record of the field work without wasting time. A form of notes, however, which saves time in the field but wastes it when it comes to plotting is not, ordinarily, an economical form to use, although occasionally it may be.

It is well to use a system which can be modified to suit different methods of field work. Thus, for example, in the second part of this chapter there is given a form of notes which not only can be adapted to any one of the four methods of running transit lines, but which can also be easily modified to suit different kinds of surveys, such as a stadia survey, a compass survey, or a transit survey. Having once become familiar with some such method of keeping notes, it is an easy matter to adapt oneself to any other method when it becomes necessary.

230. Notes Pertaining to the Transit Lines should be distinguished from all other notes because the transit lines are usually plotted first and details afterward. The field notes may be divided into two parts, viz.: (1) those pertaining to the location of transit stations, and (2) those pertaining to all other points. When transit lines are run before details are taken (Art. 193, p. 132) transit-line notes may be kept by themselves in compact form. When, however, details are taken as fast as transit stations are established the transit-line notes are not kept separate, but are scattered along mixed with other notes. In that case the notes should be kept in such a way that those pertaining to the transit line can be picked out without any trouble when it comes to plotting. One way is to *letter* all transit stations and *number* all other points. Another way is to enclose transit stations in a triangle or circle. Still another way is to keep transit-line notes in a separate column by themselves. (See Form 3, p. 172.)

231. Three General Forms of Notes. There are two methods of keeping notes which are radically different, viz.: (1) The method in which *all* measurements of angles and distances are *indicated on a sketch;* (2) The method in which no sketch is used, but measurements are recorded in some tabular form. The latter method can be used only where comparatively little description of the survey and its details is necessary. (3) A *third* method is a combination of the other two in which some of the data are recorded on a *sketch*, and the remaining data are *tabulated.* This combination method is, perhaps, the one most used.

232. The Tabulated Form of Notes without a Sketch can be used to advantage for transit-line notes when kept by themselves, or for recording a number of measurements to points or objects, all of which can be described just as well without a sketch. In most cases, however, a sketch is the easiest way of making clear the details of a survey. A number of tabulated forms are illustrated in Art. 241, pp. 169–175.

233. The Sketch Method for Field Notes. When it can be done without crowding, there is probably no better way of keeping field notes than by making a free-hand sketch with all angles and distances entered directly upon the sketch. The difficulty with this method is that a sketch made on the page of an ordinary field note-book will often be too small to provide for all the measurements in large, clear figures. In that case it is better to resort to the *combination* method.

It is not customary to draw the sketch to scale; indeed it is usually necessary to exaggerate certain portions of it to make room for recording measurements. In some forms of notes no attempt is made to sketch outlines as they appear on the ground. For example, a crooked highway appears in the sketch as perfectly straight. (See Illustration VI, p. 187.)

234. The Combination Method of Keeping Field Notes, in the opinion of many surveyors, is the best all-around method. A sketch can be made on one page and such measurements as cannot conveniently be placed on the sketch itself can be recorded on the opposite page in some one of the tabulated forms in common use. As far as possible it is well to put actual dimensions and offsets on the sketch, while angles and the corresponding distances used for locating points are put on the opposite page. For example, if a house is shown in the sketch it is usually well to put dimensions of the house on the sketch itself, while angles and distances taken to locate the corners of the house are given on the opposite page. (See Illustration III, p. 180.)

Points on the sketch are *numbered* or *lettered* when it is necessary to refer to them in the tabulated form on the opposite page. When it can be done readily it is well to number points *in order;* for example, the first one sighted at as the telescope is turned from the backsight is 1, the next 2, and so on. (See p. 134.)

235. When should the Notes Read from the Bottom of the Page up? In railway surveys or highway surveys or surveys of a similar character it is more convenient to record the notes *from the bottom of the page up.* This is especially true if the *sketch* method is used, otherwise the note-book must be held upside down to bring the points in the sketch in accord with the points which they represent on the ground. The general rule in making a sketch is to hold the book with its top toward the next transit station, and if a tabulated form on the left-hand page is used in connection with the sketch, this will read from the bottom *up* to correspond. (See Illustration V, p. 184.) On the other hand, in the notes of a closed survey, *where the entire sketch is on one page, it may be more convenient to tabulate* the corresponding notes on the opposite page, so that they will read from the top *down*. (See Illustration I, p. 176.)

236. What should the Field Notes Include? In taking notes it is well to keep in mind (1) what will be needed for plotting, and (2) what it may be well to have in the note-book even though it may not be necessary for plotting. Measurements of angles and distances, sketches and explanatory notes are of use in plotting, while the date of the survey, names of the men in the party, transit used, weather conditions, and notes indicating methods of work may be of value as a mere matter of record. Common sense should guide one in this.

In ordinary work, for example, the temperature is not recorded because there is no need of it, but in precise surveying it *is* recorded because corrections for temperature are to be made. A note which gives the methods used in a survey may indicate the limits of error, and if there is a chance that such information will be needed, it should be given in the notes, but in the majority of cases it will not be needed. On the other hand the title of the survey, the date, and the name of the man in charge of the party ought to appear in the notes of every survey.

237. Allow Plenty of Room; do not Crowd the Notes. Make sketches larger if anything than may seem necessary. Usually there will be more dimensions to put on the sketch than will at first appear; make a generous allowance for this. Especially those portions of a sketch where most of the measurements are to be shown should be large and open so that small crowded figures may be avoided. Buildings, for example, should be sketched large enough to provide for dimensions. If a fence is located by offsets from a building, make the fence in the sketch far enough away so that the offsets may be recorded in clear figures, no matter how near to the building the fence really is. Do not try to put too much on one page; use several pages if necessary for different parts of a sketch. *Note-book paper is cheap compared with time wasted in deciphering crowded notes.*

In tabulated forms it is better to skip every other line when there is room, thus leaving space for corrections.

238. Repeating Notes on Different Pages. When tabulated notes extend over two or more pages it is customary to repeat on one page enough of the notes from the preceding page to make the whole continuous. Usually this means simply to copy the preceding transit station (number or letter) and, perhaps, a transit-line angle and distance.

239. General Sketch of Transit Lines. When a survey is more or less extended and the transit-line notes are scattered through several pages, it is a good plan to start a sketch of all the transit lines on one of the first pages. This sketch, which should be completed as fast as the field work progresses, will show the relative positions of the different transit stations and will prove especially useful if there is a more or less intricate network of transit lines. It is not necessary to enter any measurements on this sketch at least until the notes are "worked up" preliminary to plotting.

240. Five Things to Make Clear in any Form of Transit Notes.

Linear Measurements. (1) Between which two points each measurement was taken.

Angles. (2) At which station the transit stood when an angle was measured.

(3) Which station or point was used for a backsight.

(4) In which direction (right or left) the angle was measured.

(5) To which point as a foresight the angle was taken.

Illustration. A point 2 in a transit survey was located from a transit line AB by an angle of 45° and a distance of 30 feet. In order to plot the point 2 the notes must make clear whether the angle was taken at A or at B. If taken at B, then A must be entered as a *backsight*, the *direction* in which the angle was measured must be understood, and finally 2 must be recorded as a *foresight*. Should the notes fail to make clear any one of these points, a serious mistake might be made, as illustrated in Art. 167, p. 117. It is especially irritating to find an angle or distance given in the notes and yet not be able to plot it.

If the sketch method is used and all distances and angles are indicated on the sketch exactly where they belong (see Illustration IV, p. 183), misunderstandings are not likely to occur in plotting—a strong argument in favor of the sketch method. In all tabulated forms the five points noted above must be made clear by the column-headings, by the system of notation, and by explanatory notes and sketches, if necessary.

In an office where the methods of keeping notes and of taking *angles are* always the same, much in the notes may be taken *for granted. For* example, if it is customary to measure all angles clockwise, it is not necessary to record the direction in which angles are measured. It is well, however, to print once for all, near the beginning of a set of notes, some such note as: "*All angles taken to the right from a backsight on the preceding station.*" (See Illustration I, p. 176.)

Suggestions. In order to make sure that everything in the notes is clear and unmistakable try to put yourself in the position of a man in the office who is not at all familiar with the ground covered by the survey, and as you take the notes examine them critically from his point of view, keeping in mind what the notes will be used for and asking yourself continually the question: "Will this be clear to the man in the office?" Even though you expect to plot the notes yourself, remember that things which seem perfectly clear to you while you are recording the notes may become very obscure and indefinite by the time you begin plotting them. It is a poor note-keeper who can plot his notes only "while they are hot."

Reread, from time to time, the general suggestions for keeping notes given in Chapter III.

Study the forms of field notes given in the next article, noting the advantages and disadvantages of each method. Study also the actual field notes reproduced on pages 175–189, noting carefully all the little devices and methods for making the notes clear and unmistakable. The best way to study these notes is to plot them. (See p. 175.)

241. Different Forms of Tabulated Field Notes. When the usual form of field note-book is used it is customary to keep the tabulated notes on the left-hand page, which is ruled in six columns for that purpose. Sketches, if there are any, are made on the right-hand page, which is cross-lined and can be used as plotting-paper if desired. Several of the forms for tabulating notes are given here. In most cases each form would be accompanied by an explanatory sketch on the opposite page, but for the present these sketches are omitted. The column headings and two or three lines of notes for each form will illustrate the different methods. The style of lettering used in these illustrations and in the reproductions of the field notes which follow is the free-hand single-stroke system explained on pages 24–30. Those forms which read from the top of the page down may easily be modified to read upward if desired.

FORM 1

Angle	Value	Line	Length		
BCD	89°29'	CD	280.1		
BC2	112°14'	C2	83.8		
		3-4	40.0		

Interpretation. Backsight at Station B, transit at Station C, foresight at Station D, angle $=89° 29'$, and distance from C to $D=280'.1$. Same backsight, same transit station, the angle to some point 2 (not a transit station) is 112° 14′, distance from C to $2=83'.8$. No angles taken to points 3 and 4, distance between them is 40′.0. In this instance *letters* are used for transit stations, numerals for all other points. Corresponding letters and numerals on a sketch (not shown here) on the opposite page of the note-book complete the notes. All angles are taken to the right. For a more complete illustration see Illustration I, p. 176.

Advantages. A simple and natural method of tabulating angles and distances. Easy to use in the field and not likely to be misunderstood in plotting. When a system of transit lines is established first, and objects located afterward, this form is a very good one for the transit-line notes, especially if there is a network of transit lines.

Disadvantages. The method of giving an angle is not self-explaining. A note should make clear once for all that the middle letter is the vertex, the first letter the backsight, and the last letter the foresight. This order is easily remembered since the backsight (first letter) is the *first* point sighted at, and the foresight (last letter) is the *last* point sighted at in measuring an angle. When stations are numbered instead of lettered the method may become cumbersome, especially if there are *plus* stations. It would be awkward, for example, to give an angle thus: (21+14) (24+30) (30+25).

FORM 2

Transit at		Sta. C,	backsight	on Sta. B	
Foresight	Angle	Line	Length		
D	89° 29′	CD	280.1		
2	112° 14′	C2	83.8		
		3-4	40.0		

Interpretation. This form differs from the preceding form in only one particular: the station at which the transit stood and the backsight are noted once for all at the head of each set of angles. The third column can be and often is omitted.

When magnetic and calculated bearings are kept the last two columns may be used for that purpose, as shown on p. 174. As in the preceding method it should be noted that all angles are taken clockwise from the backsight. Should it be desired to take angles right and left, two columns headed "Right" and "Left" respectively may replace the single column shown (see p. 173), or else R or L may be written after each angle.

Advantages. When a number of angles are taken at each station this form has, in addition to the advantages of the preceding form, the merit of being self-explaining: no doubt can arise as to which station is the backsight, which the vertex, and which the foresight. Moreover, the form can be used for plus stations. This form, or some modification of it, is a very common one. It is especially useful in topographic surveying, where a large number of side shots are taken at each station, and where vertical angles are measured. By omitting the third column three columns are available for recording vertical angles and corrected elevations or distances.

Disadvantages. When only one or two angles are taken at each station it becomes necessary to reprint the words "*Transit at Sta.* —, *backsight on Sta.* —," on almost every other line. In that case Form 1 is more compact and convenient.

FORM 3

Angle	Distance	Transit Line	Angle	Distance	
		△ Sta. D 280.1			
4=310°18'	44.6	89° 29'	1=40°30'	28.1	
3=211°11'	40.3	△ Sta. C 320.1	2=112°14'	83.8	
		108°12'			
		△ Sta. B 510.6			

Interpretation. The notes are to be read from the bottom up. All notes pertaining to the transit line are put in a column by themselves, each *distance* inserted *between the two stations* where it was measured, each *angle* directly *over the station* where the transit stood when the angle was taken, the backsight being the preceding station *below*, the foresight the next station *above*, thus: Backsight at Sta. *B*, transit at Sta. *C*, foresight on Sta. *D*, angle = 89° 29', distance from *C* to *D* = 280'.1.

All points that are not transit stations are clustered around the station at which the transit stood when they were located. Thus points 1, 2, 3, and 4 were located by angles taken at *C* from the same backsight on *B*, and by distances measured from *C*. An explanatory sketch on the opposite page is implied in this case.

Remark. Assuming that all angles were measured clockwise, the angle 40° 30' indicates that point 1 is on the *left* of the line *CD*. It would be an improvement, therefore, to enter 40° 30' and 28.1 ft. in the *left-hand* columns; then, to be consistent, the measurements corresponding to the points 2, 3, and 4 should be entered in the *right-hand* columns. This would be the relative positions of the four points if the measurements were placed directly on a sketch. (See Illustration VI, p. 187.)

Advantages. Transit-line notes are *separate* from other notes. In a survey progressing in one general direction, as, for example, the survey of a highway, this form is useful, especially if combined with a sketch (as in Illustration VI, p. 187).

Disadvantages. Notes are not self-explaining, more or less *complicated, and not* adapted to surveys where only one or two *points are located* from each station.

FORM 4

Angle	Value	Line	Length	Bearing	
	2)198°44'			Mag.	Cal.
CDE	99°22'	DE	410.8	N26°30'E	26°24
	2)148°53'				
DEF	74°26'30"	EF	328.5	N79°15'W	79°9'30"

Interpretation. This form is a modification of Form 1. Two additional columns provide for magnetic and calculated bearings. Thus, N. 26° 30′ E. and N. 26° 24′ E. are the bearings of the line *DE*. The notes indicate that the angles were doubled by repetition, every other line being left blank to provide space for the arithmetical work of division. Notes read downward.

The *advantages* and *disadvantages* are the same as for Form 1.

FORM 5. DEFLECTION ANGLES

Sta.	Deflection		Line to	Length	
	Left	Right			
E	56°24		F	379.4	
E		41°18'	1	38.3	
D	28°10'		E	220.6	

Interpretation. One of the forms used when *deflection* angles are taken to the left or right. Notes read from the bottom up. For example, backsight at *D*, transit at *E*, deflection angle to the right of 41° 18′ to point 1, distance from *E* to 1 = 38′.3.

Advantages. Right and left deflection angles are in separate columns.

Disadvantages. If the transit stands at any station (as *E*), it is assumed that the backsight is on the preceding station below (as *D*). This is not self-evident from the notes; it is, however, so well understood in practice that the disadvantage is not a serious one. The mistake most likely to occur in using this form of notes is that of entering an angle in the "Left" column when it should be in the "Right" column, or *vice versa.*

FORM 6. DEFLECTION ANGLES

Station Occupied	Point Sighted at	Deflection Angle	Distance	Bearing	
				Mag.	Cal.
+20.6	a	41°18'L	38.3		
+20.6	36	56°24'R		N36°30'E	36° 25'
32.	32+20.6	28°10'L		N20°0'W	19°59'
30	32	12°11'R		N8°15'E	8°11'

Interpretation. This form is similar to the preceding form, except that deflection angles are kept in one column instead of in separate columns, the letters L and R denoting whether they are taken to the left or to the right. Notes read upward.

Advantage. A column is saved for some other purpose. Thus, for example, two columns are left in which magnetic and calculated bearings may be entered separately if desired.

Disadvantage. If the letter R or L is omitted in any case it may cause trouble in plotting. This is more likely to occur than the mistake of entering the angle in the wrong column, in the preceding form.

FORM 7. AZIMUTHS

Line	Azimuth	Length	Mag.		
CD	190°10'	281.6	S10°15'W		
DE	280°48'	310.8	N79°15'W		
D2	291°18'	28.9			

Interpretation. This form is a modification of Form 1, p. 170, azimuths being recorded in place of angles. Magnetic bearings are also recorded as a rough check on the azimuths.

When many azimuths are to be recorded at each station a modification of Form 2, p. 171, may be used, the words "Transit at Sta. —" being placed above each set of azimuths. Backsights, of course, are not recorded in the notes.

Advantages. This form of notes is so simple and self-explaining that it (or some slight modification of it) is in general use for the azimuth method. It is especially useful for stadia notes when a large number of points are to be located.

FORM 8. BEARINGS

The form used for notes when transit lines are run by bearings (p. 125) may be a modification of Form 7, simply substituting a column of bearings as read from the vernier for the column of azimuths, and keeping the other three columns as they are.

242. Illustrations of Field Notes. In the following pages several sets of field notes are given; they illustrate not only different methods of keeping field notes, but also different methods of field work. The size of a page in the ordinary field note-book is about an inch wider and two inches longer than a page of this book.* Hence the notes are shown correspondingly reduced, but in other respects they are like the original field notes. They should be studied from two points of view: (1) To see how the methods of Chapter XII and XIII for running transit lines and locating objects are applied in actual field work; (2) To gain a clearer idea of how field notes are kept.

The student is urged to study the notes and comments in the order in which they are given and with unusual care. They have been chosen and arranged with the hope that this may prove one of the most useful portions of the book. It must be borne in mind, however, that for every method of running a transit line or locating a point given in the notes there is probably another method equally good which could have been used instead. Moreover, the methods shown of keeping notes can be and are modified and changed to suit the work in hand, and the remarks in Art. 229, p. 164, should be kept in mind.

It is suggested that a most effective way of studying the different illustrations is to actually plot the notes. If this can be done before going into the field it will be of great assistance to the student not only in keeping notes, but in choosing methods for locating details.

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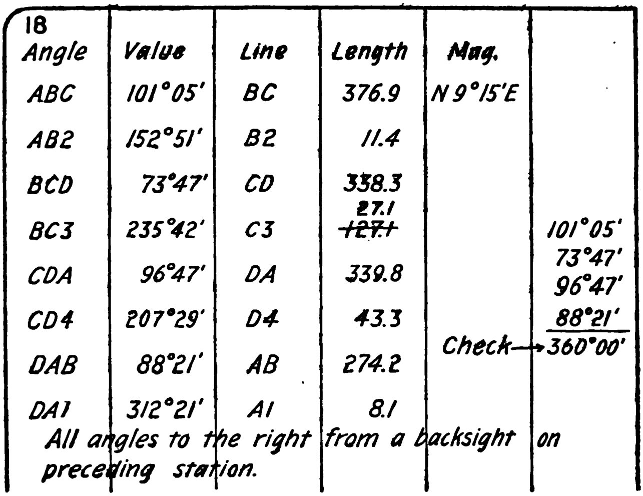
* *Note-books* can be obtained in various sizes and ruled in various ways for different kinds of work Thus, for example, books may be obtained for topographic surveying with the right-hand page specially ruled to facilitate sketching the topography. The ruling is purposely omitted in the reproductions of field notes shown in the succeeding pages of this chapter.

It sometimes happens that notes taken in the field must be sent to the office from day to day to be plotted, while the party still remains at work in the field. Duplicate notes are usually copied in this case. To avoid the work of copying, special note-books with detachable leaves are sometimes used, carbon paper being inserted in order to obtain the duplicate sheets *required for the office.*

ILLUSTRATION I



18

Angle	Value	Line	Length	Mag.	
ABC	101°05'	BC	376.9	N 9°15'E	
AB2	152°51'	B2	11.4		
BCD	73°47'	CD	338.3		
BC3	235°42'	C3	27.1 ~~127.1~~		101°05'
CDA	96°47'	DA	339.8		73°47' 96°47'
CD4	207°29'	D4	43.3		88°21'
DAB	88°21'	AB	274.2	Check →	360°00'
DA1	312°21'	A1	8.1		

All angles to the right from a backsight on preceding station.

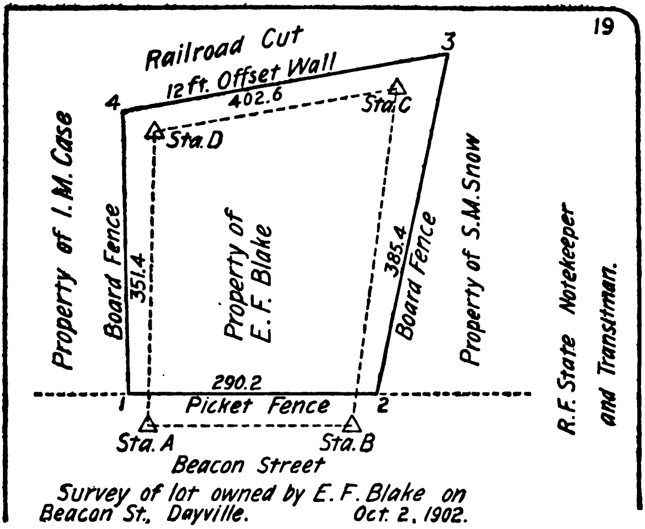
ILLUSTRATION I

Comments on the Survey. This is an illustration of a survey to determine the boundary-lines of a building lot. Transit lines are run at random within a few feet of the fence-lines, but not parallel to them. There are four transit stations, and each corner of the lot is located from the nearest station by angle and distance. Notice that in addition to transit-line measurements the length of each boundary-line was also measured, although a map of the lot could have been plotted without these four measurements. The four angles at the corners of the lot were not measured,—they can be easily calculated. See Art. 216 (*d*), p. 151.

Since angles were not doubled there was apparently no check on the work until the last or closing angle was measured. It would have been well to calculate bearings and check them by magnetic bearings (p. 126).

While the survey is complete in itself no one line seems to be referenced with respect to permanent objects, so that if any *three of the four corners should become obliterated it would be difficult to restore them.* (See Art. 260.)

ILLUSTRATION I



Comments on the Notes. All data pertaining to transit lines and to the location of corners from transit lines are tabulated on the left-hand page, according to Form 1, page 170. Actual dimensions, i.e., *lengths* of property lines, are given on the sketch on the right-hand page. In this survey the data are so few that they all could have been put on the sketch without crowding, and hence the sketch method (see the next illustration) would have been the natural one to use. Notice the method of correcting the false entry of 127.1 ft. [See p. 22, Art. 35 (5).]

It would have been well to add a note indicating the degree of accuracy with which the survey was made, as was done in the next illustration.

If any future use is likely to be made of the magnetic bearing, the declination of the needle should be given in the notes if it is not already on record in the office; in any case it should be given on the map.

ILLUSTRATION II

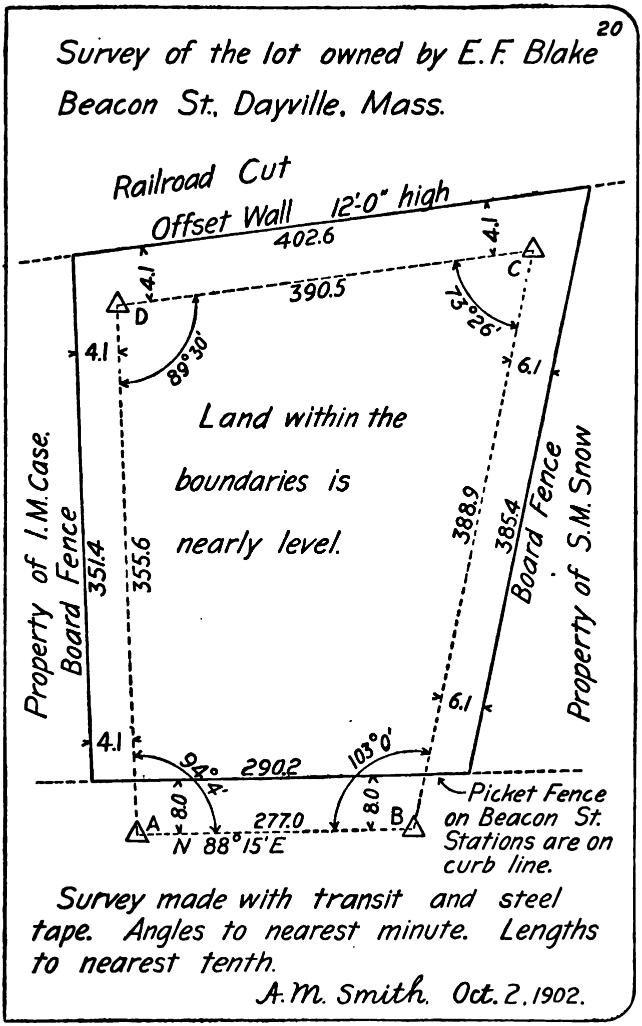
Comments on the Survey. The lot surveyed is the same as that in the preceding illustration. This time, however, the transit stations were chosen so as to make each transit line parallel to the corresponding boundary-line, and any convenient distance away from it: the angle at each corner of the lot is therefore equal to the angle between transit lines at the corresponding station. No angles are taken to the corners of the lot, but the boundary-lines are located by offsets from the transit lines, and their lengths measured as usual.

The *advantages* of this method are that the boundaries are more easily plotted by offsets than by angles, and the angle at each corner is known without computation. The *disadvantages* are in the field work itself. It takes time to get the transit lines established exactly parallel to the fence-lines even if there are no obstacles to measurements or to lines of sight. When there are obstacles, as there usually are, the extra field work involved is out of all proportion to results gained. The time of the draftsman in computing "fence angles" is usually less expensive than the time consumed by a whole transit party in measuring them. See Art. 216 (*d*), p. 151.

Comments on the Notes. This is a simple illustration of the sketch method of keeping notes. It has decided advantages over the method used in the preceding illustration when so few measurements are taken. Extra care, however, should be taken to show very definitely where each measurement belongs. This is accomplished oftentimes by the use of arrow-heads. Notice that the distance between each transit line and the corresponding fence-line is entered in two places; this shows without question that the two are parallel. Notice also that the distance between each transit line and the corresponding fence-line is greatly exaggerated in the sketch to allow space for recording the offset distances in large clear figures.

The note telling what instruments were used and the degree of accuracy with which measurements were taken is useful as a matter of record, and might have weight in relocating boundaries at some future time, or in settling a boundary dispute should one arise.

ILLUSTRATION II



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Survey of the lot owned by E.F. Blake
Beacon St., Dayville, Mass.

Land within the boundaries is nearly level.

Stations are on curb line.

Survey made with transit and steel tape. Angles to nearest minute. Lengths to nearest tenth.

A.M. Smith. Oct. 2, 1902.

ILLUSTRATION III

(See inset sheet at the end of this chapter.)

Comments on the Survey. The object of this survey was to locate the house, the barn, and the property lines. The first step was to establish transit stations where they would be of the most use. (See Art. 216, p. 149.) Station *A* was chosen in such a position that the four nearest corners of the house (*1*, *2*, *3*, and *4*), two corners of the barn (*5* and *6*), and a corner of the lot (*7*) could all be located from that one station. Stations *B* and *C* were chosen so that the transit line between them could be used for locating the river bank by offsets. Corner *D* was chosen in any convenient position near the southwest corner of the lot (*11*), where both *C* and *A* would be visible from *D*. A corner of the house (*10*) was located from *D* as a check. All points were located by angles and distances except the point *C* on *CD* and a point on *AB*, 54.9 ft. from *A*. These two points, one on a line with the front of the barn, the other on a line with the back of the house, were located by measurement as indicated on the sketch, merely as a check. The location of *C* was unnecessary, as the point *10* served as a check. It will be noticed that the transit-line angles were doubled. The calculated bearing was also kept, and the magnetic bearing observed for each transit line. This too, perhaps, was an unnecessary precaution in view of the fact that the angles were doubled. Notice also that each building was "measured up," and the lengths of three boundary-lines obtained. If for any reason the lengths of the boundary-lines could not have been measured conveniently, they could have been calculated. (See p. 151.)

The interior angles of the transit lines happen to add up to 360°. This does not mean that no error was made, for one angle may be a minute too large, another a minute too small.

The criticism on p. 176 in regard to permanently referencing some one line of the survey applies in this case also.

The transit lines might, perhaps, have been run to better advantage close to and outside of the fence-lines, provided there were no obstacles. In that case some other method might be used for locating the buildings in place of angles and distances.

ILLUSTRATION III (Continued)

Comments on the Notes. The form used corresponds to Form 4, p. 173. All angles with corresponding distances are tabulated on the left-hand page. All dimensions and offsets are given directly on the sketch. *DA* is the backsight for so many angles that, instead of repeating the letters *DA* in the first column, a vertical line is drawn. A vertical line is also drawn in the third column instead of repeating the letter *A*.

This is a good illustration of combining the sketch and the tabulated form to advantage. To put all the angles and distances directly on the sketch itself would have required considerable care to avoid crowding; that it can be done, however, is shown in the next illustration.

The explanatory notes, while not as complete as would be required in many cases, are sufficient for this particular survey. The note giving the accuracy with which measurements were taken is one often omitted, but, nevertheless, it might be of considerable value, should any dispute arise concerning the boundary-lines.

The mistake of entering 162° 12′ for 162° 42′ was probably due to an error (afterwards discovered) in reading the angle. (Omitting the half-degree, see page 77, Art. 100.) The error in recording 40.5 for 60.5 was also probably due to careless field work and not to careless note-keeping. The chainmen evidently read the tape from the wrong end and afterward discovered their mistake.

Notice the method of recording magnetic and calculated bearings; in this case they could just as well have been recorded in separate columns.

In giving offsets it would, perhaps, have been better to use the system of "*plus*" distances from *B* instead of giving distances between offsets. (See Art. 198, p. 139.) There are so few offsets in this case, however, that it makes little difference.

Reference numbers are sometimes enclosed thus (6) to avoid mistaking them for dimensions.

In this survey it makes little difference whether the notes are kept from the bottom of the page up or from the top down. Since, however, in some cases it is advantageous to keep the notes from the bottom up, many surveyors prefer to keep *all* notes in that way for the sake of uniformity.

ILLUSTRATION IV

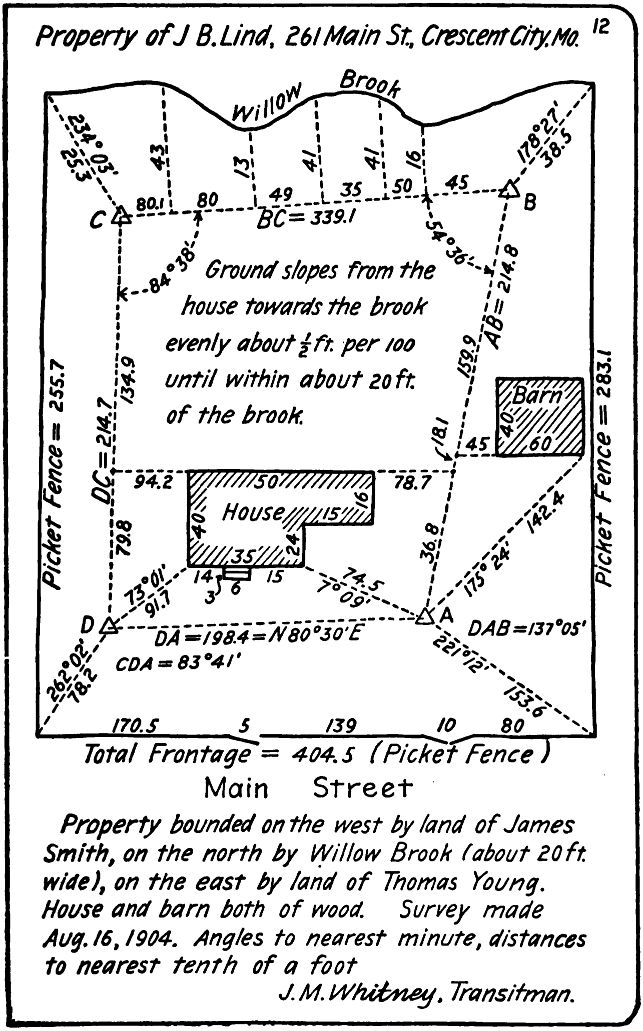
Comments on the Survey. The property surveyed is the same as that in the preceding illustration. The same transit stations were used, but the buildings were located by a slightly different method. The two rear corners of the house were located by finding two points, one on the transit line *AB*, another on *CD*, both in line with the rear of the house. The distances from each of these two points to the nearest corner of the house and to the nearest transit station (viz., 78.7, 36.8 and 94.2, 79.8) together with the dimensions of the house, are all the measurements needed to plot the house. The angles and distances to the two front corners of the house were taken as a check (rather unnecessarily). Notice the different methods of locating the two front corners of the barn. Other ways of using one or more of the methods of Chapter XIII for locating fences and buildings in place of those that were used will suggest themselves to the student. It will prove good practice to study out as many different methods as possible that might have been used in this particular case.

Comments on the Notes. In this case the sketch method is used, the sketch being made free-hand without the use of even a straight-edge. All measurements, including angles, are given on the sketch itself. Notice how the sizes of the buildings and the distances of the transit lines from the fence-lines are exaggerated to make room for recording measurements. In the same way the other distances are exaggerated, no attempt being made to draw to scale, it being far more important to have the notes open and clear. When a sketch is as clear as this illustration the notes are more easily plotted than if kept in tabulated form, but it is rather difficult to keep such a sketch from becoming crowded when there are many dimensions.

The direction, right or left, in which all angles were taken is not noted. It should be, unless previously noted elsewhere once for all.

Notice how the distance 18.1 ft. near the barn is recorded by drawing an arrow and writing the figures to one side. It is frequently necessary to do this (just as in making working drawings) when the space where the number is to go is too small for large clear figures.

ILLUSTRATION IV



Property of J B. Lind, 261 Main St., Crescent City, Mo. 12

Ground slopes from the house towards the brook evenly about ½ ft. per 100 until within about 20 ft. of the brook.

Total Frontage = 404.5 (Picket Fence)

Main Street

Property bounded on the west by land of James Smith, on the north by Willow Brook (about 20 ft. wide), on the east by land of Thomas Young. House and barn both of wood. Survey made Aug. 16, 1904. Angles to nearest minute, distances to nearest tenth of a foot

J. M. Whitney, Transitman.

ILLUSTRATION V

(See inset sheet at the end of this chapter.)

Comments on the Survey. This is a portion of the preliminary survey for an electric road. Boundaries of the highway through which the road is to run, houses, division-fences or property lines, bridges, streams, and other topographic features for a considerable distance on each side of the highway must be located. The sketch, though not drawn to scale, represents approximately a portion of the map as it will appear when plotted. Great accuracy is not required. Houses, for example, may be located to the nearest foot. Property lines should be located with more care.

In this illustration a number of different methods were employed for locating objects,—a greater variety, perhaps, than would ordinarily be used. For this reason the illustration should be studied with more than ordinary care, since it shows the practical applications of so many of the methods of Chapter XIII. The points to be noticed are:

1. Stations are numbered as in railway work. Art. 209, p. 147.
2. All angles taken are *deflection* angles. Art. 179, p. 120.
3. The boundaries of the highway were located by offsets perpendicular to the transit lines taken at irregular intervals.
4. The fence at *A* is located by finding where if produced it would strike the transit line, and by locating a point 1 (on a line with the front of the house) by angle (50° 58′) and distance (122.6). (Method (*a*), p. 134.)
5. The Bond house is located by finding where one side produced strikes the transit line (at *a*) and then measuring the tie-line (127.8). (Method (*a*), p. 139.) This was probably more convenient than to set up the transit at *c* and measure the angle. (Method (*b*), p. 135.) Point 1, already located on a line with the front of the house, serves as a check.
6. The fence at *C* was located by finding where, if produced, it would strike the transit line and, by producing the front line of the house already located, measuring the distance (95.1).
7. The Sykes house was located from Station 23 + 89 by angles and distances to two front corners (2) and (3) (p. 134).
8. The fence at *B* was located by two offsets (48.5 and 46.5) *measured* on lines produced from the front and rear sides of *the Sykes house.*

9. The fence at D was located by the tie-line method. (Method (*c*), p. 140.)

10. The four corners of the bridge (14, 15, 16, and 17) were located from Station 27+9 by angles and distances.

11. Each of the points 4 to 13 inclusive were located by two angles taken at stations 23+89 and 27+9 respectively, with the transit line between these two stations as a base-line. (Method on p. 137.)

12. The fence at E was located by the tie-line method, the point b, however, being taken on a line with the front of the Morris house.

13. The fence at F was located by producing to a point on the transit line, and locating another point 19 from Station 30+32.5 by angle and distance. Afterwards the distance to c, a point on line with the house, was measured (69.1).

14. The Morris house was located from b and c (already established on line with its front) by the distance from b, 84.4. The angle (but not distance) to one of its corners (18) was taken at Station 30+32.5, merely as a check. A better check would have been to measure from 18 to b.

Comments on the Notes. The form used corresponds to Form 5, p. 173. The notes read from the bottom up. Station 30+32.5 at the top of the left-hand page would be repeated at the bottom of the next left-hand page to make the tabulated form continuous; just as Station 20+50 was repeated from the preceding left-hand page. In thus repeating a station, only the deflection angle of the transit line is given,—no other notes are repeated.

Only angles and corresponding distances are given in the tabulated form. All other linear measurements are entered directly on the sketch. Bearings are taken for transit lines only, and each is entered between the two stations where it belongs. Thus, for example, S. 81° 15′ E. is entered between 23+89 and 27+9, and is the magnetic bearing of the line between those two stations. All bearings are understood to be *forward bearings*.

Notice that each piece of property is marked with the owner's name. In front of the Morris house a broken line indicates that there is no fence.

In this illustration the sketch covers too much ground; it would have been better to put the sketch as far as the bridge on one page and the remainder on another. Little is gained by sketching the highway as it really is instead of making two straight lines represent the boundaries. The next illustration is an improvement in both these respects.

In both this and the next illustration notice that the distances to points on the transit line at which offsets were taken are entered as "*plus*" stations instead of measuring the actual distances between successive points. This saves time in the field and in plotting since several distances can be measured or plotted at one stretching of the tape or at one setting of the scale as the case may be. When using this method the distance should be measured from the last offset point preceding each transit station to the station itself, as this is a check on the total distance previously measured between the two stations. (See note p. 139.)

Notice that at the top of the page there is a running title, as, for example, "Notes from 20+50 to 27+9." This is invaluable as an aid in finding notes quickly, especially when they are indexed properly.

ILLUSTRATION VI

In Illustration VI a portion of the notes of Illustration V have been reproduced according to the sketch method. All notes pertaining to the transit line are entered in the middle space by themselves according to Form 3, p. 172. This middle space therefore represents the transit line and may be considered *as having no width*, i.e., points on the edges opposite to each other actually coincide on the ground. Thus, for example, offsets marked 23.5 and 47.3 were really measured from the same point on the transit line. Notice how the sides of the highway, although in reality curved, are represented by straight lines. These lines were ruled, but all other lines were drawn free-hand. Compare this page of notes with those of Illustration V on the inset sheet at the end of this chapter. While it is true that one set of notes seems as clear as the other, it should be kept in mind that much more of the survey is included in Illustration V than in Illustration VI. The notes of Illustration VI, on the other hand, are probably easier to plot than those of Illustration V. The two illustrations afford an excellent opportunity of comparing the sketch method with the combination method of keeping notes.

ILLUSTRATION VI

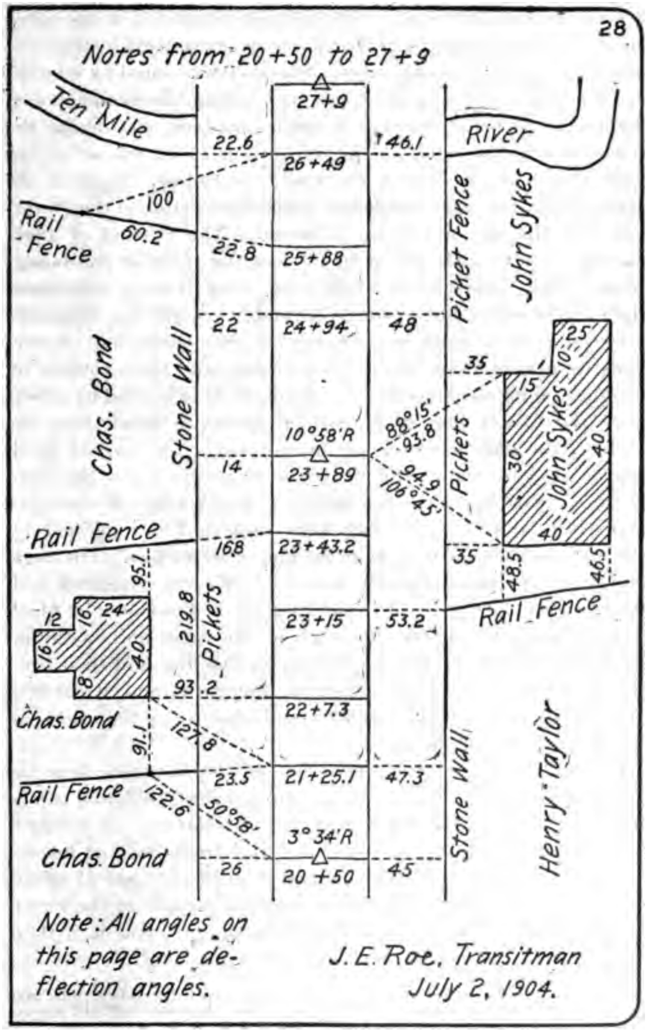


ILLUSTRATION VII

Comments on the Survey. The method illustrated in the notes on the opposite page is, perhaps, the most general method for making a survey of a city street. Station 0 was found by stretching the tape between merestones and taking the middle point. Station 3+33.4 was found in a similar manner, thus fixing the center line of the street. The substation 1+98 was set at the intersection of this center-line with a fence-line. Each of the other substations was established at the intersection of the center-line with the side of a house produced. The method of angle and distance (p. 134) was used for locating most of the house-corners, the backsight in every case being from a permanent sight on the center-line *produced* beyond Sta. 3+33.4. Distances between houses, front and rear, were also measured. Fence-lines were located by offsets from houses, and fence-corners by distances along the fence-lines. Manholes were located by offsets from the transit line, and street-car tracks by offsets from the curb to the gage-line* of the nearest rail. The curved track was located by angles and distances to points about the same distance apart on the nearer rail (p. 136). Widths of sidewalks and widths from curb to curb were measured and checked by the total measurement from street-line to street-line. The angle of the street-line on Fourth Ave. (81° 28′) was measured with the transit at Sta. 3+33.4, sighting on a monument a block away, but on Third Ave. the angle of the street-line was found by laying off 90°, measuring 100 ft. on this line, and then from this point measuring the offset to the curb- and street-line. Sometimes one of these methods and sometimes the other can be used to advantage.

Other Methods. In the above method the distances from the street-line to the fronts of houses are not measured, and can be found only by scaling the map or by calculation. A survey is often made to locate veranda-lines and front lines of houses, and in such a case the best method is probably that of offsets measured from a line run on the sidewalk *parallel* to the street-line. Sometimes offsets are taken from the curb-line or, if there are no fences in the way, from the street-line itself.

Suggestion. Where buildings are near the street-line offset-rods may be used to advantage in place of the steel tape. A stadia-rod will answer

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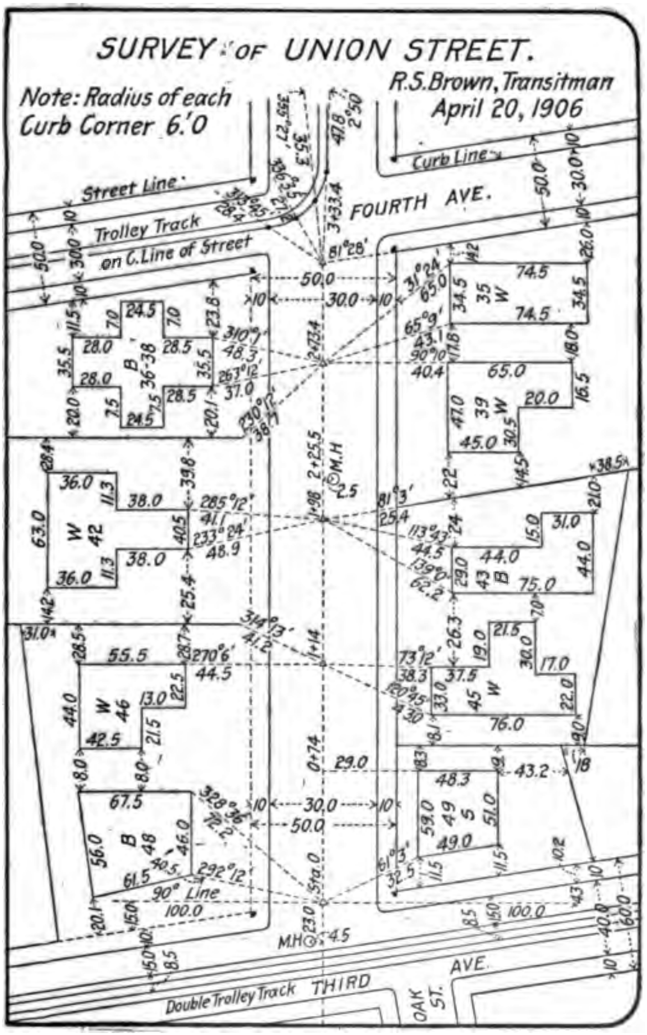
* *Gage-line is the line* on the inside of the head of the rail $\frac{1}{4}''$ below the *top.*

ILLUSTRATION VII



the purpose. One end of the rod is held against the building, measurement is read at the point where the line of sight strikes tl

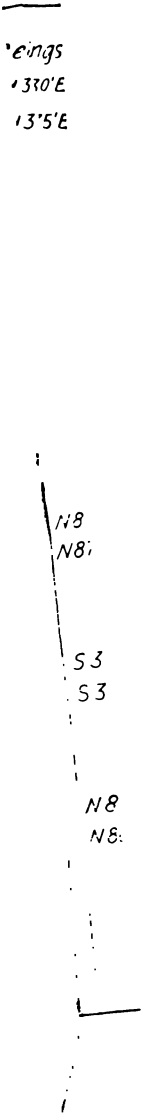
In a survey of an entire city block or of several adjacen it is often necessary to run transit lines on the sidewall the street or wherever they can be established to the best tage. Thus, for example, if a transit line were to b Oak St., it might be advantageous to have the first sta Union St. on the left-hand sidewalk, and the last sta Oak St. on the right-hand sidewalk, thus making the line between the two as long as possible.

Remark. In a series of letters to the Engineering News, Vol. the following methods for locating houses were advocated by surveyors: (1) Plane table; (2) Angle and distance (p. 134); (method (p. 138); (4) Tie-line method (p. 137); (5) Each front corn intersecting distances (p. 139); (6) By intersecting angles (p. 13 shows the diversity of opinion regarding methods of location, cates that each method has its advantages.

Comments on the Notes. This is an example of the method. Each angle is entered above a line drawn f station to the point sighted at, and the corresponding is entered below the same line. Portions of the ho exaggerated to accommodate dimensions. Each house is with its street number, and with *W*, *S*, or *B* to denote it is wood, stone, or brick.

When it is desired to distinguish between a fence along and the street-line itself the former may be indicated b line and the latter by a broken line. Thus in the sketch a fence along the right-hand side of the street, but only line along the left-hand side.

The notes present a somewhat crowded appearance this respect they are typical of city notes kept by the method. By using the combination method all angles responding distances would be transferred to the l page, thus relieving the crowded appearance, but mak little more difficult to plot the notes.



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CHAPTER XVI

TRIANGULATION

THE aim in this chapter is to give methods for simple triangulation, such, for example, as that which forms the basis of an ordinary topographic survey. Many of the suggestions and directions apply equally well to the special problems in triangulation given in Chapter XVIII. No attempt has been made to treat the subject as comprehensively as would be required in a work on geodetic surveying.

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243. Triangulation. If, in any triangle, one side and the two adjacent angles are measured, the lengths of the other two sides may be computed by the methods of trigonometry. This is the basis of all triangulation.

Illustration. Problems in finding distances to inaccessible points, distances between inaccessible points, and similar problems (see Chapter XVIII) are examples of simple triangulation.

244. In a Network or System of Triangles the side of some one triangle is measured first, and from this *base-line* and the two adjacent angles the remaining two sides of the triangle are *computed.* The *calculated* lengths of these two sides may be used as new base-lines for two other triangles, and the *calculated* lengths of the sides of these new triangles used as base-lines for still other triangles. Thus the system may be extended indefinitely. It is customary to measure all three angles of each triangle, but only one linear measurement is made—that of the original base-line in the first triangle. One side of the last triangle is usually so located that it can be measured; the agreement of the measured and the calculated length of this side is a check on the accuracy of the whole work.

(*a*) *Illustration.* A network of triangles forms the basis of nearly all surveys covering large areas, such, for example, as the coast and geodetic surveys and other extended topographic surveys.

(*b*) *Remark.* When the sides of the triangles are so long and the precision required is so great that the curvature of the earth must be taken into account, the work is no longer plane surveying,—it is geodetic surveying, p. 2.

(*c*) *How large areas may be covered by triangulation in plane surveying?* This depends, of course, upon the required limits of error. The error which results from neglecting curvature is about one foot in 34½ miles. (See Art. 5, p. 2.)

245. Sources of Error in Triangulation. (1) *Poorly proportioned triangles.* Angles less than 30° or more than 120°

should be avoided, not because it makes any difference in the field work,—one angle is as easily measured as another,—but because a given error in a small angle affects the computed lengths in a triangle more than does the same error in an angle which approaches 90°. This is because the tabular differences in a table of sines approach zero as the angle approaches 90°, while they are very large for angles near 0° or 180°. The ideal triangle is of course an equilateral triangle. It is difficult to select stations so that the triangles will be properly proportioned, yet this is a very important source of error and one too often overlooked.

(2) *Error in the measurement of the base-line.* Since any error in the length of the base-line affects every side of every triangle in the whole system, extreme precautions are usually taken to measure the base-line with great precision. (See p. 57.)

(3) *Errors in the measurement of angles.* These are reduced to a minimum by using the method of *repetition* (p. 99) or *series* (p. 100), and by taking other precautions (Art. 140, p. 101) not taken in ordinary work.

246. Stations and Signals. *Signal* is a term used in triangulation to denote any structure or object which marks the position of a station and which may be sighted upon from other stations. It includes scaffolds used to elevate either the object sighted at or the instrument used in sighting. All sorts of prominent objects are used as signals, natural and artificial. (See p. 193.) The station itself is usually a hub in the ground, an iron bolt in a ledge of rock, or some other definite mark. Permanent stations should be carefully referenced.

247. Choice of Stations. The first considerations are (1) that the triangles shall be properly proportioned, and (2) that the stations in any one triangle shall be intervisible. (3) Locate them, as far as possible, where they will be most useful either in the location of prominent features, or as starting and closing points for traversing, or for establishing secondary systems of triangles. Among other requirements are: (4) Firm ground or stable support for the instrument, free from vibration, sheltered from wind; (5) Lengths of sight suited to the telescope used; (6) The greatest possible area in view in order that a large number of substations may be controlled; (7) Freedom from *atmospheric* disturbances,—for example, if a line of sight passes *over a hill-top* close to the ground it will be affected by the "boiling" of the air, or if it passes near a chimney continually emitting smoke it may be almost useless; (8) Aim at the simplest, best-balanced system of triangles practicable.

It is very seldom that all the above requirements can be realized. Usually the surveyor has to take stations where he can get them. The most favorable conditions for triangulation are generally found in hilly country, with summits, comparatively clear and unobstructed, upon which stations may be established. The least favorable conditions are in flat wooded country, where it is either necessary to do a great deal of cutting or else to build high scaffolds upon which to set up the instrument. In cities stations may be established on the tops of buildings, and church spires or other prominent objects may be taken for substations.

A thorough reconnaissance of the ground to be covered is essential in planning any system of triangulation, and it pays to give careful consideration to all feasible schemes.

248. The Base-line. (*a*) *Location.* Try to select a place for the base-line: (1) Where the ground is level and well suited for accurate measurement; (2) Where the country is open, so that sights may be taken in any desired direction from each end of the line; (3) Where a system of well-proportioned triangles can be built up, with stations which meet the requirements of the preceding article.

Remark. Too often the surveyor will be unable to find a suitable place for a base-line, and must overcome natural obstacles as best he can. Sometimes it is necessary to lay out a broken base-line and find the direct distance between the two ends by indirect measurements. The required distance corresponds to an omitted measurement, and is found by computation. (See Part II.)

Occasionally it may be necessary to locate the base-line in a place where it is impossible to establish signal-stations at the ends, as, for example, on a railroad-track. The signals may be established elsewhere, and the ends of the base transferred by some accurate system of check measurements.* In geodetic work a comparatively short base-line is expanded to a comparatively long side of the first triangle in a similar manner by a carefully planned system of smaller triangles.† Since base-lines can be measured with accuracy greater than can be sustained throughout the triangulation, it is customary to introduce new base-lines whenever the accumulated errors are likely to exceed the limits of error.

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* Wilson's Topographic Surveying, p. 524.
† Gillespie's Surveying, Vol. II, p. 15.

(*b*) *The length of the base-line* will depend upon how large triangles are to be used. This, in turn, depends upon the ground to be covered and the points where stations are desired. The base-line must be of such a length that properly proportioned triangles can be built up. To adopt a base-line several hundred feet long and make each of the two other sides of the first triangle several miles long is an extreme illustration of what should *not* be done.

Remark. The above holds true for ordinary simple triangulation. For extended systems short base-lines may be chosen and expanded as suggested in the preceding article.

(*c*) *To measure the base-line.* The steel tape is best adapted to the measurement of base-lines, and the method of procedure is given on page 57. For extensive surveys elaborate apparatus for base-line measurement is used. Among the various forms are: (1) *Contact apparatus,* or bars fitted with a device for bringing the ends in successive contacts; (2) *Optical apparatus,* with fine lines near the ends of each bar, and microscopes for observing and setting the marks; (3) *Compensating apparatus,* intended to be so constructed that the two ends of each bar remain a constant distance apart. The steel tape, however, is fast superseding all other forms, and practically nothing else is used in plane surveying.*

249. What Angles are Measured in Triangulation? When a station is the vertex of two or more triangles several combinations of angles are possible. The angle of each triangle may be measured, then the sum-angle of any two angles or any three angles, and so on. While this multiplication of measurements furnishes more conditions for the final adjustment of all the angles at the station, it is not advisable because the extra time required is seldom warranted by the increase in the accuracy of the results. It is better to use the time available in the careful measurement of those angles only which will be used in the computations.

250. Measurement of Angles. Two general methods are in use for measuring angles with great precision: (1) the method of *series* and (2) the method of *repetition* (pp. 99-100). The former method is used in geodetic surveying where instruments of great precision are available. The method of repetition, however, is most used in ordinary work, where transits are seldom graduated to read closer than 20″. Any method of procedure should aim to eliminate instrumental errors (p. 104), and to reduce to a minimum all errors of observation. To eliminate instrumental errors the precautions of Art. 140, p. 101, should be taken. A number of methods of procedure, each of which embodies these precautions, are in common use, but the following is recommended:

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* The use of elaborate apparatus comes under the head of higher surveying. Consult Wilson, p. 495, Gillespie, Vol. II, p. 59, or Johnson's *Surveying.*

(*a*) METHOD OF REPETITION. The method, in brief, is to repeat the angle six times to the right, inverting the telescope at the end of the third repetition. Then, *without resetting the vernier*, measure the explement (360° − angle) also to the right, with the telescope still inverted for the first three repetitions, but normal for the last three. Theoretically the vernier reading should come back to the original setting, and the sum of the two angles (angle + explement) should equal 360°. The discrepancy, equal to the difference between the final reading and the original setting, represents the total cumulative error due to both accidental and constant errors, and this error has been found constant for a given instrument and given conditions within the probable error of pointing. It seems probable, therefore, that when the angle and its explement are each corrected by half the amount of the cumulative error, the errors from all sources have been as far as possible eliminated. This is confirmed by the fact that when angles measured by this method are so corrected they are in close accord, even though they are taken under conditions which give closing errors of wide range.

It is to be noted that the plates are always turned in the same direction, clockwise, and that there are in all twelve repetitions, six for each angle, but that the verniers are set only once. Both verniers should be read to eliminate errors of eccentricity.

(*b*) *Method of procedure.* (Illustrated by notes on p. 197.) *First step.* (1) Set the A vernier at 0°, or at random, and record the reading of both the A and the B vernier. (2) Take the angle to the right, record the single reading as an approximate check, and repeat two more times, three repetitions in all. (3) *Invert* the telescope and continue the repetitions for three times, or until six repetitions in all have been made. (4) Record both vernier readings.

Second step. **Without resetting the vernier, measure the explement of the angle clockwise in exactly the same way, with the telescope inverted for the first three repetitions and normal for the last three.**

These two steps constitute a "set," and from two to four sets for each angle are usually sufficient in ordinary work.

(*c*) *Practical suggestions.* (1) Great delicacy of manipulation is required, as a slightly awkward touch will often spoil a set. Keep the hands off the tripod, turn the plates gently, taking hold of the plate, *not* the telescope, and do not force the clamps. (2) Notice that in the first step the backsight is always on the left-hand station, while in the second step it is always on the right-hand station. Remember that the lower clamp and tangent-screw are used for backsighting, p. 91, and that turning the wrong tangent-screw will spoil a set. (3) Half the observations are taken with inverted telescope to eliminate, as far as possible, errors in the horizontality of the supporting axis of the telescope as well as errors of collimation. Hence do not touch the leveling-screws during the progress of a set. (4) Be sure in making all pointings to turn the tangent-screws *against* the opposing springs. (5) With the exception of the single check reading no readings are taken until the six repetitions have been completed. Read one row of graduations exclusively, p. 71. (6) This method does not involve the precaution of measuring an angle first to the right and then to the left, but the error is allowed to accumulate and then is corrected. There are several other methods of procedure in use which do involve this precaution, but the method given here is used in the U. S Coast and Geodetic Survey, and is considered one of the best of the methods of repetition.

Illustration. The following is an outline of the work that was done when the notes on the opposite page were taken: (1) Set the A vernier at 0° 0′ 0″; found the reading of the B vernier to be 180° 0′ 0″; entered these two readings on the fourth line from the bottom of the page, opposite *A*. (2) Measured the angle *ACB* once and recorded the single reading on the second line from the bottom, opposite *B*1; repeated the angle five times in addition, the last three with telescope inverted; recorded both vernier readings (252° 31′ 0″ and 72° 31′ 0″) opposite *B*6. These two readings happen to agree in minutes and seconds. (3) Allowed the verniers to remain at these two readings (which are again recorded opposite *B*, three lines up the page), backsighted on *B*, and made six repetitions of the explementary angle *BCA*; recorded the two vernier readings (0° 1′ 0″ and 180° 1′ 0″). Evidently the total error which has accumulated from all sources is 0° 1′ 0″. This completes the field work of the first set. (4) Worked out the values in the fourth column as indicated. Since the readings from both verniers agree exactly throughout the work in this first set, the values in the fourth column are exactly the same as the corresponding values in the second column except 2160° 1′ 0″, which was obtained by adding 6×360° to 0° 1′ 0″, since the indicator has moved six times around the graduated circle. [See p. 100, Art. 138(*d*).] (5) Worked out the values in the fifth column. Since the sum of 42° 5′ 10″ and 317° 55′ exceeds 360° by 10″, and since, presumably, the angle and its explement were measured with equal precision, each value was reduced 5″, the corrected values 42° 5′ 5″ and 317° 54′ 55′ were recorded in the fifth column, and the two values were finally added to make sure that their sum is 360°.

The second set was carried out in exactly the same way except that in the beginning the verniers were set at random (25° 20′ 30″ and 205° 20′ 0″) in order to avoid bias from the readings in the first set, and also to use a different portion of the limb. In this set the readings from the two verniers disagree in two instances and hence the corresponding values in the fourth column are *mean* values. The value 2185° 20′ 45″ was obtained by adding 6×360° to 25° 20′ 45″.

The values of the required angle as found in the first and second set were 42° 5′ 5″ and 42° 5′ 10″, and since no other sets were taken the final *value is the mean* of these two or 42° 5′ 7.5″, as indicated in the last *column.*

Point Sighted	Readings Vernier A	Readings Vernier B	Mean Reading	Corrected Value	Mean of Two Sets
Transit on Station C. Measured Angles ACB and its Explement,– Both Clockwise					
A 6	25°21'0"	205°20'30"	2185°20'45"		
B	277°51'30"	97°51'30"	277°51'30"		
6xBCA			6)1907°29'15"		
A 1			317°54'52.5"	317°54'50"	
B 6	277°51'30"	97°51'30"	277°51'30"		
A	25°20'30"	205°20'0"	25°20'15"		
6xACB			6)252°31'15"		
B 1	67°26' 25°20'30"		42°5'12.5"	42°5'10"	10
	42°5'30"			360°0'0"	5 15
A 6	0°1'0"	180°1'0"	2160°1'0"		42°5'7.5"
B	252°31'0"	72°31'0"	252°31'0"		
6xBCA			6)1907°30'0"		
A 1			317°55'0"	317°54'55"	
B 6	252°31'0"	72°31'0"	252°31'0"		
A	0°0'0"	180°0'0"	0°0'0"		
6xACB			6)252°31'0"		
B 1	42°5'		42°5'10"	42°5'5"	
				360°0'0"	

251. Field Notes for Triangulation. Many forms are in common use. The advantages of the form on the preceding page are: Only six columns are required, all arithmetical work is shown, and the notes are practically self-explaining. Notice the following points: (1) The first three columns are completed in the field,—the last three may be worked up in the office. (2) The vernier readings occupy only two lines for any one angle, but leave three lines at the bottom of the page and three between angles for the arithmetical work. The notes are entered from the bottom of the page up as a matter of convenience in subtracting vernier readings. (3) The fourth column is the mean of *minutes* and *seconds* read from verniers A and B. The degrees are taken from A. Ignore the degrees in the B vernier reading: they are usually not recorded at all. When they are recorded, however, they serve as a check on blunders in reading vernier A. The two readings would be 180° apart, provided there were no eccentricity. (4) The single reading in the second column is entered simply as an approximate check.

Suggestions. (1) Always enter at the head of the notes the station occupied and the angles measured. (2) Sketches of the triangulation, explanatory notes, and other data are put on the opposite page. Data pertaining to the weather, to atmospheric conditions, to the kind of instrument used, etc., are more important in triangulation than in other kinds of work. (3) When a set of notes is manifestly incorrect cross them out neatly and write across them "ABANDONED" or "INCORRECT."

252. Signals for Stations.[*] In Fig. 252 is shown a form of signal which may be used in ordinary work. The requirements are: (1) The mast should be plumb and over the station; (2) The four legs should be set firmly in the ground and be cross-braced to make them rigid. (3) The cross-bracing and the bottom of the mast should be high enough from the ground to allow the instrument to be placed underneath and the work to be carried on without disturbing the signal.

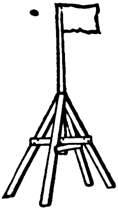

FIG. 252.

(*a*) *Practical suggestions.* (1) To anchor the legs dig holes two feet or so into the ground, and when the legs are planted drive a stake into the ground beside each leg and nail the leg to the stake. (2) Plumb the mast by observing it from four different positions, holding a plumb-line before the eye. (3) Signals may often be constructed from poles, 4″ or 5″ in diameter, cut in the woods. Old fence-rails make good braces. (4) It is sometimes difficult to set a *quadripod* signal so that its legs will not interfere with lines of sight to other stations; for this reason a *tripod* signal is often preferred. (5) Put a small hook on the bottom of the mast from which to suspend the plumb-bob.

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[*] *See Gillespie,* Vol. *II,* Johnson, or Wilson for the construction of more *elaborate signals.*

(*b*) *Practical questions.* (1) *What should be the diameter of the mast?* Make it as small as possible and yet have it distinctly visible, so that it may be accurately bisected by the cross-hair. The latter will ordinarily cover a strip 0.3 of an inch wide at a mile, or about 3 inches for a 10-mile sight. Use a pole 1″ in diameter for sights less than a mile, and poles up to 3″ in diameter for five-mile sights. A pole 4″ in diameter can be seen twenty miles with a good telescope and clear atmosphere.

(2) *What error is allowable in centering a mast or an instrument over a station?* From the table on p. 108 it is seen that in order to make an error of one second in an angle the instrument or masts must be out of position the following amounts:

$\frac{1}{32}$ of an inch for sights 500 ft. long.
$\frac{1}{16}$ " " " " " 1000 " "
$\frac{5}{16}$ " " " " " a mile long.
3 inches " " 10 miles long.

If the lengths of sights are known approximately, and limits of error for angular measurement have been adopted, the above table will serve as a guide in centering the mast, setting up the instrument, and in pointing.

(*c*) *Eccentricity of stations.*[*] It is sometimes necessary to use for substations objects which are inaccessible or over which an instrument cannot be set up. In this case one of the methods for inaccessible points (Chapter XVIII) may be used.

Illustration. A church-steeple is located from two triangulation stations. A traverse is started near the church. The spire and two traverse stations form a triangle. The length of the side between the traverse station having been measured, the other two sides may be found, p. 222. It will be necessary in addition to observe the angle between lines from one traverse station to a triangulation station and to the spire.

253. Limits of Error. In a test of a 6-in. transit reading only to 30″, sights were taken from the same station to five distant points. Angles were measured in succession, each angle being given twelve repetitions, six times with the telescope normal and six with the telescope inverted. Upon completing the circle the sum of the angles exceeded 360° by 5″, or an average error of 1″ per angle.[†]

In an example taken from actual practice eight angles of a triangulation system were measured with a 6-in. transit reading to 20″. Twelve repetitions were given each angle, following the method of procedure outlined on p. 195. Angles were adjusted by the method of logarithmic residuals, and the maximum difference between the measured value of an angle and its adjusted value was 3.98″, the minimum difference was 0.57″, and the mean difference for the eight angles was 1.86″. In a second system of eight angles the differences were: Maximum 0.83″, minimum .07″, mean 0.44″. In the survey from which these results are taken work was continued throughout the day, regardless of atmospheric "boiling" or of wind save in extreme cases, and the number of sets measured per day varied from 10 to 25, the time occupied in going from one station to another being an important factor.*

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[*] See Gillespie, Vol. II, p. 89.
[†] *Eng. News*, Feb. 14, 1901, p. 122.

The above results show what can be done with an ordinary transit. They represent a higher degree of precision than is ordinarily required in simple triangulation covering small areas. In any given case the allowable angular error is determined by the allowable error in the position of a triangulation-point, and this depends not only upon the angular error, but upon the size and proportions of the triangle and upon the precision of the base-line measurement.

254. Triangulation and Trigonometric Leveling. Let A and B be the ends of a base-line and C the third station of the triangle. If the distance AC has been found by triangulation, and the vertical angle was taken to C when the instrument was at A, the difference in elevation between A and C may be computed (p. 244). In like manner the difference in elevation between B and C may be found, and since the base-line is either level or the difference in elevation of its two ends is known this affords a check on the elevation of C. When the instrument is moved to C the vertical angles and triangulation distances to two other stations will give the elevations of these two stations, and so on indefinitely. As the horizontal angles must be measured anyhow, the only additional field work consists in measuring vertical angles; hence it is an economical method. As the elevation of each station may be determined by independent observations from at least two stations, there is a good check on the work. The results in many cases compare favorably with results obtained by spirit-leveling.

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* Eng. News, May 30, 1901, p. 402.

CHAPTER XVII

TRANSIT SURVEYING—SPECIAL WORK

THE first part of this chapter deals with certain problems which occasionally arise in transit surveying. The second part outlines methods of staking out work for purposes of construction. There is no end to the variety encountered in this special work, but typical cases, such as a young engineer is likely to meet early in his experience, have been selected. Those cases in which leveling and transit work are combined are treated in Chapter XXIII. No attempt is made to take up railway work, as that s a subject in itself, fully treated in books on railway surveying.

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255. Balancing-in the Transit, i.e., to set up a transit on line between two given points. It is assumed that neither point can be seen from the other, as this is the most difficult case, but it is also assumed that both of the given points can be seen from the required point at which the transit is to stand. Set two poles on line from 30 to 100 ft. apart, one on each side of the required transit point. Line the poles in by the method on page 55. The transit can now be set approximately on line by means of the two poles, and roughly leveled up. Sight on one of the given points, plunge the telescope, note how far to one side of the other point the line of sight strikes and move the transit accordingly. Repeat until the transit is on line.

Practical Hints. The preliminary use of the two poles (or short stakes if poles are not at hand) saves time. The nearer on line the transit is the more carefully it should be leveled up, especially the level-bulb at right angles to the line of sight. Usually after one or two preliminary set-ups the transit can be brought into line by means of the shifting head. Do not split hairs in the preliminary set-ups. Do not backsight indiscriminately first on one of the given points and then on the other. The amount to move the transit can be estimated more easily if one of the given points is always used for a backsight, the foresight striking near the other. When the transit is apparently on line it may be well to test by the method of double reverse (p. 92).

256. To Sight Parallel to a Given Line. A transit stands, for example, a few feet from the side of a building and it is desired to sight parallel to that side. By *swinging* the tape, find the *shortest* distance from the transit to the side of the building.

Suppose this distance is 3.8 ft. At any convenient distance from the transit, the farther the better, hold one end of the tape against the building, and swing the tape with the thumb or pencil held 3.8 ft. from the end. As the thumb or pencil describes an arc of 3.8 ft. radius, it will seem to the transitman looking through the telescope to move *away* from the building and then *towards* it. Keeping the cross-hairs on it by moving the telescope, the transitman should clamp just at the turn, i.e., when the thumb or pencil is farthest from the side of the building.

The same method may be used for sighting parallel to a fence or any other similar object. (For a method when the parallel lines are some distance apart, see Art. 278 (*b*), p. 224.)

257. To Find the Intersection of Two Straight Lines. Suppose that AB and CD are the two intersecting lines. Set up over A or B and set two stakes on AB from 6 ft. to 10 ft. apart. One of these stakes should be on one side, the other on the other side of the required point of intersection as judged by sighting with the eye along CD. Mark the exact points on the stakes with nails driven *part way* in. In a similar manner, setting up over C or D, set two stakes in the line CD, one on each side of the point of intersection. Find the required point by stretching two intersecting strings between the nails in the stakes.

Remark. It is evident that it is unnecessary to drive stakes on the line CD; it is often better simply to establish a single point,—i.e., where the line of sight crosses the string (or the edge of a tape) stretched between the nails on AB.

258. To Measure the Angle at a Fence-corner. It is assumed that the transit cannot be set up over either fence-line. Establish a line parallel to each fence-line as close to it as convenient (Art. 256), find the intersection of these two lines as explained in the preceding article, and setting up over this intersection, measure the angle. The angle can often be found indirectly by computation (Art. 216 (*d*), p. 151).

Remark. The above method is general; a special method requiring but one set-up and applicable in many cases is as follows: Set up over *any* point inside the fences and as close as practicable to the corner. Establish a line parallel to each fence-line (Art. 256) and measure the angle.

Where to set fences. Division fences are usually set so that the posts and rails are on one side of the property line and the pickets or boards are on the other side. Stone walls are set with centers on division-lines. *Fences* along streets should be set so that no post nor any part of the fence *will project outside of* the street-line. *Street-lines* should be marked by *monuments* or *merestones* set at the street-corners.

259. Referencing a Point. The process is the reverse of that explained in Art. 257, and consists in setting four stakes so that two lines between them will intersect at the point to be referenced. (See p. 55 for this and two other methods.)

Any point which is likely to be disturbed during construction should be referenced. In some cases the reference-stakes must be placed some distance away, in other cases close to the given point.

Illustration. A transit station may be referenced by plunging the telescope and setting stakes perhaps 100 to 200 ft. away. On the other hand, reference stakes for a merestone may be set a few feet each side of the spot where the hole is to be dug, and the exact point on each stake determined by stretching strings over the point previously located.

260. Referencing a Line. (1) Reference any two points in the line which are visible from each other and far enough apart to determine the line accurately. This may be done in two ways: (1) By offsets from buildings and other permanent structures, or by any of the other methods of referencing points. (2) By referencing one point in the line and measuring the angle which the line makes with some other line. This includes referencing a line by its azimuth or bearing.

Remark. The importance of carefully referencing at least one line of a survey is often overlooked. A map may be very accurate and complete and it may show one *point* carefully referenced, but unless some *line* is referenced also, the entire survey may be swung on the one known point as a pivot. This is the source of much of the trouble in rerunning old surveys, and gives rise to many boundary disputes. Objects used for reference should be as *definite* and as *permanent* as possible.

261. To Locate Points on the Water with Two Transits. The method used is that of intersecting angles outlined on page 137, Art. 196.

Practical hints. The base-line between the two transits should be so chosen that the angles of intersection are not too oblique or too acute. In most cases, as in taking soundings from a boat, the work must be done rapidly. Each transitman follows the boat with his telescope, keeping the line of sight on the sounding line or pole, and at a given signal from the man in the boat reads and records the angle. *It will save time* NOT *to clamp the upper plate at all.* In recording angles some system must be used so that, when the notes are plotted, the angle to a given point recorded in one transitman's book can be used with the angle to the same point recorded in the other transitman's book. It is just here that mistakes are likely to occur. One transitman, for example, misses a signal and fails to take the angle. This puts him one angle behind the other transitman, and this causes trouble in plotting. One method of avoiding this is for each transitman to record the exact time at which each angle is taken in a column reserved for that purpose. If angles are taken very rapidly, record the time, say, of every fifth angle. Of course the watches of the two transitmen should be set to keep the same time. As an additional precaution, flags of different colors may be used by the boatman, as, for example, a white flag each time a new set of 10 soundings is begun, a red flag each time a new set of 100, and a green flag for all other soundings.

262. To Establish a Base-line by Means of a Supplementary Broken Base-line. When local conditions make it impossible to establish a base-line of the required length by the usual methods, two points may be chosen to serve as ends of a base-line, a series of transit lines may be run connecting these two points, and the length of the required base-line may be computed by the methods explained in Part II for calculating omitted measurements. The calculated length may be checked by repeating the work, using a different or supplementary broken base-line. This problem is practically the same as one which often arises in mine surveying and tunnel work.

263. To Run in an Arc of a Circle with the Transit and Tape. It is assumed that the radius is so large that points on the arc cannot be found by swinging the tape. The method here given is the one used in railway surveying.

(*a*) *Geometrical and trigonometrical principles.* In Fig. 263 AB and EF are two lines connected by the arc of a circle beginning at B and ending at E. Each of the lines AB and EF is *tangent* to the arc, and the two lines are produced until they intersect at G.

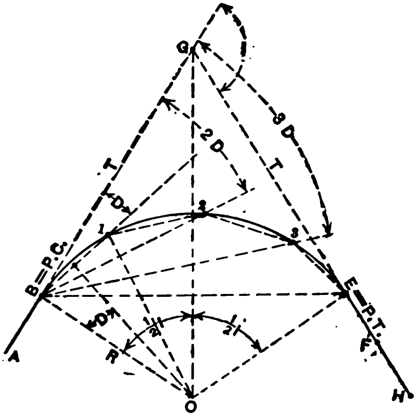

FIG. 263.

DEFINITIONS AND NOTATION

$BG = GE$ are called *tangents* (denoted by T).

Angle at G is the *angle of intersection* (denoted by I).

$BO = OE$ is the *radius* of the curve (denoted by R).

The angle GB1 is the deflection angle (denoted by D).

B-1 = 1-2 = 2-3 is the *chord* used in the field work (denoted by C).

B = point of curvature (denoted by P.C.)

E = point of tangency (denoted by P.T.).

(b) GEOMETRICAL PROPERTIES OF A CIRCLE

(1) $GB = GE$ and $GBE = GEB = \frac{1}{2}I$.

(2) $GBO = 90° = GEO$.

(3) $I = BOE$ and $BOG = \frac{1}{2}I = EOG$.

(4) $GB1 = \frac{1}{2}BO1$, $GB2 = \frac{1}{2}BO2$, $GB3 = \frac{1}{2}BO3$, etc.

(5) $1B2 = \frac{1}{2}1O2$, $1B3 = \frac{1}{2}1O3$, $1BE = \frac{1}{2}1OE$.

(6) If B-1 = 1-2 = 2-3, then $BO1 = 1O2 = 2O3$.

(c) FORMULAS *

(7) Given R and I to find T: $T = R \times \tan \frac{1}{2}I$.

(8) Given I and T to find R: $R = T \times \cot \frac{1}{2}I$.

(9) Given R and C to find D: $\sin D = \frac{1}{2}C \div R$.

(10) Given D and C to find R: $R = \frac{1}{2}C \div \sin D$.

(11) Given I, T, and C to find D: $\sin D = (C \times \tan \frac{1}{2}I) \div 2T$.

(12) Given I, D, and C to find T: $T = (C \times \tan \frac{1}{2}I) \div 2 \sin D$.

Formulas for chords 100 ft. long:

$\sin D = 50 \div R$; $\sin D = (50 \times \tan \frac{1}{2}I) \div T$;

$R = 50 \div \sin D$; $T = (50 \times \tan \frac{1}{2}I) \div \sin D$.

To find the number of chords in the length of a curve:

(13) $N = \frac{1}{2}I \div D$.

If D is not contained an even number of times in $\frac{1}{2}I$, the fraction of a chord remaining is called the *sub-chord*, and the corresponding deflection angle is called a *sub-deflection* angle. Thus in Fig. 263 the chord 3-E is not quite as long as the others, 3-E is a *sub-chord* and $3BE$ a *sub-deflection* angle.

(d) *Length of curve* is usually understood to mean the length as found by measuring the chords and *not* the length measured on the curve itself. Thus if the standard chord used in running in the curve is 100 ft., and the number of chords (N) found from (13) is 7.813, then the *length of curve* is 781.3 ft.

(e) *Degree of curve* is understood to mean the number of degrees at the center subtended by a 100-ft. chord. Thus in Fig. 263, *if* B-1 = 100 ft. and $BO1 = 6°$, the curve is a 6° curve; in a 5° curve $BO1 = 5°$; in a 1° curve $BO1 = 1°$, and so on. From Fig. 263, $R \times \sin \frac{1}{2}$ degree of curve $= \frac{1}{2}B$-1, or if *deg.* represents degree of curve, $R = \frac{1}{2}C \div \sin \frac{1}{2}$ deg., or for 100-ft. chords

(14) $R = 50 \div \sin \frac{1}{2}$ deg.

This formula is practically the same as formula (10).

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* The trigonometrical relations in formulas (7) to (12) will become evident from a study of *Fig. 263.*

(*f*) *Field work of running in a circular curve.* In Fig. 263 it is desired to connect two lines AB and HF by the arc of a circle which shall be tangent to AB at a *given* point B (P.C.), and tangent to HF at some *unknown* point E (P.T.).

(1) Produce AB and HF until they intersect and measure I.

(2) Measure the length of T (BG) and establish E so that $EG = BG$.

(3) From (11) calculate the deflection angle D.

(4) Set up at B, backsight on G, turn off $GB1 = D$ and set the point 1 100 ft. from B. Keeping the backsight on G turn off $2D$ (i.e., $GB2$), and by swinging the tape with one end held at 1 set the point 2 100 ft. from 1. Turn off $3D$ (i.e., $GB3$) and set the point 3 100 ft. from 2. Since 3–E is less than 100 ft., the additional angle to turn off (i.e., $3BE$) is less than D. This *sub-deflection* angle $3BE$ must be the same part of D that the *sub-chord* 3–E is of C (i.e., of 100 ft.).[*] The length of 3–E may be found from equation (13). E as located from B by deflection angles should check with E as located in the first place by chaining from G to E.

(*g*) *To pick up a tangent.* It often happens that some of the points on the curve cannot be seen from P.C. It then becomes necessary to move the transit from P.C. to the last point on the curve set from P.C. It may even be necessary to set up at several points before the curve is completed. The two following rules should be employed in backsighting and foresighting:

(1) *When sighting at any point of the curve the vernier reading must be equal to the total deflection angle used for locating that point.*

(2) *When sighting along a tangent to the curve at any point occupied, the vernier reading should be equal to the total deflection angle for that point.*

Illustration. In the above rules the total deflection angle for any point means the sum of all the deflection angles from the original tangent to the point. This is sometimes called the index reading. Suppose in Fig. 263 it is necessary to set up at 2 and again at 3.

Set up at 2, backsight on B with vernier at zero (since the deflection at B is zero), plunge and turn off $3D$, since the deflection for 3 is $3D$. This locates 3. Set up at 3, backsight on 2 with the vernier set at $2D$ (since the deflection for 2 is $2D$), plunge the telescope and turn off $\frac{1}{2}I$ (since the deflection for E is $\frac{1}{2}I$).

The student should prove the above method. Prove also that, in accordance with the second rule, if a transit is at 3, the backsight at 2 and the vernier at $2D$, the line of sight may be made tangent to the curve by setting the vernier at $3D$.

(*e*) *Practical suggestions.* (1) In the preceding description of the field work it was assumed that 100-ft. chords were used. This is customary in railway work, but if for any reason shorter chords are used the principles still hold good.

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[*] *This is not strictly* correct, but accurate enough for most purposes.

(2) The problem has numerous variations. For example, it may be required to run a curve of a given degree. Equation (14) gives the radius, and T can be found from (7). Again, the point of intersection G may be inaccessible. These variations, as well as problems dealing with reverse curves, spiral curves, and other curves, will be found explained in field books for railway work. For anything likely to arise outside of railway work the foregoing principles should be sufficient.

(3) The total deflection or index reading for each point of the curve should be worked out and entered in the note-book before beginning to run in the curve. As a check notice that the index reading for P.T. is equal to $\frac{1}{2}I$.

264. To Lay Out a Circular Curve with the Tape.

(*a*) *By tangent offsets.* This method simply consists in laying off each deflection angle by measuring an offset from the tangent instead of turning it off with a transit. Thus, for example, point 1 is located by laying off BC, found by calculation, and C-1, also found by calculation. Theoretically C-1 is perpendicular to BC, but in practice it is better to establish C and then find the intersection of C-1 with the chord B-1, the length of which is known, than to construct the right angle at C. The formulas needed in computation are easily derived from trigonometry. If D = the deflection angle and R the radius, then:

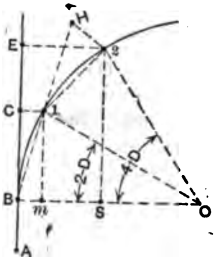

FIG. 264 (*a*).

Distance along tangent	Offset from tangent
$BC^* = m\text{-}1 = R \sin 2D$	$C\text{-}1 = Bm = R$ versine $2D$
$BE = S\text{-}2 = R \sin 4D$	$E\text{-}2 = BS = R$ versine $4D$
For N chords $= R \sin 2ND$	For N chords $= R$ versine $2ND$

More convenient formulas to use where C = length of chords may be found by substituting for R its value $\frac{1}{2}C \div \sin D$:

Distance along the tangent $= (\frac{1}{2}C \sin 2ND) \div \sin D$.
Offset from the tangent $= (C \sin^2 ND) \div \sin D$.

In computing, the logarithms of $(\frac{1}{2}C \div \sin D)$ and $(C \div \sin D)$ remain constant, which facilitates work.

(*b*) *By offsets from chords produced.* Produce B-1 to H (Fig. 264 (*a*)), making $1\text{-}H = B\text{-}1$. Then $BO : B\text{-}1 = B\text{-}1 : H\text{-}2$ or $R : C = C : H\text{-}2$.

Hence $H\text{-}2 = C^2 \div R$, or in the case of 100-ft. chords $= 100^2 \div R$.

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* In most cases no appreciable error is involved by calling BC = B-1.

Note that this offset equals twice the tangent offset.

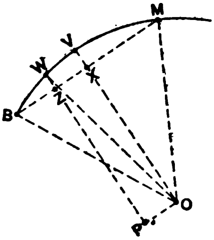

FIG. 264 (*c*).

(*c*) *By offsets from a long chord.* Suppose that two stations, B and M, are given and the length of the curve BVM is known. Then from equation (13), p. 205, $BOM = 2ND$ and the middle ordinate $XV = R$ versine $\frac{1}{2}BOM$

$= R$ versine ND.

If $N = 2$, then $BOM = 4D$ and

$XV = R$ versine of $2D$.

Another form is

$$XV = \tfrac{1}{2}BM \tan \tfrac{1}{2}BOM;$$

or still another form:

$$XV = OV - OX = R - \sqrt{R^2 - \left(\frac{BM}{2}\right)^2} = R - \sqrt{R^2 - \overline{BX}^2}.$$

For any other ordinate WZ a distance XZ from the middle ordinate:

$$WZ = WP - ZP = \sqrt{\overline{WO}^2 - \overline{PO}^2} - XO = \sqrt{R^2 - \overline{ZX}^2} - \sqrt{R^2 - \overline{BX}^2}.$$

(*d*) *By distances scaled from a map.* (1) Plot the curve to a scale large enough so that distances can be scaled as closely as will be required for measurements in the field. (2) Scale ordinates or offsets to some assumed base-line, and enter these distances on a map or a specially prepared sketch for use in the field. (3) Lay out the base-line in the field, and lay off the measurements as indicated on the sketch.

This is a very useful method for work in which extreme accuracy is not required, such, for example, as landscape work.

265. Giving Lines for Masonry. (See also Art. 224.) Excavation is required for most masonry structures. The first step, therefore, is to set stakes which will guide the contractor in digging. Such stakes are set roughly and often very hurriedly, especially if the contractor is already "on the job" and waiting to begin work. Before the excavation reaches its full depth the surveyor usually sets "batter-boards." These batter-boards are horizontal boards nailed to posts or to some other support, such as the wall of an adjacent building. The upper edges of the boards are usually set at some known elevation, say an even number of feet above or below the required elevation of some well-defined line of the structure. They are also *set in such a position* that by stretching strings from one to another the mason can get the most important lines of the structure (neat lines). The exact points where the strings should be fastened to the boards are marked by nails or by notches cut in the board. Additional suggestions for staking out work are given on p. 159.

Remark. The next article gives in detail the work of staking out a house, and Arts. 272 and 273 give the methods of staking out a retaining-wall and a bridge abutment. These typical cases will suggest methods of procedure in other cases. When a masonry structure is on a grade the work usually involves considerable leveling, examples of which are given in Chapter XXIII on *Leveling—Special Work.*

266. To Stake Out a Building means to establish points which will guide the contractor in building the foundation. It would be of little use to set a stake at each corner of the building, for these stakes would be knocked out in excavating. The usual method is to set batter-boards or corner-boards a few feet outside, where they will not interfere with the work of construction, and to cut notches or drive nails in the upper edges, so that the mason can get his lines by stretching strings between the points thus established. Usually the upper edges of the boards are set at the elevation of the top of the foundation-wall or water-table (or some whole number of feet above or below the top), and thus serve as points from which to measure the depth of the cellar, the heights of walls, and other elevations. The architect or owner usually furnishes the surveyor with a blue-print plan of the building, and with other necessary data, such as its location with respect to street-lines, boundary-lines, and adjacent buildings; also the elevation of the top of the foundation-wall or water-table with respect to some datum, such as a city bench-mark, a merestone, or the water-table of an adjacent building.

General method. The aim should be to accomplish the work with as few set-ups and as few measurements as is consistent with the certainty of correct results. In most cases the best method is to stake out temporarily a reference rectangle, to make sure that it is correct, and then to use the sides of this rectangle as base-lines for locating each corner of the building. The more irregular the outline of the building the more necessary it is to have base-lines. The following illustration shows one method of staking out a building. Study it to see if some other method equally good or even better could have been used.

(*a*) *Illustration.* In Fig. 266 (*a*) the front of the building is parallel to and 10 ft. from the street-line; it is 30 ft. from the western boundary-line.

(1) Located the corners *A*, *B*, *C*, and *D* roughly with the tape so that the contractor could set posts for the batter-boards.

(2) The line of sight along the street-line was obstructed, hence set up 2 ft. out from a merestone at the nearest street-corner and established stations (2) and (3) on a line parallel to and 2 ft. outside of the street-line. Station (2) is 30 ft. from boundary line, and (3) is 40 ft. from (2).

(3) Set up at (2) and backsighting on a near-by bench-mark, marked a horizontal line on each post 3 ft. above the elevation of the water-table. The contractor nailed the boards with upper edges on these lines. (Batter-boards represented by double lines in the figure.)

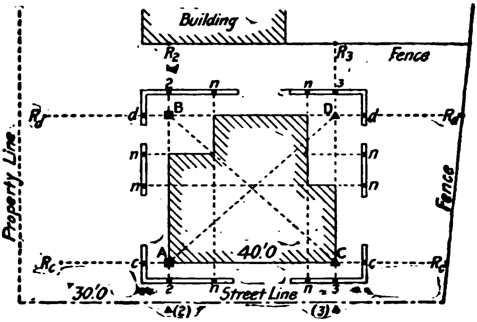

FIG. 266 (*a*).

(4) Turned off 90° from the line through (2) and (3), and established temporary stakes *A* and *B*; also two nails on batter-boards marked 2, and a reference-point on another building marked R_2.

(5) Set up at (3), turned off 90° from line through (2) and (3), and established temporary stakes *C* and *D*; also two nails marked 3 and a reference-point on a fence marked R_3.

(6) Measured the diagonals *BC* and *AD* of the reference rectangle as a check. Found one of them 0.01 ft. longer than the other. Corrected the error.

(7) Set up at *C*, sighted at *A*, established a reference-hub R_c and a nail *c*, plunged, and established a reference-point on the fence R_c and another nail *c*.

(8) Set up at *D*, backsighted on (3), turned off 90° as a check,—line of sight passed slightly to the left of *B*,—error small enough to be ignored. Brought line of sight on *B*, established a reference-stake *Rd* and nail *d*, plunged and established a reference-point on the fence *Rd* and another nail *d*.

(9) Set the intermediate nails marked *n* by measuring with a tape.

(*b*) *Practical suggestions.* (1) The reference-stakes should be placed where they are not likely to be covered up by building materials, and where they will be of the most use in re-establishing points should the batter-boards be disturbed. (2) It is often necessary to set a new building with its front on line with one already built. If the latter is not parallel to the street-line, stations (2) and (3) should be set on a line parallel to and a definite distance from the front of the old building. (3) Occasionally a building must be oriented with respect to the true north, and then it may be necessary to establish a true meridian.

267. To Stake Out a House-lot. Procure the deed of the land, or consult the town or city records, and from the descriptive clause in the deed stake out the lot, using a method similar to that of the preceding article. When, as is often the case, the descriptive clause is so indefinite as to be of little value, the surveyor must resort to other methods. (See article on boundary surveys.)

Practical suggestions. An iron rod or half-inch iron pipe set in the ground with about 3 ft. of it below frost-line will serve as a permanent monument for each corner. Stone posts with tops a few inches below the surface of the ground are much better. In making a record of boundary-lines at least one line should be carefully referenced with respect to permanent landmarks (Art. 260, p. 203).

268. Farm Survey. The following method is recommended: Run transit lines conveniently near the boundaries by the azimuth method, locating stations to the best advantage (p. 149). Take side shots to each corner, and if boundaries are straight, chain the lengths of boundaries as well as of transit lines. If boundaries are not straight, locate them by offsets from a transit line. The azimuths and lengths of the transit lines and side shots to corners as found above are all the data required for the office work of calculating the azimuths and length of each straight boundary line. (The measured length of a boundary-line serves as a check.) From the azimuths and lengths of the boundary-lines thus calculated the area may be computed, as explained in the chapter on *Areas*, Part II.

Note. For a quick rough survey run over the boundary-lines with compass and chain or tape. (See chapter on Compass Surveying.)

269. Survey for an Architect. A map from which an architect is to get the data for locating and planning a building should show a number of things not found on an ordinary map. Moreover, everything of importance should be given by dimensions and explanatory notes similar to a working drawing, so that the architect will not have to depend upon scaling the map. For a city lot the map should show as a rule:

(1) Outlines of the lot with lengths of all boundary-lines and the values of all angles plainly marked on the map. One side at least should be shown carefully referenced with respect to two or more permanent points (Art. 260, p. 203).

(2) Street and curb lines and the line of the street sewer.

(3) Adjacent buildings, large trees, and other objects which might affect the location of the building or the location of windows in the building.

(4) Direction of the *true* north shown by an arrow to enable the architect to plan rooms for exposure,—the *true* bearing of a boundary-line is even better, for it helps to reference that line.

(5) Elevations of the flow-line of the sewer, elevations and grades of all sidewalks in front of or adjoining the lot, and the elevation of the ground itself. If the surface is irregular, contour-lines of perhaps one- to two-foot intervals will best show *the elevations of the* ground.

Sometimes in addition to the above it will be desirable to give the elevations of water-tables of adjoining buildings, and the location, height, thickness, and plumb of adjacent walls, especially if the proposed building is to fit exactly between two existing buildings;* location and depth of existing foundations; size and location of water, gas, and electric mains, and the location of points where service-pipes come through the curb.

When the building is planned by an out-of-town architect the engineer can often furnish useful information, such as a copy of the city ordinances pertaining to buildings, the nature of the soil where the building is to stand and its bearing power, the city water pressure, and other local conditions.

270. To Locate Machinery, Lines of Shafting, and Similar Objects. The usual methods of field work are employed. In much of the work, however, such, for example, as the alinement of shafting, leveling must be combined with transit work; i.e., the line must be given vertically as well as horizontally. Establish one or more reference-lines for transit work (similar to the reference rectangle in staking out a house, p. 210), and one or more permanent bench-marks for leveling. Establish the location and elevation of all important or ruling points, such as bed-plates and foundations for machines and hangers for shafting. (See Chapter XXIII for method of leveling for a line of shafting.) The data for this work are usually obtained from a previously prepared plan, although occasionally the surveyor is called upon to prepare this plan also.

Surveying methods are now used to a considerable extent in the construction of large machines. (See "The Use of Surveying Instruments in Machine-shop Practice," *Engineering News*, Dec. 11, 1902.)

271. To Stake Out a Road or Highway. Stakes are usually set at least every 50 ft. For an ordinary road, such as a macadam road, the first step is to run the center-line, setting the stakes, however, on either side so that there are two rows of stakes, one for each edge of the road. These stakes are usually located by offsets from the center-line, and if set some whole number of feet (say 3 or 4 ft.) outside of the edges of the road they are less likely to be disturbed.

The second step is to go over the line with the level, establishing grades, and marking the stakes 'Cut' or 'Fill.'

Note. For the methods of establishing grade-points, marking grade-stakes, and staking out city streets and pavements, see Chapter XXIII.

Practical suggestions. (1) For an ordinary country highway stakes should not vary from line or grade more than one tenth of a foot. Of course greater variations might be allowed, depending upon the location of the road, material used, and methods of payment for the work. (2) If extensive cutting or filling is to be done, it often pays to set slope-stakes, as in railroad work. (3) Curves may be staked out according to one of the methods in Arts. 263, 264, p. 204–208. The fourth method (scaling distances), is much used for irregular highways. (4) Where the road curves sharply or the grade changes rapidly it may be necessary to set stakes at shorter intervals than fifty feet. (5) If the highway has not already been laid out, it will be necessary to set stakes for fence-lines or boundaries of the road. (6) Sometimes a separate set of stakes is used for grade, but one set for both line and grade is preferable.

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* The author has in mind a case where it became necessary to practically redraw the plans of a large building because the distances between *existing buildings* on either side were incorrectly given on the map.

272. To Give Lines for a Retaining-wall. (See also Arts. 224 and 265.) In Fig. 272 the section shown of a retaining-wall is only a few feet in length, but the method may be used for a wall of any length. (1) Set stakes *A* and *B*, or if more convenient stakes *C* and *D*. If possible, each stake should be some *whole* number of feet from the nearer end of the wall.

Either line, *AB* or *CD*, will be sufficient for purposes of excavation. The contractor can get the widths *W* and *W'* from the blue print and set his own stakes for the outside lines of the foundation no matter how much the latter may be broken up by steps. The lower course is often concrete and it is immaterial whether the contractor gets the outside lines exactly right or not, so long as he makes this concrete base extend the *minimum* distance each way as called for by the plan; if he should make it a little wider, no harm is done.

(2) When the excavation is well under way, if not before, set batter-boards at a known elevation, and by nails or notches give points for both lines *AB* and *CD*. Very often these lines must be established parallel to some street-line or property line.

Practical suggestions. (1) There are several well-defined lines at the top of the wall and several at the bottom, but it would only lead to confusion to give more than the two lines *AB* and *CD*. These two neat lines are chosen because they are definite lines and moreover determine the finished face of the wall. (2) In giving lines for excavation, it is often more convenient to give parallel lines that are offset a whole number of feet from the required lines. (3) In giving lines and grade for the finished face it may be convenient to use plank set on edge across the trench, as in sewer-work (Chapter XXIII). Sometimes nails driven in the concrete base are used for the lower line. (4) Nearly all walls are so long that they are built in sections. Stakes and batter-boards are set in ad-

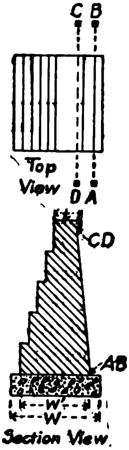

FIG. 272.

vance of the work as they are needed, and strings are drawn from the stakes or batter-boards to the finished wall, or an offset line may be used at a safe distance from the excavation. (5) When the wall is on a curve the stakes must be set near enough together so that the wall, though really built in a series of straight lines, will appear as a smooth curve. Thus, for example, for a curve of 200 ft. radius points should not be more than, say, 10 ft. apart. There are four ways of locating the points: (*a*) By locating the center of the curve and swinging the arc with the tape,—only possible for true arcs of circles of small radius. (*b*) By deflection angles, p. 204. (*c*) By offsets from tangent or long chord, p. 207. (*d*) By ordinates from a convenient base-line, the ordinates having been previously determined either by calculation, or by plotting to a large scale and scaling off such ordinates as it is desired to reproduce upon the ground. The latter method is very useful for irregular curves, such as those which occur in landscape work.

273. To Give Lines for a Wing Abutment. In Fig. 273 the main part of an abutment is shown, but the concrete foundation is not shown. The work is similar to that of giving lines for a retaining-wall as outlined in the preceding article. As suggested in that article, the best line to give is where the face of the main wall joins the foundation, or in this case the line *AB*.

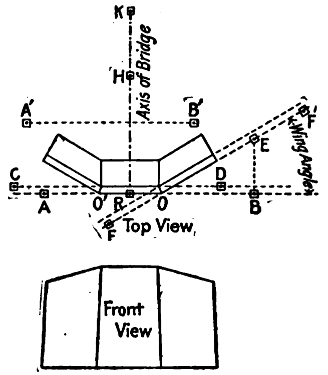

FIG. 273.

(1) Set a stake *R* on the axis of the bridge.

(2) Set up over *R*, and set stakes *A* and *B*; also establish points *O* and *O′*.

(3) Set up over *O*, and turning off the angle of the wing set a stake in line, as *E*. Repeat the operation at *O′*.

Practical suggestions. (1) It may be necessary to set points *A* and *B* some distance from *R*, perhaps on piles or coffer-dam, if construction has already begun. (2) If *AB* comes in the water, it may be necessary to set offset stakes, as *A′* and *B′*, instead of *A* and *B*. The offset distance should be some whole number of feet if possible, and should be plainly marked on the stakes. (3) If for the same reason it is inconvenient to establish *R*, set two points, as *H* and *K*, on the axis of the bridge. The contractor can produce the line through these points with sufficient accuracy for starting the piling, coffer-dam, etc. If *the stream is narrow,* one of these points may be set on the opposite shore. (4) If O is inaccessible, the line EO may be established by measuring off either a tangent distance from AB, as BE, or a tangent distance from the parallel offset line $A'B'$. The right angle at B may be laid off by eye, as the length of the wing is usually short compared with the distance OE. Since O cannot be marked by a stake, set some point, as F, on the opposite shore, or if the stream is too wide, set F' on the same shore. (5) Stakes will answer for preliminary construction, but they should be replaced by batter-boards giving elevations as well as lines.

274. To Give a Line for a Tunnel. The field work involved is simple or complicated according to local conditions. The simplest case is when a preliminary straight line can be run over the surface of the ground between hubs on the center-line of the tunnel, one at each entrance. It may be necessary to run a traverse in order to locate this line. Once the line is established it may be prolonged a few hundred feet at each end, and permanent targets established which will be used as backsights when, in the progress of the work, it becomes necessary to set up the transit inside of the tunnel. The line may be prolonged from a backsight by the method of double reverse.

In many cases the preliminary straight line between the ends of the tunnel cannot be run because of obstacles, such as a river or a high cliff. In that case the direction and length of the axis of the tunnel must be ascertained either by triangulation or by running a traverse, the closing side of which is not measured, but found by the methods explained in Part II for calculating omitted measurements. The omitted side is of course the required line of the tunnel.

When, as is often the case, the tunnel is on a grade the methods for giving a grade explained in the chapter on leveling are used. Even when the tunnel is level it will be necessary to establish the two ends at the same elevation by running lines of levels between them before the tunnel is started.

When the tunnel is on a curve the work becomes still more complicated, the curve being run in by the method of Art. 263, p. 204, or by some similar method.

275. To Locate Piers for a Bridge.* It is assumed that it is impracticable to measure from pier to pier with the tape. The problem is a double one: (1) To find by triangulation the total distance from a point on one shore to a point on the opposite shore, both points lying in the axis of the bridge; (2) To locate at least one point for each pier by calculating two angles such that when these angles are turned off from different base-lines or from two different points on the same base-line, the two lines of sight will intersect at the required point. The amount of work involved in the second half of the problem depends upon the number of piers to be located.

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* Chapter XVI, on *Triangulation* should be read in connection with this article. For a more complete treatment of the subject the student is referred to a chapter on triangulation for bridge-piers in "De Pontibus" by J. A. L. *Waddell (John Wiley & Sons).*

(a) *The ideal method.* (1) Establish a base-line on each shore exactly perpendicular to the axis of the bridge, extending above and below the bridge. (2) Find by triangulation the perpendicular distance *AB* between base-lines on opposite shores. (3) Having determined where the piers are to be located, lay off above and below the bridge on each base-line the distance from that base-line to the nearest pier, and then in succession the distances to other piers, so that each pier will be located by two intersecting lines making angles of 45° with the base-lines on opposite shores as well as with the axis of the bridge. Any one of the intersecting lines can be established at any time by simply sighting from a point on one shore to a point on the opposite shore *without turning off an angle.*

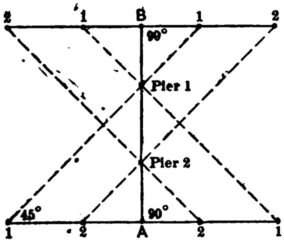

FIG. 275 (a).

Remark. The chief advantage of this method lies in the fact that after the 90° angles at *A* and *B* have been laid off and the four angles measured in step (2) to determine the length of *AB*, no further measurement of angles is necessary.

(b) *General method.* It is seldom that the ideal method can be employed because the conditions do not often permit base-lines to be laid out on both shores in the manner indicated. There are two alternatives:

(1) Establish base-lines on opposite shores, both above or both below the bridge.

(2) Establish two base-lines on the same shore, one above and one below the bridge.

Question. Two other locations for base-lines are possible: (1) Both base-lines on the same shore and on the same side of the bridge. (2) The two base-lines on opposite shores, one above, the other below the bridge. Why would each of these conditions be unsatisfactory?

Base-lines should be laid off *approximately* at right angles to the axis of the bridge, but it is not worth while to aim at exact right angles, for much time in the field may thus be wasted, while the time saved in computation is very little. It is better to lay off the base-line in the most favorable location, and let the angle of its intersection with the axis of the bridge be oblique. A general method of procedure is given in the following illustration.

(*c*) *Illustration.* The following steps were taken in locating the center-points *A* and *B* of two piers.

(1) Established a base-line *SH* on the south shore intersecting the longitudinal axis of the bridge at *S*; also a similar base-line, *NM*, on the north shore.

(2) Carefully referenced hubs *S*, *H*, *N*, and *M*, p. 203.

(3) Measured the length of each base-line a number of times and found its probable value and the probable error, pp. 57–59.

(4) Measured the angles of the triangle *MSN*; also the angles of the triangle *NSH*. Six sets (p. 194) were taken for each angle and its probable value and probable error determined, p. 19.

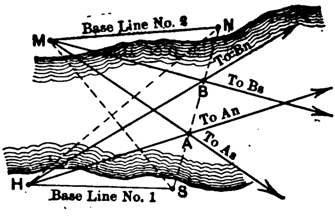

FIG. 275 (*c*).

(5) Computed the distance *SN* from the triangle *MSN*; also from the triangle *NSH*. The two computed lengths checked within the limit of error adopted. (A discrepancy of less than 0.05 ft. (or ⅝″) per 1000 ft.)

(6) Computed the angles *BHS* and *AHS*; also the angles *BMN* and *AMN*.

(7) Made out and blue-printed a triangulation-sheet showing all computed angles and distances as well as measured angles and distances.

(8) Laid off the computed angles *BHS*, *AHS*, *BMN*, and *AMN*. The check on the intersection-points *A* and *B* was tried by sighting from *S* to *N*: this line passed through *B* almost exactly but was ¼″ from *A*, which was within the allowable error adopted for this work, i.e., ½″ for all intersections. Each angle was turned off twelve times.

(9) Set targets *An* and *Bn* on the north shore, and *As* and *Bs* on the south shore, so that the intersecting lines could thereafter be established without turning off angles. Carefully referenced these points.

(*d*) *Practical suggestions.* *Base-line.* (1) Choose its location with a view to the proper proportions of triangles, p. 193. This will make each base-line approximately equal to the total length of the bridge,—not less, say, than three quarters of that length. (2) Use substantial hubs and supporting stakes, and properly protect all hubs, p. 57. (3) Use a 50-ft. or a 100-ft. tape: it may be well to send it to the bridge company for comparison with their standard. (4) Choose the end of the base-line where all piers are visible, as it is preferable to locate all piers from the same point. When, however, it is impracticable to do this, either because of obstacles to sight or because of too acute intersections, intermediate hubs may be established on a base-line for locating different piers. (5) If shore conditions are unfavorable, it may be necessary to run a broken base-line, or a shorter base-line may have to be expanded, p. 193.

Measuring angles. (6) The transit should be graduated to 20″ or in important work to 10″. (7) The angles should be taken in sets by some such method as that explained on p. 196, and the most probable value and probable error of each angle determined, p. 19. (8) The error in the sum of the most probable values of the three angles in each of the main triangles should not exceed, as a rule, 5 seconds, and may be reduced to, say, 2 seconds in important work. An error of one second in an angle means an error of $\frac{1}{16}$ inch at 1000 ft. (p. 198). (9) If the error in the sum of the angles is too large, determine from the field notes which angle was measured under the most unfavorable weather conditions, and remeasure that angle first: it may save measuring the other angles.

Laying off angles. (10) Lay off the angle for locating a pier and set a wooden target, usually on the opposite shore, where it is not likely to be disturbed. Turn off the angle ten to twenty times *continuously* around the limb, marking the line each time where the line of sight strikes the target. This will result in a series of lines, the average of which, assumed to be correct, is marked conspicuously on the target. Repeat the operation for each intersecting line used in locating each pier. Reference each target. (11) When piers are built in an open coffer-dam, or when staging is used, points on lines may be placed on the dam or the staging and occasionally tested to see that they have not been disturbed. When, however, the pier is built in a movable caisson the latter should be kept in place by daily tests; no angles are turned off, but sights are taken to the targets. (12) For round piers only one point is given (the center), but for rectangular piers an additional point near the periphery must be given to detect any rotation of the pier about its vertical axis during the sinking of the caisson.

CHAPTER XVIII

TRANSIT SURVEYING—PROBLEMS

There are certain geometrical and trigonometrical problems which occasionally arise in transit surveying, but which are of too infrequent occurrence to be considered a part of the routine field work. These problems have been grouped for reference in this chapter. Many of them are problems in simple triangulation to which the methods of Chapter XVI apply.

276. Problems in Triangulation. On pages 64 to 66 are given the so-called obstacle problems. Most of these problems are solved in transit surveying by triangulation. While it is not often necessary to resort to the precise methods described in Chapter XVI, the points emphasized in that chapter should be kept in mind, namely, that the accuracy of any triangulation depends to a large extent upon the *proportions* of the triangles and that very oblique intersections should be avoided. Usually it will be sufficiently accurate to measure each angle with an ordinary transit reading to minutes, doubling each angle if desired, but more accurate methods should be adopted when they are warranted by the importance of the work. If two lines starting from the same point make an angle of one minute with each other, they will be approximately 0.03 ft. apart at 100 ft. from their intersections and 0.3 ft. apart at 1000 ft., i.e., *the error for one minute of arc is 0.03 ft. per 100 ft.* See p. 105. These quantities, if kept in mind, will often suggest the precision with which angles should be measured in order to meet given requirements.

277. Obstacles to Transit Surveying. *Obstacles to alignment* and *obstacles to measurement* give rise to a special set of problems in transit surveying. Many of the methods already given for obstacle problems in chaining (Chapter VII) can be used in transit surveying, and with greater precision. The problems treated in this chapter are those most likely to arise in practice. Two or three constructions are explained for each problem—enough, it is thought, to suggest other methods not only for problems given here, but also for those problems which occur so seldom in practice that they have been omitted.*

Choosing methods. In choosing between the methods given for each problem two things are to be considered: *accuracy* and *time.* When one method seems as accurate as another and it is simply a question of which is the quicker, it should be noted that constructions which require the least time when worked out on paper sometimes require the most time when worked out in the field. Thus, for example, the intersection of two straight lines is quickly found on paper, but not so quickly in the field. Perpendiculars and parallels are quickly drawn on paper, but corresponding operations in the field may involve considerable work.

OBSTACLES TO ALIGNMENT

(*a*) *To prolong a straight line through an obstacle.* Three methods are shown in Fig. 277 (*a*). In all three methods each

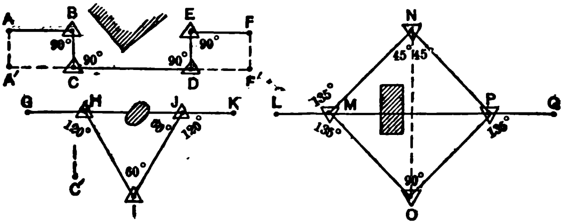

FIG. 277 (*a*).

point at which the transit was set up is indicated by a small triangle, and the angles actually laid off are noted on the figure. The lines to be prolonged are respectively *AB*, *GH*, and *LM*.

First method. Lay out a rectangle *BCDE*, the fourth side of which, *BE*, is in the required line.

Second method. Lay out an equilateral triangle, *HIJ*, the third side of which, *HJ*, is in the required line.

Third method. Lay out a square *MNPO*, the diagonal of which, *MP*, is in the required line.

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* For a very complete collection of such problems see Gillespie's *Surveying, Vol. I, pp. 245 to 257.*

The three methods compared. The rectangle method is less accurate than either of the others when the right angles at C, D, and E have to be turned off from short backsights. When, however, the lines $A'F'$ and EF are established by *offsets* this method is very satisfactory, especially when long sights parallel to the original line can be taken. Thus A' and C having been established by offsets, the transit may be set at either point, preferably A', and two points D and F' established in the line $A'C$ produced. E and F may then be found by measurement, assuming that the offsets are short. This is about the only method that is used when the space is too narrow to permit triangles to be laid out. Another method of avoiding short backsights in the rectangle method is to set some point C' in BC produced, and to use this point as a backsight in turning off 90° at C. Similarly set some point in DE produced, as far away as convenient, and use this point as a backsight in turning off 90° at E.

The equilateral-triangle method requires fewer set-ups than either of the others, and the distance HJ is known without calculation. It is a good method to use especially if an equilateral triangle can be run on the other side of the object as a check. The third method has the advantage that no linear measurements are necessary, the work being done wholly with the transit. If, however, the distance from M to P is required, either the diagonal NO must be measured or else MP must be calculated from the sides of the square.

Practical Hints. (1) Of course convenient distances arbitrarily chosen may be used for the sides of the figures, but if possible they should be at least from 100 to 200 ft. long to avoid short backsights. For the same reason 120° was measured at H and J and 135° at M, instead of 60° and 45° respectively, it being presumed that the longest backsight could be had in the direction of G or I in one case and in the direction of L in the other.

(2) There are other methods than the three given which are equally good. Of course any system of triangulation may be used, but it is well to avoid the calculation involved in the use of oblique triangles.

(*b*) *To run a line between two given points, when an obstacle intervenes.*

First method. C is any convenient point which is visible from the two given points A and B. Make $CD = AC$ and $CE =$ CB. Measure ACB and CDE. Then lay off the angle $CAB = CDE$, and the angle $CBA = 180° - (ACB + CDE)$. (Or the value of CBA can be found by measuring CED. This affords a check.) Transit points and angles actually measured or laid off are indicated on the figure by arrows.

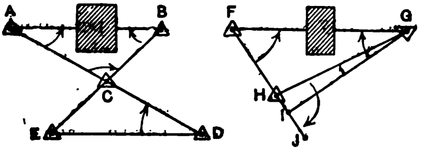

FIG. 277 (*b*).

Another method. Measure AC and BC, also the angle ACB, and solve for the two angles CAB and CBA by trigonometry.

Second method. Choose H, any convenient point visible from the two given points F and G. Produce FH to some point J

and measure GHJ. Turn off at G the angle $HGI = 90° - GHJ$. This makes GIF a right angle. Hence $\tan HFG = GI \div FI$. The angle thus calculated is laid off at F, and its complement at G, the angle at G being laid off first to save a "set-up."

Practical hints. The first method requires four set-ups, the second three, but trigonometrical computation is involved in the second. The second method can be used sometimes where there is not room on the ground to lay out the first.

OBSTACLES TO MEASUREMENT

(c) *To measure between two points when one is inaccessible.* The methods given for solving this problem with the tape (see page 64) can be used also with the transit. There are several other methods, three of which are given here.

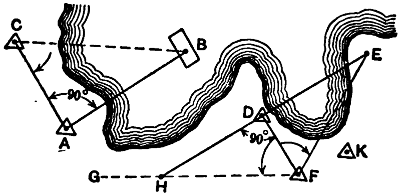

FIG. 277 (c).

First method. Lay off a base-line AC perpendicular to the required line AB, and of *any* convenient length. Then $AC \times \tan BCA = AB$.

Second method. Lay off DF perpendicular to the required distance DE, and of *any* convenient length. Measure DFE and lay off $DFG = DFE$. Then $DH = DE$.

Third method. When both ends of the lines are accessible a third point K may be chosen. In the triangle DKE measure enough parts to obtain DE, as, for example, two sides and included angle, or one side and two adjacent angles.

Practical hints. The second method appears on paper to be the easiest, as no calculation is involved. Considerable work is involved, however, in laying off accurately the angle DFG and in finding the precise point of intersection at H, i.e., the intersection of FG with ED produced. Moreover, many times there is not room to lay off DH.

To check either the first or the second method repeat the process, using a perpendicular or base-line of different length. To check the third method, choose a new point in place of K, and repeat the process. Sometimes it is better to check one method by using an entirely different one.

(*d*) *Special case. To measure the distance between two given points when one is invisible from the other and is also inaccessible.*

One solution. Choose two auxiliary points visible from both of the given points and from each other. The inaccessible point and the two auxiliary points form a triangle, and by measuring the angle at each of the auxiliary points and using the distance between them as a base, the sides of the triangle can be calculated. An auxiliary point, the given point, and the inaccessible point form a triangle one side of which has just been found by calculation, another side (from the given point to the auxiliary point) can be measured, and the angle between these two sides can also be measured. The third side (the required distance) can then be found from these two sides and the included angle.

Remark. This is an example of numerous special cases which now and then arise in practice, the solutions of which depend on various forms of triangulation. The student should draw a figure illustrating the method here given.

(*e*) *To measure between two points when both are inaccessible.* Let AB, Fig. 277 (*e*), be the required distance. Lay off any convenient base-line, as CD, and measure its length. At C measure the angles ACD and BCD. At D measure ADC and BDC. In each of the triangles ACD and BDC one side and two adjacent angles are now known, and hence the distances AC, AD, BC, and BD can be calculated. AB can then be found from either of the triangles ACB or ADB, one computation serving as a check on the other. (Two sides and included angle. See formulas in the back of the book.)

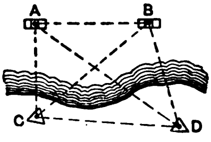

FIG. 277 (*e*).

(*f*) *To measure the height of an inaccessible point.*

General method. In Fig. 277 (*f*), B is the point the height of which is required. The transit is set up at any convenient point A, and the vertical angle BAC is measured, AC being a horizontal line.

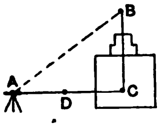

FIG. 277 (*f*).

Case I. *When AC can be measured*

$$BC = AC \times \tan BAC.$$

Case II. *When C is inaccessible* find AC by one of the methods of Art. 277 (*c*) and proceed as in Case I.

Practical hints. Note that the resulting height *BC* is the height of *B* above the *supporting axis of the telescope.* The height of *B* above any point on the ground can then be found by the usual methods of leveling or by the vertical-angle method just explained. There are other methods of solving the problem, one of which is to establish a point in *AC* as *D*; from the triangle *ABD* find *AB* and then determine *BC*. This method is simple in theory but difficult in practice, i.e., it is difficult to measure the angle *BDC*. (Why?)

278. Perpendiculars and Parallels.

(*a*) *To establish a perpendicular to a given line from any given point.*

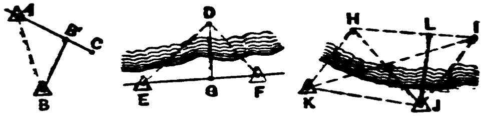

FIG. 278 (*a*).

Case I. *When the given line AC and the given point B are both accessible.* At *any* point *A* on *AC* measure *CAB*, and at *B* lay off $ABB' = 90° - CAB$.

Case II. *When the given line EF is accessible but the given point D is inaccessible.* At *any* two points *E* and *F* measure *DEF* and *EFD*. Then $EDF = 180° - (DEF + EFD)$ and $EG = [EF \times \sin EFD \times \cos DEF] \div \sin EDF$.

Case III. *When the given point J is accessible but the given line HI is inaccessible.* Choose some point *K*, and with *KJ* as a base-line find the angle *JHI* by the method of Art. 277 (*e*), p. 223. Turn off $HJL = 90° - JHI$.

(*b*) *To establish a line through a given point parallel to a given line.*

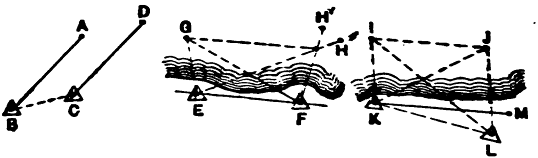

FIG. 278 (*b*).

Case I. *When the given line AB and the given point C are both accessible.* At *any* point *B* measure *ABC*, and at *C* lay off $BCD = 180° - ABC$.

Case II. *When the given line EF is accessible but the given point G is inaccessible.* At *any* two points *E* and *F* measure

GEF and *EFG*. At *F* lay off *EFH'* = *GEF*, and at *E* lay off *FEH* = *EFG*. The intersection of *EH* and *FH'* is a point in the required line through *G*.

Case III. *When the given point K is accessible but the given line IJ is inaccessible.* Choose some point *L*, and with *KL* as a base-line find the angle *IJK* by the method of Art. 277 (*e*), p. 223. Lay off *JKM* = *IJK*.

CHAPTER XIX

USE OF THE LEVEL AND THE LEVELING-ROD

MANY of the directions for setting up the transit and for using its telescope (Chapter IX) apply equally well to the use of the level, so that only a few additional suggestions are necessary. Before undertaking any work in leveling, however, the student should practice reading the leveling-rod following the directions given in this chapter. Special attention is called to the "danger-points" or places on the rod where mistakes in reading are most likely to be made.

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279. Setting up the Level. In setting up the level the sole object is to get the axis of the telescope level, and to keep it there, so that the line of sight will always revolve in the same horizontal plane. The process is easier than that of setting up a transit, since it is not necessary to center the instrument over any given point. Plant the tripod legs in the ground so that the head of the instrument is approximately level as judged by the eye. Turn the telescope until it is above two *diagonally* opposite thumb-screws, and bring the bubble to the center of the tube by turning the screws (see p. 88). Turn the telescope until it is over the other pair of screws and level up as before. Turn it back until it is over the first two screws and repeat the process, first over one pair of screws and then over the other, until the bubble stays in the center of the telescope no matter in what direction the latter may be pointed.

Practical hints. (1) The bubble will move in the direction in which the left thumb is turned.

(2) Observe the first four hints for manipulating the tripod given on p. 87, Art 114, and the hints for using the thumb-screws given on p. 88, Art. 114.

(3) Observe the suggestions as to *where* to set up given in Art. 322, p. 251, and note the errors (Art. 349) which arise from setting up carelessly. It is especially important that the level should not settle.

(4) If the level is out of adjustment, it may be impossible to set it up so that the bubble will remain in the center for all positions of the telescope, but errors from this source may be eliminated by making backsight and foresight equal.

280. Use of the Telescope. Suggestions for focusing and sighting have already been given on page 85. Additional precautions are necessary in leveling, for the accuracy of the work depends to a large extent on keeping the line of sight truly horizontal. Anything, therefore, which tends to throw the telescope in the least out of level should be avoided.

The whole instrument, and especially the telescope, should be handled with a delicacy of touch that will preclude any serious disturbance, such, for example, as that which may come from the downward pressure of the hand upon the focusing-screw while sighting through the telescope.

(*a*) *An experiment.* The results of the following simple experiment will surprise even experienced levelmen who have never tried it, and will impress upon the beginner the great sensitiveness of a good level. Carefully set up and level the instrument. (1) Rest the hands rather heavily upon two legs of the tripod and watch the bubble "move out," returning to place when the hands are removed. (It will often move out three or four divisions, which might make an error of 0.15 ft. in a rod-reading if the rod were held 300 ft. from the instrument.) (2) The same results will follow if the weight of the hand rests on the focusing-screw of the object-glass or the eyepiece at the rear of the telescope. *Moral. Keep the hands off the instrument* after it is focused on the leveling-rod. (3) Place several fingers on one end of the glass bubble-tube and observe how the warmth of the fingers will "draw" the bubble. *Moral.* Very accurate work cannot be done when the sun shines on one end of the tube and not on the other.

281. Reading the Leveling-rod. There are two methods, viz.: (1) The rodman moves the target up or down until a signal from the levelman tells him that the center of the target is exactly covered by the horizontal hair in the level; (2) The levelman observes the division on the rod exactly covered by the horizontal hair, and gets the reading directly through the telescope without the use of a target.

The method of reading the target depends upon whether the target is provided with an ordinary *scale* or with a *vernier.* Both cases are explained in the succeeding pages.

Note. In explaining how to read the leveling-rod (Arts. 282–283) it is taken for granted that the target is used. Once having learned to take target readings correctly, it is an easy matter to read the rod directly through the telescope.

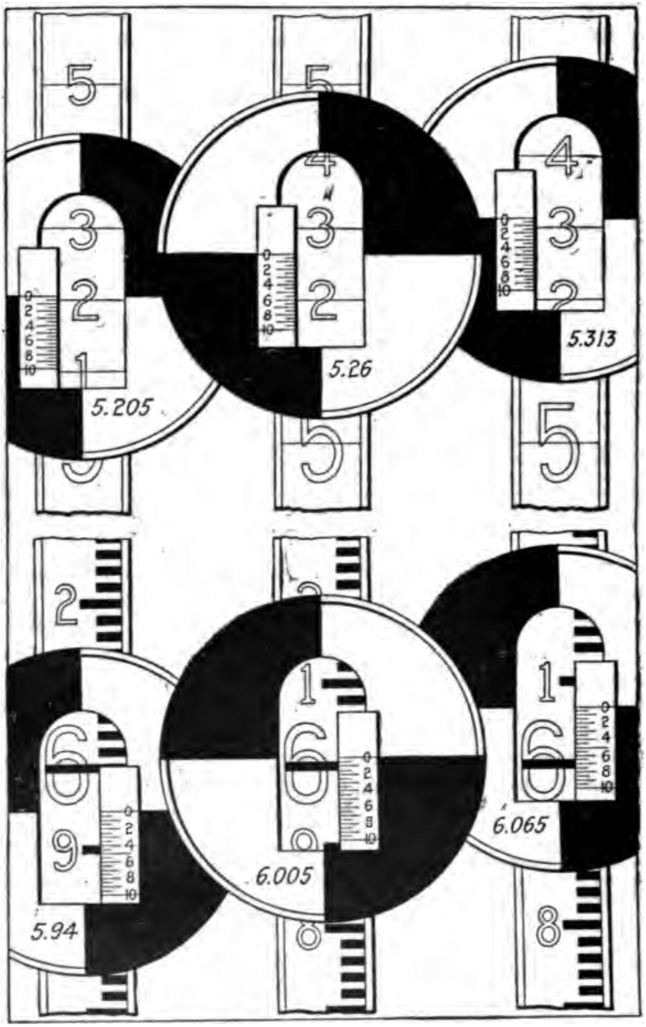

Fig. 282.

282. Targets with Ordinary Scales. The figures on the opposite page illustrate two forms of the *Philadelphia* rod. The scales on all six targets are exactly alike, i.e., each is a tenth of a foot, subdivided into hundredths and half-hundredths (0.005 ft.). In the three upper figures the smallest division on the rod is *one tenth* of a foot, while in the three lower figures each tenth is further subdivided into ten parts, or hundredths of a foot. This latter subdivision is for reading without a target. In target-readings *no attention is paid to hundredths of a foot on the rod.* In the three lower figures readings are taken to the *upper edge of a black mark* instead of to a fine line, as in the upper figures: this is the only difference between the two forms of rods so far as target-readings are concerned. Note that the target-scales *are not verniers.*

(*a*) *To read the rod.* (1) Note the nearest *foot below* the zero or target-center.

(2) Note the nearest *tenth of a foot* below the target-center.

(3) Note the division on the scale opposite the tenth of a foot observed in (2).

(4) If no division on the scale coincides with the tenth of a foot-mark on the rod, note the nearest division on the scale *above* the mark on the rod and estimate the thousandths of a foot thus: 5.31 + 0.003 (estimated) = 5.313. The method of getting the correct reading for each rod (printed on the target) is given below.

Feet	+	Tenths	+	Hundredths	+	Thousandths	=	Reading
5	+	2	+	0	+	5	=	5.205
5	+	2	+	6	+	0	=	5.260
5	+	3	+	1	+	3	=	5.313
5	+	9	+	4	+	0	=	5.940
6	+	0	+	0	+	5	=	6.005
6	+	0	+	6	+	5	=	6.065

(*b*) *Common mistakes.* (1) To omit a zero from a reading; thus, 5.25 instead of 5.205, or 6.05 instead of 6.005, or 6.65 instead of 6.065.

(2) To omit two zeros from the reading; thus, 6.5 instead of 6.005.

(3) To take the nearest numbered division *above* the target-center instead of *below* it; thus, 6.94 instead of 5.94, or 6.105 instead of 6.005, or 6.165 instead of 6.065.

Remark. A study of the figures on the opposite page will show that the above mistakes are *natural* ones for the beginner to make.

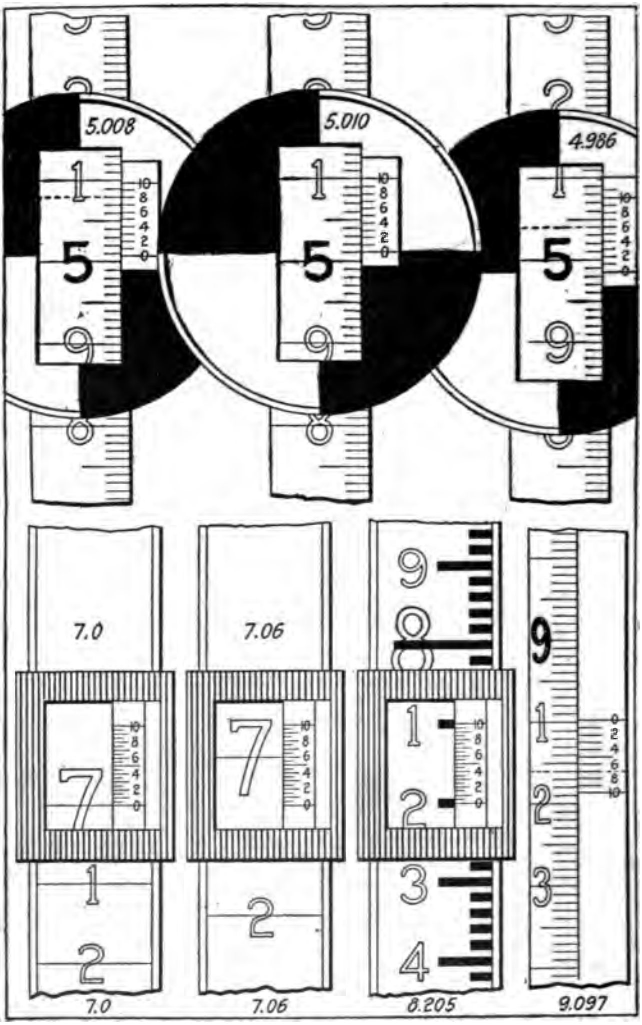

FIG. 283.

283. Targets with Verniers. Hundredths are read on the rod itself instead of on the target, as in the preceding illustration. Thousandths are read from the vernier, not estimated. The graduations on the *New York* rod (three upper figures, opposite page) do not differ from those on the second form of *Philadelphia* rod, except that hundredths of a foot are marked with fine lines. On the target, however, ten divisions correspond to nine on the rod (see middle figure), or each division is $\frac{1}{1000}$ ft. shorter than a division on the rod.

To read the rod. (1) Read feet, tenths, and hundredths by observing the position of the vernier zero. (2) Read *thousandths* by observing what mark on the vernier coincides with *any* mark on the rod.

Feet	+	Tenths	+	Hundredths	+	Vernier	=	Reading	Mistaken readings
5	+	0	+	0	+	8	=	5.008	5.08 or 5.8
5	+	0	+	1	+	0	=	5.010	5.10
4	+	9	+	8	+	6	=	4.986	5.986 or 5.16

284. Extended Rods. When a reading is over 7 ft. the front half of the rod is extended up, and the reading is taken from the back of the lower half. Each of the first three lower figures on the opposite page represents the back of a *Philadelphia* rod, while the fourth figure represents a *New York* or *vernier* rod. The graduations on the back of the rod are numbered from the top *downwards*, or in just the reverse direction from the numbering on the front of the rod. Consequently, in reading the back of the rod, observe the nearest foot and tenth *above* the zero of the scale instead of below it, i.e., read *downward* instead of upward; this is the only difference between reading the back and the front of a leveling-rod.

(*a*) *To use a high rod.* The first step is to set the center of the target exactly opposite the 7-ft. mark on the front of the rod. Before extending the rod the reading on the back should be exactly 7 ft. (see first figure), and the target should be exactly 7.0 ft. from the base of the rod. (See Art. 286 for precaution.) If the front half of the rod with the target fixed at 7 ft. is then raised, say, 0.06 ft., the reading on the back should be 0.06 (see second figure). The reading for the third rod is 8.205. (Mistaken readings 8.25, 7.25.) For the vernier rod the correct reading is 9.097. (Mistaken readings 9.97, 9.107, 8.107.)

(*b*) *Remark.* The chief source of error in reading a "high rod" is to read the back of the rod upward instead of downward. The *Boston* rod (a form not shown here) must be inverted end for end when used as a high rod. On many rods the last graduation on the front and the first on the back is 6.5 ft. instead of 7 ft.

(*c*) *Note.* The upper half of leveling-rods should be graduated so that, if desired, the extended rod can be read directly through the telescope. Some rods are jointed near the bottom so that practically the whole front half of the rod is extended for a high rod. In this case the surveyor can graduate the *back* of the upper half for himself if the maker has failed to do so.

285. The Difference between Methods of Reading the ordinary rod and a vernier rod may be summed up as follows:

Target with a scale	Target with a vernier
(1) It is not necessary to observe the *hundredth* of a foot on the rod; take the nearest tenth *below* the zero of the scale (or *above* it for extended rod).	(1) The hundredth of a foot-mark on the rod just *below* the zero of the vernier (or *above* it for extended rod) must be observed.
(2) Thousandths are estimated.	(2) Thousandths are read from the vernier.

286. To Test a High Rod-reading. When using any rod for the first time the following test should be made: Clamp the target at the highest mark on the front of the rod, set the zero at the corresponding mark on the back of the rod, measure from the base of the rod to the target-center with a steel tape or another leveling-rod, and see if this measurement agrees with the reading (6.5 ft. or 7.0 ft.). If it does not agree, make a fine scratch on the face of the rod to indicate where the target should be set. Set the target at this scratch, extend the rod to any reading, clamp, measure to the target-center, and compare readings as before. Repeat this last step two or three times; it not only shows exactly where to set the target, but it settles all doubt as to how the rod is read when extended. If an error is found, attach a marked tag to the rod or mark the rod in some other way as follows: "The target on this rod should be set at 6.995 instead of 7.0 ft. for all high rod-readings."

Remark. The error described above is very often found in leveling-rods, especially if they have been used for some time. It is often caused by letting a high rod down "on the run," thus jamming the small piece at the upper end of the rod, and gradually knocking it upward. Out of a large number of school rods examined by the author over 75% contained this error.

A second test. Ascertain, by measuring with a tape, the reading at which the scale or vernier on the *back* of the rod should *be set in order that* readings taken on the front of the rod *above* the joint shall be correct. It is not safe to assume that when the joint is closed in perfect contact the readings above it will be correct. (Why is it better to have this joint near the top of the rod than near the bottom?)

287. Use of the Hand-level. Hold the level to the eye and raise or lower the end until the reflection of the bubble is exactly bisected by the cross-hair. The horizontal line of sight thus established is at the elevation of the eye. There are two ways of leveling with a hand-level.

(1) *To stand erect.* In this case the "height of instrument" is found by adding to the elevation of the ground upon which the observer is standing the height of the eye above the ground. Any other point which the line of sight strikes is of the same elevation as the eye. Selecting such a point and moving forward to it, the observer stands with his feet on a level with it, and establishes a new "height of instrument" as before.

Note. The above method is used when leveling *up* hill. In leveling down hill the process is reversed. The leveler moves down each time to a point where he can stand and have the horizontal line of sight strike the point which he just left. He can then find the elevation of the ground where he is standing by subtracting the height that his eye is above the ground from the elevation of the point sighted at.

(2) *To rest the level on a rod* of known height, say five feet. This is more accurate than the preceding method, as the height of instrument is always exactly five feet above the ground upon which the rod rests.

The hand-level is intended to give only approximate results, and is used mostly for locating contours. In this work it is usually necessary to start with the hand-level at the elevation of one of the required contours. This is done by backsighting on a bench-mark or point of known elevation, as explained in the following illustration.

Illustration. It is required to locate contours at five-foot intervals. (See Chapter XXVII.) The elevation of the bench-mark nearest to the starting-point is 93.684 ft. The hand-level, held on a five-foot rod, is moved up or down hill until the line of sight strikes 1.3 ft. (or 95–93.7) from the base of a rod held on the bench-mark. Then the height of the line of sight must be 95 ft., and the elevation of the ground at the base of the rod supporting the hand-level is 95 – 5 = 90 ft.

CHAPTER XX

LEVELING—THEORY

IN this chapter the *theory* of leveling is explained without entering upon the practical details of the field work. Leveling terms are defined and forms given for level notes. Practical directions and suggestions for field work will be found in the next chapter.

288. Leveling. A branch of surveying the object of which is:

(*a*) To ascertain the difference of elevation between two or more points.

For example, to find how much a given point on the earth's surface is higher or lower than another given point.

(*b*) To establish one or more points at the same or at different elevations with respect to a fixed point.

For example, to establish points all on a level or on a slope for grading, or to fix the heights for different parts of a structure.

289. Datum. In any given survey all elevations are referred to some common *datum.* This datum is usually an imaginary *level* surface, all points of which are assumed to have an elevation of *zero.* The elevation of any other point is its distance above or below this surface or datum.

290. Datum Line and Datum Plane are two terms whose meanings, though obvious, are less general and comprehensive than the simple term datum. In fact, as pointed out on page 1, a level surface is not a plane surface but a curved surface, hence a level line is not a straight line but a curved line. This distinction is not often made, however, in plane surveying, since curvature of the earth's surface is ignored.

The mean sea-level affords a convenient datum the world

and elevations are commonly given as so much above or
v sea-level. It is often more convenient, however, to
ne some other datum, especially if only the *relative* eleva-
of points are required.

1. Bench or Bench-mark. A fixed point of reference
e elevation with respect to some assumed datum is known;
used either as a starting-point for leveling, or as a point
which to close as a check. In a survey of any extent
are usually a number of such bench-marks, established
respect to the same datum and at points where they are
needed.

stration. A city establishes an elaborate system of bench-marks so
y starting at any one of them a surveyor can determine elevations
espect to the same datum that the city uses in its engineering work,
it is more convenient to end at a different bench from that at which
rted, he will still have a check upon his work.

ark. The elevation of the *first* bench-mark is sometimes arbitrarily
ed, as, for example, an even hundred feet, but this is equivalent to
ing arbitrarily a datum one hundred feet below that particular
-mark instead of assuming the sea-level as the datum. This would
ne only when the *relative* elevations are required regardless of the
ions above sea-level.

2. Stations. Points whose elevations are to be ascer-
d or points that are to be established at a given elevation.
e that a station is where the leveling-rod is held and *not*
the level is set up.

ark. In transit surveying the transit is usually set up directly over
ion. In leveling, on the contrary, the level is practically never set
er a station, but is placed at any convenient point from which the
s can be seen.

3. Plane of Sight. If a level is in adjustment and prop-
et up, the *line of sight* will revolve in a *horizontal* plane,
t will always lie in a horizontal *plane of sight* no matter in
direction the telescope is pointed. (See Fig. 294 (*a*).)

ark. Since this plane is perpendicular to a plumb-line, it is tan-
o a level surface at the point where the instrument stands, but, as
y pointed out, the distinction between a level surface and a horizon-
ne is not an important one in plane surveying.

4. The Two Essential Steps in Leveling. After setting
level the first step is to ascertain the elevation of its *plane*
ht by measuring *up* to this plane from some point *whose*
ion with respect to the assumed datum is known; then the
ion of any other point may be found by measuring *down*
the plane of sight. These two steps, (1) *measuring up to*
2) *down from the* horizontal plane of sight, should be the

first conception of spirit-leveling, for it is one most helpful in practice, and comprises practically the whole theory.

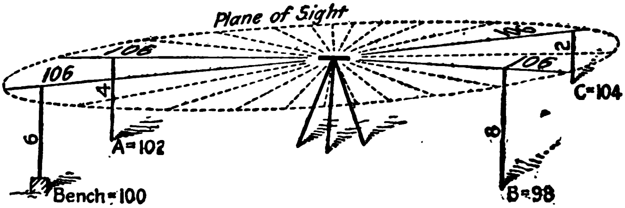

FIG. 294 (*a*).

(*a*) *Illustration.* Let the elevation of the bench-mark in Fig. 294 (*a*) be 100 ft. above some assumed datum. It is required to ascertain the elevations of the ground at *A*, *B*, and *C* with respect to the same datum.

The first step after setting up the level is to ascertain the elevation of its horizontal plane of sight. The line of sight strikes a leveling-rod held on the bench 6 ft. *up* from the base of the rod. The elevation of the plane of sight is therefore 100+6=106 ft.

If the telescope is then directed to *A* and it is 4 ft. *down* from where the line of sight strikes the rod to the base of the rod, then the ground at *A* must be 4 ft. below the plane of sight, or the elevation at *A* is 106−4=102 ft. Likewise if the ground at *B* is 8 ft. below the plane of sight, its elevation at *B* is 106−8=98 ft.; if at *C*, the ground is 2 ft. below the plane of sight, its elevation at *C* is 106−2=104 ft. Thus the elevations of any number of points may be obtained.

Remark. The leveling-rod is graduated in only one direction, from the bottom up, and so there is really no difference between measuring *up* and measuring *down*, but it is well to think of measuring *up* to ascertain the elevation of the plane of sight, and measuring *down* to ascertain the elevation of points on the ground.

295. Height of Instrument. The elevation of the plane of sight with respect to the assumed datum. Thus, for example, 106 ft. is the height of instrument in Fig. 294 (*a*).

Remark. Notice that height of instrument does *not* mean the height of the telescope above the ground where the level stands, nor does it mean its height above the last station backsighted upon unless, as is rarely the case, the elevation of this last station is the same as the assumed datum.

296. Backsight. A sight taken on a rod held at a point of *known* elevation to ascertain how far the plane of sight is *above* that point, thus establishing the *height of instrument* with respect to the assumed datum. For example, the sight on the bench, Fig. 294 (*a*), is a backsight.

Remark. Backsighting is equivalent to measuring *up* from the point of known elevation to the plane of sight.

297. Foresight. A sight taken on the rod held at a point of *unknown* elevation to ascertain how far that point is *below* the *plane of sight,* thus determining the elevation of that point with respect to the assumed datum. Thus, for example, in Fig. 294 (*a*) sights on A, B, and C are foresights.

Remark. Foresighting is equivalent to measuring down from the plane of sight to the point whose elevation is required.

Confusion of terms. The terms backsight and foresight are unfortunate terms because they imply direction. They have nothing whatever to do with direction. A backsight is not necessarily on a point behind the level, and a foresight may be anywhere in front, to one side, or behind, as shown in Fig. 294 (*a*), where all the sights except that on the bench are foresights.

An effort has been made to avoid this confusion of terms by adopting the term *plus sight* in place of backsight, since the backsight reading is always *added* to a known elevation, and *minus* sight in place of foresight, because the foresight reading is always *subtracted* from the height of instrument. The chief objection to this change is that a *minus* foresight already has one meaning, i.e., a foresight taken to some point *above* the plane of sight, as occasionally happens, for example, in tunnel work or in leveling for a line of overhead shafting.

It will not be necessary for the beginner to ask the usual question: "How can you tell which is a *backsight* and which is a *foresight?*" if he will keep in mind that the object of backsighting is to establish the height of instrument (plane of sight), while in foresighting the object is to ascertain the elevation of some point.

298. Abbreviations. B.S. for *backsight,* F.S. for *foresight,* H.I. for *height of instrument,* El. for *elevation* of a station, T.P. for *turning-point* (see Art. 301), B.M. for *bench-mark,* and Int. for *intermediate stations* (Art. 300).

299. Theory of Leveling. (*a*) From what has preceded it is seen that the two steps in leveling for any one set-up are: (1) To find how far the plane of sight is *above* a point of known elevation by adding to that elevation the backsight rod-reading, or

$$(1)\quad \text{El.} + \text{B.S.} = \text{H.I.}$$

(2) To find how far *below* this plane of sight points are whose elevations are required, by subtracting foresights from the elevation of the plane of sight (i.e., height of instrument), or

$$(2)\quad \text{H.I.} - \text{F.S.} = \text{El.}$$

(*b*) *Note.* When the points are above the plane of sight the above rules still hold good provided minus signs are given to the sights. Thus, for example, a point on the ceiling of a room is used as a bench-mark, and its elevation above some assumed datum is known to be, say, 112 ft. The leveling-rod, held bottom end up for convenience, gives a backsight reading of 6 ft., or the plane of sight is six feet below the ceiling, or its elevation is obviously 106 ft., i.e., $112 + (-6) = 106$. Suppose that the foresight reading to some point just below the ceiling is 5 ft. (rod bottom end up), then the elevation of this point is $106 - (-5) = 111$ ft.

Minus sights seldom occur in leveling except in special kinds of work.

(*c*) If all the elevations required could be found from one set-up of the level, no further explanation of fundamental principles would be required, but even when the level is frequently moved the work consists simply in repeated applications of the two steps in a sort of endless chain. When it becomes necessary to move the level it may be set up in a new place, the *new* height of instrument ascertained by backsighting on a point whose elevation was found from the preceding set-up, and from this new height of instrument the elevation of new points may be found. Thus the process may be continued indefinitely. The point to be emphasized is *that for each new set-up a new height of instrument must be ascertained.*

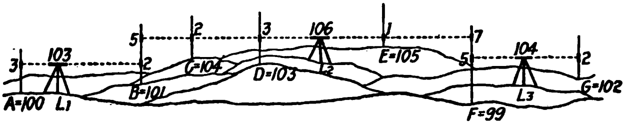

FIG. 299 (*d*).

(*d*) *Illustration.* In Fig. 299 (*d*) it is not intended to represent stations in a straight line,—they may be anywhere to one side or the other of the level as in the preceding figure, p. 236. Let the horizontal broken lines represent *planes* of sight. Let the target-readings at each station be indicated by the number at the point where the line of sight (dotted line) strikes the rod. Either the elevation of the first station must have been previously determined, or, if simply *relative* elevations are required as is often the case, it may be *assumed* as any convenient number, say 100.

Then $100+3=103=$ H.I. when level is at L_1 $103-2=101=$ El. of B.
$101+5=106=$ H.I. " " " " L_2 $106-2=104=$ El. " C
$106=$ H.I. " " " " L_2 $106-3=103=$ El. " D.
$106=$ H.I. " " " " L_2 $106-1=105=$ El. " E.
$106=$ H.I. " " " " L_2 $106-7=\ 99=$ El. " F.
$99+5=104=$ H.I. " " " " L_3 $104-2=102=$ El. " G.

(*e*) *Remark.* The two steps in leveling are not likely to be forgotten if the student will form a mental picture similar to Fig. 294 (*a*) or Fig. 299 (*d*). This is much better than depending upon the memory to retain the formulas given at the beginning of this article. Always fall back on the original conception of leveling, p. 235: (1) getting the distance of a plane of sight *above* some point of known elevation, and (2) getting the distances of points of unknown elevation *below* that plane.

300. Intermediate Stations. A point taken for the sole purpose of ascertaining its elevation, or for the purpose of establishing it at a given elevation. Thus, in Fig. 299 (*d*), *C*, *D*, and *E* are intermediate stations.

Remark.—The distinguishing characteristic of an intermediate station is that only one sight is taken on it,—a *foresight.*

301. Turning-point. A point upon which a backsight is taken in order to establish the height of instrument. Thus, in Fig. 299 (*d*), *A*, *B*, and *F* are turning-points.

(*a*) *Remark.* A turning-point is the only point upon which a backsight is taken, and this is its distinguishing characteristic. It probably got its name, however, from the fact that *two* sights are taken on it, a foresight from one set-up and a backsight from the next set-up; it thus acts as a turning-point without which no relation could be established between the *two planes of sight* corresponding to the two set-ups.

Either a regular station or a bench-mark may be used as a turning-point, or, some temporary point which is neither a regular station nor a bench-mark may be used.

(*b*) The first and last points sighted at are usually considered turning-points. Thus *G* is a turning-point on the assumption that if the work were continued a backsight would be taken on it. *A* is a turning-point because a backsight was taken on it; moreover, a foresight must have been taken at some previous time in order to ascertain its elevation. (Assuming its elevation would be equivalent to foresighting.)

302. Importance of Turning-points—Compared with Intermediate Stations. An error made at a turning-point affects every succeeding elevation, but an error at an intermediate station affects the elevation of only that station.

Illustration. Suppose that in Art. 299 (*c*), Fig. 299 (*d*), the backsight at *B* is read by mistake 6 instead of 5. The new height of instrument would become $101+6=107$, and the elevations of all the remaining points would be one foot too high. On the other hand, should a mistake be made at *C*, reading 3 instead of 2, the elevation of *C* would become $106-3=103$, or a foot too low, but since there is no backsight at *C* (i.e., no height of instrument is ascertained from *C*) none of the other elevations would be affected.

303. Forms of Notes. The forms for level notes differ somewhat for different kinds of work. Standard forms are given in this article. Study these forms in connection with Fig. 299 (*d*). The six columns correspond to the six columns on the left-hand page of the field-book, the right-hand page being used for descriptions of stations and for other notes.

FORM A FOR LEVEL NOTES.

(Left-hand page.) (Right-hand page.)

			Foresight.			
Sta.	B.S.	H.I.	T.P.	Int.	El.	
A	3.0				100.	
		103.0				Descriptions of stations go on this page. All bench-marks and important stations should be so carefully described that they can be found at any time without difficulty.
B	5.0		2.0		101.0	
		106.0				
C				2.0	104.0	
D				3.0	103.0	
E				1.0	105.0	
F	5.0		7.0		99.0	
		104				
G			2.0		102.0	
	13.0		11.0			
	11.0	Check				
	2.0					

(*a*) *Comments on Form A.* In this form of notes, and in all other forms as well, it is necessary to enter the foresights and backsights during the work in the field. The heights of instrument and the elevations of different stations may be left to be worked out in the office unless elevations are needed in the field for checking the work or for other purposes. Even then it is necessary many times to work up only the notes for turning-points.

In this form each H.I. occupies a line by itself. There is no great advantage in this except that in looking over the notes it is more evident, to the beginner at least, which H.I. was used in calculating any given elevation. Thus all elevations recorded between any two successive values of H.I. were found from the upper H.I. if the notes read down the page; from the lower H.I. if the notes read up the page.

Notice also that the foresights on turning-points and the foresights on intermediate stations are kept in separate columns. This is so that the foresights on turning-points may be added together by themselves in checking the notes by the method of Art. 304. The advantage thus gained is to a great extent offset by the additional precautions necessary to avoid entering the foresights in the wrong column.

FORM B FOR LEVEL NOTES.

(Left-hand page.) (Right-hand page.)

Sta.	B.S.	H.I.	F.S.	El.	
A	3.0	103.0		100.0	Descriptions of stations go on this page. All bench-marks and important stations should be so carefully described that they can be found at any time without difficulty.
B	5.0	106.0	2.0	101.0	
C		"	2.0	104.0	
D		"	3.0	103.0	
E		"	1.0	105.0	
F	5.0	104.0	7.0	99.0	
G		"	2.0	102.0	

(*b*) *Comments on Form B.* This form differs from form A in two respects. (1) Each H.I. does not occupy a line by itself, but is put on the line with the B.S. and El. from which it was figured. (2) Foresights are all in one column. Turning-points may be easily distinguished in the notes, as they have numbers in the backsight column while intermediate stations do not. (Art. 301 (*a*).) Many surveyors prefer to skip every other line in entering notes, thus affording more room for corrections and for the descriptions of stations on the right-hand page.

By comparing these notes with the arithmetical work on page 238, Art. 299 (*d*), the method of working out the different H.I.s in the third column and the elevations in the last column *will become* clear. Thus each B.S. is added to the elevation *recorded on the same* horizontal line to get the H.I. from which the elevations are figured for all succeeding stations up to and including the next turning-point.

This form is more compact than the preceding form and is, perhaps, the one most often used. For profile leveling Form C given below is in many respects a better form.

FORM C FOR LEVEL NOTES.

B.S.	T.P.	H.I.	F.S.	El.	Sta.
3.0	100.0	103.0			A
5.0	101.0	106.0	2.0		B
			2.0	104.0	C
			3.0	103.0	D
			1.0	105.0	E
5.0	99.0	104.0	7.0		F
			2.0	102.0	G

FORM D FOR LEVEL NOTES.

Sta.	B.S.	F.S.	Dif.	El.	Check.
A	3.0			100.0	3.0 2.0
B	5.0	2.0	+1.0	101.0	+1.0 100.0 101.0
C		2.0	+3.0	104.0	
D		3.0	−1.0	103.0	5.0 7.0
E		1.0	+2.0	105.0	−2.0 101.0
F	5.0	7.0	−6.0	99.0	99.0
G		2.0	+3.0	102.0	

(*d*) *Comments on Form C.* This form differs from the preceding form in the order of arrangement of the columns, and in having a separate column for elevations of turning-points. It is not as common a form as the others, but in many respects it is more convenient. Its chief advantage is: Quantities that are to be added or subtracted are adjacent. Thus corresponding quantities in the first two columns are added to obtain a corresponding quantity in the third column, and from quantities in the third column are subtracted quantities in the fourth column to obtain quantities in the fifth column. It is also convenient to have stations and their elevations in adjacent columns.

(*e*) *Comments on Form D.* This form is entirely different from the other three forms. The first three columns are filled out in the field in the usual manner. The column of differences is found by subtracting each F.S. from the preceding F.S., except in the case of the first F.S. after each turning-point, and this is subtracted from the B.S. on that point. The elevations in the fourth column are found by adding *algebraically* each difference to the preceding elevation. The checks in the last column are found by subtracting from the B.S. on one turning-point the F.S. on the next turning-point, and adding this difference algebraically to the elevation of the first in order to check the elevation of the second.

This form is not much used. Its chief advantage is the check afforded for the elevation of each *intermediate* station. There is no such check in the other three forms. For this reason this *method of differences* is especially useful in working up notes for profile leveling, since the computation for every point on the profile is checked once and turning-points are checked twice.

Another form of notes is shown on p. 256, where the method of keeping level notes is illustrated by several examples from practice.

304. Rules for Checking Level Notes. (*a*) Add all backsights together; add all *turning-point* foresights together; the *difference* between these two sums should equal the difference between the elevations of the first and last stations. The foresight on the *last* station should be included in the sum of foresights although there is no corresponding backsight. The last station *is* a turning-point in the sense that should the work be continued a backsight would be taken on it (Art. 301(*b*), p. 239). In leveling around a closed figure, moreover, the first and the last station coincide, and the difference between the two sums would equal zero were it possible to do perfect field work.

(*b*) Notice that this is a check on office work but not on field work. It enables one to detect numerical mistakes made in working out heights of instrument and elevations of turning-points, but it does not disclose mistakes in using the level, reading the rod, or entering the rod-readings. Moreover, it is not a check on the computed elevations of intermediate stations, because no backsights are taken on such stations.

(*c*) *Illustration.* In form A, sum of backsights = 13; sum of turning-point foresights (including that of G) = 11; thus, $13-11=102-100=2$ (check).

(*d*) *Note.* Another method of applying the same rule: (1) Add the sum of all the backsights to the elevation of first station. (2) Subtract from this result the sum of the *turning-point* foresights: the result should be the elevation of the last station. Thus, $(100+13)-11=102$ (check). See also the level notes on page 257 for another illustration.

305. Differential Leveling. Leveling in which the object is simply to find the difference in elevation between two or more points without regard to distances between the points.

306. Profile Leveling. Leveling in which the object is to get the elevations at *known distances apart* and thus obtain the profile of the surface along a given line. A profile should show all elevations and depressions along the line as if seen from a side view.

307. Flying Levels. A line of levels to determine approximate elevations, run with more rapidity and less precision than usual.

308. Cross-levels. Levels taken to show heights on either side of the main line. Used in railway and highway work to determine cross-sections at right angles to the general direction of the road-bed.

309. Reciprocal Leveling consists in finding the difference of elevation between two points by *two* sets of observations as follows: (1) Sights are taken on both points from a level set up *near one* of them; (2) then the level is set up near the *other* point, in such a position that the longer and shorter sights on the two points are about equal in length to the longer and shorter sights respectively of the first set-up. The mean of the two results obtained from the first and second set-ups should be the true difference of elevation. This process eliminates error due to lack of adjustment of the level, and in precise leveling corrects for curvature and refraction.

310. Establishing Points at Given Elevations is the reverse of the process explained in the preceding articles. It is in leveling what "staking-out work" is in transit surveying, and in a great deal of construction work the two must be combined. Setting grade-stakes and fixing heights for different parts of steel and masonry structures constitute a large part of the leveler's work. For a detailed explanation of such work see Chapter XXIII.

(*a*) *Grade.* The degree of inclination from the horizontal, as, for example, a rise or fall of one foot measured *vertically* for each 100 ft. measured *horizontally* (a *one per cent* grade), or a grade of 2 ft. per 100 ft. (2% grade), or of 5 ft. per 100 ft. (5% grade), or no rise or fall (a level grade).

(*b*) *Grade-rod.* A *calculated* rod-reading found by subtracting the *required* elevation of a point from the height of instrument, i.e., it is the distance to measure *down* from the line of sight to establish a point at a given elevation.

Illustration. In Fig. 310 (*b*) it is required to drive a stake at each of the points A, B, and C, so that the tops of the stakes will be respectively 98 ft., 102 ft., and 104 ft. above some given datum. The height of instrument, found by backsighting on some point of known elevation, is 108 ft. The *grade-rod* for A is therefore 10 ft., for B 6 ft., and for C 4 ft.

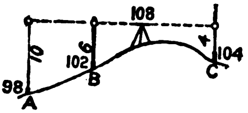

Fig. 310 (b).

311. Trigonometric Leveling. Leveling by means of vertical angles and horizontal distances. The theory is based on the trigonometric properties of a right triangle.

FIRST METHOD

(*a*) *Illustration.* In Fig. 311 (*a*) let T be the center of a transit telescope. After measuring the horizontal distance BT and the vertical angle BTA, the height BA is found by trigonometry, $BA = BT \times \tan BTA$. In like manner $CD = TD \times \tan CTD$. The distance TE from the center of the supporting axis to the ground having been measured, the relative elevations of A, E, and C are determined. Thus the elevations of any number of points above or below T (or E) may be found.

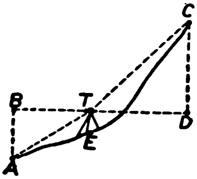

FIG. 311 (*a*).

Remark. The point T is the intersection of three axes, viz., (1) through the line of sight, (2) through the center of the supporting axis, and (3) through the plumb-bob line. This method is the same as that of Art. 277 (*f*), p. 223.

(*b*) If the elevations of points are required with reference to some datum, first find the elevation of T (i.e., the height of instrument) by backsighting on some point of known elevation, as, for example, some bench-mark.

(*c*) When for any reason the *horizontal* distance from T to C (i.e., TD) cannot be measured, TD may be found by triangulation, treating C as an inaccessible point (see 277 (*c*), p. 222), or in some cases it may be possible to measure TC and then proceed as explained in the next two illustrations.

SECOND METHOD

A second method often used, especially in stadia work, is one in which the hypothenuse is measured instead of the base.

(*d*) *Illustration.* Let T in Fig. 311 (*d*) represent the center of a transit, and a a point on the ground underneath T. Let a' represent a target set as far above the bottom of the rod as T is above a (i.e., $a'G = aT$). Then $Ta' \times \sin a'TD = a'D = Ge$, or the elevation of G above a

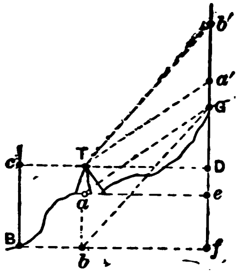

FIG. 311 (*d*).

(*e*) *Illustration.* Let B represent some point of known elevation *not* under the instrument, as, for example, some bench-mark. With the telescope horizontal set the target at c. Let b be an imaginary point underneath T, of the same elevation as B (i.e., $Tb = cB$). Without changing the target hold the rod at G and let b' represent the target for this position of the rod. Then $b'G = Tb = cB$ and $Tb' \times \sin b'TD = b'D = Gf$, or the elevation of G above B.

(f) *Caution.* Since all the methods illustrated are in common use, it is very necessary in keeping field notes to make clear what point is taken as a reference-point, T the center of the supporting axis, a a point on the ground under the instrument, or B some point not under the instrument; also whether the base or the hypothenuse is measured.

312. Leveling with the Gradienter. The gradienter is a tangent-screw attached to the supporting axis of the telescope. By means of a micrometer-head small vertical angles may be measured, or large angles may be read with the vertical arc supplemented by the micrometer-screw. As a rule the value of the screw-thread is such that one complete revolution of the screw will move the horizontal hair over a space of one foot on a vertical rod at a distance of 100 ft., so that angles are measured *in terms of the tangent.* If the distance to a rod at any point is known, and the vertical angle is found by the gradienter, the elevation of the point may be determined by trigonometry, as already explained. The distance from the transit to the rod may be found also with the gradienter by turning the screw through one complete revolution. If it passes over one foot of the rod, the latter is 100 feet away, 1½ ft. 150 ft. away, and so on in the same proportion. This method of obtaining distances is similar to the stadia method. Thus, if necessary, the entire work of finding the elevation of a point may be done with the gradienter alone, without using a tape. This is also true of the stadia.

313. Barometric Leveling. Since the barometer is an instrument for measuring atmospheric pressure and since this pressure is less for high altitudes than for low altitudes, differences in elevation may be estimated by differences in barometer-readings. Roughly speaking, the average reading of the barometer at sea-level is 30 inches and the barometer falls *about* one inch for every 900 ft. of ascent above the sea-level.

If the decrease in the density of the air were *exactly* uniform and if the pressure at a given elevation were always the same, barometric leveling would give very satisfactory results.

(a) *Atmospheric sources of error.* The pressure of the atmosphere at any point is constantly changing, due to changes in meteorologic conditions. This change may amount to as much as two inches or more. In other words the elevation of the same place as indicated by the barometer at two different times may change as much as 1800 feet. For the same reason barometer-readings taken simultaneously at two different places may be exactly the same and yet the elevations of the two places be quite different. The error from this source may be largely eliminated by taking barometer-readings over a long period of time at different seasons of the year and hours of the day, and thus obtain a mean value, but this method, of course, cannot be used in ordinary surveying.

(*b*) *Temperature* also affects barometer-readings in two ways: (1) it affects the barometer itself, and (2) the weight of the air, for the warmer the air the lighter it is. In a mercurial barometer the height of the column of mercury is affected by the temperature. It is usually claimed that aneroid barometers are compensated for temperature, and though this is not strictly true it is customary to make no correction for the temperature of the barometer itself, as in the case of a mercurial barometer. In using either barometer the greatest uncertainty arises from the method of determining the average temperature of the air-column. Thus, in finding the difference in altitude between two stations, the temperature of the air taken at each station is necessarily the temperature of a stratum of air near the earth, and the common assumption that the average temperature of the air-column, upon which the difference of pressure between the two stations depends, is the average of the observed temperatures at these stations, is an erroneous one.

(*c*) *Barometric formulas.* Numerous formulas, differing chiefly in experimental constants, have been developed for establishing a relation between altitude and barometer-readings.* These formulas may be divided into two classes: (1) *statical* and (2) *dynamical.* Statical formulas assume that the air is in statical equilibrium and are more commonly used. Dynamical formulas take into account the fact that the air is more or less in motion, but require extended observations to obtain the required data. There is little difference in accuracy between the two classes of formulas.

In ordinary work those terms in the formulas which depend upon the temperature of the barometer, humidity, latitude, gradient, and other still smaller sources of error may be ignored, leaving only the mean temperature of the air to be taken into account. *Ferrel's formula*,* which is perhaps one of the best, will then reduce to

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* For a discussion of these formulas see Baker's Engineer's Surveying *Instruments, Chapter XII.*

$$\log H = \log(\log B' - \log B) + \log 60521.5[1 + .001017(t' + t - 64°)];$$

H = difference in altitude between the two points whose observations are taken, B' = observed height of the barometer at the lower station (in inches), B the height of the barometer at the upper station, t' and t the temperatures of the air at the lower and upper stations respectively.

If the mean temperature is assumed as 50°, then $t' + t = 100$ or $t' + t - 64° = 36°$. If the mean barometer reading at the sea-level is approximately 30 inches, the above equation may be written

$$H = 60521.5(1 + .001017 \times 36)\left(\log \frac{30}{B} - \log \frac{30}{B'}\right)$$

$$= 62737 \log \frac{30}{B} - 62737 \log \frac{30}{B'}.$$

Tables may be computed from the equation $A = 62737 \log \frac{30}{B}$ for different values of B, the corresponding values of A being the altitudes above a datum corresponding to a barometric pressure of 30″ (or approximate sea-level), when $t + t' = 100°$. For other than the mean temperature of 50° a correction may be applied as follows:

$$H = (A_1 - A_2)(1 + C),$$

where H = the corrected difference in altitudes of the two stations observed, A_1 and A_2 the altitudes of these stations as taken from Table I, and C a correction for various values of $t + t'$ as taken from Table II.†

For the method of using the aneroid barometer see Art. 343, p. 265.

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* See Appendix 10, *U. S. Coast and Geodetic Survey Report* for 1881.

† These two tables are given in the back of the book.

CHAPTER XXI

LEVELING—FIELD WORK

The theory of leveling, explained in the preceding chapter, is exceedingly simple, but no branch of surveying requires greater care in the field work. Accurate results can be obtained only by observing certain precautions which tend to eliminate mistakes and errors. These precautions are so well understood that the methods of field work are more uniform in leveling than in almost any other branch of surveying. It is, therefore, somewhat easier to give definite directions and suggestions, but, as in all field work, it is not desirable to lay down hard-and-fast rules, nor to follow blindly such directions as are given.

314. A Level Party usually consists of a levelman and a rodman. In rough country a third man is sometimes needed, to act as axeman. In very accurate work two rodmen may be used to advantage, and as many as five men may be needed in the party.

315. Equipment for a Level Party. (Things most likely to be forgotten are put first.)

Rodman. Tacks or brads, crayon, stakes, hatchet, tape, note-book, and leveling-rod.

Levelman. Necessary notes from transit party, datum, etc., note-book, adjusting-pin, level.*

Both men should provide themselves with hard pencils, pencil-erasers, and scribbling-pads.

316. Signals. When the levelman wishes the rodman to raise the target he holds his hand *above* the shoulder. The hand held *below* the shoulder means to lower the target. In either case the nearer the target approaches to the line of sight the nearer the hand should approach the level of the shoulder. Dropping the hand to the side means to stop moving the target. A gentle motion of the hand up or down from the shoulder means to move the target *slowly* in the direction indicated. When the target is in the correct position, bring the hands together over the head, which means "all right." Holding the right arm straight up means to "plumb the rod to the right"; the left arm means "plumb to the left."

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* Accurate work in leveling may be done with a transit provided it has a good level attached to the telescope and is kept in good adjustment, a fact which students are slow to realize. Of course the methods of work are practically the same as for the level.

Remark. Any other set of signals may be agreed upon, but they should be clear and unmistakable at several hundred feet away, and should enable the rodman to get the target into position in the shortest possible time. It is essential that the signals to move the target up or down should indicate to the rodman *about how much the target should be moved.* A common fault with levelmen is that of holding the arm horizontal and moving the fingers in such a way that the rodman cannot tell whether the target is to be moved up or down. *Avoid all shouting.*

317. Some Things to Know Before Beginning Work. (1) How to set up a level (p. 226). (2) How to use the telescope to get accurate results (p. 227). (3) How to read the rod (p. 227). (4) What points of the rod are danger-points, i.e., points where mistakes in reading are likely to occur (p. 229). (5) How to test the rod for a high rod-reading (p. 232). (6) What is meant by *datum, bench-mark, station,* and *plane of sight* (p. 234). (7) What the two essential steps in leveling are (p. 235). (8) What is meant by *height of instrument, backsight, foresight,* and *turning-point* (p. 236). (9) Why it is important to take readings with greater care on turning-points than on intermediate points (p. 239). (10) How to keep notes (p. 239), and how to check them (p. 242).

Remark. As soon as the student has become familiar with the fundamental principles and operations in leveling, he should begin a study of the sources of errors and the methods of eliminating or correcting them. (See the next chapter.) This is the quickest and surest way of learning to do accurate and reliable work.

318. Two Methods of Getting the Rod-reading. (See also p. 227.) (1) In many cases, especially at intermediate stations, the levelman can read the rod from the instrument, no target being used. (2) For turning-points, or for any station where the reading is required to the hundredth of a foot, it is usually necessary to use the target. After a target is clamped the rod should be held a *second* time to make sure that the target did not slip in the process of clamping.

Remark. It is a great saving of time to read the rod directly from the instrument; that this is accurate enough in the majority of cases is shown on p. 254.

319. The Starting-point or First Backsight. If only the elevations of stations with respect to each other are required, establish any convenient bench-mark to start with, and assume its elevation as 100 or 1000 or any convenient number great enough so that there will be no minus elevation, i.e., so that the lowest station will have an elevation *above* zero. This is equivalent to assuming a datum regardless of sea-level or of any other datum previously used.

If elevations are required with reference to some fixed datum, it is necessary before starting out to obtain the elevation of some near-by bench-mark previously established from that datum, and then to use that bench-mark as a starting-point. Or, if no bench-mark of known elevation is at hand, one may be assumed, and the levels run; afterward the true height of the assumed bench-mark can be ascertained by running levels from some bench-mark of known elevation, and the false elevation previously obtained can easily be reduced to correspond to the datum.

Illustration. In a city survey, no city bench-mark being conveniently near, a bench-mark was established on the top of a hitching-post, its elevation being assumed as 100 ft. Afterward, by running levels from the nearest city bench-mark, the elevation of the top of the hitching-post with respect to the city datum was found to be 54.6 ft. The elevations already obtained, using 100 as the assumed elevation of the post, were easily corrected by subtracting from each elevation 45.4 ft. (or 100 − 54.6). (See also Form G, p. 257.)

320. The Field-work Routine in differential leveling and in profile leveling is much the same. The chief difference is that in differential leveling all points are turning-points, while in profile leveling there are intermediate stations.

(*a*) *Differential leveling.* The levelman sets his level up at any convenient point where he can sight on a rod held at the starting-point. The rodman holds his rod, moving the target up or down according to the signal from the levelman, and clamps the target at the signal "all right." *He then holds the rod a second time,* and if the levelman again signals "all right" he reads the rod, and, as he passes the level on his way to the next turning-point, the levelman checks the reading. Both men enter the reading, each in his own note-book. A turning-point is then chosen at such a distance from the level that the length of the foresight shall be about equal to the length of the backsight just taken. The target is set as before, a second sight being taken after clamping. The levelman takes up the level and moves on. As he passes the rodman on the way to the next set-up, he checks the foresight reading, and each enters it in his note-book. This completes one step; from there on it is but a repetition of this process.

Remark. It is not always customary in practice for both men to keep the notes and read the rod, but it is very desirable that students should adopt this precaution, especially when they are new to the work.

(*b*) *Profile leveling.* The work is similar to differential leveling except that distances between stations, if not already known, must be measured, and a number of foresights from the same set-up are often taken on intermediate stations. Make it a rule after backsighting on one turning-point to foresight *immediately* on the next turning-point before the instrument has a chance to "get out"; afterward the foresights on the intermediate stations between the two turning-points may be taken.

321. Lengths of Foresights and Backsights. In ordinary work, for the best results, the lengths of sights should not exceed 300 feet nor be much less than 100 feet. On level ground the tendency is to choose turning-points too far apart, thus making the sights too long. On steep slopes the tendency is the other way, but too short sights may be avoided by choosing turning-points so that they "zigzag." (See the next article.) *For each set-up endeavor to make the backsight on one turning-point and the foresight on the next turning-point approximately equal.*

322. Where to Set Up a Level. Off to one side where it is not easily disturbed or jarred, and where, if possible, solid footing can be had for the tripod. Set up *half-way* between turning-points. In going up or down hill one can usually take a much longer sight on the down-hill side than on the up-hill side, especially if a high rod is used, but resist the temptation to do this, and keep the sights on turning-points nearly equal. This can often be done by zigzagging, i.e., working so that a turning-point on the up-hill side, while not much higher up the hill than the instrument, is far enough to one side to make the up-hill sight about equal to the down-hill sight.

On steep slopes the beginner almost invariably tries to take too long sights. Going up-hill he finds that the line of sight is too high to strike the down-hill rod; going down-hill he finds the line of sight too low to strike the up-hill rod. Result, loss of time in setting up the level again.

Nevertheless, in going down-hill, set up the level so that the line of sight will strike near the bottom of the rod. A good way to do when in doubt is to set one leg of the tripod on the ground, and taking the other two legs, one in each hand, quickly bring the telescope into about the position it will occupy when the level is set up; and then sight along the telescope to see where the line of sight will strike.

323. Choosing Turning-points. From the two preceding articles it is seen that the *location* of turning-points is largely determined either by the lengths of backsight and foresight or by the position of the level itself. In choosing the object upon which to rest the rod, keep in mind that it should be something firm and solid, easily described and found, definite and unmistakable.

(*a*) ***Remark.*** More trouble comes from choosing unsuitable turning-points than from almost any other cause. For example, why choose for a turning-point a small stone in loose soil that may settle half an inch while the rod is being held on it, and then read the rod to the thousandth of a foot? Moreover, turning-points should be found easily, so that if a mistake occurs it will not be necessary to go back to the last bench-mark, but to the last turning-point. Choose some *definite* point, so that the base of the rod will rest always on the same spot. It is well to mark the spot with chalk. Sometimes it may be necessary to drive a stake solidly into the ground and to use the top of it for a turning-point.

(*b*) *A steel turning-point* may be bought or constructed. One form consists of a steel triangular plate about one quarter of an inch thick, with its corners bent down at right angles. The rodman forces these three sharp points into the ground with his foot, and holds the rod on a round rivet-head that is in the center of the plate. The objection to using such a point is that it has to be moved at each new set-up, and thus the preceding turning-points are all destroyed.

324. Holding the Leveling-rod. It is not easy to hold a leveling-rod *plumb* and *still* at the same time, especially if the wind is blowing. If one stands squarely behind the rod and holds it with both hands, he is in the best position to hold it still. Some surveyors, however, prefer to have a rodman stand facing the *side* of the rod, since from this position he can see better whether or not the rod leans toward or away from the instrument, while the levelman can tell by means of the vertical cross-hair when it is out of plumb in the other direction. The vertical edge of a building or some other vertical line may often be used by the rodman as a guide. In addition wave the rod as directed on p. 270; *failure to observe this precaution may result in an error as great as 0.1 ft. for a single sight.*

325. Moving the Target. The signals from the levelman should indicate whether the target is to be moved much or little. When it is nearly correct the rodman will save time by moving it slowly but steadily,—not jumping first too high and *then too low.*

326. Establishing Bench-marks. (*a*) *Permanent.* If a bench-mark is to be permanent, as, for example, a city bench-mark, it should be established with great care, and referenced to several surrounding points. A careful description should be recorded, together with its elevation as obtained by several sets of levels run from other bench-marks. Such bench-marks should be established where they are not likely to be disturbed. If a stone monument is used, it should be set well below the frost-line. Some cities publish a list of bench-marks arranged according to streets.*

Illustration. City Bench-mark, Grove St., near College St., New Haven, Conn. The highest point of the middle post of the three stone hitching-posts on the north side of Grove St., in front of Sheffield Hall. Elevation is 33.006.

(*b*) *Temporary.* If a bench-mark is to be used temporarily, as, for example, during the progress of a survey, definite, solid, and comparatively permanent objects, easily described and found, may be used. A tack in a blaze on the root of a tree is a common example, the letters B.M. and the elevation being marked with crayon in a blaze on the trunk of the tree directly over it.

Other examples. Corner of a stone over a catch-basin, tops of stone hitching-posts, gate-posts, door-steps, highest point of a solid rock, or definite points on solid masonry.

(*c*) *Where to establish bench-marks.* Near the beginning and end of a line of levels; wherever in running levels work is temporarily stopped, as at noon-time or at night; at intervals of several hundred feet; wherever a branch line of levels is likely to be started; at the bottom and top of any high hill passed over; at the crossing of a railway or highway; on each bank of a stream crossed; in a city, near the street corners.

Special kinds of work call for special bench-marks. Thus in making a survey for a building they may be required at the nearest sewer-manhole, on the curb in front of the building site, and on the water-tables of adjacent buildings.

327. Sights on Intermediate Stations may be taken usually to the nearest tenth of a foot; and without using a target. In this matter one is governed by tne requirements of the work. It is foolish, for example, to hold a rod on rough, uneven, or soft ground, where a slight movement one way or the other may raise or lower the rod a tenth of a foot, and then read to thousandths or even hundredths of a foot. On the other hand, in some cases, as in giving a grade for a curb, intermediate stakes must be set with greater accuracy.

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* A *Dictionary of Altitudes*, by Mr. Henry Gannett, published by the Department of the Interior, U. S. Geological Survey, gives elevations to the nearest foot of many bench-marks throughout the United States. Stations of the U. S. Coast and Geodetic Survey and bench-marks established by the different railroads are often of great value as starting-points or as points upon which to close for a check.

328. Numbering Stations. Stations are usually numbered in order from the beginning of the work. In differential leveling, every station being a turning-point, there can be no break in the numbers. In profile leveling a turning-point is not numbered as a rule unless it happens to be a regular station on the profile line, but instead it is marked simply T.P. in the notebook, with, perhaps, a description of it on the opposite page.

In profile leveling it is often customary, especially in railway work, to let the number of a station indicate its distance from the starting-point. Thus: 0+50, 1, 1+50, 2+75, and 140+67.8 mean 50, 100, 150, 275, and 14067.8 feet respectively from 0+0 (the first station).

329. Checking Levels. On page 242 is given a method for checking notes, but the only method of checking the field work is by rerunning the levels. It is often customary, near the close of a day's work, to run a set of flying levels back over the line to the starting-point, checking on a few of the most important bench-marks, and finally on the one at the starting-point. In class work, where there are several parties, they should check on common bench-marks established by the instructor at intervals of several hundred feet.

Rerunning a line in the opposite direction will usually result in a greater discrepancy than if it is rerun in the same direction, but the mean result is more nearly correct because certain cumulative errors are eliminated.

330. How Close to Check. In practice the limit of error is determined by the requirements of the work. Results are affected by the instruments used, by the nature of the ground covered, by the time in which the work must be done, by the personal equations of the rodman and the levelman, and by the atmospheric condition.

The error depends to a large extent on the number of "set-ups," and the distance covered. In class work a party, at first, ought to run half a mile and back with a difference of less than 0.10 ft. between the assumed elevation of the starting-point and its elevation as found upon returning to it. This is assuming eight or ten set-ups each way. A difference of 0.03 ft. would indicate excellent work for the twenty set-ups, ten each way, but with good instruments and under favorable conditions students should soon be able to come within this limit using ordinary precautions. This does not mean that the absolute error is only 0.03 ft., for some of the errors are compensating, while the cumulative errors are to a large extent eliminated by running in opposite directions.

As a rough check it may be assumed that under ordinary conditions the total error E should not exceed $0.02\sqrt{s}$, where s is the total number of set-ups. If the number of set-ups is not known, it may be obtained by dividing the total distance by *twice* the average length of sight.

The customary limits of error are treated more in detail in the next chapter.

331. Errors in Leveling. In no branch of surveying is a knowledge of the sources of errors and of the methods of eliminating errors more important than in leveling. This subject is treated in the next chapter.

332. Speed. Rapid leveling consists in not wasting time. Time may be wasted in setting up in the wrong place, in setting up too high or too low to strike the rod, in slow manipulation of the level, in getting the target into position, in reading the target, and in waiting for a rodman to slowly saunter from station to station. The rodman should guess about where to move the target or the rod, and should have it there by the time the levelman is ready to sight. He should watch signals closely, and these signals should show whether the target must be moved much or little. Avoid moving the target in jumps first too high, then too low. Often a levelman can call out the approximate reading, thus helping the rodman to set the target quickly.

Remark. At first, speed is not so important as accuracy. The student should not forget, however, that in practice the two should be combined. So much depends on the requirements of the work, on the nature of the ground covered, and on the instruments used, that it is difficult to give an idea of what the speed should be. It may be said in a general way that in class work a level party working under favorable conditions should soon be able to set up and take the backsight and foresight in ten minutes, or, on an average, at the rate of six stations an hour. If the average length of sight is 200 ft., i.e., 400 ft. between turning-points, this means about half a mile an hour.

FORM E

21 Sta.	B.S.	H.I.	F.S.	El.	
B.M.16	6.102	124.318		118.216	
31			5.2	119.1	
+50			6.4	117.9	
T.P.	5.813	126.315	3.816	120.502	
32			8.9	117.4	
B.M.17			7.235	119.080	

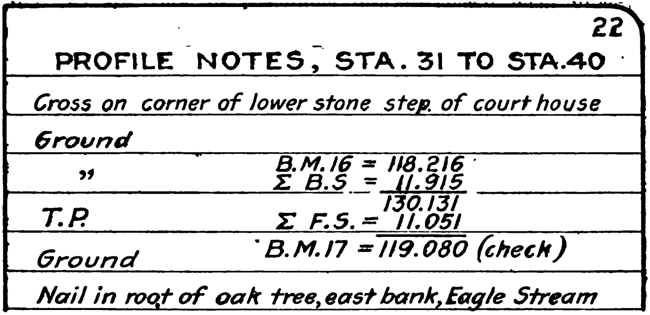
FORM F

21 +S.	B.M. and T.P.	⊼	-S.	El.	Sta.
6.102	118.216	124.318			B.M.16
			5.2	119.1	31
			6.4	117.9	+50
5.813	120.502	126.315	3.816		T.P.
			8.9	117.4	32
			7.235	119.080	B.M.17

FORM G FOR DIFFERENTIAL LEVELING

(1) 159 59.253		59.253		59.253	
+10.250		5.700		5.825	
69.503	+3.745	64.953	+3.747	65.078	+3.747
-6.505		1.953		2.078	
(2) 62.998		63.000		63.000	
+6.872		7.089		7.164	
69.870	+3.818	70.089	+3.817	70.164	+3.817
-3.054		3.272		3.347	
(3) 66.816		66.817		66.817	

FORM E



22

PROFILE NOTES, STA. 31 TO STA. 40

Cross on corner of lower stone step of court house

Ground

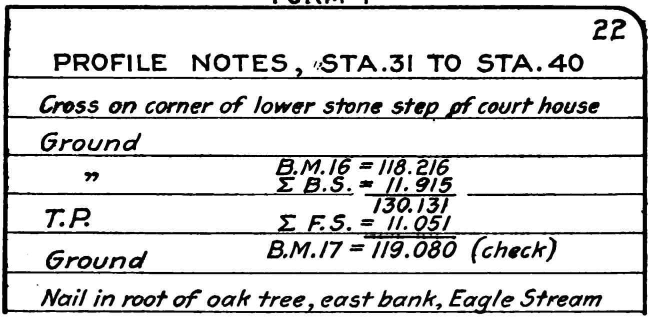
" B.M. 16 = 118.216
Σ B.S = 11.915
130.131

T.P. Σ F.S. = 11.051

Ground B.M. 17 = 119.080 (check)

Nail in root of oak tree, east bank, Eagle Stream

FORM F



22

PROFILE NOTES, STA. 31 TO STA. 40

Cross on corner of lower stone step of court house

Ground

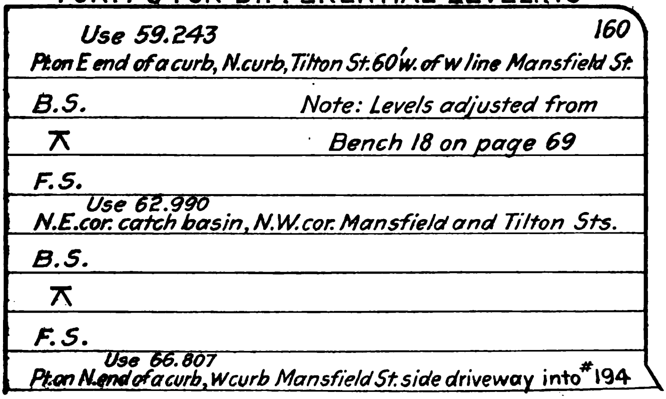
" B.M. 16 = 118.216
Σ B.S. = 11.915
130.131

T.P. Σ F.S. = 11.051

Ground B.M. 17 = 119.080 (check)

Nail in root of oak tree, east bank, Eagle Stream

FORM G FOR DIFFERENTIAL LEVELING



Use 59.243 160

Pt. on E end of a curb, N. curb, Tilton St. 60' W. of W line Mansfield St.

B.S. Note: Levels adjusted from

⊼ Bench 18 on page 69

F.S.

Use 62.990

N.E. cor. catch basin, N.W. cor. Mansfield and Tilton Sts.

B.S.

⊼

F.S.

Use 66.807

Pt. on N. end of a curb, W curb Mansfield St. side driveway into #194

333. Field Notes. Different forms of field notes have already been discussed and compared in Art. 303, p. 239. Forms E and F, shown on pp. 256, 257, correspond to forms A and C on pp. 239, 241. Form G shows the notes for three independent runnings of a line to establish a series of benches for a sewer. The notes for a single running are kept almost entirely in one column, the difference between the B.S. and F.S. at each station being noted in an adjacent column. The form is self-explaining; it affords an excellent comparison of the three results obtained for each B.M., since these results are side by side on the same horizontal line, and it suggests how special forms of notes may be devised to suit special kinds of work. It will be noticed that the final elevations used are 0.01 ft. less than the mean values obtained. This comes from correcting the elevation of the first B.M. to accord with elevations previously used on p. 69 of the field-book as indicated by a note on the right-hand page.

Remark.—Follow the general directions for keeping notes given in Chapter III.

334. Setting Grade-stakes. This consists in setting stakes which shall indicate how much cut or fill is required to bring the surface of the ground to a given elevation or grade. Grade-stakes are also set for curbs, pavements, sewers, foundations, etc. (See Chapter XXIII.) The height of the instrument having been established, and the required elevation of any point being known, the grade-rod for that point can be calculated. (See p. 243.) This is simply reversing the usual operations of leveling. There are several ways in which stakes may be set. (1) *Each stake may be driven so that its top is "at grade."* This is the ideal method, but in many cases it cannot be used, since the top of a stake would come either below the surface of the ground or too far above it. (2) *Each stake may be driven so that its top is one foot, two feet, or any whole number of feet above or below grade.* This is a method often used. (3) *Each stake may be driven approximately to grade, and then a horizontal line may be drawn across one side or the top of the stake may be sawed off, either at grade or some whole number of feet above or below grade.* This can be done when stakes are not easily driven to the desired height. (4) *Each stake may be driven without regard to grade, the elevation of its top afterward ascertained, and the amount of cut or fill from the top marked on the* *stake.* This method makes it necessary for the contractor to measure to decimals of a foot, and hence mistakes are more likely to occur. (5) *A stake may be driven and marked cut or fill so much, with the understanding that the cut or fill must be measured from the* EXISTING *surface of the ground.* This method could be used only for rough grading. Thus, for example, in railway work the preliminary grading can be done from such stakes, but the road-bed is finished and the rails are laid from stakes driven with tops at grade.

(*a*) *General method of procedure.* The required elevation at each point where a stake is to be set must be determined. It is well to do this in the office before going into the field. These elevations are entered in the level notes in the usual way. After the instrument is set-up and the height of instrument found, the grade-rod should be figured for each stake that is to be driven from that particular set-up. If method (1) is used set the target at the grade-rod for any stake, and drive the stake until, when the rod is held on its top, the center of the target is covered by the horizontal hair. If method (2) is used proceed in the same way, except use a "field" grade-rod which shall bring the top of the stake a convenient distance above ground, and a whole number of feet above or below grade.

(*b*) *Illustration.* The required elevation of a point is, say, 120 ft. The actual elevation of the ground at that point, given in the note-book or found at the time by taking a foresight, is 128.4 ft. The H.I. has been found to be 133.6 ft. The *true* grade-rod is, therefore, 133.6 − 120 = 13.6 ft. As the surface of the ground is 128.4 − 120 = 8.4 ft. above grade, the top of the stake should be about 9 ft. above grade or about 0.6 ft. above the ground. Hence the *field*-rod used in setting the stake is 13.6 − 9.0 = 4.6 ft. The stake is marked "Cut 9.0 ft." and the contractor is told to measure from the tops of stakes.

A similar method of procedure may be used for method (3). In method (4) the cut or fill marked on the stake is simply the difference between the elevation of the top of the stake and the grade elevation at that point; in (5) it is the difference between the elevation of the ground and the grade elevation.

(*c*) *Practical suggestions.* (1) First drive a stake *approximately*, being careful to stop before it is too low. Hold the rod and let the levelman indicate how much more it should be driven. (2) Don't split hairs. Keep in mind how close the contractor can work, and drive stakes accordingly. It is folly, for example, to drive stakes to the nearest hundredth of a foot for ordinary grading, the nearest tenth will often do. On the other hand, stakes for curbs and pavements should be set with greater accuracy. (3) Time can often be saved by not using a target at all, i.e., by reading the grade-rod directly from the instrument. (4) Sometimes, as in the case of sewers, some other device than stakes must be used for giving grade (see Chapter XXIII). (5) Stakes should be close enough together to enable the contractor to do his work, but not so close as to be in his way. (6) There are two very common mistakes which occur in setting grade-stakes. One is to observe the wrong horizontal cross-hair (i.e., a stadia-wire), the other is to set the target incorrectly for the grade-rod. The only absolute check is to test the stakes by setting up the instrument in a different position, but errors may often be detected by the eye alone. If the tops of three or more stakes should be on the same line, and any one of them is enough above or below this line to affect construction, the error will be apparent to the eye in sighting across the tops of the stakes. Hence form the habit of *frequently sighting by eye, to make sure that stakes are properly set.*

(7) *The field notes* are kept in the usual form, except that when they have been worked up, additional data will appear in the notes, such as a column of grade-rod values, grade elevations, height of ground above grade, and height of grade-mark above ground. A study of the notes for giving a grade for a sewer (Chapter XXIII) will suggest forms for similar work in setting grade-stakes.

(8) The transit is very convenient for setting grade-stakes (Art. 336.)

335. Marking Grade-stakes. If a stake is at grade it is customary to mark it "Grade," if cut or fill is required it is marked "Cut — ft." or "Fill — ft." The number of feet should be plain and unmistakable. In addition the number of the stake is often marked on another side, as, for example, "20+50" or "A-23." Sometimes when the cut or fill is measured from a line drawn across the stake, an arrow drawn downward for cut and upward for fill is used in place of the words "cut" and "fill." Thus, an arrow drawn downward from the line and marked 10 ft. means "measure down 10 ft. from this line to get a point on grade." Stakes should be marked with crayon which will not be washed out by rain. Too much of the surveyor's crayon on the market will not stand this test.

336. Shooting in Grades. This consists in establishing a line of sight *parallel* to the grade, so that one grade-rod will answer for all the points. It is most easily done with a transit, although it may be done almost as easily with a level. The first step is to establish two fixed points at grade. These are usually stakes, one at each end of the line, or if that brings them too far apart for accurate sighting, extra stakes are set at intervals of, say, 400 ft. The method of procedure for shooting in a grade between any two stakes is described on p. 281. (See also *Leveling with the Gradienter*, p. 245.)

337. Summary—Directions and Suggestions for Spirit-leveling.

(a) *Suggestions for the Levelman.*

1. Choose a good place for setting up the level, away from passing wagons or cars, where there is a firm footing, and where sights on turning-points will be equal. If turning-points are established, set up half-way between them. Avoid sights of less than 100 ft. or more than 300 ft.

2. *On steep slopes* resist the temptation to take unequal

sights, and estimate how far to set the level up or down hill from the backsight and still have the line of sight strike the rod.

3. Be sure the level is set up correctly before sighting. All four thumb-screws should bear firmly, and the bubble should be given time to settle, especially if it is at all sluggish. If between sights the bubble "gets out" slightly, bring it back before the next sight. Watching the bubble closely should become instinctive with a levelman.

4. It is not necessary to clamp the spindle. Usually it is better not to do so.

5. Keep the horizontal cross-hair truly horizontal.

6. If the sun shines into the object-glass, and there is no regular sunshade, roll up a piece of paper and tie it around the front end of the telescope instead of trying to shade the glass with a hat.

7. Avoid unnecessary walking around the instrument, and keep the hands off the instrument and tripod when sighting.

8. Make your signals to the rodman definite, and when the target is to be moved indicate whether to move it much or little.

9. In reading the rod without a target avoid errors of even feet or tenths of a foot.

10. Do not split hairs on intermediate stations when only approximate elevations (to the nearest tenth) are wanted.

11. Keep the level in adjustment. Test every day.

12. In ordinary work effects of heat and wind cannot be avoided, though it is just as well to set the level up in the shade. In precise work shade the level.

13. Protect the level at all times. Never go away leaving it standing where there is any chance of its being knocked over.

(b) *Suggestions for the Rodman.*

1. Choose suitable turning-points, avoiding above everything else objects that settle or are easily moved. There should be a definite point for the rod to rest on, and do not through carelessness hold the rod on any other part of the object. Mark turning-points or describe them so that they may be easily found again.

2. In choosing a new turning-point go as far from the level as the level is *from the* preceding turning-point; if this is too

far up or down the hill for the line of sight to strike the rod go off to one side (zigzagging).

3. Keep the rod plumb, especially in the direction toward and away from the level—the levelman can tell when it is out of plumb in the other direction. The vertical edge of a building or other vertical object may sometimes be used as a guide to the eye. *Hold the rod still.*

4. Be on the alert and move the target much or little, according to the signals. When the target is nearly right do not move it by jumps, first too high and then too low. A good rodman should have the target nearly in position before the levelman is ready to sight.

5. After clamping the target hold the rod for a second sight, to see if setting the clamp has disturbed the target.

6. A rod should be tested before using it for high-rod reading (p. 232). In using the high rod extra care is necessary (1) to set the target correctly, (2) to read the back of the rod in the right direction.

7. Look out for errors of an even foot or an even tenth in reading the rod, and especially the error of omitting the zero when there are no tenths, as, for example, 4.35 instead of 4.035.

8. Do not lean the rod against a tree or any other object where it is likely to be knocked down or blown over—it is safer to lay the rod on the ground. Careless handling of the rod is likely to result in bending the clamp-screw to the target. *Never let a high rod down "on the run,"* as it is one of the most common ways of spoiling a rod—and one of the surest.

9. When a leveling-rod is not at hand rough work can be done with a transit-pole. When the differences in elevation of only two or three points are required rubber bands (one for each point) may be used as targets on any straight stick, the actual differences in elevation being shown by the final positions of the bands. The bands furnish excellent sights, especially if paper is wrapped about the stick underneath each band.

(c) *Suggestions for both the Levelman and the Rodman.*

1. It is worth while, for beginners at least, to check each *other's rod*-reading and for each to keep separate field notes.

2. *Make backsights* and foresights equal. When this is im-

possible, as, for example, in crossing a wide river, use the reciprocal method. (See p. 243.)

3. Take sights on both turning-points before taking intermediate stations. Describe the local bench-marks carefully, not forgetting to establish bench-marks at regular intervals and wherever they may be of use later. (See p. 253.) If anything happens to throw the work out go back to the nearest bench-mark known to be correct and start over again. Write the word "Abandoned" across the portion of the notes that are incorrect. Look out for mistakes in recording. (See p. 274.) If even a single numeral of a reading as entered in the notes is incorrect do not erase, but draw a line through the *whole* reading, and enter the correct reading directly above it. Follow the other directions for taking notes in Chapter III.

Use discretion and common sense in carrying out directions and suggestions. Keep the general requirements of the work in mind, learn what portions of the work can be slighted and what cannot be slighted and still meet the requirements. Not all portions of the work need be done with the same care, as, for example, sights on intermediate stations may be taken less accurately than those on turning-points. (See p. 253.)

338. Duties of Levelman. To run the level, to keep it in adjustment, to care for it and protect it. To keep a general run of the work, seeing that no stations are omitted, that turning-points are properly chosen, bench-marks established, and all work done as expeditiously as possible. To keep notes if it is desirable (and it is, at least in class work), and to "work up" the notes with the rodman each day after the field work is done.

339. Duties of the Rodman. To hold the rod plumb and still, and exactly on the point where it should be held. To be alert and attentive to signals, moving the target to position in as little time as possible, and having set the target to give a second sight. To read the rod, enter the reading in the note-book, and show the rod to the levelman for a check-reading. To choose turning-points, establish bench-marks, and describe them in the notes. To care for the leveling-rod. To work up notes with the levelman at the end of each day's work.

340. Instructions for very Accurate Leveling, based on customary methods for eliminating constant errors, are given in the next chapter.

341. Trigonometric Leveling. *First method.* (See also p. 244.) (1) Measure the horizontal distance from the transit to the point where the elevation is required. (2) Sight on a pole held at the required point and read the vertical angle. (3) Multiply the *horizontal distance* by the *tangent of the vertical angle.* The result is the difference in elevation between the *supporting axis* of the telescope and the point sighted at *on the pole.*

(*a*) *Practical hints.* The difference in elevation usually required is that between two points *on the ground,* one at the transit and one at the sight-pole. Hence as soon as the transit is set up hold the pole at the transit and mark on it the height of the supporting axis of the telescope above the ground. (Wrap paper around the pole and use a rubber band.) Sight on this mark when measuring vertical angles. A level-rod with the target set at the height of the supporting axis is still better.

The horizontal distance may be measured (1) with the tape, (2) with the stadia, or (3) by triangulation, as in the case of an inaccessible height (see p. 223).

If the elevations of many points are to be reduced from vertical angles the use of a reduction table or a special form of slide-rule will save much computation.

(*b*) *Illustration.* A transit stands at Station A. A sight-pole is held so that the height above the ground of the supporting axis of the telescope may be marked upon it. The cross-hairs are then brought on this mark when the pole is held at a second point *B*. The vertical angle as read at the transit is 5° 20′. The horizontal distance from *A* to *B* is known to be 421.6 ft. Tangent of 5° 20′ is 0.09335. Then $421.6\times0.09335=39.36$ ft., the difference in elevation of the *ground* at *A* and at *B*.

(*c*) *The accuracy of the method* may be judged by the following results: If the vernier on the vertical arc reads to minutes the error in reading the vertical angle should not exceed 1′. If, in the above illustration, the heights are computed for three readings, 5° 19′, 5° 20′, and 5° 21′, the three results are respectively 39.234, 39.356, and 39.483, or a difference for one minute of 0.12 for 421.6 ft. If the vertical angle had been, say, 20° larger, the error for one minute would still be approximately the same. Thus, for three readings, 25° 19′, 25° 20′, and 25° 21′, the three results would be respectively 199.438, 199.590, and 199.741, or a difference of about 0.15 ft. in 421.6 ft. It is thus seen that the error which arises from reading the vertical arc is comparatively small. By the use of the gradienter this error can be made much less (see p. 245). *It must be remembered, however, that there are other sources of error.* The level-bulb on the telescope may not be sensitive, and it may also be out of adjustment, i.e., the line of sight may not be horizontal when the reading on the vertical arc is zero. There may be an error in the horizontal distance. Thus, if in the illustration the distance had been called 420.6 instead of 421.6, the error in height for 5° 20′ would be 0.09 ft., and for 25° 20′ it would have been 0.47 ft. It is seen, however, that the method is sufficiently accurate for most purposes, and can be used a great many times in place of a level and level-rod. The method is used for determining the height of an inaccessible point (see p. 223).

(*d*) *Second method.* In topographic surveying the second method (p. 244) is more often used, the hypothenuse of the triangle being measured in place of the base by means of the stadia.

342. The Aneroid Barometer. (See p. 245 for the theory.) The aneroid barometer is less accurate than the mercurial *barometer,* but it is far more portable and convenient, and *therefore it is used* almost exclusively in surveying. It consists of a metallic box from which the air has been exhausted. The varying pressure of the atmosphere acting upon a thin corrugated top of the vacuum-chamber moves it in or out, and this motion is transmitted by multiplying levers, chains, and springs to a pointer moving around a dial which is graduated so that atmospheric pressure is recorded in *inches* to correspond to the varying heights of a column of mercury. In addition the altitudes in *feet*, corresponding to various pressures, are usually given by a second set of graduations—less accurate than the first because dependent upon barometric formulas for reducing pressure to altitude.

In addition to the atmospheric sources of error discussed on p. 245, the aneroid barometer is subject to many instrumental errors such as: (1) Varying sensitiveness or lack of sensitiveness of the corrugated top; (2) incorrect graduations; (3) failure of chains and levers to respond quickly to movements of the corrugated top. The instrument is not intended for accurate work and is used chiefly for reconnoissance or exploration in hilly country, and for rough topographic surveys where it is desired to sketch in contours quickly without great regard for accuracy.

343. Leveling with the Barometer. There are three methods: (1) Single observations; (2) simultaneous observations; (3) extended series of observations.

The method of single observations is the one most used for ordinary work. The barometer is carried from point to point and a single reading taken at each station. The readings thus obtained involve all atmospheric errors, especially those due to the changes in the atmosphere which take place during the interval between observations.

The altitude scale should not be used except for very rough work, and only for the purpose of ascertaining *differences* of altitudes. If the scale is adjustable the barometer may be taken to some bench-mark or point of known elevation and set at the altitude of that point, then the elevations of other points may be read directly from the altitude scale. This means, however, that temperature and other sources of error are ignored, and only approximate results can be expected. A better method is to use the scale corresponding to inches and correct for temperature as follows:

(a) Method of procedure. (1) Take the barometer-reading

(in inches) at the first station or bench-mark, and note the temperature of the atmosphere.

(2) Take the barometer-reading and note the temperature at the second station.

(3) Find from Table I the altitude corresponding to each of the barometer-readings, and subtract one from the other, the result is the difference in altitude of the two stations *uncorrected for temperature.*[*]

(4) Add the two temperatures and from Table II find the correction coefficient C for that sum.

(5) Multiply the difference obtained in (3) by $(1+C)$, *giving to C its proper algebraic sign* ($-$ if $t+t'<100°$ and $+$ if $t+t'>100°$). The result is the approximate difference in altitude of the two stations.

(*b*) *Suggestions:* (1) Notice that the erroneous assumption in regard to the average temperature of the air-column (p. 246) is involved, as are all the other sources of error mentioned on p. 245. (2) Readings for altitudes should not be taken just before or after or during a storm. (3) The shorter the time between the readings at two different stations the better. (4) At the beginning of a day's work compare the barometer-reading at some station with the reading at the same station on the day before, and thus get an approximate correction for the atmospheric changes if any have taken place. It is well to do this at intervals during the day, as the pressure at a given point may change very much in a short time. (5) Hold the barometer in the same position when reading it, always vertical or always horizontal, preferably vertical. (6) The barometer is apt to "catch" or "hang"—tap it gently with the finger before reading it. (7) The barometer "drags," sometimes registering at one moment the elevation where it was twenty minutes before. Hence, in ascending a hill consecutive readings are likely to be too low—in descending too high. This dragging is a most common source of error.

(*c*) *Simultaneous observations*, i.e., taking simultaneous readings at different places with barometers which have been previously compared. The aim of this method is to eliminate as far as possible error due to changes in atmospheric conditions during the time which elapses between the observations at the first and second stations. One barometer is kept at the base or starting-point, and it is assumed that whatever change is indicated by this barometer, as read at intervals previously agreed upon, should also be allowed for in reading the other barometer read at the same intervals. This assumption may or may not be true, but the method is the most satisfactory one which the engineer can use when stations are widely separated, since he can seldom carry on a long series of observations at any one station to determine the mean reading.

(*d*) *Extended series of observations* are usually taken with mercurial barometers and cover long periods of time. The mean readings thus obtained often give excellent results, but of course the method cannot be used to any extent in surveying.

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[*] Tables I and II are in the back of the book.

(*e*) *Limits of error.* Errors may be expected in barometric leveling of from one to ten per cent, though many instances may be cited where the error was much less than one per cent, and sometimes the error is greater than ten per cent. In ordinary work single observations taken on the same day at frequent intervals will vary from three to five per cent from results obtained by spirit-leveling. If temperature is ignored, that alone may introduce an error of from one to ten per cent. (See correction in Table II for $t+t'=180°$.) By comparing barometers with each other and by taking other similar precautions the errors may be corrected or reduced. In the ordinary topographic surveying where bench-marks have been established, the best method is to start at one bench-mark and close as frequently as possible on other bench-marks; thus, by comparing with the elevations obtained by spirit-leveling, one can get the general run of errors to be expected in the use of the barometer under the given conditions.

CHAPTER XXII

ERRORS IN LEVELING

IN this chapter sources of errors in leveling are pointed out, the relative importance of errors from different sources is discussed, and methods of correction or elimination are given. Limits of error are suggested, the precautions which are necessary in accurate leveling are outlined, and a method of procedure embodying these precautions is given. The general discussion of errors in Chapter II should be studied before beginning this chapter.

344. Sources of Errors in Leveling. (1) *Instrumental errors.* (*a*) Errors of adjustments, and (*b*) defects in the construction of non-adjustable parts.

(2) Mistakes in manipulating the level and leveling-rod.

(3) Mistakes in reading the leveling-rod.

(4) Errors in sighting.

(5) Errors due to changes in the position of the level or rod.

(6) Errors due to natural sources.

(7) Mistakes in recording and computing.

(8) Personal errors.

345. Instrumental Errors. (*a*) *Errors due to imperfect adjustment.* Errors are *compensating;* they are practically eliminated if the lengths of the foresight and the backsight are made exactly equal. The adjustments of the level (explained in Part III) are for the purpose of making the line of sight generate a *horizontal* plane of sight (p. 236) when the level is properly set up. If the line of sight is not parallel to the bubble-axis (the principal error of adjustment), it will generate the surface of a cone the axis of which is vertical, and hence rods held equally distant from the instrument will be intersected at the *same* level by an imaginary base of the cone, and the error due to the fact that the line of sight does not move in a plane is eliminated.

(*b*) *Sluggish bubble.* If the bubble is sluggish in its movement, an error may result from the bubble coming to rest in the wrong position, even though it may creep back to the correct position while the sight is being taken. The error is *compensating* and partially avoided by observing the bubble *after* the target is set to see if it is still in the center of its tube, but such a bubble is a constant source of annoyance and delay.

(*c*) *Error in the movement of the object-glass slide.* Error is *compensating,* and is entirely eliminated by making the backsight and foresight equal, since in that case the focus is not changed and hence the slide is not moved.

(*d*) *Error in rod-graduation.* Not likely to be appreciable in ordinary work if the rod is graduated by a reputable maker, but every new rod should be tested.

(*e*) *Error due to a defective joint.* An extension-rod with an abutting joint should be thoroughly tested at frequent intervals by the method explained on page 232. The wear and tear from letting the rod down "on the run" and from other sources may seriously affect this joint (see Remark, p. 232), and failure to test the rod may result in a large *cumulative* error.

346. Mistakes in Manipulation. (*a*) *The level.* Mistakes in setting up, unnecessary clamping of the spindle, errors in centering the bubble, failure to watch the bubble, resting the hands on the tripod or telescope. (See "*An experiment,*" p. 227.) Precautions should be taken to keep the bubble in the center of its tube. If it moves out slightly, bring it back with the leveling-screws. Sometimes in work of great precision mirrors are used (Art. 349 (*b*), p. 272), or a bubble-tender brings the bubble to position with a slight pressure of the fingers (p. 279).

(*b*) *Note.* The levelman should know the error caused by a displacement of the bubble judged by the division-marks on the glass tube. The value of a single division varies greatly for different instruments, but it may be ascertained easily for a given tube. (See Part III.) The value is commonly given in units of angular measure, and in 18-in. levels it is usually from 20″ to 30″; in some levels it is 10″ or even less (p. 277), while in others it will be found to exceed 60″ or 1′. Below is given a table which shows the errors in feet to be expected with instruments of different sensitiveness and for sights of different lengths. "D" in the table stands for one division on the tube.

ERRORS IN FEET CORRESPONDING TO DIFFERENT DISPLACEMENTS OF THE BUBBLE

Sensitiveness of Level		D = 10″			D = 30″			D = 1′		
Deviation of Bubble....		½D	D	$\frac{3}{2}$D	½D	D	$\frac{3}{2}$D	½D	D	$\frac{3}{2}$D
Distance from Rod to Instrument	50	.001	.002	.003	.003	.006	.009	.006	.012	.018
	100	.002	.004	.006	.006	.012	.018	.012	.024	.036
	150	.003	.006	.009	.009	.018	.027	.018	.036	.054
	200	.004	.008	.012	.012	.024	.036	.024	.048	.072
	250	.005	.010	.015	.015	.030	.045	.030	.060	.090
	300	.006	.012	.018	.018	.036	.054	.036	.072	.108

(*c*) *The leveling-rod held so that it leans towards or away from the instrument* will make a reading too large. Error is compensating on turning-points. (Why?) Avoided by swaying the rod slowly backward and forward after the target is set. If the target *center* does not move *above* the line of sight, and coincides in its highest position with the horizontal cross-hair, the target was set when the rod was vertical. (Why?) Otherwise it must be reset. (Satisfy yourself, by trial, of the importance of holding the rod plumb.)

Question. Why is the error not compensating when leveling entirely up hill or down hill?

Remark. For any reading less than 2 ft. it is probably more accurate to have the rodman plumb the rod rather than wave it, for the graduated face may be *raised* when the rod is inclined away from the instrument, since the face is not directly over the point of support. This error is inappreciable for large rod readings. (See Eng. News, Vol. II, p. 35, 1906.)

A better way to make sure that the leveling-rod is held plumb is to attach a plumb-line or, better still to use a special attachment devised for that purpose. Some rods are furnished with angle-targets in which the right-hand half of the target is at right angles to the left-hand half; the horizontal dividing-line is carried over these two surfaces and appears as a continuous and unbroken line only when the rod is vertical.

(*d*) *Allowing dirt or grit to accumulate* on the base of the rod or in either abutting joint is another source of error. (On what part of the rod would readings be affected by dirt in the joint?)

347. Mistakes in Reading the Rod are illustrated on pages 229 and 231. The common mistake of dropping a zero from the reading may be avoided by considering as "danger-points" the first tenth of each foot, and on rods more finely graduated the first subdivision of each tenth of a foot ($\frac{1}{100}$ ft.). Every rodman is likely to have his own "danger-points" which he will do well to keep in mind and to guard against by extra care when a reading is near one of them.

348. Errors in Sighting.

(*a*) *The difficulty of telling when the cross-hair exactly coincides with the center of the target* leads to an error which is hard to avoid; it is, however, *compensating.* The coarseness of the cross-hair, the form of the target, and the eyesight of the levelman, all have to do with such an error. It may help the levelman after looking steadily at a target until he thinks it is in correct position, to close the eye momentarily and then look again before giving the signal to clamp. He should see to it also that the horizontal cross-hair of the level is kept truly horizontal. (See adjustments of the level.)

Sights should be neither too long nor too short (from 100 to 300 ft., see p. 251.)

A common blunder is to mistake one of the stadia wires for the horizontal hair.

Experiment. The following experiment was tried to ascertain the probable error of setting a target at different distances. A level was set up with two leveling-screws in the line of sight. An instructor held an umbrella over the instrument and kept the bubble in the center of its tube, while eighteen students who had used the level just enough to become accustomed to its use, set the target, each in turn, at three different distances, the instructor reading the rod for all settings. The probable errors for a single observation were as follows:

Conditions	Instrument	Length of sights		
		100	200	300
Sun shining,	Level No. 1	0.00196	0.00196	0.00370
Level in the shade,	" " 2	0.00147	0.00229	0.00327
Air slightly boiling at 200 and 300 ft.,	" " 3	0.00130	0.00157	0.00213

Levels No. 1 and No. 2 were 18″ wye-levels, $1\frac{3}{8}$″ objective, magnifying power about 35 diameters. Level No. 3 was a 15″ wye-level, $1\frac{1}{4}$″ objective, magnifying power about 30 diameters. The results for levels 1 and 2 were obtained by one group of students; the results for level 3 were obtained by another group of the same number of men (18). The above results indicate that, for beginners at least, under favorable conditions the chances are even that the error of sighting and of setting the target will not exceed, say, 0.002 ft. for a 100-ft. sight, 0.0023 for 200 ft., and 0.0037 for 300 ft.

The error of reading a rod directly without the use of a target is not as great as is commonly supposed. From a large number of readings taken by a class of over 100 beginners, using "speaking" rods of various styles with graduations more or less dim, the probable errors for a single observation at different distances were found to be (in feet):

Length of sight	50	100	150	200	250	300
Probable error	0.0024	0.0035	0.0038	0.0048	0.0062	0.0065

Comparing these results obtained under unfavorable conditions with those above obtained under favorable conditions, it is seen that target readings are but slightly more accurate than direct readings. Under certain conditions they have been found to be less accurate.

349. Errors Due to Changes in Position of the Level or the Rod.

(*a*) *Settling of the instrument* between the backsight on one turning-point and the foresight on the next. *Cumulative,* tending to make all elevations too high because the foresight is shortened, and the calculated elevation of the second turning-point will therefore be too high. Avoided by setting up the instrument solidly on firm ground, and taking the foresight on the second turning-point immediately after backsighting on the first. For this reason two rods and two rodmen are sometimes used (p. 277). Rerunning in the opposite direction tends to eliminate this error also. On warm winter days when the ground is thawing the error due to settling may be very large.

(*b*) *The level disturbed or jarred* by passing trains, teams, or by the levelman walking around it. Error most likely to occur on loose soil, or on a smooth surface where the tripod-shoes do not catch. On some instruments mirrors are placed so that the levelman can watch the bubble without leaving his position behind the instrument, and in precise leveling he can observe the cross-hair and the bubble at the same time. As a precaution in ordinary work, set two legs of the tripod in a line parallel to the general direction of the stations, and see that all three legs are reasonably firm before leveling up.

(*c*) *Settling of a turning-point* between the foresight and the backsight on it. Cumulative, tending to make elevations too high because the backsight reading will be increased, thus increasing the calculated height of instrument. Easily avoided by choosing proper turning-points. (See p. 252.) Rerunning in the opposite direction tends to eliminate this error.

350. Errors Due to Natural Sources.

(*a*) *Changes in the instrument due to the effect of sun and wind.* The sun shining on a level may cause its parts to expand unequally. For example, in running towards the sun the front wye will expand, making the foresight too high, hence the elevation will be too low. On backsighting the other wye will expand, making the backsight too small, hence the height of instrument too low. The error is therefore apt to be cumulative, making all elevations too low. Again, if the sun heats one end of a bubble-tube more than the other the bubble will move *towards* the heated end. (Try breathing on one end of a tube which is level, or warm one end by holding the fingers on it, watching the bubble to see if it is drawn over.) Errors due to the sun may be avoided by shading the instrument with an umbrella, or working on a cloudy day. The effect of wind and heated air-currents is to make the target seem to tremble or dance, and often renders accurate work out of the question. In precise leveling a tent is sometimes used to shelter the level from both sun and wind. The least wind is usually at daybreak, a time when the atmosphere is also in the best condition for long sights.

(*b*) *Change of length of level-rod* from variations in temperature. Inappreciable in ordinary work, but in more precise leveling it is quite possible that errors from this source may exceed the errors arising from the leveling itself.*

(*c*) *Errors from curvature and refraction.* (1) *Curvature.* The difference between a horizontal line of sight and a level line is about 8 in. per mile, and varies as the square of the distance, i.e., the curvature of a level surface is 0.66725 ft. multiplied by the square of the distance in *miles*. (See foot note p. 2.) The effect of curvature is to *increase* the rod-reading.

(2) *Refraction.* The effect of refraction is the same as if the line of sight were curved downward, or concave towards the earth's surface, and hence the rod reading is *decreased*. The curve is irregular because of varying atmospheric conditions, but for average conditions it is assumed to have a diameter about seven times that of the earth. The correction for refraction is $\left[\frac{(\text{Distance in feet})^2}{\text{Mean radius of earth in feet}}\right] \times 0.07$, where 0.07 is the mean value of the coefficient of refraction.

(3) *Combined curvature and refraction.* The effect of refraction is to reduce the correction for curvature. The following table gives approximate values for the correction for combined curvature and refraction.

The distance *D* and the correction *c* are both in feet except in the last column, where the distance is given in miles.

CORRECTION FOR COMBINED CURVATURE AND REFRACTION †

D	*c*	*D*	*c*	*D*	*c*	*D*	*c*	*D*	*c*	Miles	*c*
300	.002	1300	.035	2300	.108	3300	.223	4300	.379	1	.571
400	.003	1400	040	2400	.118	3400	237	4400	.397	2	2.285
500	.005	1500	.046	2500	.128	3500	.251	4500	.415	3	5.142
600	.007	1600	.052	2600	.139	3600	.266	4600	.434	4	9.141
700	.010	1700	.059	2700	.149	3700	.281	4700	.453	5	14.282
800	.013	1800	.066	2800	.161	3800	.296	4800	.472	6	20.567
900	.017	1900	.074	2900	172	3900	.312	4900	.492	7	27.994
1000	.020	2000	.082	3000	.184	4000	.328	5000	.512	8	36.563
1100	.025	2100	.090	3100	.197	4100	.345	5100	.533	9	46.275
1200	.030	2200	.099	3200	.210	4200	.362	5200	.554	10	57.130

† From Johnson's Surveying, p. 597.

An empirical formula giving approximate results is: Curvature and refraction, in feet $= 0.574$ (distance in miles)^2.

The error due to curvature and refraction may be eliminated by making the backsight and foresight equal for each set-up.

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* Wright's *Adjustment of Observation*, p. 372.

351. Mistakes in Recording and Computing. (*a*) *Mistakes in recording* will sometimes vitiate the otherwise careful field work of a whole day. Entering the reading with figures interchanged, as 3.45 instead of 3.54, entering backsights and foresights in the wrong columns, or neglecting to enter a backsight at all, are some of the mistakes which may be avoided, especially in class-work, if the rodman and the levelman will each keep an independent set of notes.

(*b*) *Errors in computing* can usually be discovered by the check given on p. 242.

352. Personal Errors. *The personal equation.* Errors which arise from causes peculiar to the individual are not easily avoided. They may not amount to much in ordinary work, but they are taken into account in precise leveling, especially in comparing results. A man may habitually read the rod too high or too low, or he may manipulate the level in some peculiar manner so that slight errors of the same sign (cumulative) occur. Running levels in opposite directions, the rodman and the levelman changing places before going over the ground a second time, or, in running a single line of levels, changing places at every other set-up, helps to eliminate errors of this kind.

353. Elimination of Errors,—Recapitulation. (*a*) *Mistakes.* Keep in mind the "danger-points" in reading the rod (Art. 347, p. 270). Mistakes in reading the rod and in recording the readings are best avoided by having two men keep independent notes, reading the rod independently, and comparing notes *after* both have read the rod. The levelman should avoid mistaking a stadia-wire for the middle horizontal cross-hair.

(*b*) *Constant errors.* By making backsight and foresight equal for each set-up several constant errors are eliminated, such, for example, as instrumental errors and the error due to curvature and refraction. The level, however, should be frequently tested for adjustment. When it is impossible to make backsight and foresight equal reciprocal leveling may be employed (p. 243). Rerunning a line in the opposite direction tends to eliminate certain other constant errors. The error due to the settling of the turning-point or to holding the rod on slightly different points for foresight and backsight should be avoided—it is usually inexcusable; in many cases steel *turning-points are used to* eliminate such errors. The error due to settlement of the instrument is reduced by taking the foresight as soon as possible after the backsight, and in precise leveling two rodmen are used for this purpose. The error due to the bubble being out of center is avoided by instinctively glancing at the bubble just before and after sighting, by the use of mirrors on levels of precision, and sometimes by employing a bubble-tender. Errors due to the sun shining on the instrument are avoided by shading the level or working on a cloudy day. The error due to the boiling of the atmosphere is avoided by suspending work during midday hours. Rod errors are avoided by testing the rod graduations, by keeping the base of the rod and the abutting joints free from dirt and grit, by examining the setting of the target as well as the vernier reading for an extended rod, and when the rod is closed by examining the vernier reading to see that it reads 6.5 ft. or 7.0 ft.. not depending upon the abutting end to bring it back to place. These last errors are eliminated altogether by using a "one-piece" rod—many surveyors will use no other in important work. The error due to not plumbing the rod is avoided by waving the rod, and in precise work by using an attachment devised for that purpose. For an illustration of leveling in which many of the foregoing precautions were adopted see page 277.

(c) *Accidental errors,* such as small errors in sighting and errors resulting from small disturbances of the instrument, can be reduced only by adjusting independent or related observations.

354. Customary Limits of Error. Probably in no branch of surveying does the law of accidental errors (that the total error will vary as the square root of the number of observations (Art. 71, p. 45)) accord more closely with experience than in leveling. This is because constant errors are more easily eliminated and hence accidental errors are likely to predominate, even in ordinary work. There are two ways in which the precision of leveling may be judged. (1) By comparing the two elevations of the same point as found from independent lines of leveling,—the difference is not an error but a discrepancy. (2) By the difference between the *assumed* elevation of some starting-point and the elevation found by completing a circuit, i.e., returning to the same starting-point. This is a *true error, called* the closing error of leveling.

In either case the total error of a line of levels may be expected to vary about as the square root of the number of set-ups. When, however, the number of set-ups per mile is practically constant it is more convenient to express the error in terms of the distance; thus,

$$\text{Error in Feet} = \text{Constant}\sqrt{\text{Distance in Miles}}, \text{ or } E = C\sqrt{M}.$$

The value of C will depend, of course, upon the purpose of the work, the period of time in which it must be done, the expense warranted, and other local conditions. There seems to be, however, a more general agreement as to the allowable error in leveling than there is in regard to limits of error for chaining or for angular measurements. The most common value for C is 0.05. This is for ordinary work where reasonable precautions have been taken to eliminate constant errors, and where speed will average, say, two or three miles per day, or where a single instrument duplicates from 1 to 1½ miles per day. This is the coefficient adopted by the officers of the U. S. Geological Survey "after careful consideration of the needs of the work and of the standards set by other organizations." In precise leveling a good average value for C is 0.02, and 0.02 ft. $\sqrt{M}$ is practically the limit adopted by the U. S. Geodetic and Coast Survey and the Mississippi River Commission. Under favorable conditions the error may be kept within this limit working with a good wye-level, but it will be necessary to adopt a method of procedure somewhat similar to that used in precise leveling. (See p. 277 for an illustration of the precautions taken to attain a degree of precision of $0.02\sqrt{M}$ or better). The maximum permissible error for leveling in the Chicago Sanitary District was about 0.012 ft. $\sqrt{M}$.

Note. In an example of railway location the discrepancies between elevations obtained in the preliminary location and those obtained in the final location are given for a number of stations at varying distances apart throughout about 70 miles of "66 feet country." * The average difference between the divergence at one point and that at another is stated as 0.028 per 100 stations. This, however, is misleading as the total discrepancy at any one station, found from the values given, if substituted for E in $E = C\sqrt{M}$. gives in some cases a value as high as 0.17 for C with perhaps an average value of about 0.12.

The character of the ground affects the precision of leveling. For example, the degree of precision attained in precise leveling in India and also in Switzerland varied from about 0.013

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* *Beahan's Railway Location*, p. 106.

ft. for nealy level ground to about 0.033 ft. for gradients entirely between 1 in 20 and 1 in 10, the quantity in each case being the average discrepancy per mile between two observers.*

Beyond a certain limit a small increase in precision requires a large increase of effort, and the difference in precision between ordinary leveling and precise leveling is not at all in proportion to the difference in the skill and care required.

The following table, computed from the formula $E=C\sqrt{M}$, gives the permissible error in *feet* for different distances in miles for different coefficients of precision.

Permissible Errors in Leveling for Different Coefficients and Different Distances

Miles.	0.05	0.04	0.03	0.02	0.01
0.5	0.035	0.028	0.021	0.014	0.007
1	.050	.040	.030	.020	.010
2	.070	.056	.042	.028	.014
3	.086	.069	.052	.035	.017
4	.100	.080	.060	.040	.020
5	.112	.089	.067	.045	.022
6	.122	.098	.073	.049	.024
7	.132	.106	.079	.053	.026
8	.141	.113	.085	.057	.028
9	.150	.120	.090	.060	.030
10	.158	.126	.095	.063	.032
15	.194	.155	.116	.077	.039
20	.224	.179	.134	.089	.045

Illustration of the use of the table. What discrepancy between two lines of levels is permissible at the end of four miles if 0.05 is the adopted value of *C*? Answer 0.1 ft. What is the permissible error of closure of a circuit 6 miles long if 0.02 is the value adopted for *C*? Answer 0.049 ft.

355. Instructions for Accurate Leveling. The following instructions and method of procedure were taken from the report on spirit-leveling along the line of the Barge and Canal Survey, by the state engineer and surveyor of New York. A wye-level was used of 16½″ focal length; clear objective aperture of 1¼ in.; magnifying power 35 diameters; value of one division on the bubble-tube 7.04 seconds.

The instructions embody most of the precautions already given for eliminating errors, and they indicate the care required to attain a degree of precision of $0.02\sqrt{M}$ or better.

INSTRUCTIONS FOR LEVELING

(1) All lines forward and backward shall be run with two rodmen.

(2) Each rodman shall keep separate notes of rod-readings on all turning-points and bench-marks he holds on, and compute the elevations of

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* See Baker's *Surveying Instruments*, p. 283.

the same when furnished with the height of instrument by the recorder. Each rodman, when he receives the signal "all right" from the instrument man, shall at once read the rod and record the reading in the book provided. The recorder shall always read the rod after the rodman, make the necessary calculations, and compare the results with the rodman. If the results differ, each shall again read the rod before comparing results, and if the readings of the rod differ, another setting of the target shall be made by the instrument man.

(3) Work must not be attempted during high wind or when the air is "boiling" badly. During very hot weather an effort should be made to begin work very early and remain out late, rather than to work during midday.

(4) Foresights and backsights should be of equal length, and no sights over 300 ft. shall be taken unless unavoidable circumstances necessitate the same, as in the case of crossing rivers or deep ravines. In such cases extra precautions must be taken, and the average of repeated readings at changed positions of rod and instrument taken.*

(5) If it be impracticable to take equal foresights and backsights, as soon as the steep slope or river crossing is passed, enough unequal sights shall be taken to make each set balance.

(6) Distance along railroads may be taken by counting rails; at other times stadia or pacing may be used.

(7) The instrument must always be leveled exactly before setting the target. After setting it and before giving the signal "all right," the bubble must be examined. One rod, No. 1 or No. 2, should always be read first, so that one rod is used as foresight first at one set-up and as backsight first at the next set-up. The bubble-tender shall always stand on the same side of the level-tube when the reading is made, moving around the tripod as the level-tube is reversed.

(8) The level must be examined daily or oftener, if necessary, for adjustment, the especially important adjustments being the line of collimation and the level-bubble.

(9) The steel pegs as furnished must be used as turning-points in all cases. These shall be driven firmly in the ground, and the backsight peg shall not be removed until the foresight reading is completed, and the recorder and rodman have compared results on the backsight.

(10) Plumbing-levels must always be used and kept in adjustment.

(11) Bench-marks or turning-points left at the termination of work at night, or for rain or other causes, must be selected with great care, and located in such a manner that there will be no danger of their being disturbed or tampered with in order that the rod may again be held on the exact spot.

(12) Permanent bench-marks shall be clearly described, not only with reference to the nearest base-line station, but also to existing and easily identified features of the ground. A sketch shall be made showing the bench-mark and the reference-marks referred to.

(13) All circuit closures or checks by duplicate lines shall be distinctly noted, and a reference made to check levels.

Duplicate lines of levels shall be run forward and backward, and the error of closure of the two runnings shall fall within

$$.020 \text{ ft.} \sqrt{\text{distance in miles between benches}},\dagger$$

or the lines shall be re-run.

Bench-marks shall be established at intervals of one-half to one mile.

On the Champlain Canal a single line shall be run from the Erie Canal to Whitehall. The error of closure with the U. S. D. W. shall fall within $.050 \text{ ft.} \sqrt{\text{distance in miles between the benches}}$ or the lines shall be re-run.‡

The number of men in each party shall consist of five,—an instrument

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* See p. 243 for reciprocal leveling.

† After the levels had been run from Albany to Herkimer, the results were so good that the limit of error was reduced to

$$.016\sqrt{\text{distance in miles between benches}}.$$

‡ *See p. 276 for the* explanation of this formula.

man, a recorder, two rodmen, and an umbrella man. The instrument must always be shaded from the sun, both during the set-ups and in moving from point to point.

PROCEDURE OF WORK

Starting from a bench or turning-point, the instrument man paced along the towpath from 200 to 250 ft., and set up the level, protecting it by the umbrella or wind-shield as occasion required.

Rodman No. 1 remained at the bench. and rodman No. 2, starting at the same bench, paced to and beyond the instrument till he reached a point as many paces beyond the istrument as the instrument was from rod No. 1, at which point he drove the steel pin.

Having carefully leveled the instrument, the leveler set the target on rod No. 1 as a backsight, and then, avoiding both haste and delay, turned the telescope to rod No. 2, and set the target as a foresight. The bubble-tender kept the bubble constantly in the middle of the tube by slight pressure of the fingers on the leveling-plate of the instrument. To balance errors due to defective vision of the bubble-tender, or differences in the light on the bubble, the bubble-tender moved around the tripod when the telescope was turned.

The recorder remained with rodman No. 1 until both he and the rodman had read, recorded, and checked the rod-reading, when he walked rapidly to rod No. 2, checking the paced distances on the way. The recorder then read, recorded, and checked the reading of target No. 2, and signaled "All right" to the instrument man, who repeated the signal to rodman No. 1, when they both moved forward. Rodman No. 1, going to rod No. 2, read, recorded, computed, and compared results with rodman No. 2 and the recorder, the leveler having at the same time paced up to point No. 2 to check the pacing, and then paced past point No. 2 the proper distance and set up the instrument. Rodman No. 1 paced up to the instrument from point No. 2, and then an equal distance beyond it, and drove steel pin and set target. Thus this alternation occurred: On first set-up rod No 1 on backsight is set first, and rod No. 2 on foresight is set last; on second set-up rod No. 1 on foresight is set first, and rod No. 2 on backsight is set last.

CHAPTER XXIII

LEVELING—SPECIAL WORK

This chapter deals with methods of giving grade-points for curbs, pavements, sewers, and roads; of setting stakes for grading land areas; of field work for estimating cut and fill in ordinary grading; and of giving grade-points for vertical curves. The typical cases of special work selected for this chapter will suggest methods to be followed in any work of a similar nature that is likely to arise in the experience of the young engineer. The article on *Setting Grade-stakes*, p. 258, should be studied in connection with this chapter.

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356. To Determine the Profile of a Street. Mark points on the curbs, or drive stakes, at regular intervals, close together where the grade is changing rapidly, farther apart where the grade is uniform, and treat these points thus marked as intermediate stations. Sometimes a profile of the center-line of the street is all that is wanted, but often in addition the profiles of curb-lines and street-lines are also required. Elevations should be taken at all street-corners, catch-basin covers, bridges, and other points which should appear upon the profile; also at any abrupt change in the surface of the ground and at the top and bottom of all slopes.

Practical suggestions. (1) Measurements between points may be taken with a tape or with the stadia; sometimes in rough work they may be paced. (2) Bench-marks should be established at intervals of several hundred feet, and at other points where they might prove useful (see p. 253). (3) Usually the best method is to number points as described on p. 254. Enough stakes or points on the curb should have numbers actually marked on them to render unlikely any confusion or omission of points. (4) Don't split hairs,—read the rod on intermediate points without using the target; often the nearest tenth will do. Why read to hundredths when the rod is resting on uneven or loose soil?

357. To Set Stakes on a Grade between Two Fixed Points. (*a*) The most general case is when the stakes are at unequal intervals, and when the ground is too uneven to permit the use of the "shooting-in" method. The grade will be the hypothenuse of a right-angled triangle, the base of which is the horizontal distance between the two fixed points; the difference in their elevations will be the altitude. By similar triangles the elevation of any point on the grade will then be proportional to the distance of that point from one of the fixed points. The elevations of the points having been calculated, the stakes may be set as explained in Art. 334, p. 258.

(*b*) *Illustration.* The elevations of two fixed points *A* and *B*, 360 ft. apart, are respectively 226.42 ft. and 233.12 ft. The finished grade is a uniform slope between the two points. It is required to set stakes 40 ft., 120 ft., and 300 ft. from *A*. The difference in elevation is 6.7 ft. Hence $\frac{40}{360}\times 6.7$ ft. $=0.74$, and $0.74+226.42=227.16$ ft. = elevation at 40 ft. from *A*. In the same way $2.23+226.42=228.65$ = elevation at $A+120$, and $5.58+226.42=232$ = elevation at $A+300$. As a check the difference in elevation for the remaining 60 ft. is $\frac{1}{6}\times 6.7=1.12$, and $232.0+1.12=233.12$, which agrees with the elevation of *B*.

(*c*) *Practical suggestions.* (1) See p. 259 for suggestions for setting stakes. (2) When practicable, set stakes at equal intervals so that the difference between successive grade-rods will be a constant. [See Art. 358 (*a*).]

(*d*) *To shoot in a grade between two fixed points.* If the two points are not already established, drive a stake to grade at each point by the grade-rod method. The next step is to establish a line of sight at a known distance above grade and *parallel to grade.* Either the transit or the level may be used. If the level is used it is best to set it up with one pair of leveling-screws in line with the two given points, so that these screws can be used for inclining the telescope. Method of procedure: (1) Set up near one stake, a foot or so behind it. (2) Hold the rod on the stake and set the target at the height of the center of the object-glass. (3) Hold the rod on the second stake; incline the telescope until the horizontal hair covers the center of the target. This brings the line of sight parallel to grade, provided the height of the object-glass has not been changed appreciably by inclining the telescope. (4) Hold the rod on the first stake, and set the target over again if the object-glass has been changed. (5) Hold the rod at the second stake, change the line of sight slightly if necessary, and *establish a permanent foresight* at the given distance above the stake. (6) Set the intermediate stakes without changing the target. This cannot be done if the surface of the ground is very irregular, hence this method is suitable only for comparatively uniform slopes.

(*e*) *Note.* When the transit is used it is often better to set up directly over one of the stakes, and fix the target at the height of the supporting axis, thus avoiding the readjustment in inclining the telescope. For the same reason some surveyors prefer to set the level directly over a stake instead of a foot or so behind it

(*f*) *Practical suggestions.* (1) Make sure that the line of sight is parallel to grade before setting intermediate stakes, no matter how many times the target must be reset,—two trials are usually sufficient. (2) For a permanent foresight, wrap a strip of paper about a pole, slip a rubber band over it, stick the pole behind the stake, move the rubber band until it coincides with the line of sight. Then any change in the line of sight which may occur during the setting of intermediate stakes may be readily detected and corrected by the levelman. (3) The error involved in setting the target opposite to the center of the object-glass, without looking through *the telescope,* is very small, often less than that which ordinarily occurs in setting the target at a distance of 300 ft. or 400 ft. (4) If the surface of the ground is too far above or below the required grade to drive the stakes to grade, the method can still be used, provided all the stakes can be driven some constant distance above or below grade. (5) See p. 259 for other practical suggestions for driving stakes.

358. To Set Grade-stakes for a Curb. A transit adjusted for leveling is the best instrument for this work. (1) Find two points known to be on the curb-line, and if the curb is straight, line in stakes at intervals of, say, 25 ft. to 50 ft., driving each stake a few inches into the ground. (2) Shoot in these intermediate stakes to grade if practicable; if not, figure the grade-rod for each stake (see p. 243). If the grade is on a vertical curve there is no difference in the field work,—it is simply a matter of fixing upon elevations at stated intervals (see p. 289).

(*a*) *Grade-rod method, illustration.* A curb 376.1 ft. long is to be set to a 2% grade. The lower end begins at a point of 129.64 ft. elevation. Stakes are set on line every 50 ft. The grade-rod will therefore be 1.0 ft. *less* for each succeeding stake as the grade ascends. The 3+50 stake is 26.1 ft. from the end, so that .02×26.1=0.522 must be subtracted from the grade-rod used for that stake to get the grade-rod for the end-stake; i.e., if the grade-rod for the first stake is, say, 8.94, for the 3+50 stake it is 1.94, and for the last stake it is 1.42.

(*b*) *Practical suggestions.* (1) When practicable set the instrument midway between the ends, and high enough to control the last or highest stake. (2) The curb-line may be marked by tacks in the top of the stakes *after* the latter have been set to grade, or the stakes may be driven with their edges on line. See suggestions, p. 259, for setting stakes. (3) In resetting old curbs a stone may be removed wherever it is desired to drive a stake, or an offset-line may be run on the sidewalk, two or three feet from the curb-line. Points on this line at intervals of about 25 ft. may be marked by spikes driven flush in the sidewalk, or if this be impracticable, by scratches on the surface. The curb-setter should be furnished with a memorandum giving the distance that each point is above or below the grade of the curb opposite to it.

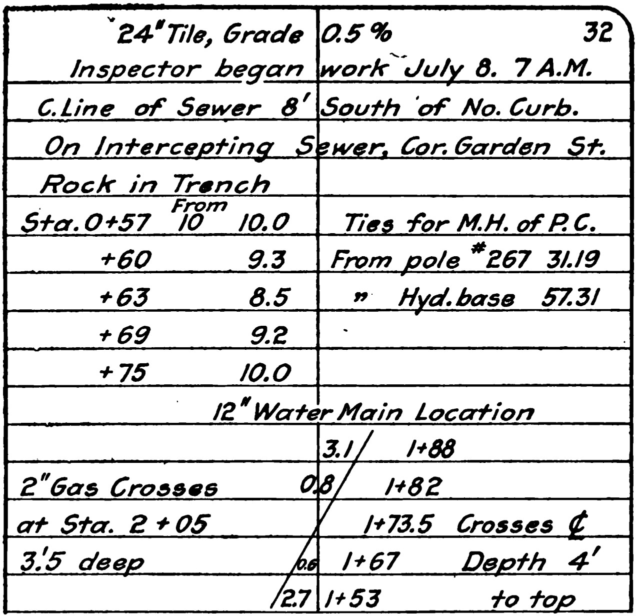
359. To Lay Out Grades for Pavements. (1) Set stakes for curbs as in the preceding article. After the curb has been set, mark points on the face of the curb where the surface of the pavement will strike the curb. These marks may be made with a chisel at intervals of from 25 to 50 ft. A row of stakes may also be set on the center-line of the street, especially if it is a wide street, to indicate the top of the crown of the roadway. These stakes and the marks on the curb are established with level and leveling-rod by the grade-rod method or by "shooting in." If it is necessary to estimate quantities before the pavement is laid, profiles should be made of the center-line and of the gutter-lines, and also in many cases of the sidewalk lines and property lines. The engineer often marks out the templet used in laying a pavement. Unless the street is *very wide* this templet corresponds to one half of the cross-*section; its lower* edge is an arc of a vertical circle or parabola starting from the gutter-line and rising to the center of the street to give a proper crown to the pavement.

360. To Give Grades for a Sewer. The center-line of the trench is first staked out on the ground. As soon as a portion of the trench is well under way, have the contractor set planks about two inches thick on edge across the trench at intervals of about 25 or 30 ft., the ends sunken into the banks flush with the ground surface. Nail upright strips (grade-slats) about two inches wide to the plank so that a line along the edges of the strips will be directly over the center of the sewer. The transit used for lining in the strips may now be used for "shooting in" the grade, or the grade-rod method may be applied. The grade is marked on the uprights, either by driving a nail horizontally into the edge of each "slat," or by cutting a notch. The nails should be driven only part way in, so that by sighting along the nails any error may be detected by the eye. The nails or notches on the uprights are usually placed a whole number of feet above the flow-line of the sewer (the lowest line along the *inside* surface of the sewer). This holds good for pipes as well as for built sewers.

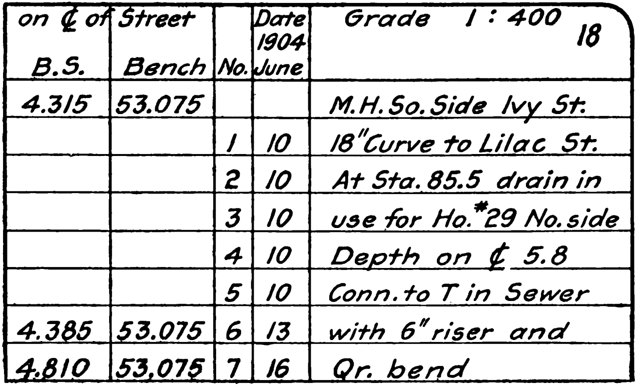
(*a*) *Practical suggestions.* (1) Grades should be given to the nearest hundredth of a foot, and the flatter the grade the more accurately the sewer should be laid. (2) When the ground is very uneven it may be necessary to set the nails or notches at *unequal* distances above the flow-line, but the distance for each notch should be some *whole* number of feet. It is well to mark a "crow-foot" on each grade-slat, with the apex at the nail or notch, and the amount of cut marked between the lines. (3) When nails or notches are a constant distance above grade and one is set by mistake far enough above or below the others to affect construction, the error will be apparent to the eye—hence form the habit of sighting frequently along the nails or notches. (4) The planks may be disturbed by blasting, or the uprights themselves be knocked out, especially if a cable trenching-machine is used. For this reason, if for no other, bench-marks should be established near the end of each stretch, or at distances not exceeding, say, 400 ft. (5) A straight, stiff grade-pole should be made for the use of the contractor and inspector, say of 2″×2″ pine, with a bracket at the bottom extending at right angles to the pole. Measuring from the bottom of the bracket, cut grooves to mark each foot. These grooves should be accurately cut across the face of the pole so that when the cord is stretched between the nails or notches on the uprights any groove may be fitted to the cord at any point, thus making the bottom of the bracket the corresponding number of even feet below the cord. (6) To avoid mistakes, work in one direction, say from down hill up hill, so that the target will always be moved downward at each new setting. (7) As sewers are usually laid in streets, it is easier to measure the distances between grade-slats along the surface of the ground, and find the proportion of the total rise or fall for each of these distances. (See (2), p. 286.) The *per cent.* for grade thus found does not accord with the definition in Art. 310 (*a*), p. 243, but the difference, numerically, is small. On rough uneven ground or very steep slopes it may be more convenient to measure between grade-slats by horizontal stretches, as in ordinary chaining, although a final "O. K." measurement should be made parallel to the sewer, since the contractor will be paid for the length of sewer as thus measured, and not for its horizontal projection.

31	Albany Ave. Sewer.			July 10 1905	
Sta.	B. S.	H. I.	Rod	El.	Cut
B.M. = 0-01.30	55.610 3.207	58.817	13 3.203	.614	10 .7 9.3
−1.30 0	C.M.H			.006 45.62	
25.16 0+25.16			13 3.071	.126 .746	10 .5 9.5
30.01 +55.17			12 2.921	.150 .896	10 .6 9.4
29.99 +85.16			2.771	.150 46.046	10 .5 9.5
30.02 1+15.18	55.15 60.03		2.621	.150 .196	10 .4 9.6
29.78 +44.96	58.61 59.32		2.472	.149 .345	10 .1 9.9
28.86 +73.82	42.36		2.328	.144 .489	10 .2 9.8
29.63 2+03.45	275.47		2.180	.148 .637	10 .3 9.7
29.73 +33.18			2.032	.148 .785	10 .5 9.5
30.03 +63.21			11 1.882	.150 .935	10 .6 9.4
5.13 +68.34	C.M.H @ P.C.			.025 .960	
7.22 +75.56			1.821	.036 .996	10 .8 9.2
2+75.47	O.K. July 10 1905		11.15 A.M.	R.N.C.	W.T.T.

17	Newhall St.		18″ Tile	Sewer	
Sta.	Grade	H. a. G.	E. G. P.	Rod	H. I.
0	40.125				57.390
47.15	.118 40.243	12	52.243	5.147	
40.34 87.49	.101 40.344	12	52.344	5.046	
39.67 127.16	.099 40.443	12	52.443	4.947	
40.05 167.21	.100 40.543	12	52.543	4.847	
39.58 206.79	.099 40.642	12	52.642	4.748	
39.74 246.53	.099 40.741	12	52.741	4.719	57.460
39.90 286.43	.100 40.841	12	52.841	5.044	57.885



24" Tile, Grade Inspector began	0.5 % 32 work July 8. 7 A.M.
C.Line of Sewer 8'	South of No. Curb.
On Intercepting S	ewer, Cor. Garden St.
Rock in Trench	
Sta. 0+57 From 10 10.0	Ties for M.H. of P.C.
+60 9.3	From pole #267 31.19
+63 8.5	" Hyd. base 57.31
+69 9.2	
+75 10.0	
12" Water	Main Location
	3.1 1+88
2" Gas Crosses 0.8	1+82
at Sta. 2+05	1+73.5 Crosses ℄
3'.5 deep 0.6	1+67 Depth 4'
2.7	1+53 to top



on ℄ of B.S.	Street Bench	No.	Date 1904 June	Grade 1 : 400 18
4.315	53.075			M.H. So. Side Ivy St.
		1	10	18" Curve to Lilac St.
		2	10	At Sta. 85.5 drain in
		3	10	use for Ho. #29 No. side
		4	10	Depth on ℄ 5.8
		5	10	Conn. to T in Sewer
4.385	53.075	6	13	with 6" riser and
4.810	53.075	7	16	Qr. bend

(*b*) *Illustration.* (See field notes for Albany Ave. sewer, pp. 284,285.) *Data.* The blue-print called for the following flow-line elevations: At the center of the manhole (or Sta. 0), El. = 45.62 ft.; at 268 ft. upstream (or Sta. 2+68), El. = 46.96 ft.; a straight grade between these two points or a rise of 46.96 − 45.62 = 1.34 ft. in 268 ft., or, approximately, a 0.5% grade (for 1.34 ÷ 2.68 = 0.5). An outline of the different steps that were taken in giving the grade is as follows:

(1) The contractor set planks on edge across the trench, at intervals of about 30 ft. The engineer lined in the uprights or grade-slats. As a slat could not be set over the manhole at Sta. 0, one was set directly back of it (−1.3 ft.).

(2) Measured the partial distances between slats, 25.16, 30.01, 29.99, etc., and entered these distances in column 1, written in small figures just below the ruled lines. As the grade of the street was slight, these distances were measured along the surface.

(3) Added each partial distance to the number of the preceding station to get the number of the station at the succeeding grade-slat, thus: (0)+25.16 = Sta. 0+25.16; (0+25.16)+30.01 = Sta. 0+55.17; (0+55.17)+29.99 = Sta. 0+85.16, and so on, thus completing column 1. Observed that the last station checked, very closely, with the total distance.

(4) Multiplied each partial distance by 0.5 (per cent.) and put the results in column 5, written in small figures just below the ruled lines, thus: 25.16 × 0.5 = 0.126; 30.01 × 0.5 = 0.150, and so on.

(5) Starting with the known elevation of 45.62 for Sta. 0, found the elevation of the flow-line at each station by adding to the elevation of the preceding station the quantity in small figures just below, thus: 45.62 + 0.126 = 45.746; 45.746 + 0.150 = 45.896, and so on. Noticed that 46.96, the elevation thus obtained for Sta. 2+68.34, checked with the elevation called for on the blue-print for that station. This completes column 5.

(6) Backsighted on a bench-mark to get H.I. For convenience, 55.610, the elevation of the B.M., and 3.207, the B.S., were both recorded in the second column. Recorded 58.817 (their sum) in the third column.

(7) Subtracted each of the calculated elevations in column 5 from H.I. to get the grade-rod and entered the results in the fourth column, the whole number of feet at the top of the space, the decimal part on the ruled line, thus: 58.817 − 45.746 = 13.071; 58.817 − 45.896 = 12.921, and so on.

(8) Found, by trial, that it would be most convenient to cut the notches on the grade-slats 10 ft. above the flow-line, hence each grade-rod was reduced by 10 ft., giving "field-rods" of 3.071, 2.921, etc., thus completing column 4.

(9) The notch on each grade-slat was then cut, a "crow-foot" marked on the slat with its point at the notch, and "10" marked between the lines of the crow-foot. As the work progressed, the last column was filled out. Opposite Sta. 0+25.16, for example, there are three quantities recorded, i.e., 10 ft., the difference between the grade-rod and field-rod, or the distance of the notch on the slat above flow-line; 0.7 ft., the distance of the notch above the ground, and 9.3 ft. (or 10 − 0.7), the actual cut at that station. This column shows the actual depth of earth which the contractor has to remove; it is also useful in plotting the profile of the finished sewer.

(10) Finally, measurements were taken between notches, skipping every other slat so as to make each measurement cover two intervals. These measurements of 55.15, 60.03, etc., in column 2, were added up, and their sum or the "O. K." measurement was compared with the length obtained in the first column from the partial measurements. As might be expected, it was slightly shorter. The contractor is paid for the length of sewer as thus measured, i.e., as measured parallel to grade.

(11) Explanatory notes and records of various underground structures and rock measurements were put on the right-hand page. Thus, for example, rock in the trench began at Sta. 0+57, 10 ft. down, measured from the cord stretched between notches on the grade-slats; at Sta. 0+60 the top of the rock was 9.3 ft. below the cord; at Sta. 0+63, 8.5 ft. below, and so on. The direction of the 12″ water-main is given by offsets from the center-line of sewer; thus at Sta. 1+88 the offset is 3.1 ft, at 1+82 it is 0.8 ft, and at 1+73.5 it crosses the center-line, with its top 4 ft. below the cord stretched between notches on the grade-slats.

(*c*) *Illustration.* (See notes for Newhall St. tile sewer.) The first and second columns correspond to the first and fourth columns, respectively, of the preceding illustration. The third column is the "height above grade" of the notches or nails in the slats and corresponds to the first part of the last column in the preceding illustration. The fourth column contains the "elevation of grade points" or actual elevations of the notches or nails. The fifth column gives the field-rods. The third and fifth columns are combined in column 4 in the preceding illustration.

(*d*) *Suggestions.* (1) The heading of the notes for sewers should give: name of street; size of sewer and material; rate of grade and date. (2) Manholes and other important points should be referenced, and notes made of curves, house-connections, and bends, as illustrated on p. 285.

361. To Give Elevations for a Line of Shafting. The controlling points, say at each hanger, must be lined in first, hence the transit used for that purpose may also be used for leveling. The elevations of the controlling points may be established by the grade-rod method in the usual way. Since the points are usually above the instrument, *minus* grade-rods must be used, and hence it will be more convenient, although by no means necessary, to hold the rod upside down. A bench-mark should be established from which all elevations may be obtained.

Illustration. The elevation of the line of shafting at some hanger-point is to be, say, 12 ft. The height of instrument is, say, 5 ft. Then $5-12=-7$ ft. is the grade-rod, i.e., the required point is 7 ft. *above* instead of below the line of sight. Set the target at 7 ft., and hold the rod upside down. The base of the rod will then be 7 ft. above the line of sight and hence will give the required point.

362. To Estimate Cut and Fill for Grading Divide the area to be graded into squares, as in Fig. 468, p. 432. Find the elevation of the ground at the corner of each square, as in profile leveling, and calculate the cut or fill according to the method explained in Part II. *Borrow-pits* may be staked out and the contents estimated by this method.

Practical suggestions. (1) Establish one or more permanent bench-marks, according to the size of the plot. (2) Make the squares as large as possible, and yet have the surface of the ground in any one square approximately a plane. The more uniform the surface the larger the squares; very rough ground, small squares. (3) Take extra rod-readings to cover hollows and mounds within a square, which otherwise would cause appreciable errors in results. (4) Chain-pins can be used to mark corners, but stakes are better, for the number or letter corresponding to the corner may then be marked on the stake. (5) Use some system of marking corners like that shown in Fig. 402 (*c*), p. 341. (6) It is often accurate enough to line in stakes by eye, measuring the distances between with a tape, or sometimes by pacing. Another method is to lay a tape on the ground first along one line of corners and then along another line; the rod is held at every 10-ft. point or 20-ft. point, or whatever the proper interval may be and thus the use of stakes or chain-pins is avoided.

363. To Set Stakes for Grading Land. Divide the plot into squares, drive a stake a few inches into the ground at the corner of each square, and mark each stake according to some system like that shown in Fig. 402 (*c*), p. 341. If the cut and fill is to be estimated before the stakes are set to grade, the elevation of the ground at each stake is found in the usual manner, and the computations made as explained on pp. 429–434. When the finished grade has been decided upon, it remains to drive each stake either by using a grade-rod or by "shooting in" the grade. There are three general cases, according to whether the finished grade is (1) to be level, (2) to slope in one direction, or (3) to slope in two directions.

Remark. In each of the three cases it will be assumed for purposes of explanation that the surface of the ground is such that all stakes can be driven to grade. When, as is usually the case, this cannot be done, the customary method is to drive temporary stakes marked with the cut or fill *measured from the existing surface.* (See (5), p. 259.) When the rough grading is completed, the final set of stakes may be driven with tops to grade (see (1), p. 258), or with tops, say, one foot above grade (see (2), p. 258), or with lines marked on the stakes (see (3), p. 258). Occasionally it will be necessary to set stakes once for all before grading is begun, and to use different "field-rods" for different stakes. (See illustration (*b*), p. 259.)

(*a*) *Case I. To set stakes so that the finished grade will be level.* It is required to grade to an elevation of 100 ft. (actual or assumed). (1) Set up the level and find H.I. Say it is 106.45. (2) Set the target at the grade-rod, in this case 6.45. (3) If the ground is already nearly level it may be possible to drive all stakes to grade, using this grade-rod, or, if more than one set-up is required, by figuring a new grade-rod for each new set-up of the level.

(*b*) *Case II. To set stakes so that the finished grade will slope in one direction only.* In this case it is best to set the lines of stakes parallel to the direction of the slope. Then each line may be set in a manner similar to that explained for setting curb-stakes (p. 282). A quick method, when the surface of the ground is fairly even, is to "shoot in" one row of stakes in the direction of the slope, and then set the cross-rows at right angles, with all the stakes in any one row at the same elevation. In rough ground it is usually necessary to set stakes by the use of "field" grade-rods, as explained above.

(*c*) *Case III. To set stakes so that the finished grade will slope in two directions.* In this case, if the two directions of slopes are at right angles to each other, the work is similar to the preceding case, the stakes in one direction being set parallel to the slope in that direction; the cross-lines in the other direction will then be parallel to the other slope. When the ground is nearly at the required grade, a quick way is to calculate the elevations of the four corner-stakes, set these stakes, "shoot in" the outside rows, and then "shoot in" the intermediate stakes of each row from the two extreme stakes in that row. If, however, the ground is too uneven to set the stakes without using different grade-rods, it then becomes necessary to calculate the elevation of the finished grade at each stake, and from these elevations find the grade-rod to use for each stake, as in the preceding illustration.

When the two directions of slope are not at right angles to each other, the problem is somewhat more complicated. As a general rule, several points may be chosen to be used as controlling points. The elevations of these points having been determined, by calculation or by some other method, stakes may be set on a straight line between any two controlling points in the usual manner. Controlling points should be selected with a view of simplifying the work of calculation as well as the actual field work.

364. To Stake Out a Vertical Curve. Once the elevations of points on the curve are known, the work of setting stakes does not differ from any of the work already explained in which the grade-rod method is used. The elevations on the curve are usually taken at regular intervals, and may be determined in two ways: (1) by calculation, and (2) by plotting the curve and scaling the ordinates (elevations). If the latter method is used, the vertical scale is usually much greater than the horizontal scale,—large enough so that ordinates may be scaled to the limit of error fixed upon for setting stakes. In case this limit of error is very small, it is usually necessary to calculate elevations. The parabola is often used for vertical curves, because (1) it is well adapted for a gradual and easy change from an up grade to a down grade, or vice versa, and (2) the calculation of elevations is very simple.

(*a*) TO CALCULATE THE ELEVATIONS OF POINTS ON A VERTICAL PARABOLA

Let I be the point of intersection of an up grade AI with a down grade IB. Let d = the distance between stations, and let A be n stations on one side of I, and B be n stations on the other side, i.e., $AE' = E'B'$, and hence $AE = EB$. Let g represent the rise of AI per d ft., and g' the fall of IB per d ft. These two grades may be unequal. Required: To find the elevation of points on a vertical parabola passing through A and B, tangent to AI and BI. There are two methods: (1) To calculate tangent corrections y_1, y_2, y_3, etc., at regular intervals; this corresponds to tangent offsets for circular curves (p. 207). (2) To find the chord gradients for successive chords. Since $AE=EB$, IE is a diameter of the parabola, and the distances of the curve in a vertical direction from stations on the tangent AI are as the squares of the number of stations, or if $x=$ the number of stations and $c=$ a constant, $y=x^2c$. (From analytic geometry.) It can be shown that

$$c=\frac{g-g'}{4n}. \qquad \qquad (1)$$

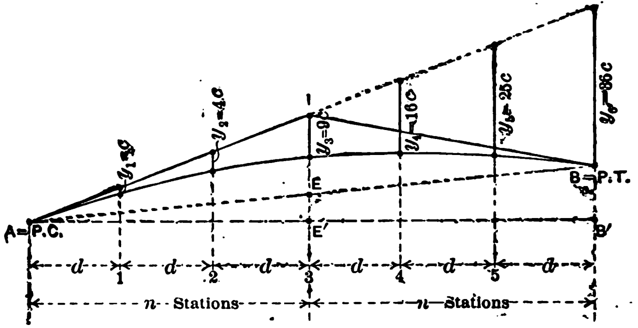


FIG. 364 (*a*).

I. First method, *tangent deflections*	II. Second method, *successive chord gradients*
d ft. from A or $P.C.$ $y_1=c$	First chord gradient $=g-c$
$2d$ " " " " " $y_2=4c$	Second " " $=g-3c$
$3d$ " " " " " $y_3=9c$	Third " " $=g-5c$
$4d$ " " " " " $y_4=16c$	Fourth " " $=g-7c$
Nd " " " " " $y_N=N^2c$	Nth " " $=g-(2N-1)c$

Remark. The plus sign may be given to an ascending grade and a minus sign to a descending grade. If strict attention is paid to algebraic signs in determining the value of c, the above formulas hold good for all six possible combinations of grades, including the cases when both grades are rising or both falling.

(*b*) *Elevations of special points.* The elevation of a point between stations may be found by treating the distance of the point from the preceding station as a decimal part of d. (See remark, p. 292.)

The elevation of I is the elevation of $P.C.$ plus ng. The elevation of $P.T.$ is the elevation of I plus ng'.

The elevation of the point on the curve directly above or below I may be found by adding algebraically n^2c to the mean of the elevations of $P.C.$ and $P.T.$

FIELD WORK OF STAKING OUT A VERTICAL PARABOLA

(*c*) *Method of procedure for tangent corrections.*

(1) Calculate c from formula (1), p. 290, being careful to give to g and g' their correct algebraic signs. (See remark, p. 290.)

(2) Calculate the uncorrected elevation of each station as if it were on the given grade through $P.C.$ by adding g *algebraically* to the elevation of the preceding station.

(3) Calculate the tangent corrections y_1, y_2, y_3, etc.

(4) Subtract *algebraically* the values of y_1, y_2, y_3, etc., found in (3) from the corresponding uncorrected elevations found in (2). This gives the actual elevations of points on the curve.

(5) Set stakes for the various points by the grade-rod method (pp. 243, 258–259).

(*d*) *Method of procedure for chord gradients.*

(1) Calculate c as in the preceding method.

(2) Calculate the gradient for each chord by subtracting $2c$ *algebraically* from the gradient of the preceding chord. (The gradient of the first chord is $(g-c)$. See p. 290.)

(3) Beginning with the elevation of $P.C.$, add *algebraically* the gradient of each chord to the elevation of the station at the beginning of the chord to find the elevation of the station at the end of the chord.

(4) Set stakes for the various points by the grade-rod method.

(*e*) *Methods compared.* The method of chord gradients is preferable to the other, since it involves not only less arithmetical work, but if the elevation of $P.T.$ comes out correct, it is a check on all of the calculations.

(*f*) *Illustration.* A grade ascending at the rate of 1 ft. per 100 ft. is to be connected by a parabola with a grade descending at the rate of 3 ft. per 100. Stakes are to be set 100 ft. apart. $P.C.$ and $P.T.$ are each to be 200 ft. from I. Then $d=100$, $g=1$, $g'=(-3)$, $n=2$, and $c=\frac{1-(-3)}{4\times 2}=+0.5$. Suppose the elevation of $P.C.$ is 128.16, then the following table will illustrate the two methods.

BY TANGENT CORRECTIONS

Stake	Uncorrected elevation	$y=x^2c$	Corrected elevation
$P.C.$	128.16	− 0.0	= 128.16
	+1.0		
1	129.16	− 0.5	= 128.66
	+1.0		
2	130.16	− 2.0	= 128.16
	+1.0		
3	131.16	− 4.5	= 126.66
	+1.0		
$P.T.$	132.16	− 8.0	= 124.16

BY GRADIENTS

Line	Gradient	Corrected elevation
— to $P.C.$	1.0	128.16
	+0.5	+0.5
$P.C.$ to 1	$g-c=+0.5$	1 = 128.66
	+1.0	−0.50
1 to 2	$g-3c=-0.5$	2 = 128.16
	+1.0	−1.5
2 to 3	$g-5c=-1.5$	3 = 126.66
	+1.0	−2.5
3 to $P.T.$	$g-7c=-2.5$	P.T. = 124.16

Check. From (*b*) the El. of $I=128.16+2\times1=130.16$. El. of $P.T.=130.16+2\times(-3)=124.16$ (check). El. of curve at a point below I is $(128.16+124.16)\div2+2^2\times0.5=126.16+2.0=128.16$ (check).

Remark. The elevation of any substation say 140 ft. from $P.C.$ (1.4 stations) $=128.16+1.4g-(1.4)^2c=129.56-0.98=128.58$.

Both grades ascending or both descending. Method still holds good.

Exercise. Find the corrected elevations in the above illustration when g' is +3. (The elevation of $P.T.$ should come out 136.16.)

(*g*) *To make the vertical parabola connecting two grades pass through a point at a given distance above or below I.* From p. 290 $c=\frac{y}{n^2}=\frac{g-g'}{4n}$. Hence y for a point above or below I (i.e., n stakes from $P.C.$) is given by $y=\frac{n(g-g')}{4}$.

(1) Substitute for y the amount the curve must pass above or below I, insert values for g and g', and solve for n. (2) Find c and proceed as usual.

Illustration. A grade descending at the rate of 2.0 ft. per 100 ft. is to be connected with a grade ascending at the rate of 3.6 ft. per 100 ft. by a vertical parabola. The curve must pass through a point 1.4 above I. Stakes must be set at 25 ft. intervals. $g=-2$ ft. per 100 ft. $=-0.5$ ft. per 25 ft. $g'=+3.6$ ft. per 100 ft. $=+0.9$ ft. per 25 ft.

By the conditions $y=1.4$, hence $1.4=\frac{n(-0.5-0.9)}{4}$, and $n=4$. Hence $c=\frac{-0.5-0.9}{4\times4}=-0.0875$. Suppose that the elevation of $P.C.=98.6$.

BY TANGENT CORRECTIONS

Stake	Uncorrected elevation		$y=x^2c$		Corrected elevation
P.C.	98.6		0.0		98.6
	−0.5				
1	98.1	−	−0.0875	=	98.1875
	−0.5				
2	97.6	−	−0.35	=	97.95
	−0.5				
3	97.1	−	−0.7875	=	97.8875
	−0.5				
4	96.6	−	−1.4	=	98.0
	−0.5				
5	96.1	−	−2.1875	=	98.2875
	−0.5				
6	95.6	−	−3.15	=	98.75
	−0.5				
7	95.1	−	−4.2875	=	99.3875
	−0.5				
P.T	94.6	−	−5.6	=	100.2

BY GRADIENTS

Line	Gradient	Corrected elevation
— to P.C.	−0.5	98.6
	−0.0875	−0.4125
P.C. to 1	−0.4125	98.1875
	−0.1750	−0.2375
1 to 2	−0.2375	97.9500
	−0.1750	−0.0625
2 to 3	−0.0625	97.8875
	−0.1750.	+0.1125
3 to 4	+0.1125	98.0000
	−0.1750	+0.2875
4 to 5	+0.2875	98.2875
	−0.1750	+0.4625
5 to 6	+0.4625	98.7500
	−0.1750	+0.6375
6 to 7	+0.6375	99.3875
	−0.1750	+0.8125
7 to 8	+0.8125	100.2000

Check. El. of $I=98.6+\times4(-0.5)=96.6$. El. of $P.T.=96.6+4\times0.9=100.2$ (check). The elevation of a point on the curve above I is $(98.6+100.2)\div2+4^2\times-0.0875=99.4+(-1.4)=98.0$ (check). (See also p. 368.)

(*h*) *When the rate of change c and the two grades g and g′ are given.* The first step is to find n from $n=\frac{g-g'}{4c}$. Then proceed as in the other cases already illustrated. (See books on Railway Curves for other special cases.)

CHAPTER XXIV

COMPASS SURVEYING

SINCE the theory of transit surveying already explained includes practically all of the theory of compass surveying, it is necessary in this chapter to give only a few additional directions and suggestions for the use of the compass. The first part of Chapter XI, which deals with the subject of bearings, should be studied in connection with this chapter.

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365. Compass Surveying. The transit has taken the place of the surveyor's compass to such an extent that now the latter is used for rough work only, where speed and not accuracy is the first requirement. For such work the compass is a very useful instrument, and surprisingly good results may often be obtained with it. The positions of any two points with respect to each other are determined by the *bearing and the length* of the line between them, and this is the basis of compass surveying. The angle between two lines may be calculated easily if the bearing of each line is known. (See chapter on *Calculation of Bearings.*) The clumsy and inaccurate chain formerly used in compass surveying is being gradually replaced by the steel chain-tape. (See remark, p. 33.)

The objections to the use of the compass are: (1) Under the most favorable conditions bearings cannot be read with any degree of certainty closer than five or ten minutes. (2) The needle is unreliable, and under certain conditions, as for example in city surveying, local attraction may render it practically useless.

The advantages are: (1) The compass is light and portable; and while it is less accurate than the transit, it requires much less time for setting up, sighting, and reading. (2) An error in the direction of one line does not, necessarily, affect other lines of the survey. (3) The compass is especially adapted to running straight lines through woods and other places where obstacles are likely to interfere with the line of sight. Thus, for example, when a foresight strikes a tree the instrument may be set up beyond it at a point, found by trial, such that the backsight on the tree will have the same bearing as the foresight. (4) Since the compass was the instrument in general use before the introduction of the theodolite and the transit, it is especially adapted to retracing old surveys.

366. Use of the Compass. *Practical suggestions.* (1) Set up over the station and level up. If the instrument is fitted with a ball-spindle only, it is made level by grasping the main plate *near the compass-box*—not near the sight-vanes—and moving it until both bubbles come to position, where it is held by friction on the ball. If there are leveling-screws, the instrument is made approximately level on the ball and the leveling perfected by the leveling-screws.

(2) The upright sights or sight-vanes are usually unlike. For example, the one nearer the north point of the compass-box may have graduations marked on it for vertical angles, while that at the south end is not graduated but has attached to it two little revolving disks or peep-holes. If there is a counter for keeping tally in chaining, it is usually placed on the plate near the south sight. Fix in mind once for all which of the two uprights is the *south* sight, so as to tell it at a glance, always keep this next to the eye in sighting, and always read the *north* end of the needle, as explained on p. 102. Use a magnifying-glass in reading, and look along the axis of the needle to avoid parallax. Read to the nearest half-degree and estimate to the nearest five minutes. It is good practice to glance at the *south* end of the needle as a check, especially if one moves around the instrument to take the reading, for in changing positions any source of local attraction about the person is likely to be detected by the corresponding change in the needle reading.

(3) Protect the pivot in every way possible, for unless the pivot is sharp and perfect the needle may be sluggish and unreliable. Hence lift the needle off the pivot before starting to carry the compass to another point, and keep it off when not in use. Avoid excessive vibrations, after setting up, by turning the main plate before releasing the needle until the latter is nearly in the magnetic meridian; the same result will be obtained if the vibrations are checked by slightly raising the needle, but the longer the needle is in coming to rest the more accurate will be its final position.

(4) Keep sources of attraction such as axes and chain-pins away from the compass; bits of steel about the person, such as *pocket-knives,* wire in a hat brim, a steel suspender buckle,

spectacle-cases, sometimes affect the needle. The glass of the compass-box may become electrified from rubbing it—wet it with the finger or breathe upon it.

(5) If the needle is sluggish it may need to be remagnetized by bringing it into the magnetic field of an electric light or a dynamo. In doing this the poles may be reversed; test by putting the needle back on its pivot, and if reversal is found put the needle back into the field the other end about. A more tedious method is to lay the needle down and rub each end from the center out with that end of a bar-magnet which attracts the end of the needle that is being rubbed. In passing the magnet back from end to center, lift it some distance above the needle to avoid counteracting the rubbing.

(6) For important lines take duplicate readings. After the first reading, disturb the needle by lifting it or by tapping the box gently, and see if it comes to rest at the same reading as before. Another way is to turn the sights off the point sighted at and to bring them back again to check the original reading; this takes more time, but it is the surest check.

(7) To guard against local attraction read the bearing of important lines at two different points—usually at the two ends.

(8) When, because of obstructions, it is inconvenient to take the bearing of a line, take the bearing of a parallel line, established by offsets as near as practicable to the required line.

(9) In ordinary work make sure that the declination arc is set at zero. When, however, it is desired to run lines by true bearings instead of by magnetic bearings, set off the declination arc (Art. 185, p. 124). In retracing old surveys the change in the declination of the needle may be ascertained and the declination arc adjusted so that the lines may be run from the bearings given by the original surveyor. (See p. 298.)

(10) When the compass is put away, allow the needle to swing to the magnetic meridian and then lift it—it retains its magnetic strength longer in this position.

367. Compass Surveying,—General Method. (*a*) *Reference lines* corresponding to transit lines, Chapter XII, are established and the length and bearing of each ascertained. (See also Art. 188, p. 125.)

(*b*) *Where to run lines.* The suggestions in Art. 216, p. 149, hold good. It is often possible, however, to make compass lines coincide with boundary-lines, where in transit surveying it would be impossible. When this can be done it saves much unnecessary work (Art. 216 (*a*), p. 150). Avoid choosing stations near sources of local attraction.

(*c*) *Methods of locating details.* The methods of Chapter XIII hold good, except that bearings of lines are read in place of angles in those methods which involve angular measurements. Because of the inaccuracy of the compass, methods involving linear measurements only (p. 138) are used even more than in transit surveying. The old rule that "when distances and direction to the same point disagree distances govern" is more true in compass surveying than in transit work.

(*d*) *Practical suggestions.* (1) *Equipment* nearly the same as given in Art. 204, p. 145, with the compass in place of the transit. (2) *Referencing stations* by marks on trees, etc. See Art. 211, p. 148. (3) *Relation between office and field work.* See Art. 215, p. 149.

368. Detection and Elimination of Local Attraction. In this article the term *local attraction* is used in its strict sense, and does not include the deviation of the needle due to things about the person or to other avoidable sources.

(*a*) *Detection.* When the bearings read at the two ends of a line do not agree, *local attraction* probably exists at one or possibly both ends, provided the compass is in good condition and no mistakes are made in observations. To ascertain the correct bearing, set up at a third point on the line, and see with which of the first two readings the third agrees. If it agrees with neither, try a fourth point, and so on until two readings do agree.

(*b*) *Elimination.* The angle formed by two lines can be calculated from the bearings of the two lines regardless of local attraction, provided the bearings are both read at the same station—i.e., where the lines intersect. This fact suggests a method of eliminating local attraction from the bearings of a number of connected lines, as illustrated below.

Compass at	Line	Bearing	Correction	Corrected bearing
A	*AD*	N. 62° 40′ E.	+50′	N. 63° 30′ E.
	AB	S. 69° 0′ E.	+50′	S. 68° 10′ E.
B	*BA*	N. 67° 0′ W	−1° 10′	N. 68° 10′ W.
	BC	N. 41° 0′ E.	−1° 10′	N. 39° 50′ E.
C	*CB*	S. 39° 50′ W.	0	
	CD	N. 57° 0′ W.	0	
D	*DC*	S. 57° 0′ E.	0	
	DA	S. 63° 30′ W.	0	

Illustration. The bearing of each line in Fig. 368 (*b*) was read at both ends of the line. Glancing down the results in the third column above, it is seen that in only one case (the line *CD*) do the bearings agree. This indicates that there is no local attraction at either *C* or *D*. Hence the bearing of *DA* = S. 63° 30′ W. must be correct. But at *A* the reading is N. 62° 40′ E., indicating an error of +50′ due to local attraction at *A* (+ = clockwise, − = counterclockwise). Applying this correction to the erroneous reading S. 69° 0′ E. of *AB*, the correct bearing of *AB* is S. 68° 10′ E. But the reading at *B* is N. 67° 10′ W., indicating an error of −1° 10′. Applying this correction to the other reading taken at *B*, i.e., *BC* = N. 41° 0′ E., the correct bearing of *BC* is N. 39° 50′ E. This checks with *CB* = S. 39° 50′ W., taken at *C*, where there is no local attraction. Thus local attraction may be eliminated from any number of connected lines provided the correct bearing of one line is known.

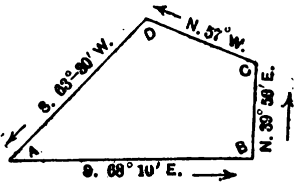

FIG. 368 (*b*).

Note. When the bearing of not even one line can be taken as correct, assume, for temporary purposes, that the correction at some one station, say the first station, is zero, and correct all the other bearings accordingly by the method just illustrated. It will usually be found that for a number of stations the correction is the same. It is therefore pretty safe to assume that at these stations no local attraction exists, and the bearings may again be corrected on this assumption.

369. Field Notes and Method of Procedure. The field notes are usually kept by the sketch method (p. 165). Thus, for example, if the compass had been used in the survey illustrated on p. 183, the bearing would be recorded along each line in place of the angle. When simply the bearings of a few lines are taken and no sketch is required, the notes may be tabulated in a form similar to Form 9—a modification of Form 7, p. 174. Be careful to record whether the bearings are true or magnetic.

FORM 9

Station	Line	Bearing	Length		
D	DE	S.1°20′E	229.7	All bearings are Magnetic.	Declination = 10°58′W July 23, 1906
	DC	S.72°55′W			
C	CD	N.72°55′E	155.0		
	CB	N.8°45′E			
B	BC	S.8°45′W	87.3		
	BA	N.22°10′W			
A	AB	S.22°10′E	107.8		

Method of procedure. Set up at A, took the bearing of AB. Set up at B, took the back bearing of AB, i.e., the bearing BA,—found that it checked with the forward bearing. Took the forward bearing of BC, and moved on to C. While this was being done the chainmen measured the lines between stations. Continued the work by taking the forward and back bearing at each station, as indicated in the notes.

Remark. The first column is unnecessary and may be omitted provided it is understood that the first letter of a line is the station at which the compass stands when the bearing is taken. In case there are side shots for locating details, these may be entered in the tabulated form as in transit surveying, and an explanatory sketch made on the right-hand page—a method resorted to sometimes when there is not room on the sketch. In compass surveying, however, details are usually located by linear measurements only, and these should be put directly on the sketch itself.

If the methods of keeping notes for transit surveys have been mastered, it is an easy matter to adapt them to compass surveys.

370. Sources of Error. (1) Local attraction. (2) Errors of observation. (3) Instrumental.

The first source of error is annoying, but it can be eliminated by the method of the preceding article. The second includes errors in sighting and in reading,—errors from this source can be avoided by following the suggestions in Art. 366, p. 294. The chief source of error is the compass itself. Not only is it difficult to read the needle closely, but different compasses will often give different bearings for the same line, even when the instruments seem to be in good adjustment.

Limits of precision. The maximum error of bearing should not exceed ten minutes if a good compass graduated to half-degrees is used. Since 1 in 57.3 corresponds to 1°, this is equivalent to 1 in 344. If the bearing is correct to five minutes, which is as close as the compass can be read with a reasonable degree of certainty, the error is 1 in 688. This is much less accurate than the results which may be reasonably expected in measuring distances with a chain. (See p. 49.) Of course better results are often obtained with the compass, but a higher limit of error should not be required.

Rerunning Old Surveys. The chief use of the compass, in many communities, is in rerunning former surveys, especially in finding boundaries which have become more or less uncertain. The original surveys were made, say, from 40 to 100 years ago, and the large tree or the pile of stones which once marked a corner have entirely disappeared. In such a case the surveyor can proceed only from such evidence as he can procure by examining town records, and by searching for existing bounds. If the owners who are interested will not accept his results, it is then a matter for the courts. The surveyor cannot, of course, settle a boundary dispute, though many persons seem to think so. He can, however, give expert testimony, and frequently he can persuade the disputants to accept a boundary-line which he has run out without resorting to the courts.*

Assuming that the lengths and bearings of the boundaries are known, it will be necessary to ascertain the change in the magnetic declination which has taken place since the original survey was made. If *two* points on any boundary-line are still preserved, the most direct method is to sight along this line and to change the declination arc until the bearing agrees with the original. When, however, only one point on a boundary-line remains, it will be necessary to find the change in magnetic declination in some other way. This change may often be calculated, provided the date of the original survey is known.† When tracing deeds in town records, however, it is not easy to know when one has gone far enough back, i.e., to be sure that one has reached the owner for whom the original survey was made. When the date has been ascertained and the change in the magnetic declination has been figured, the declination arc should be adjusted accordingly, or, if there is no provision for this adjustment, proper allowance should be made in the bearing of each line. The importance of dating a map and of giving the corresponding magnetic declination is obvious.

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* It is not within the province of this book to enter upon the difficulties encountered in rerunning old surveys, or to cite cases illustrating the existing laws. For this part of the subject the student is referred to standard works of reference, especially to a *Manual of Land Surveying* by F. R. Hodgman, Climax, Mich., published by the author.

† When there is no local record of the change in magnetic declination from year to year, it may often be obtained from the "U. S. Declination Tables and Isogonic Charts for 1902" by L. A. Bauer, published by the U. S. Government (Coast and Geodetic Survey). This volume embraces some 8000 observations on magnetic declinations in different states.

CHAPTER XXV

STADIA SURVEYING

In this chapter the principles of stadia surveying are explained, formulas for reducing inclined sights to horizontal and vertical distances are derived, methods of determining the stadia interval and of graduating rods are given, methods of procedure in field work are outlined, sources of error are pointed out, and limits of error are suggested.

372. Stadia, Stadia-wires, Stadia-rod. The stadia method of measuring distances * consists in observing how much of a rod held at one end of a line appears to be included between two parallel cross-hairs (or wires) as the observer looks through a telescope at the other end of the line. The farther the rod from the telescope, the more of the rod will be included between the two hairs. *Stadia-wires* in the telescope of the transit or the level are horizontal, one above and one below the regular horizontal cross-hair. In some telescopes they are a fixed distance apart; in others the distance can be changed, i.e., the stadia-wires are adjustable. *Stadia-rods* are usually from four to six inches wide and from one half to three quarters of an inch thick. There are many patterns in use, the main object being to secure graduations on the face of the rod which can be read easily at a distance and yet be fine enough to indicate definitely and accurately the distance of the rod from the stadia-wires. Leveling-rods are often used as stadia-rods. The term *stadia*, formerly used to denote the rod, is now used to include both the rod *and* the wires; it also stands for the method itself: thus, for example, measuring with the stadia, i.e., rod and wires, and measuring by the stadia, i.e., by the stadia method. *Telemeter* is a term often used in place of *stadia*. *Tachymeter* is sometimes used to denote a transit equipped with stadia-wires, vertical circle, and telescope-level.

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* *Said to have been* invented in 1770 by no less a man than James Watt.

373. The Principle of the Stadia is based on the geometric principle that in two similar triangles corresponding sides and altitudes are proportional. In practice the distance of a stadia-rod from a transit or a level is found by observing how much of the rod appears to be exactly included between the two stadia-wires as the observer looks through the telescope. For example, if at 100 feet away a *foot* of the rod is thus included, at 200 feet away *two* feet of the rod will be included, at 300 feet *three* feet, and so on. This is not, however, strictly true, for a *constant*, which differs slightly for different telescopes, must be added to each reading.

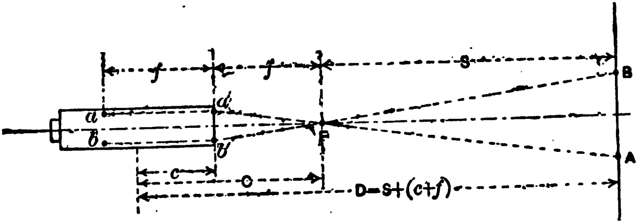

FIG. 373 (*a*).

(*a*) *Theory involved.* Let *a* and *b* in Fig. 373 (*a*) represent the position of the upper and lower stadia-wires. Let *AB* be the portion of a stadia-rod which appears to be exactly included between *a* and *b*. Any ray of light from *A* which passes through *F* will become parallel to the axis of the telescope after passing through the lens.* Since all rays from *A* converge at *a*, they may be treated as a single line *AFa'a*. Likewise the cone of rays from *B* gathered by the lens and brought to a focus at *b* may be considered as one line *BFb'b*.

Let *a'* and *b'* be the points where the oblique rays are changed to parallel rays. Let *c* be the distance from the center of the instrument to the center of the object-glass—a slightly variable quantity, dependent on the focusing. Then from the similar triangles *a'Fb'* and *AFB*, $a'b':f::AB:S$, or $S=\frac{f}{a'b'}\times AB$. Since $a'b'(=ab)$ is the interval between the stadia-wires (denoted by i), and since $AB=R=$ the rod-reading, then $S=\frac{f}{i}R$. The ratio $\frac{f}{i}$ for any given telescope is constant, therefore the distance S is always equal to the rod-reading multiplied by this ratio, no matter where the rod may be held. But the distance usually required is the distance from the rod to the *center* of the instrument. This distance $D=S+(c+f)$. The quantity $(c+f)$ for all practical purposes is a constant C. The quantities c and f may be found with much greater accuracy than is necessary, as follows: (1) Focus on a point 100 ft. or more away. (2) Measure from the center of the object-glass to the center of the instrument to find c. (3) Measure from the center of the object-glass to the cross-hairs to find f. (Since the cross-hairs are approximately at one of the principal foci or as far behind the lens as the other principal focus, F is in front of the lens.)* (4) Add these two measurements to find the total correction C. Notice that the constant C, once determined, is the same for all readings; it is *not* twice as much for 200 ft., three times as much for 300 ft., and so on.

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* From the theory of lenses explained in Part III.

(*b*) *Remark.* It makes little difference whether the telescope is focused on a distant point or on a point a few feet away, since the extreme range of the object-glass slide is less than one inch for the average telescope. This extreme variation in the distance c may be ignored when it is considered that stadia readings are usually taken to the nearest foot. Moreover, in changing the focus from say 100 ft. to the longest distance ever read, the variation is only a very small *fraction* of an inch.

The total correction C varies from about 0.9 ft. for a short telescope to about 2.5 ft. for the telescope of an 18″ wye-level. For transit work it is usually close enough to add an even foot to each reading.

374. To Test the Stadia Interval. The stadia-wires in most instruments are so placed that $\frac{f}{i}=100$; hence when one foot of the stadia-rod is intercepted the rod is $(100+C)$ feet from the center of the instrument; when 2 feet of the rod are intercepted it is $(200+C)$ feet away, and so on. The stadia interval and the value of C (determined as explained in the preceding article by measuring the telescope) may both be tested as follows:

Test. Set up the instrument on level ground. Hold the rod at such a distance away that the stadia-wires intercept a foot of it. Suppose this distance, measured with the tape, is 101.2 feet. This would indicate that the constant C is 1.2 feet. To make sure, however, hold the rod so that two feet are intercepted. The distance as measured should be 201.2 (*not* 202.4). If three feet are intercepted, the distance as measured should equal 301.2. A series of such observations should be made before beginning any important work with the stadia. (See p. 304.)

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* *From the theory* of lenses explained in Part III.

375. To Adjust the Stadia-wires. When stadia-wires are movable they may be adjusted as follows: (1) Set two targets on a leveling-rod exactly one foot apart, and hold the rod at a distance of $(100+C)$ ft. from the center of the instrument. (2) Sight at the rod, moving the telescope up or down with the slow-motion screw until the middle horizontal hair is exactly half-way between the two targets. (3) Make one stadia-wire bisect one target and the other wire bisect the other target by turning the screws which control the stadia-wires. Watch the middle hair to see that it does not move off the middle point, and if it does bring it back again. (4) Hold the rod at $(200+C)$ ft. from the instrument, and test each stadia-wire separately to see if it bisects one target when the middle hair bisects the other. (5) Test both hairs at the same time by setting the targets two feet apart. (6) Repeat the test for the wires at $(400+C)$ ft.

Practical suggestions. (1) The method just described makes the stadia-wires equal distances from the middle cross-hair. This is not essential so long as the interval between the stadia-wires is correct, unless, as is sometimes the case, distances are measured by observing the space included between one stadia-wire and the middle cross-hair. (2) In place of the regulation targets it may be more convenient and more accurate to make targets of cardboard having well-defined ink-lines at half-foot intervals, or three separate targets to be fastened to the rod at required intervals. (3) Notice that rods may be constructed and stadia-wires spaced so that units other than feet may be used, as, for example, units in the metric system. (4) One advantage of the standard interval is that the ordinary leveling-rod may be used as a stadia-rod when desired. (5) The screws for adjusting the stadia-wires are usually two capstan-headed screws, close to the four screws for adjusting the cross-hair ring. (See Adjustment of Cross-hairs in the chapter on Adjustments.) (6) The middle hair should never be disturbed in adjusting stadia-wires,—it is supposed to be in adjustment.

376. Fixed Stadia-wires. For the most accurate work stadia-wires should be *non-adjustable.* Most instruments having fixed stadia-wires are intended to be used with standard rods graduated to feet. A series of tests will often show, however, that though the maker has fixed the interval $\frac{f}{i}=100$ with accuracy sufficient for ordinary work, yet in order to obtain results better than say $\frac{1}{500}$, either (1) a special rod must be graduated to suit the instrument, or (2) an *interval factor* must be used for correcting readings.

(*a*) *To graduate the rod.* (1) Establish a point F a distance C in front of the instrument. (2) Measure accurately 100 ft., 200 ft., 300 ft., and 400 ft. from this point F. (3) Hold a rod with cardboard targets at each of the above points in succession, and determine as accurately as possible what interval on the rod corresponds to 100 ft. (4) Lay off these intervals on the rod as if they were feet, and again test at 100 ft., 200 ft., 300 ft., and 400 ft. (5) When the intervals are correct, the subdivisions may be painted on, according to any desired pattern.

(*b*) *The interval factor.* When it is desired to use a rod with standard graduations, and the fixed stadia-wires are found to be slightly in error, it will be necessary to multiply each reading by a constant factor to reduce it to the true reading. This interval factor may be determined by a series of tests similar to that given for graduating the rod.

Illustration. Suppose that the average intercept on a rod, as determined from a number of observations at different distances, repeated several times is 1.0042 ft., then all distances read would be too long and should be reduced by dividing by 1.0042. If the distances are inclined, this must be done before correcting for the inclination. If the interval is too short, say 0.9981, then all readings are divided by 0.9981.

(*c*) *Practical suggestions.* (1) Fixed stadia-wires are not likely to be displaced, and for this reason are used by many surveyors in preference to adjustable wires, especially where results better than $\frac{1}{500}$ are desired. The adjustable wires are apt to change their distance apart with every change of temperature. (2) When results no better than $\frac{1}{300}$ are required it usually will be found unnecessary to use either a special rod or an interval factor. (3) For very accurate work the tests for graduating a rod or for determining the interval factor should be made not at any one hour of the day, but during all hours of field-work, so as to approximate as closely as possible to the average conditions. (See p. 313.) (4) It is probably better in an extended survey to use a standard rod and to correct readings by the interval factor than to use special graduations, for the following reasons: Subsequent tests of intervals can be made without repainting or regraduating the rod; rods can be interchanged among transits or used with new transits; rods can be used for leveling without computation; leveling-rods can be used as stadia-rods; rods can be used by observers having different personal equations.

377. Method of Reading the Rod. The reading of the rod is learned more easily from actual practice than from any printed directions. The main object is to determine as accurately as possible how much of the rod is included between the stadia-wires. In doing this, sight at a point on the rod about as far above the ground as the telescope is above the ground. Move the telescope slightly up or down until either the upper or lower stadia-wire exactly strikes the nearest even-foot mark on the rod, and then observe how far from that even-foot mark the other stadia-wire strikes. *The graduations on the rod should* *be read in terms of distance.* Thus one foot on the rod stands for 100 feet in distance, $\frac{1}{5}$ of a foot stands for 20 feet, $\frac{1}{50}$ of a foot for 2 feet, and so on. Be careful not to mistake the regular horizontal cross-hair for a stadia-wire; do not use the former at all, except in special cases. In some instruments this danger is avoided by placing the stadia-wires so that when they are visible (in focus) the regular horizontal hair is invisible, and vice versa.

(*a*) *Common mistakes.* (1) To read between the stadia-wire and middle cross-hair, thus getting *half* the correct distance. Avoided by habitually estimating distances by eye, and thus checking the stadia reading; also by purposely reading and entering the half-distance in the notes, and then reading and entering the full distance independently. (2) Omitting an even hundred feet or multiples of ten feet, as 189 ft. in place of 289, or 129 in place of 139.

Practice. Before beginning the regular stadia survey, take a number of sights varying from 50 ft. or less to 300 ft. or more, and check each reading by measuring with a tape.

378. Distance and Elevation when the Line of Sight is Inclined. In the preceding articles it has been assumed that the line of sight in each case has been nearly horizontal. In hilly country it is necessary to incline the telescope. For such sights two corrections are involved: (1) If the line of sight is inclined, the distance obtained will correspond to the distance found by measuring on a slope, and this distance must be reduced to a *horizontal* distance. (2) If the rod is held plumb (vertical), as it usually is, it is not perpendicular to the line of sight, hence more of the rod appears to be included between the wires than would appear were it perpendicular (out of plumb). This false reading must be reduced to the true reading. Both corrections depend upon the vertical angle; hence, unless the notes are plotted in the field, it is usually sufficient to record the vertical angle and the stadia reading as it actually appears on the rod. The reductions are then made in the office by the use of tables or diagrams or slide-rule, based upon the formula derived in the next article. It will also appear that the stadia method is useful for determining elevations by trigonometric leveling.

379. Inclined Readings. In Fig. 379, let the line of sight be inclined at an angle *a*. If the rod were held *perpendicular* to the line of sight the stadia reading *b′e′* would give the distance *Fm*, which, when added to the constant *C*, would be equal to the distance *Bm*. This could be reduced to the required distance *BE* by multiplying Bm by cos *a*. But it is not practicable to incline the rod until it is perpendicular to the line of sight, and even if it were complications would arise in getting elevations, hence the general practice is to hold the rod plumb. This introduces another error, since it makes the reading *be* longer than the true reading *b′e′*. The angle *bb′m* is

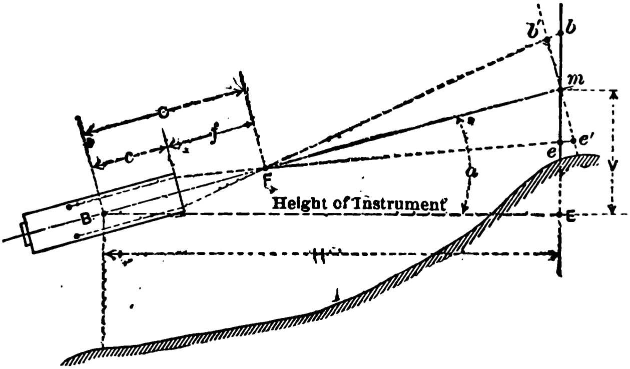

Fig. 379.

approximately a right angle, and $bmb'=a$, hence $be\cos a=b'e'$ (very nearly). In other words, the distance actually read must be multiplied by $\cos a$ to get *Fm*. Hence

$$Bm=C+Fm=C+\frac{f}{i}(be\cos a). \quad . \quad . \quad . \quad (1)$$

But $BE=Bm\cos a$, hence

$$BE=C\cos a+\frac{f}{i}be\cos^2 a. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If R represents the actual reading $\left(\frac{f}{i}be\right)$, then equation (2) may be written:

Horizontal distance: $BE=H=R-R\sin^2 a+C\cos a. \quad . \quad . \quad (3)$

In finding elevations, the vertical distance *Em* is required. $Em=Bm\sin a$. Substituting the value of *Bm* from equation (1),

$$Em=V=\frac{f}{i}be(\cos a\sin a)+C\sin a;$$

and remembering that

$$\cos a\sin a=\tfrac{1}{2}\sin 2a$$

we get

Vertical distance: $V=R\tfrac{1}{2}\sin 2a+C\sin a. \quad . \quad . \quad (4)$

380. The Reduction of Inclined Stadia Readings. In using formulas (3) and (4), keep in mind the fact that $C \cos a$ is practically equal to C for any vertical angle likely to occur in practice, and that $C \sin a$ is a negligible quantity. The quantities to be found therefore are $R \sin^2 a$ for *horizontal* distances, and $R\frac{1}{2} \sin 2a$ for *vertical* distances. These reductions are usually made in the office, although it will be necessary occasionally to reduce distances in the field. The use of stadia reduction tables or geometrical diagrams greatly facilitates the work. (See chapter on Working up Field Notes, Part II.)

381. To Measure Horizontal Distances with the Stadia. (*a*) *On level ground.* Keeping the telescope as nearly horizontal as possible, bring one of the stadia-wires to coincide with the nearest even-foot mark on the rod, and read the other wire *in terms of distance*, as directed on p. 305.

(*b*) *On sloping ground.* (1) Sight at a point on the rod about as far above the ground as the telescope is above the ground. (2) Bring a stadia-wire on the nearest foot-mark, and read and record the distance as before. (3) Read and record the vertical angle which the line of sight makes with the horizontal. (4) The true distance may be reduced in the office or in the field.

(*c*) *Practical suggestions.* (1) It is not essential to make one wire bisect an even-foot mark—it merely simplifies reading the rod. Clamp the telescope and use the slow-motion screw when the stadia-wire is nearly on the mark. (2) When the distance between two stations is too great for accurate reading, assume a point about midway between the two, read the distance to this point from each station, and add the two readings. The objection to this method is that no check on the total distance can be had by two readings, one from each end of the line.

(*d*) *Practical questions.* (1) *How accurately can distances be measured?* This depends upon the magnifying power of the telescope, the fineness of the stadia-wires, the graduations on the rod, the atmospheric conditions, and the length of sight.

With the average telescope and atmospheric conditions the error in a 300-ft. sight should not exceed 1.0 ft. With high-grade instruments and favorable conditions, sights up to 700 ft. may be taken with an average error of less than 0.5 ft. or $\frac{1}{1400}$. Even better results have been obtained. (See also *Limits of Error*, p. 313.)

(*e*) *How long sights should be taken?* This depends not only on the conditions mentioned above, but also upon the accuracy required for the work in hand. In ordinary work best results are obtained when the length of sight does not exceed 300 ft. to 400 ft. At greater distances the stadia-wires will appear coarse as compared with the rod graduations. Occasionally, however, when the map is to be plotted to small scale and the limits of error are comparatively large, very much longer sights may be taken—sights, for example, of 2000 ft.

(*f*) *When may vertical angles be ignored?* This depends upon the accuracy required for the work in hand. The longer the sight the greater the absolute but not the relative error. The following table gives approximate values of the errors which will result for sights from 100 ft. to 500 ft., if vertical angles from 1° to 5° are ignored.

	100	200	300	400	500
1°	0.03	0.06	0.09	0.12	0.15
2°	0.1	0.2	0.4	0.5	0.6
3°	0.3	0.5	0.8	1.1	1.4
4°	0.5	1.0	1.5	2.0	2.5
5°	0.8	1.5	2.3	3.0	3.8

If, for example, 300 ft. is the longest sight taken, the maximum error involved by ignoring angles of 3° and less is 0.8, while for a 500-foot sight it is 1.4. For 300-ft. sights and under the maximum error involved by ignoring angles up to 5° is 2.3 ft. Hence the common rule that in ordinary stadia-work, when sights average from 150 ft. to 300 ft., vertical angles up to 3° may be ignored; and in some work the limit is put at 5°.

382. To Obtain Vertical Distances or Elevations. There are two methods,—one is used when the elevation of one point is required with reference to another point very near the instrument, as, for example, the station under the plumb-bob; the other is used when the point of reference is not near the instrument, as, for example, some bench-mark too far away for the first method to be used. The two methods are explained on pp. 244 and 264; they differ only in the method of setting the target at the start.

(*a*) *Reference-point very near the instrument.* (1) Stand the rod up beside the transit with its base on a level with the point of reference, and set the target at the height of the supporting axis, or opposite the center of the object-glass when the telescope is level. (2) Hold the rod on the point of required elevation, direct the telescope, and read the distance as usual, choosing that even-foot mark which brings the middle hair nearest to the target center. (3) Make the middle hair exactly bisect the target, signal to the rodman to move to the next point, read and record the vertical angle. (4) The elevation may be reduced in the field or in the office.

(*b*) *Reference-point not near the instrument.* (1) Clamp the telescope in a horizontal position, bringing the bubble exactly to the center by the slow-motion screw. (2) Backsight on the bench-mark or point of reference, setting the target so that it is bisected by the middle hair. (3) Without disturbing the target, proceed exactly as directed in the preceding paragraph.

(*c*) *Practical suggestions.* Once the target has been set, any number of elevations may be taken, but it is very essential to state definitely in the field-notes what point is taken as a reference-point; otherwise it may be uncertain which of the two methods was used.

(*d*) *Practical questions. How accurately can elevations be obtained by this method?* This depends upon the accuracy with which the vertical angle is taken, the magnitude of that angle, the accuracy with which the inclined distance to the rod is measured, and the amount of that distance.

The following tables give the errors in elevation due to angular errors of 2′, 6′, and 16′ for 100-ft. sights, and also the error due to an error of one foot in distance for different angles. Angles of 1° and 26° are taken because they are the two extremes, vertical angles rarely exceeding 26° in practice—14° is approximately half-way between the two. The extreme error in vertical angles is not likely to exceed 16′.

Errors in Elevation per 100 ft. due to Errors of 2′, 6′, and 16′

	1°	14°	26°
2′	0.06	0.05	0.04
6′	0.18	0.16	0.11
16′	0.47	0.41	0.29

Errors in Elevation due to an Error of 1 ft. in Distance

1°	14°	26°
0.017	0.235	0.394

Notice that the larger the angle the *less* the error due to angular error, and the greater the error due to linear error. Thus an error of 16′ results in an error of 0.47 ft. if the angle is 1°, while it is only 0.29 ft. if the angle is 26°. On the other hand, an error of one foot in distance causes an error of only 0.017 ft. if the angle is 1°, but an error of 0.394 if the angle is 26°.

As vertical angles can be read easily to within 1′ and distances to within 0.5 ft., it follows that for a 100-ft. sight the maximum error need not exceed from 0.2 ft. to 0.3 ft., and will usually be less. Thus, for errors of 2′ and of ½ ft., the total error for a 1° angle is 0.06+0.009=0.069 ft., while for a 26° angle it is 0.04+0.197=0.237 ft (See also *Limits of Error*, p. 314.)

(*e*) *How close should distances and vertical angles be measured?* If the vertical angle is estimated to the nearest 15′ the angular error will not exceed 6′ or 7′, and if the stadia is read to the nearest foot the linear error will not exceed ½ ft. Hence for a 100-ft. sight the total error of elevation due to errors of 6′ and ½ ft. is for a vertical angle of, say, 14°, 0.16+0.118=0.28 ft., or about *0.3* ft.; for 26° it is 0.11+0.19=*0.3* ft. For errors of 6′ and 1 ft., the total error for 14° is 0.16+0.236=0.39, or about *0.4* ft. For a 300-ft. sight errors of, say, 6′ in a 14° angle and 1 ft. in distance would result in a total error in elevation of about 0.48+0.236=*0.72* ft.

(*f*) In most stadia work, especially in topographic surveying, elevations are required to the nearest foot only, and hence for such work *it is not necessary to read the vernier for vertical angles; estimate to the nearest 15′, reading directly from the vertical circle itself. Read distances to the nearest foot.* It is seen from the above that this should give elevations, as a rule, to within 0.5 ft. (See also *Limits of Error*, p. 314.)

383. Stadia Surveys are those in which the stadia is used for linear measurements instead of the chain or tape. This is the only difference. The same methods are used for running transit lines (Chapter XII) and for locating details (Chapter XIII), as in other kinds of surveys, and the suggestions for choosing stations, running transit lines, and carrying on field-work given in Chapter XIV hold good. Additional suggestions pertinent to stadia surveying are given below.

Practical suggestions. (1) Always read the distance between stations twice—once when the transit is at one station, a second time when the transit has been moved to the other station. *Record both readings, even if they agree.* (2) Important side shots may be checked by taking a half reading, using the middle hair and a stadia-wire, and then a full reading, using the two stadia-wires. Enter both readings in the notes. (3) Since the stadia method is best adapted for comparatively rough work, and since such work is not usually done in the city, but outside in places where local attraction is not so likely to exist, the azimuth method (pp. 120 and

156) is a favorite method for stadia surveying. (4) When taking azimuths or angles to important points, such as transit stations, hold the rod edgewise towards the transit. (5) Keep in mind the limits of error, and do not waste time on needless refinements in setting up, leveling up, and manipulating the transit. (6) Read the stadia first, then signal to the rodman to move to the next point, and read the vertical angle while he is going there.

384. Forms of Notes. The forms of notes given in Chapter XV may be modified easily to suit stadia surveying. The principal addition is a column for vertical angles, and frequently another column for corrected distances. When elevations are taken, columns are also added for vertical distances and elevations. When these last two columns and the corrected distances are computed in the office, the values may be inserted in red ink. The form of notes will depend largely upon whether the azimuth method, the angle method, or some other method of field-work is used. Some of the forms of tabulating notes are given below, and these forms will suggest others. The notes shown are those on the left-hand page of the note-book, and it is understood that these are supplemented by sketches and explanatory notes not shown here, but placed on the opposite page in the field book.

FORM 10

Line	Azimuth	Stadia Distance	Vertical Angle	Corrected Distances	
				Horizontal	Vertical
	S 50° 30′ E				624.76
AB	129° 28′	286.0	+8° 30′	279.8	+41.81
					567.25
A2	184° 54′	121.5	-7° 30′	119.4	-15.7

(a) *Interpretation.* Form 10 is similar to Form 7, p. 174. Unless the notes are plotted in the field the last two columns may be worked up in the office. The bearing S. 50° 30′ E. checks the azimuth 129° 28′ as closely as the needle can be read, this bearing was recorded because *AB* is a line between stations, and it is well for the notes to indicate that the azimuths of important lines, such as transit lines, were checked by needle readings. The azimuth of *A*2 may have been checked also by glancing at the needle, but it was hardly worth while [illegible] this bearing, as *A*2 is a side-shot.

The elevation of the bench-mark from which the rod was set (582.95 ft., see Form 12) should be entered in the notes—on the right-hand page if there is no place for it on the left-hand page. The distance 286.0 appears in the notes only once, but if the notes were continued to include those taken at Sta. *B* the check value would be recorded as *BA*, since the stadia reading is taken from *B*. The elevations 624.76 and 567.25 for *B* and 2 respectively, if worked out in the office, would better be entered in red ink.

FORM 11

Angle	Value	Line	Stadia Distance	Vertical Angle	Corrected Distance
DAB	2|101°20' 50°40'	AB	286.0	+8°30'	279.8
DA2	106°06'	A2	121.5	-7°30'	119.4

(*b*) *Interpretation.* This form is the preceding form modified to suit the direct-angle method (pp. 120 and 154). Vertical distances are not shown in the notes. When these are desired, another column, corresponding to the last column in the preceding form, may be added; space for this column may be gained by omitting the third column, provided it is understood that the last two letters in the first column indicate the line measured.

FORM 12

Point Sighted	Azimuth	Vertical Angle	Distance	Vertical Distance	Elevation
Transit at Station A. Target set for rod					
held on B.M. No.24 Elevation 582.95					
B	S 50°30' E 129°28'	+8°30'	279.8 286.0	+41.81	624.76
2	184°54'	-7°30'	119.4 121.5	-15.7	567.25

(c) *Interpretation.* The magnetic bearing is entered directly above the corresponding azimuth, and the corrected distance directly above the stadia distance. This leaves the last two columns for vertical distances and elevations. In topographic surveying, where a large number of side shots are taken in locating contours, this is an excellent form. The station occupied and the elevation of the reference-point are carefully noted at the head of the page. A sketch is put on the opposite page with points numbered (usually in the order in which the side shots are taken), and the names of party, explanatory notes, and all data which may be of future use complete the notes.

As a rule, the azimuth to some transit station will be the first entry after setting up. (Why? See precautions, p. 152.)

(d) *Sketches.* The sketches are fully as important in stadia surveying as in other kinds of surveying, because, as a rule, the stadia is used when a large number of points are located from each station, and the relative position of these points should be clearly shown. Suggestions for making sketches have been given on p. 167. In some kinds of work, notably topographic surveying, notes are often plotted in the field (p. 347).

385. Sources of Error in Stadia Surveying. In addition to the errors of ordinary transit work (Chapter X), stadia-work is subject to many of the errors discussed in connection with leveling (Chapter XXII). For example, it is important that the rod should be plumb, that the rod and wires should be in focus, and that the rod should be correctly graduated. The errors discussed here are those peculiar to stadia-work.

1. *Error due to change in the wire interval.* Best avoided by having *fixed* stadia-wires, which are not affected by changes of temperature and are not easily displaced.
2. *Mistakes or blunders in reading the rod.* Avoided by estimating the distance by eye; by taking a half-reading and then a full reading; and by reading from each end of a traverse line.
3. *Error in estimating the space on the rod intercepted by the stadia-wires.* This error is affected by the magnifying power of the telescope, the fineness of the stadia-wires, the character of the rod graduations, the length of sight, and atmospheric conditions. For very long sights a rod with two targets will reduce the error; for short sights a rod without targets is better.
4. *Error in determining the stadia interval.* This is one of the most important sources of error because it is cumulative. As shown in the next paragraph, atmospheric conditions affect the accuracy with which the stadia interval may be determined.
5. *Refraction.* Mr. L. W. Smith of the University of Wiscon- has shown that "differential refraction" (i.e., unequal

refraction due to varying densities in different strata of air) is a source of cumulative error in stadia-work.* The stratum of air within three or four feet of the ground is of greater density than the air above. Between the hours of 7.00 A.M. and 9.00 A.M., and between the hours of 2.30 P.M. and 7.00 P.M., this difference in density is comparatively slight, and does not seriously affect stadia readings; but during the midday hours of from 9.00 A.M. to 2.30 P.M. the density of the lower stratum is increased *upward* by the radiation of the earth's heat. The depth of this stratum varies for different hours and seasons, but is never over four feet. A ray of light above this stratum is not greatly affected, but a ray of light passing through it is bent upward. Hence the space on a rod included between the upper and lower rays is less at noon than in the morning, and the error is enough to seriously affect stadia-work where results of, say, $\frac{1}{1000}$ or better are desired. Hence the following:

Practical conclusions. (1) A stadia interval determined during midday hours will give distances too *long* when these distances are read during early or late hours, and, *vice versa*, if the interval is determined during early or late hours of the day, distances read during midday hours will be too short. (Why?) Hence do not attempt accurate work during hours which do not correspond to your interval determination.. (2) If work is to be carried on all day, determine the stadia interval by tests made at different hours throughout the day. (3) During midday hours do not take sights which require the line of sight to pass nearer than 3 ft. to the ground,—for long sights use a half-reading on the upper portion of the rod.

386. Limits of Error. Two questions arise in discussing limits of error in stadia surveying, viz.: (1) With what degree of precision can a single horizontal distance be measured or a single difference in elevation be found? (2) What error may be expected in running long lines where the errors from different set-ups are likely to be compensating?

(*a*) *Errors of separate horizontal measurements.* From a series of tests by 125 of the author's students it would appear that the probable error of sighting is not likely to exceed one foot up to a distance of 400 ft. The mean of ten probable errors *of a single observation* for each of four distances is given below. Each probable error was found from at least 25 readings by different students, several transits being used. The mean of the probable errors involves therefore approximately 250 readings for each distance.†

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* See Eng. News, June 6, 1895.

† See a paper by the author on "Some Tests in Sighting at Leveling-rods and Stadia-rods," *Proceedings of Connecticut Society of Civil Engineers, 1907;* also Engineering News, March 21, 1907.

Distance	90–125 ft.	185–210 ft.	285–300 ft.	415–435 ft.
Mean of probable errors	0.51 ft.	0.72 ft.	0.83 ft.	0.93 ft.

These results do not indicate the *discrepancies* to be expected between tape measurements and stadia measurements, since such discrepancies will involve constant errors, which are not involved in the mere act of reading the rod, such, for example, as errors in the wire interval and errors in rod graduations. In order, therefore, to determine what the discrepancies really were, each of the distances was measured with a tape. Each quantity in the following table is the mean of approximately 50 discrepancies, i.e., the readings of approximately 50 students at each transit and for each distance were compared with the tape measurement for that distance and the mean of the 50 discrepancies entered in the table.

MEAN DISCREPANCIES BETWEEN STADIA AND TAPE MEASUREMENTS

Distance	90–125	185–210	285–300	415–435
Transit No. 1	0.58	0.66	1.37	1.77
" " 2	0.65	1.30	1.09	1.30
" " 3	0.57	0.65	0.73	1.87
" " 4	0.44	0.63	0.93	0.99
Mean	0.56	0.81	1.03	1.48
Minimum	0.44	0.63	0.73	0.99
Maximum	0.65	1.30	1.37	1.87

The above results were obtained under *unfavorable* conditions.

With reasonable care the discrepancies in practice should rarely exceed the mean discrepancies of 0.5, 0.8, 1.0, and 1.5 ft. respectively, and will frequently be less than the minimum values of 0.4, 0.6, 0.7, and 1.0 ft.

(*b*) *The error in a single difference of elevation* is due to two principal errors, viz., (1) the error in the stadia distance and (2) the error in the vertical angle. The error in the vertical angle seldom exceeds 5′ of arc, and if necessary it can easily be reduced to less than one minute. The mean discrepancy in distance may be taken from the preceding table as 0.6 ft. for a 100-ft. sight, 1.0 ft. for a 300-ft. sight, and 1.5 ft. for a 400-ft. sight. The total errors given below are based on these discrepancies, combined with errors in vertical angles of 1′ and 5′ respectively.

	3°		5°		10°		15°		25°	
Distance.	1′	5′	1′	5′	1′	5′	1′	5′	1′	5′
100±0.6	.05	.15	.08	.20	.15	.25	.20	.25	.25	.35
300±1.0	*.15*	.45	.20	.55	.25	.60	.35	.70	.50	.80
400±1.5	*.20*	*.60*	.25	.75	.35	.85	.45	.90	.70	1.05

The above results indicate the *maximum* errors likely to occur in ordinary work with an instrument in good adjustment.

(*c*) *Limits of error for a series of lines.* The accuracy with which a single horizontal distance may be measured and a single elevation obtained by stadia methods has been indicated above. Since a stadia traverse is a combination of many measurements, and since most errors of stadia work are compensating, the total error may be expected to vary about as the square root of the length of the traverse (p. 45). For ordinary work the total error in feet should not exceed $0.04\sqrt{D}$ (in which $D=$ total distance in feet), and with proper precautions, such as those indicated in the last article, this limit can be greatly decreased. Smith, in his experiments measuring a base-line nearly a mile and a half long, got as the average error of 13 measurements $\frac{1}{2250}$, these measurements being taken under ordinary field conditions and at different hours of the day. The average error of five afternoon measurements was as low as $\frac{1}{8185}$.

Note. The error of closure depends upon angular as well as linear measurements (p. 160), so that results are not purely those of stadia-work. A very complete comparison of stadia and base-line work is given in the Engineering News, June 28, 1900, in an article on the surveys of the Oswego-Mohawk Ship Canal Route. The total errors of stadia measurements as compared with those made with the steel tape are given for varying distances. By selecting different distances and using the corresponding results as given in that article, one can obtain different ratios of precision. Thus for one distance of 2000 ft., selected at random, a ratio of $\frac{1}{2850}$ is obtained, while for another distance of 29 580 ft. the ratio is found to be $\frac{1}{1850}$. These ratios were calculated from the discrepancies between stadia measurements and tape measurements of the same lines. The discrepancies in elevation found by comparing stadia elevations with those obtained with the spirit-level were 0.04 ft. for the 2000 ft., and 0.49 ft. for the 29 580 ft. It is to be noted that the ground covered was very favorable for leveling operations.

Limits of error for given conditions may be determined after a short time in the field, and this is the best course to pursue if the survey is extensive enough to warrant it.

387. Practical Suggestions,—Summary.

1. Determine the constant $C=c+f$ and do not forget to add it to each reading.

2. Adjust the stadia-wires before beginning a survey. If fixed wires are used, see that the rod is graduated to correspond, or else determine an *interval factor*. Slight errors in the rod may be ignored in rough work.

3. As a rule, sight at a point about as far from the bottom of the rod as the telescope is above the ground, and then bring either the upper or lower wire to coincide with an even-foot mark. When the stadia-wires are coarse, lower the telescope until the even-foot mark can just be seen over one wire, and then

look just above the other wire, thus getting the distance more exactly than if the marks were entirely covered by the wires.

4. In reading the rod do not mistake the middle hair for a stadia-wire ; do not omit even hundred feet or multiples of ten feet—as, for example, 189 in place of 289, and 129 in place of 139. Form the habit of estimating distances by eye to guard against mistakes of an even 100 ft. and the mistake of reading the middle cross-hair.

5. The best results for single sights are obtained when the distance does not exceed 300 ft. When the total length of a line is found by adding a number of readings together, compensating errors affect the result, and longer sights are often better.

6. When the distance between two stations is too great for a single sight, assume a point about midway between, read the distance to this point from each station, and add the two readings. (What is the chief objection to this method?)

7. When only a small portion of the rod can be seen, take a half-reading and multiply by two. This cannot be done unless the middle hair is half-way between the other two.

8. Do not forget to read and record the vertical angle when the line of sight is inclined more than from 3° to 5°.

9. Make it a rule to read distances between stations *twice*, once from each station, and enter both readings in the notes.

10. Important side-shots may be checked by taking a half-reading and then a whole reading. If the middle hair is not half-way between the other two, take three readings, two of which added together should equal the third.

WHEN ELEVATIONS ARE REQUIRED

11. Assuming that the target has been set, either by holding the rod beside the supporting axis of the telescope or by back-sighting on some point of known elevation, observe the following order of procedure: (1) Read the distance. (2) Bring the middle hair to the target center. (3) Signal the rodman to move to the next point to be taken. (4) Read the vertical angle.

12. Do not forget to make the field-notes very clear as to the point of reference for all elevations taken at a given set-up.

13. In obtaining elevations the longer the sight the more carefully the vertical angle should be read. The larger this vertical angle, the less important are small angular errors and the more important are small errors in distance. In work where elevations are required to 0.5 ft. and the lengths of sight do not exceed 200 or 300 ft., read the distance to the nearest foot, and the vertical angle to the nearest fifteen minutes without using the vernier.

14. Any error in the adjustment of the telescope level-bubble affects materially the measurement of vertical angles; hence special attention should be paid to this adjustment when elevations are required.

15. Finally, in all stadia-work keep the limits of error in mind, and do not waste time on needless refinements in setting up, leveling up and manipulating the transit.

CHAPTER XXVI

PLANE-TABLE SURVEYING

In this chapter five methods of plane-table surveying are explained, practical suggestions for the use of each method are given, and the different methods are compared. General suggestions for the use of the plane-table are made, practical questions of field work are discussed, and four methods of solving the "three-point problem" are explained.

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388. The Plane-table consists of two parts: (1) the *table* or *board* supported by a tripod, and (2) the *alidade* with telescope attached. The edge of the alidade and the axis of the telescope are parallel.

The traverse-board is a rough form of plane-table, with upright peep-sights in place of the telescope.

389. Plane-table Surveying is peculiar in this respect—the field work and the plotting are carried on simultaneously. The board, with drawing-paper stretched upon it, is brought into a horizontal plane and kept there. The alidade, resting on the board, may be moved from place to place on the paper. When the telescope is pointed to any object, the *direction* of the line of sight is transferred to the paper by drawing a line along the edge of the alidade. Linear measurements, obtained by chaining or by the stadia, are laid off to any desired scale. Thus, as the field work progresses, it is reproduced in map form on the board of the plane-table, step by step, without the use of other instruments, and often without any direct measurements, angular or linear.

The plane-table is not intended for extremely accurate work, yet surprisingly good results may be obtained with it. It is especially useful for plotting indefinite details, such as roads, streams, and edges of woods, while at the same time more definite details, such as buildings, bridges, and fences, can be located accurately enough for most purposes. Thus many surveys may be made wholly with the plane-table, as, for example, surveys of parks and private estates, but its chief use is for filling in the details and contour-lines of large topographic surveys where the primary stations have been established with the transit.

Advantages of the plane-table method. (1) It dispenses with all field-notes (since the survey is plotted as fast as it progresses), and hence mistakes in recording measurements are avoided. (2) Since the map is made in the field, with the area surveyed in plain view, it is easy to see what data are necessary and whether or not any have been omitted which should be obtained. (3) By means of check-lines (p. 325) errors in measurements or in plotting are readily detected. (4) Inaccessible points are easily located without trigonometric calculation (p. 325). (5) The position of a point with respect to three known points may be quickly determined (Method 7, p. 4, three-point problem). (6) More ground can be covered in a given time than by almost any other method,—and with reasonable accuracy.

> "The special advantages of the plane-table as a mapping instrument are due to the rapidity with which it obtains results by the method of graphic triangulation and to the facility it affords the topographer in determining his position at an unknown point by the graphic solution of the three-point problem.
>
> When the latter method is applicable—that is, when the country is open and signals can be easily seen—its superiority over a system of traverse lines is manifest. The topographer is then at liberty to choose his ground without reference to his last station or to the one succeeding. He is not tied down to a backsight nor restricted by the conditions imposed by a foresight. He need not set up his instrument on an area barren of detail, nor cut his way through obstacles (bushes, hedges, trees) to establish a station at a commanding point of view." (*Wainwright* in *A Plane-table Manual.*)

Disadvantages. (1) The plane-table is cumbersome, awkward to carry, and requires many accessories for plotting. (2) Under certain conditions of climate and weather, the work of plotting is disagreeable if not impossible. (3) Angles are not recorded in degrees, hence are of little value for the calculation of areas or for other office computations. Areas may be determined by the planimeter from a plane-table map, but ordinarily the plane-table is not used for ascertaining areas.

390. To Set Up the Plane-table. Two things are aimed at: (1) to get the board level; (2) to get some point on the board over a corresponding point on the ground and, at the same time to have the board properly oriented.

(1) *To get the board level.* Set the tripod approximately level

to start with, two legs down-hill. (See hints for manipulating tripod, p. 88.) If the plane-table has leveling-screws, they are worked like those on the transit. (See p. 88.) When there are only three screws, one may be turned without turning either of the others—an advantage especially when the board is approximately level. When the table has a cup-joint (ball and socket), it is leveled by moving the board with the hands; when it is nearly level it may be moved slightly by tapping with the finger. During the process a level-bulb rests on the board; it is placed parallel first to one edge of the board and then to a second edge at right angles, occupying alternately these two positions until the two edges of the board are both horizontal.

(2) It is a little difficult (though not so difficult as it is sometimes represented to be) to set up a plane-table so that, after it is oriented, a point on the map representing the position of the plane-table shall be exactly over the corresponding point on the ground, unless the point on the map happens to be over the axis upon which the board revolves. In most work accuracy in this respect is not essential. By alternately orienting and shifting the table one can ordinarily in two or three trials get the position near enough for all practical purposes. A device for suspending the plumb-bob from any point on the board is usually furnished with the instrument.

Practical suggestions. (1) First get the table approximately level, orienting it roughly by eye, or, better still, by the compass; then pick it up and move it bodily over the station. (2) The tendency is to set the board too high—bring it a little below the elbow, low enough so that all parts of the board may be reached easily, and yet high enough so that sights may be taken with comfort. (3) How close must the point on the table be over the point on the ground? The answer depends upon the limit of error adopted and the scale of the map. (See p. 105.) Thus, for example, a displacement of one inch would cause an angular error for a 500-ft. sight of only about 30 seconds.

In the greater part of plane-table work the scale is so small that the whole board may be considered as a point.

391. Five Methods of Work. Let B be the station occupied and b a point on the board directly over B. When the edge of the alidade is placed in coincidence with b, and the telescope is directed first to a distant point A and then to a distant point C, if the board is level the angle formed by the two lines drawn on the paper is the *horizontal* angle ABC. Thus the angle is represented *graphically* instead of being measured and recorded in degrees, as in transit surveying. In *all other respects* the methods of locating points are the same.

and the analogy between transit work and plane-table work should be kept in mind. Linear measurements may be made by chaining or by the stadia. In two of the five methods used in plane-table work, linear measurements are not required.

1. *Radiation.* Method of angles and distances, p. 134.
2. *Progression or Traversing.* Corresponds to the azimuth method, p. 120.
3. *Radio-progression.* A combination of 1 and 2.
4. *Intersection.* Corresponds to triangulation, p. 137.
5. *Resection.* A modification of 4.

The only linear measurement required in either of the last two methods is the length of one base line.

Note. In the first five figures on the following pages the actual boundaries are represented by the large polygon, and these boundaries are shown drawn to smaller scale on rectangles representing the plane-table.

In studying these figures bear in mind that the size of the plane-table is very greatly exaggerated. Thus, for example, it appears in Fig. 391 (I) to be larger than the house. It is customary to represent a point on the ground by a capital letter, and its plotted position on the plane-table by the corresponding small letter.

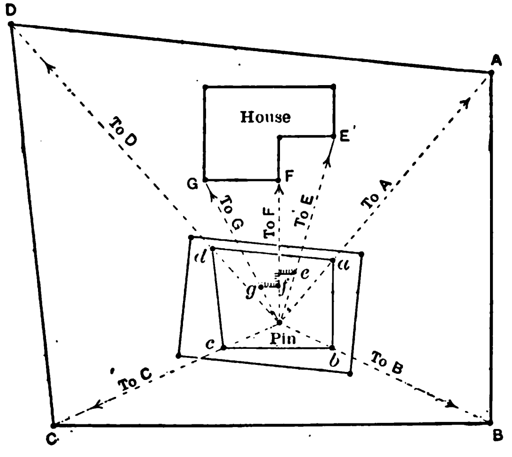

FIG. 391 (I).

I. **RADIATION.** *Method.* Set up the plane-table in a position from which all points to be located can be seen. Clamp the table so that it cannot revolve. Stick a pin in the board at any convenient point, and let this pin represent the point on the ground at which the plane-table stands. With the edge of the alidade against the pin, draw radial lines to all points to be located, and mark off the distances to these points to scale. (Method corresponds to that illustrated in Fig. 195 (*a*), p. 134.)

Illustration. In Fig. 391 (I) the radial lines are drawn to the four corners of the lot and to three corners of the house. The distances to these points as fast as they were measured were laid off to scale on the corresponding radial lines.

Practical suggestions. (1) Choose a position for the pin such that no point to be plotted will fall outside of the limits of the paper. (2) In the illustration shown it would be well to measure the lengths of the boundary-lines and scale the corresponding distances *ab*, *bc*, *cd*, and *da* to see if they agree. (3) The house is, of course, "measured up" to locate the remaining three corners.

Comments. This method is employed chiefly in combination with some other method; otherwise its use would be restricted to a comparatively small area, i.e., the ground which can be covered from a single set-up.

II. PROGRESSION OR TRAVERSING. In this method the plane-table may be moved from one station to another. After setting up over one station, the first thing to do is to *orient* the table.

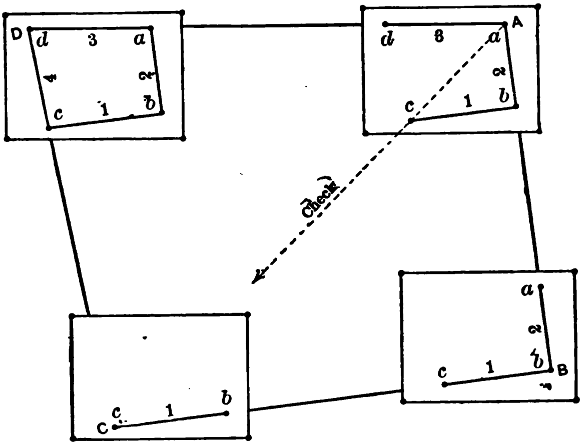

FIG. 391 (II).

This is done by making the edge of the alidade pass through the plotted positions of two stations, *one of which must correspond to the station at which the plane-table stands;* the table is then revolved until the line of sight is directed to the other station. This usually amounts to backsighting on the preceding station, as in transit surveying; hence this method corresponds to the amizuth method (p. 120).

Illustration. Required to plot the boundary-lines *ABCD* in Fig 391 (II). Set up at any corner as *C*. Choose a corresponding point *c* on the board, and draw *cb* with the alidade directed to *B*. Measure the distance from *C* to *B* and lay it off to scale, thus establishing *b*. Set up at *B*, make the edge of the alidade pass through *b* and *c*, turn the table until the line of sight strikes *C*, and clamp. The table is now oriented. Sight at *A*, draw *ba*, measure *BA*, lay off the distance to *a*. Set up at *A*, orient as before by sighting along *ab* to *B*, draw *ad*, measure *AD*, and lay off the distance to *d*. Thus any number of stations may be plotted one after the other. In Fig. 391 (II), the lines on the board are numbered in the order in which they were drawn.

Practical suggestions. (1) When the area covered is small and the scale adopted is large, it may be necessary to set up so that the station at which the plane-table stands is exactly under the corresponding point on the board (p. 320). (2) Choose the first line *cb* with due regard to the limits of the paper. (3) After orienting at *A*, if the alidade is made to pass through *ac*, the line of sight should strike *C*. Such checks as this should be used constantly. (4) It is not necessary to set up at the last station *D* except as a check. (5) The house or any other detail could be located from the most convenient station by the preceding method of *radiation*, or by the next method of *intersection*. (6) If it is impracticable to set up over boundary corners (as is usually the case), establish stations and corresponding reference-lines inside or outside the boundaries, as in transit surveying. (See pp. 141 and 150.)

Question. How could the work have been started at *C* so as to plot the four corners in two set-ups, counting the one at *C*? How could the positions of *d* and *a* be checked in this case?

Comments. This method requires all stations to be accessible. It is used for the main lines of a survey (corresponding to transit lines), the details being located in some other way. It is especially adapted to surveys of roads, streams, etc., the stations being chosen at the bends, while the sides of the road or banks of the stream are located by offsets,—buildings, fence-corners, etc., by radiation. Frequently offsets may be sketched in by the eye. The method is often used for filling in details of a topographic survey, the primary stations of which have been previously established by triangulation with more accurate instruments. These primary stations are a valuable check on the plane-table work.

III. RADIO-PROGRESSION. A combination of the first two methods just explained. Characteristic features: (1) The table is always set up with its center directly over the station occupied; (2) for all sights at all stations the edge of the alidade passes through the same point on the paper, i.e., a point over the axis around which the table revolves; (3) the direction of any line of sight is transferred from the center of the board to the point representing the station occupied, by drawing through this point a line parallel to the edge of the alidade.

Illustration. In Fig. 391 (III) lines of sight are represented by broken lines,—the lines actually drawn on the table by full lines. Set up at *A*, with the center of the table over the station. Stick a pin in at the center of the table, and with the edge of the alidade against the pin, sight at D. Assume a point *a* and draw the line *ad* parallel to the edge of the alidade, In like manner sight on *B* and *E* and draw the corresponding parallel lines *ab* and *ae*. The distances *AD*, *AE*, and *AB* are measured, and the corresponding distances *ad*, *ae*, and *ab* are laid off to scale. Move the board to *B*, orient by sighting at *A* (edge of alidade parallel to *ba*). As a check, sight at *D* and see if a line *parallel to the edge of the alidade* passes through both *b* and *d*. Determine points *c* and *f* by drawing from *b* lines parallel to the corresponding lines of sight in the usual manner. It is not necessary to go beyond this station, except to check the work, but in this case the plane-table was again set up at *C*, the location of *d* checked by sighting at *D*, and a third corner *g* of the house was plotted.

Practical suggestions. (1) Parallel lines may be drawn by placing one edge of a triangle in perfect contact with an edge of the alidade, and then sliding this triangle along the edge of a second triangle, as in ordinary drafting. If the two edges of the alidade are parallel, place the triangle against either edge as may be most convenient. (2) In the illustration shown, the house could have been located wholly from *A*, the most convenient station. Two or three corners having been located, the others are found by measuring the sides of the house. The lengths of the boundary-lines should be measured also. (3) If in orienting at *B* it is necessary to make the edge of the alidade parallel to *ba*, extra work is involved; moreover, *ba* may be too short for accuracy. It is better to draw a long line through the *center* of the table before leaving *A*, by pointing the alidade to *B*, and use this line for orienting at *B*. This means that a number of such lines will eventually be drawn through the center of the table for purposes of orienting it: hence it is better to omit the center portion of each line, drawing the two ends only.

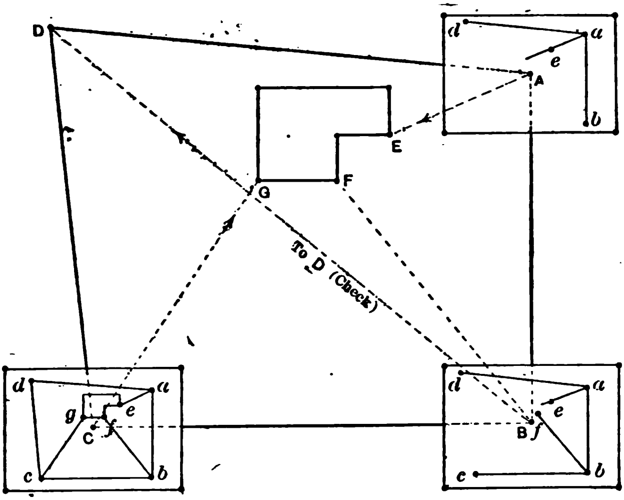

Fig. 391 (III).

Comments. The radio-progression method is especially adapted to mapping small areas to large scale, i.e., when one must be particular to have a given point on the table directly over the station occupied. When many points are to be plotted from any one station, much time is wasted in drawing parallel lines,—it is better to set the station on the board directly over the station on the ground and use the first two methods.

IV. **Intersection.** This method corresponds to triangulation (see Fig. 196, p. 137); one distance being known, no further linear measurements are required. The first thing to do after setting up the plane-table is to orient it as in the preceding methods. The work then consists in drawing radial lines to the points which are to be located. The distances will not be determined until these lines are intersected by corresponding lines to the same points, drawn at some other station.

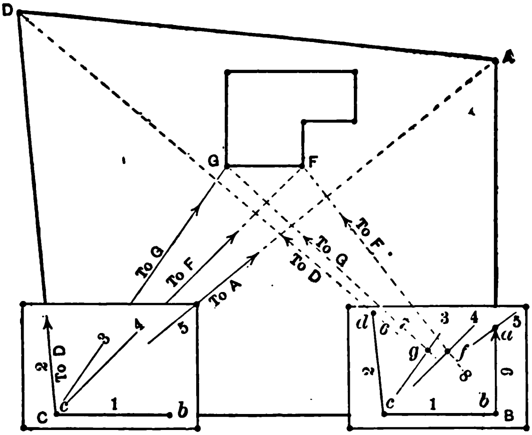

Fig. 391 (IV).

Illustration. Figure 391 (IV) is practically self-explaining. The table was first set up at *C*, the length of *CB* measured, and the line *cb* plotted. Lines 2, 3, 4, and 5 of indefinite length were then drawn, the table moved to *B*, oriented, and lines 6, 7, 8, and 9 drawn to determine the points of intersection *d*, *g*, *f*, and *a*. Had it been necessary to locate other points not visible from *C*, but visible from both *B* and *A*, the line *ab* could have been used as a base-line in exactly the same way. Thus the plane-table can be moved from station to station until all the ground is covered.

Suggestions. (1) As in all triangulation, stations should be chosen so that intersections are definite and not too oblique. (See p. 191.) (2) As this method is usually used for large areas plotted to small scale, it is seldom necessary to set up so that the point on the table is exactly over the station on the ground. (3) If a point cannot be located by a definite intersection from two given stations, consider if it cannot be located more definitely from two other stations,—this suggests a method of check-lines, i.e., locate points occasionally from at least three stations. (4) It is often desirable to sight on a church-steeple, the top of a tree, or some other prominent landmark merely as a check, even though the location of the point itself is of no consequence. Having oriented a table, it is very reassuring to direct the line of sight at a distant object and to find that the edge of the alidade passes through the corresponding point already plotted from other stations.

Comments. This method is especially useful for plotting *inaccessible* points, and for mapping banks of streams, edges of woods, and other indefinite topographic features. As it is difficult to choose stations so that all intersections on the paper will be definite, this method is of greatest value when used in combination with some one of the preceding methods,—it is especially valuable as an occasional check on distant points.

V. Resection. This method is a modification of the method of intersection. Its chief characteristic is that the point determined by the intersecting lines is *the station occupied by the plane-table* at the time those lines are drawn. This means that a plane-table can be set up anywhere and the corresponding point on the map can be found by *resection,* provided three points already plotted are visible from the plane-table. This method requires one linear measurement—a base-line.

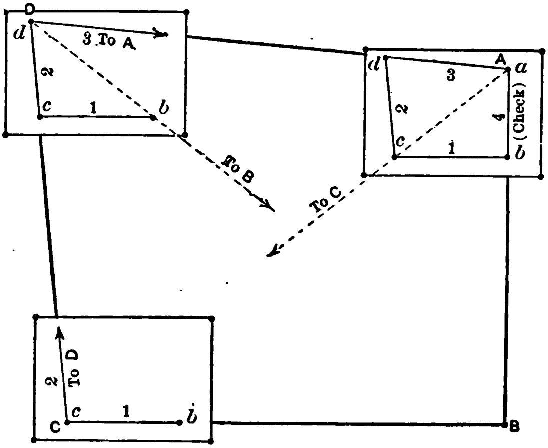

Fig. 391 (V).

Illustration. In Fig. 391 (V) set up at *C*, assume *c*, direct the alidade to *B*, measure *CB*, lay off *cb* to the scale adopted, and draw an indefinite line 2 towards *D*. Set up at *D*, orient the table by sighting at *C* with the edge of the alidade along the line 2; then move the alidade to *b*, sight at *B*,—where the edge of the alidade cuts the line 2 will be the point *d*. Draw an indefinite line 3 towards *A*. Set up at *A*, orient the table, center the alidade at *c*, sight to *C*, and find the intersection *a* with the line 3. As a check, center the alidade at *a*, sight to *B*, and see if the edge of the alidade cuts *b*.

Practical suggestions. (1) In setting up the plane-table, estimate about *where* the station to be plotted will come, and set this point approximately *over the point on the ground.* (2) When there are more than four sides *to the survey a station* may be plotted by sighting *across* the inclosure to *any* station previously plotted. This often gives a choice of two or more sights,—it is well to use one for locating the point on the paper, and the other for a check-sight. (3) Details are best located by radiation or intersection.

Comments. The method of resection is not a very useful method except in cases where it becomes necessary first to occupy a station and then to locate it on the map. The well-known *three*-point problem and the *two*-point problem are problems in *resection*. (See pp. 329 to 335.)

392. The Five Methods Compared—Summary.

1. *Radiation.* An accurate method, but restricted to one set-up. Used chiefly in combination with other methods.

2. *Progression.* Requires all stations to be accessible. Used only for locating *stations*. Corresponds to traversing. Useful for establishing main lines of reference.

3. *Radio-progression.* Adapted to plotting small areas to large scale, since the center of the table is always over the station. Not much used if many points are to be plotted from one station.

4. *Intersection.* Requires no linear measurements except a base-line, and is especially useful in plotting inaccessible points. Method is inaccurate when intersections are too oblique. Method used chiefly in combination with other methods, especially for check-sights.

5. *Resection.* Used chiefly for locating the station occupied.

393. Practical Suggestions for Plane-table Work.

(*a*) (1) *The paper* must be selected and handled with care in accurate plane-table work. Paper which has a good surface for drawing hair-lines and is least affected by atmospheric changes should be used,—for very accurate work double-mounted paper, for ordinary work single-mounted,—where there is much dampness celluloid sheets. Tinted paper is less trying to the eyes than pure white. (2) Most paper except the double-mounted paper can be rolled and kept in a metal cylinder or carrying-case,—diameter not less than three or four inches. It is preferable, however, to cut sheets the size of the board and to keep them *flat* in a leather or canvas carrying-case. (3) Paper should be so firmly fastened to the board that it is practically a part of it, so that neither wind nor friction of the alidade will disturb it. For this purpose spring-clamps, thumb-tacks, or screw-tacks are used. Spring-clamps are unreliable, and interfere with the free movement of the alidade; thumb-tacks are better, but the double-screw tacks with which the plane-table boards of the Geological Survey are provided are perhaps the best for the purpose. In any case the paper must be stretched tight and smooth, and occasionally it may be worth while to turn the paper over the edges of the board and paste it underneath. A waterproof cover for protecting the paper on the board in case of sudden rain is almost indispensable. If caught without it, turn the board upside down. (4) When more than one sheet is used it is sufficient in ordinary work simply to transfer to one sheet at least two points from the preceding sheet, say the last two stations occupied. In addition it is well to transfer such other points as are likely to be needed for check-sights. Points may be pricked through from one sheet to another, or if greater accuracy is required they may be transferred by any of the well-known methods used in drafting.

(*b*) *Manipulation of the plane-table.* (1) If there is no slow-motion screw for turning the table *after* it is clamped, it may be turned slowly *before* it is clamped by tapping a corner sidewise with the finger. After clamping look again to see that the cross-hairs still cover the point sighted at. (2) In turning the alidade, steady the standard with the left hand, while the right swings the rear end of the ruler. A fine needle may be stuck at the point on the paper representing the station, and the edge of the alidade kept against it. (3) Be careful not to disturb the board after it is oriented by leaning heavily on it, or by contact with its edge. The less the weight of the alidade rests near the edge of the board the better.

(c) *Plotting.* (1) The pencil used should be *hard*, sharpened at both ends (round point and chisel point). A smooth stone may be used in place of a sand-paper pad, but it is better to have the latter always at hand, attached to the board by a string. A needle for pricking points and a pencil-eraser are also needed. A triangular scale is recommended, and distances are laid off by laying the edge of the scale along the line. The method of transferring distances by dividers is a poor one. (2) *Draw as fine lines as possible, as few lines as possible, and as short lines as possible.* Coarse lines make accurate work out of the question, and the paper soon becomes covered with a network of lines which is bad enough when reduced to a minimum. Instead of drawing a line the full length of the alidade, draw a short line approximately where the point is to be plotted. Points on contours and similar points often can be plotted without drawing lines by marking off distances along the edge of the alidade while it is yet in position. Instead of drawing a long line, as, for example, when one is to be used for orienting, draw a short line at each end of the alidade. In intersections the point is simply marked or pricked where the second line crosses the first,—the second line is not drawn. (3) Several points on the same straight line may be located quickly by plotting the two ends of the line, and then intersecting this line by sights to the remaining point. (4) Draw all lines close to the straight-edge, so that each is hardly visible until the straight-edge is moved away; but if any part of the edge is raised off the paper, be careful that the pencil does not run under it, thus deviating from a straight line. (5) Above all, *have some good system of keeping track of points and lines*, so that those plotted at one station may be easily identified when the table has been moved to some other station. This is especially important in the method of intersections.

(d) *General suggestions.* (1) Colored glasses relieve the strain upon the eyes. (2) Rules for cleanliness: Keep the leavings of the pencil-eraser and dust of all kinds off the paper. Keep the triangles and the bottom of the alidade clean. Do not slide the alidade unnecessarily over the paper,—lift it up and set it down again to avoid rubbing. (3) At the first set-up in a survey draw a north and south line by means of the compass, using the edge of the box; at the second set-up check it. (4) The first thing to do after setting up and orienting the table is to take a check-sight to some distant point which has been previously plotted,—the last thing to do before leaving the station, and at other times when not employed, is to sight at the same point to see if the table has been displaced. One of the most important precautions in all plane-table work is to take check-sights frequently.

(e) *Vertical angles.* Vertical angles may be measured if the telescope has a vertical circle; this is most necessary when the stadia is used and vertical angles are required for obtaining elevations and for reducing inclined sights to the horizontal, as in transit surveying.

394. Practical Questions Relating to Field Work.

(a) *Choice of scale.* If the plane-table work is carried on in connection with a transit survey, it may be desirable to adopt the same scale as that used in the office for plotting the transit notes. This is not essential, however, as the plane-table map can be transferred easily to a map of different scale by means of the pantograph or by some other method. Assuming, however, that the choice of scale does not depend upon other work, the following suggestions are made:

1. Choose a scale in common use. For ordinary work some multiple of ten, as 10, 20, 30, 40, 50, 60, 80, 100, or 200 ft. to the inch. For government work standard scales are adopted, such, for example, as $\frac{1}{12000}$ and $\frac{1}{24000}$. (See pp. 488 and 489.)

2. If the size of the map is limited, as, for example, to the size of the board, estimate the extreme dimensions,—this will still further limit the choice of scale.

3. Choose a scale consistent with the limit of s, for example,
if measurements to the nearest foot are want ft. to the inch
is as small a scale as should be used; 50 f ould be better,
since it is rather difficult to plot to

(*b*) *Choice of stations.* The suggestions given on p. 149 for choosing stations hold good, remembering, however, that in plane-table work it is well to have each station visible from at least three others, though this is not essential. When a plane-table survey is made in connection with a transit survey, it should begin at one of the main stations and be "tied in" with other transit stations as completely as may be necessary for definite checks.

(*c*) *Methods of locating details* explained in Chapter XIII may be used, modified, of course, to suit plane-table work. Thus, for example, offsets often may be paced, and details sketched in by eye. The suggestions on p. 156 hold good.

(*d*) *What details to take.* In a large survey it should be clearly understood what details are to be taken by the plane-table party, and what are to be left for other parties.

(*e*) *Limits of error* depending upon the nature of the work in hand (see p. 159) should be kept constantly in mind.

(*f*) *Sources of error.* (1) Error in the position of the table. (2) Error when the board is not truly horizontal. (3) Error in sighting. (4) Errors of adjustment. (5) Errors in drawing lines. (6) Errors in laying off distances to scale. (7) Errors due to the expansion and contraction of the paper.

395. The Three-point Problem. *Given: Three points, A, B, and C, visible from a station P occupied by a plane-table; also a, b, and c, the plotted positions of the three visible points. Required: To plot p, the position of the station occupied.*

Remark. This is a special case of the method of resection where the position of the station occupied is the point required, but, unlike the general case (p. 326), no line known to pass through p, the required point, has been drawn at some previous set-up, hence there is no direct method of orienting the table as is the case on p. 326. The problem, then, amounts to finding some point p such that when the edge of the alidade is made to pass through p and any one of the points a, b, or c, the table may be correctly oriented by sighting at the corresponding point A, B, or C. Thus the solution of the three-point problem determines the location on the sheet of the station occupied and orients the table simultaneously. As in all forms of triangulation, the definite location of the point depends upon well-proportioned triangles and sharp intersections.

Solutions. (1) Mechanical. (2) Graphic. (3) By trial.

I. *Mechanical solution.* (1) Fasten a piece of tracing-paper to the board, and assume a point p to represent the station occupied. (Assume it as near its true position as can be estimated by eye.) (2) Center alidade on p, and sighting successively on the three visible points A, B, and C, draw lines of indefinite length along the edge of the alidade. (3) Shift the tracing-paper until the three lines pass through the points a, b, and c previously plotted from other set-ups, and prick the point p through to the paper. (4) With the edge of the alidade passing through p and a, orient the table by sighting at A. (5) Take check-sights through b and c to B and C respectively. If, because of little inaccuracies, the lines do not meet at p, the triangle of error will usually be so small that a slight movement of the board will correctly orient the table.

Comments. This method is quick, and, under favorable circumstances, easy to use and sufficiently accurate for most purposes. It is, however, somewhat cumbersome, and under unfavorable circumstances, especially in a high wind, not very practicable.

II. *Graphic methods.** There are several graphic methods, but only two will be explained.

(*a*) BESSEL'S METHOD. Fig. 395 (*a*) represents the table in three positions, but *over the same station.* (The three positions correspond to the three principal steps.) Let *A*, *B*, and *C* represent the three visible points, and *a*, *b*, and *c* their plotted positions.

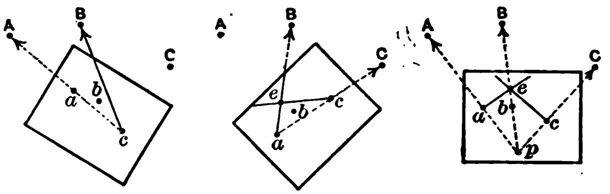

FIG. 395 (*a*).

First step. (1) With the edge of the alidade through *c* and *a*, sight at *A* and clamp the board. (2) Sight at *B* and draw a line of indefinite length.

Second step. (1) Unclamp; with the edge of the alidade through *a* and *c*, sight at *C* and clamp. (2) Sight at *B* and intersect the first line drawn, thus obtaining the point *e*.

Third step. Draw *eb*, and unclamp; with the edge of the alidade through *b* and *e*, sight at *B* and clamp. Center on *a* sight at *A*, and intersect *eb* at *p*.

Check. Center on *p* and see if *c* and *C* are on line.

Comments. The objection to Bessel's method is that the points *e* and *b* frequently fall so near together that the direction of the line between them is practically indeterminate; or else the point of intersection *e* comes off the board altogether. In many cases this difficulty may be overcome by expanding or contracting the triangle formed by the three fixed points as follows: (1) Draw a line parallel to *ac*, and find its intersection with each of the lines *ab* and *bc*. (2) Through the two points thus found, draw lines parallel to *ae* and *ce*. These lines will intersect in a *third point* of *eb*.

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* For a very complete treatment of the three-point problem, consult the U. S. Coast and Geodetic Survey Report for 1880, also *A Plane Table Manual*, by D. B. Wainwright, in the Report of 1905. See also *Handbuch der Vermessungskunde*, by Jordan, and Bauernfeind's *Vermessungskunde*. *Vol. II.*

Bessel's solution is based on the geometric principle that *in any inscribed quadrilateral the angle made by one of the sides with one of the diagonals is equal to the angle made by the opposite side with the other diagonal.* The board of the plane-table *is treated as a point,* i.e., compared with the area covered by the survey it has no dimensions.

(*b*) LLANO'S METHOD.* The three visible points, *A*, *B*, and *C*, and their plotted positions, are taken in the same relative positions as in Fig. 395 (*a*) of the preceding method.

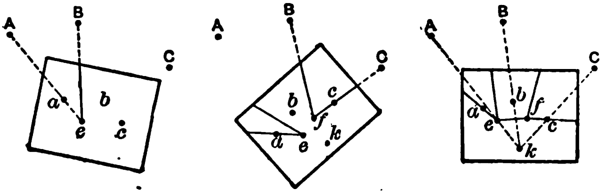

FIG. 395 (*b*).

First step. (1) Draw a line perpendicular to the line *ab* at its middle point, place the alidade along this line, sight at *B* and clamp. (2) Center on *a*, sight at *A*, and find the intersection *e*.

Second step. (1) Draw a line perpendicular to *bc* at its middle point, place the alidade along this line, unclamp, sight at *B* and clamp. (2) Center on *c*, sight at *C*, and find the intersection *f*.

Third step. Find *k* (the required point) at the intersection of two arcs, one with *e* as a center, and a radius *ea*, the other with *f* as a center, and a radius *fc*.

Fourth step. With the edge of the alidade through *b* and *k*, *orient* by sighting at *B*, and *check* by sighting through *a* to *A* and through *c* to *C*.

Comments. If more convenient, find *k* by drawing from *b* a perpendicular to the line *ef*; *k* is on this perpendicular and as far on one side of *ef* as *b* is on the other. Llano's method fails when *e* and *f* fall so near together that the location of *k* becomes indefinite.

III. *Methods by trial* are those in which the position of the required point is alternately assumed and corrected until its proper position is determined. The most satisfactory method of procedure is that known as the *Coast Survey method.*

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* See *Engineering News*, Dec. 29, 1904.

(*c*) COAST SURVEY METHOD. (1) Orient the table as nearly as can be guessed, first using the compass to bring it approximately into position, and then estimating the position of the point *p* with respect to *a*, *b*, and *c*, guided by the rules given in this article. (2) Clamp, and draw lines through *a*, *b*, and *c* by sighting at the corresponding points *A*, *B*, and *C* respectively. (3) If the table has been oriented correctly (which is unlikely) these three lines will meet in the required point *p*; otherwise they will intersect, forming a small *triangle of error*, or else two of the lines will be parallel and be intersected by the third. The position of *p* can now be estimated more nearly than before, the table newly oriented, and the process repeated. Thus the method is a series of approximations, the triangle of error becoming smaller and smaller as the orientation becomes more nearly correct. This method is used extensively, especially by the Government topographers, and is quick and accurate, more so than would at first appear.

Certain rules for guidance in estimating the position of *p* taken from *A Plane Table Manual*,* by Mr. D. B. Wainwright, are given below.

The relative positions of the three fixed points with reference to the new station have an important bearing on the strength of its determination.

In the following statement in regard to the different groupings of points met in practice, for the sake of brevity, the term "fixed points" will be understood to mean points already determined and plotted on the sheet; the "great triangle" referred to is one formed by the three fixed points, and the "great circle" is the circle passing through them.

When the new station is outside the great circle, the strength for determination of a position will be weak when the middle point as seen from the new station is the farthest of the three and the angles are small. (See Illustration 395 (*c*), Fig. 3.) If the new station is located outside the circle, and some distance below it, the angles are small and the determination correspondingly weak.

The determination increases in strength for given angles as the middle point approaches the new station. (Fig. 1.)

When one angle is small or 0° (points in range), the determination will be strong, provided the two points making the small angle or range are not too near each other when compared to the distances to the new station and to the third point; provided also the angle to the third point is not too small. (Fig. 2.)

When the new station lies on or near the great circle, its position is indeterminate. (See Illustration 395 (*c*), Fig. 3.)

When the new station is within the great circle, the strength of its determination increases as it approaches the center of gravity of the great triangle. (Figs. 3, 4, 5.)

There are a number of graphic solutions, but all save three are better suited to the drafting-room with its appliances than to the conditions which exist in the field.

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* *Department* of Commerce and Labor, U. S. Coast and Geodetic *Survey, Appendix No.* 7, Report for 1905.

Lehmann's method of solution is the simplest and most direct, and applies under all circumstances. The directions are stated in the form of rules.

The term "*point sought*" will be understood to mean the true position on the sheet of the projected point of the station occupied. The surveyor is assumed to be facing the signals, and the directions right and left are given accordingly.

Rule 1. The point sought is always distant from each of the three lines drawn from the three fixed points in proportion to the distances of the corresponding actual points from the station occupied, and it will always be found on the corresponding side of each of the lines drawn from the fixed points.*

The simplest case for the application of this rule occurs when the station to be determined is within the triangle formed by the three fixed points; the point sought must then be within the triangle of error to satisfy the conditions. (See Illustration 395 (*c*), Fig. 5.)

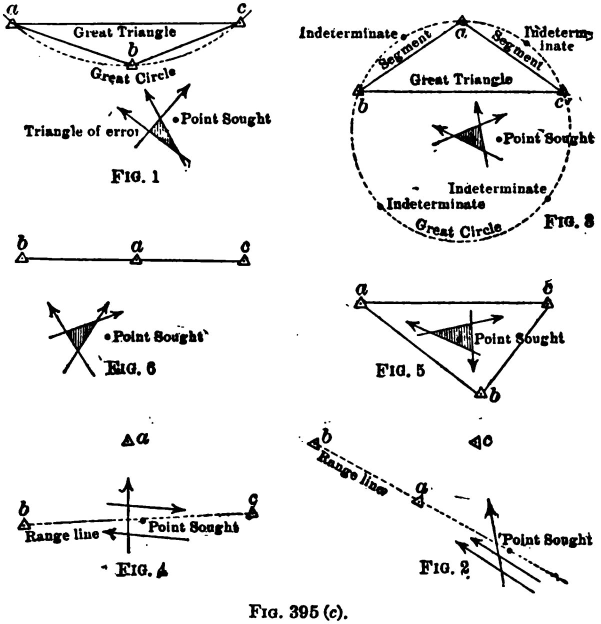

FIG. 395 (*c*).

Although Rule 1 is sufficient in itself for the solution of the problem, there are two subordinate rules which materially assist the topographer in reaching a decision as to the proper location of the point sought with reference to the lines from the fixed points.

Rule 2. When the point sought is without the great circle, it is always on the same side of the line from the most distant point as the intersection of the other two lines. (See Illustration 395 (*c*), Fig. 1.)

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* That is, if it is on the right side of one line, it is on the right side of each one of the other two, and if on the left side of one, it is on the left side of each one of the other two.

Rule 3. When the point sought falls within either of the three segments of the great circle formed by the sides of the great triangle, the line drawn from the middle point lies between the point sought and the intersection of the other two lines. (See Illustration 395 (*c*). Figs. 3, 4, 6.)

Application of rules. In practice the topographer first decides the relation of the new station with reference to the fixed points, whether it is within the great triangle or in one of the segments or outside the great circle. He then determines the position of the point sought with reference to one line (if within one of the segments or without the great circle by Rule 2 or 3); it then follows from Rule 1 that it must be on the corresponding side of the other two lines. Finally, he estimates the relative distances of the three actual points from him and marks the position of the point sought a proportionate distance from the three lines.

EXAMPLES

Illustration 395 (*c*). Fig. 1. When the point sought is without the great circle, the intersection of the lines from *B* and *C* falls to the right of the line from *A*, the most distant point; therefore (Rule 2) the point sought must be on its right, and also (Rule 1) on the right of the line from *B* and *C*. Its exact position is then estimated according to Rule 1.

Illustration 395 (*c*), Fig. 2. When the point sought is on or near the prolongation of a range-line, it must be outside the parallel lines on the side of the line to the nearest fixed point of the range. In the figure it will be seen that the point sought must be outside the lines from *A* and *B*, and to their right to satisfy Rule 1, and also to the right of the line from *C*.

Illustration 395 (*c*), Fig. 3. When the point sought is on the circle passing through the three fixed points, the position is indeterminate, as the three lines will intersect at one point, although the table is imperfectly oriented. Another selection of points must be made.

Illustration 395 (*c*), Fig. 3. When the point sought falls within one of the segments of the great circle, the line drawn from *A*, the middle point, is to the right of the intersection of the lines from *B* and *C*; therefore (Rule 3) the point sought must be on its right side, and also (Rule 1) to the right of the line from B and from *C*. Locate it exactly according to Rule 1.

Illustration 395 (*c*), Fig. 4. When the point sought is on or near the range-line between the fixed points, the point sought must be between the parallel lines to satisfy the conditions of Rule 1. Its position with reference to the intersecting line follows from the same rule. In the figure the point sought, being between the lines from *B* and *C*, is to the right of each, therefore it is to the right of the line from *A*.

Illustration 395 (*c*), Fig. 5. When the point sought falls within the great triangle, it must fall within the triangle of error. No other position would satisfy the conditions of Rule 1.

Illustration 395 (*c*), Fig. 6. When the three fixed points are in a straight line. In this case the points are considered as being in the circumference of a circle of infinite diameter and the point sought always lying in one of the segments of the great circle. The treatment of this case is then identical with that of Illustration 395 (*c*), Fig. 3.

The preceding cases are all examples of the conditions which may occur when the table is deflected to the right. By turning the printed side of the illustration to the light and looking at the figures through the paper, they will appear reversed, and they will then be examples of conditions which may occur when the table is deflected to the left.

Repetition. When the true point has been estimated and marked on the sheet in accordance with the foregoing rules, a new orientation is made. If the lines from the three stations now intersect at that point, it proves the estimate to have been correct, and the position is determined. If a new triangle of error is formed, it indicates an erroneous estimate and the operation must be repeated.

(*d*) TWO-POINT PROBLEM

The occasion may arise where it is desirable to place the table in position at a given point from which only two determined points are visible. This may be done by the following methods:

One method possesses the virtue of requiring no linear measurements, and demonstrates in a very satisfactory manner the effectiveness of the table in determining position by resection.

Illustration 395 (*d*) (all four figures). Two points, *A* and *B*, not conveniently accessible, being given, by their projections *a* and *b*, to put the plane-table in position at a third point, *C*. (The capital letters refer to points on the ground, and the small ones to their corresponding projections.)

Select a fourth point, *D*, so that the intersections from *C* and *D* upon *A* and *B* make sufficiently large angles for good determinations. Put the table approximately in position at *D*, by estimation or by compass, and draw the lines *Aa* and *Bb*, intersecting at *d*; through *d* draw a line directed to *C*. Then set up at *C*, and assuming the point *c* on the line *dC*, at an estimated distance from *d*, and putting the table in a position parallel to that which is occupied at *D*, by means of the line *cd*, draw the lines from *c* to *A* and from *c* to *B*. These will intersect the lines *dA* and *dB* at points *a'* and *b'*, which form with *c* and *d* a quadrilateral *similar* to the true one, but erroneous in size and position.

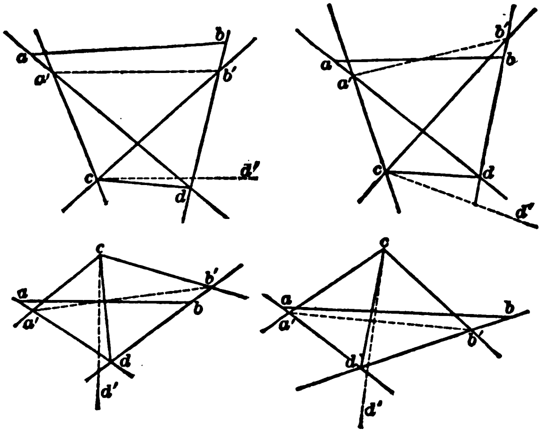

Fig. 395 (*d*).

The angle which the lines *ab* and *a'b'* make with each other is the error in position. By drawing through *c* a line *cd'*, making the same angle with *cd* as that which *ab* makes with *a'b'*, and directing this line *cd'* to *D*, the table will be brought into position, and the true point *c* can be found by the intersections of *aA* and *bB*.

Instead of transferring the angle of error by construction, we may conveniently proceed as follows, observing that the angle which the line *a'b'* makes with *ab* is the error in the position of the table. As the table now stands, *a'b'* is parallel with *AB*, but we want to turn it so that *ab* shall be parallel to the same line. Place the alidade on *a'b'* and set up a mark in that direction, then place the alidade on *ab* and turn the table until it again points to the mark; then *ab* will be parallel to *AB*, and the table is in position.*

Where it is possible to get the two signals *A* and *B* in range, it is easy to determine the position of a third point by a method long practiced by topographers.

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* See Wainwright's *Plane Table Manual* for a second method.

Set up the table anywhere on the range-line, and orient by the latter. Resect on the unknown point, drawing the line anywhere on the sheet most convenient. Leave a signal at the occupied point on the range-line and set up the instrument at the unknown point. Orient by the line drawn when at the station on the range-line, sighting on the latter station. The table will now be in a parallel position to that when on the range line, which is the true position, and the unknown point may be determined by resection upon the two fixed points and their projections.

CHAPTER XXVII

TOPOGRAPHIC SURVEYING

In topographic surveying, methods of leveling are combined with methods of horizontal location. So many combinations are possible that the subject cannot be comprehensively treated in a book of this kind.* The aim, therefore, is to explain thoroughly fundamental principles, and then to discuss briefly the most important questions which arise in ordinary topographic surveying. Practically every surveying instrument may be used to advantage at one time or another in topographic work, and hence a thorough knowledge is required of all of the methods of horizontal and of vertical control explained in preceding chapters.

396. Contour. An imaginary line on the earth's surface containing all points which have the same elevation above or below some assumed datum, e.g., the sea-level.

397. Contour Interval. Since there are an indefinitely large number of elevations there can be an indefinitely large number of contours. It is customary in surveying, however, to work with contours which differ in elevation by even-foot intervals. This difference in elevation between two successive contours may be one foot, two feet, five feet, ten feet, or any other fixed interval assumed to meet the requirements of the work. Different intervals, however, are not used in the same survey, i.e., the *contour interval* for any survey once adopted remains the same throughout the work.

Illustration. Imagine the earth's surface to be partly covered with water. Suppose that the surface of the water is lowered five feet at a time, leaving contact-marks at the end of each interval. These marks correspond to *five-foot* contours. If the surface of the water be lowered one foot at a time, the contact-marks correspond to *one-foot* contours.

Suggestion. Select a small plot of ground with an irregular surface, by means of the level find a number of points of the same elevation from 30 to 50 ft. apart, mark each point with a stake or a chain-pin and imagine a contour passing through these points. This is an excellent way for the beginner to see how a contour winds in and out along the surface of the ground; ultimately he must learn to follow with the eye the approximate windings of a contour which has *not* been staked out; he will be helped in this by imagining the ground intercepted by a water surface, as explained in the preceding illustration.

398. Use of Contours. A map without contour-lines simply shows the relative position of points without regard to elevations, but when contour-lines are added the approximate elevations of all points are at once evident from inspection. Such a map is called a topographic map,—it represents the configuration of the earth's surface.

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* For a comprehensive treatise on the subject, *Topographic Surveying*, by H. M. Wilson (John Wiley and Sons), is recommended.

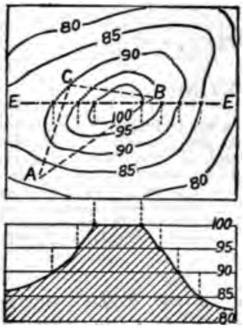

FIG. 398.

Illustration. In the upper portion of Fig. 398 the approximate elevations of the points *A*, *B*, and *C* are shown by the contour-lines. Thus it is seen that the elevation of *A* is somewhere between 80 ft. and 85 ft., *B* is somewhere between 95 ft. and 100 ft., while *C* is 90 ft.

The lower portion of the figure is a vertical section at *EE*. There is nothing shown by this section which is not evident from a study of the contour-lines. On the contrary the contour-lines show the exact shape of the hill better than the section. From the latter, one might suppose the ground to run in the form of a ridge, but from the contours it is seen to be a cone-shaped hill.

Question. If the hill were a perfect cone, would the contour-lines be perfect circles with a common center?

399. Contour-lines, Fundamental Principles.

1. *All points on the same contour-line have the same elevation.*

2. *Two contour-lines of different elevations cannot cross each other.* If they did, the point of their intersection would have two different elevations, which is absurd. (How can an overhanging cliff or a cave be an exception to this rule?) Contour-lines of different elevations sometimes appear to merge in one line as in the case of a vertical cliff.

3. *Two different contour-lines of the same elevation cannot merge and continue as one line.* For the single line would indicate a knife-edge ridge or depression, something which does not occur in nature. Different contours of the same elevation, however, often approach very near to each other.

4. *Contour lines near together indicate a steep slope,—far apart a gentle slope,—equally spaced a uniform slope.* It is evident that the horizontal distance between successive contour-lines indicates the rate of slope.

5. *A contour passing through any point is perpendicular to the line of steepest slope at that point.* This agrees with (4), since the perpendicular distance between contour-lines is the shortest distance. Ridge and valley lines cross contours at right angles.

6. *A closed contour-line with one or more higher ones inclosed indicates a hill* (*Fig.* 398); *with one or more lower ones inclosed it indicates a depression without an outlet* (Fig. *402 (b), p. 340).*

7. *A contour-line cannot have an end within the limits of the map; it must either close upon itself, or else it must begin at some point on the edge of the map and extend continuously until it disappears at some other point on the edge of the map.*

8. *The contours at the top of a ridge or at the bottom of a valley either close or go in pairs.* For the highest horizontal plane that cuts a ridge must cut it on both sides; the same is true of the lowest horizontal plane that cuts a valley.

9. *No single contour-line can intervene between two higher or two lower contour-lines.* This follows from (8).

10. *No contour line should be drawn across a stream. The contour turns upstream until it disappears in coincidence with the line defining the stream or ravine.* Questions: Why cannot a contour turn *downstream*? If the bed of the stream or the bottom of the ravine rises above the plane of a contour is there an exception to the rule, i.e., does the contour-line cross? Is (10) an exception to (7)?

400. Ridge Lines and Valley Lines indicate largely the general character of the surface of the country.

Ridge lines or watershed lines: Lines along the tops of ridges dividing watersheds, i.e., on one side of a ridge line water will flow in one direction, on the other side in the opposite direction.

Valley lines or thalwegs: Lines along the bottom of valleys where the two sides of the valley meet. They are often indicated by streams and rivers which follow them.

The sharpest bends or curves in contour-lines occur at points on ridge lines and valley lines, the convex side of the curve being toward the lower ground for ridges, toward the higher ground for valleys. (See Fig. 398, p. 338, and Fig. 402 (*b*), p. 340.)

401. Topographic Surveying. The field work may be divided as follows: (1) Establishing stations and main lines of reference. (2) Locating contours. (3) Locating roads, buildings, boundary-lines, streams, and other details. (4) Taking field notes and sketching topography. The work may be further subdivided into two parts: (1) that which pertains to *horizontal* control (horizontal measurements), and (2) that which pertains to *vertical* control (leveling or hypsometry). In small surveys all of this work may be carried along simultaneously and by one party, but in more extensive surveys it is usually divided up, one party establishing the stations or principal points of horizontal control, another party establishing the elevations of stations, of bench-marks, and of other points of vertical control, while other parties follow, filling in contours and topographic details. No new principles are involved.

Points of horizontal control are established by traverses (Chapter XII) or by triangulation (Chapter XVI), while details are located by the methods of Chapter XIII.

Points of vertical control are established by spirit-leveling, trigonometric leveling, or by the use of the aneroid barometer (Chapters XIX–XXI). The distinguishing characteristic of topographic surveying, however, is the location and sketching of contours, and this part of the work calls for further explanation.

402. Locating Contours. There are two general methods: (1) locating points on each contour near enough together to determine the contour-line, and (2) locating points at random, but so chosen that when they are plotted, contour-lines can be interpolated. In either case each point must be located with respect to some line between stations (horizontal control), and its elevation determined with respect to the assumed datum (vertical control).

(*a*) *Illustration, Method* (*a*). In Fig. 402 (*a*) the dots represent points located in the field. These points were taken first on one contour and then on the next. Thus a number of points at irregular intervals, as 1, 2, 3, 4, 5, and 6, each having an elevation of 100 ft., were located with respect to stations *E* and *F*, and the 100 ft. contour-line sketched through these points. Then a number of other points at irregular intervals, each having an elevation of 105 ft., were located and the 105 ft. contour sketched in. In a similar manner the other contours were successively established by locating the points indicated by the dots. The routine of the field work will be further explained in Art. 411 (*a*), p. 348.

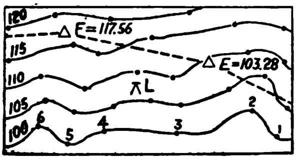

FIG. 402 (*a*).

(*b*) *Illustration, Method* (*b*). The dots (Fig. 402 (*b*)) represent points taken at random in the field. Each point was located with respect to stations *M* and *N*, and its elevation determined. These points were then plotted and used to locate the points on contour-lines (indicated by circles). The latter were found by interpolation. Occasionally, by chance, a point located in the field happens to be a point on a contour-line. It will be noticed that a point indicated by a dot can be used for locating more than one contour-point. Thus, for example, circle (2) was interpolated between dots (1) and (4) while circle (3) was interpolated between (1) and (5). The elevations from the field notes were (1)=63.5, (4)=67.3, and (5)=57.5. For the methods of interpolating contours see page 497.

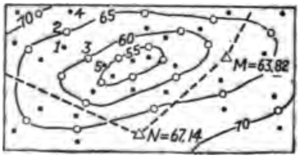

FIG. 402 (*b*).

(c) *Method (b) modified.* For a small area, as a building-lot, or a city park, or a field to be graded, Method (b) is often used in connection with a system of squares. These squares are simply a convenient method of locating points. The method is used also for more extended surveys.

Illustration. A building-lot about 160 ft. × 190 ft. is shown in Fig. 402 (c). A rectangle 160 ft. × 200 ft., subdivided into 20-ft. squares, was staked out, and elevations taken at the four corners of each square (at points indicated by dots). From these points contour-lines were sketched in by interpolation, as in the preceding illustration. When required by any sudden change in contours, intermediate points other than corners of squares may be taken. Each corner of a square is known by the letter and number corresponding to the intersecting lines. Thus the dot indicated by the circle is C8.

Size of squares. In any given case make the squares as large as possible, yet small enough to conform to the inequalities of the ground and to the accuracy required. In rough surveys over extensive flat areas, squares several hundred feet on a side may be used.

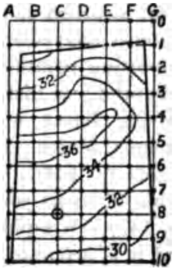

FIG. 402 (c).

403. Methods of Horizontal Control. (a) *Location of reference-lines.* In an extensive survey the first step is to establish the most important or *primary* stations. This is usually done by triangulation, and in Chapter XVI suggestions are given for locating the base-line, choosing stations, building signals, and carrying on the work of triangulation. Starting from any triangulation station, traverses may be run to establish *secondary* stations, each traverse closing on some primary station, though not necessarily upon the one from which it started. In this way the lines between triangulation stations form a skeleton upon which the survey is built, and this skeleton is further developed by traverse-lines, until every portion of the ground is covered. Any of the methods of Chapter XII may be used for running traverses. The azimuth method is, perhaps, the best, except for work in cities and in other places where the needle is useless as a check. The compass or the plane-table is often used, either in connection with transit traverses or in place of them.

Remark. For small areas triangulation is unnecessary, and only one system of reference lines is used, but for very large surveys it is not unusual to establish primary stations by triangulation, secondary stations by running transit-lines, and still other stations by the compass method or by the use of the plane-table. (See seventh illustration, p. 352.)

(b) *Reference meridian.* In extended topographic surveys it is customary to establish a true meridian (see Chapter XXVIII) and to "tie" the skeleton of the survey to this meridian. The most direct method of tying to the meridian is to measure the angle between this meridian and some main line of the survey.

then the true bearing of the remaining lines may be calculated. The angle may be measured by repetition, as explained on p. 195, several sets being taken. (See also p. 389, Art. 445 (3).)

(*c*) *Location of details.* Details are located from the nearest or most convenient stations by the methods of Chapter XIII. Important points near stations are usually located by angle (or azimuth) and distance, but for much of the work methods involving linear measurements only are used. Thus roads, streams, fences, and even buildings may be located wholly by offsets (p. 138). For indefinite lines, such as edges of woods, banks of rivers, and irregular roads, the method of intersections (p. 137) is a rapid method, especially if a plane-table is used, and it is accurate enough for the purpose.

(*d*) *Location of contours.* As far as horizontal control is concerned, it makes no difference whether the points to be located are on the contours themselves (Method (*a*), p. 340), or whether they are taken at random for interpolating contours. Any point may be located by some one of the seven methods given on p. 4, but probably the method most used is the angle-and-distance method. For angular measurements the azimuth method is used perhaps more frequently than the direct-angle method, because of the ease with which readings may be checked by the needle. For linear measurements, the stadia is more useful than the tape, since it is quicker, and the accuracy required for side shots is not great. Other methods of horizontal control include dividing the area into squares or rectangles (Method (*c*) p.341; see also fourth illustration, p. 350), by locating contour points on lines at right angles to the transit-line (Offset method; see fifth illustration, p. 351), and by intersection (a method used principally in plane-table work).

(*e*) *The method of procedure* at any one station is outlined on page 154 for the direct-angle method, and page 156 for the azimuth method. If traverse-lines and other reference-lines have been established, some of the steps pertaining to the location of stations may be omitted—simply follow, the directions for taking side shots. In such a case the work of locating contours may be begun at any station, but in the case of the azimuth method, the following precaution must be observed.

(*f*) *Precaution.* In starting at any station the transit must be oriented by backsighting along some main line of the survey, the vernier being set at the azimuth (or back azimuth) *already determined for that line*, so that all azimuths will be measured from lines parallel to the same reference meridian. Sometimes when the main lines have not been run by the azimuth method, but it is desired to use that method for side shots, the bearing of the backsight has to be calculated (p. 126), and this bearing changed to the corresponding azimuth. When the reference meridian is a *true north and south* line, it is well to set off the declination arc (p. 124).

(*g*) *Precautions in establishing substations.* When it becomes necessary to establish a substation more care should be exercised than for an ordinary side shot. Thus, for example, the azimuth should be checked by the needle, and, if the stadia is used, the distance should be read from each end of the line. The correct location of all points taken at a substation depends of course upon the correct location of the substation itself.

(*h*) *Reading horizontal angles to contour-points.* Side shots are rarely longer than 500 ft. An error of 10 minutes amounts to a displacement of not more than 1½ ft. at 500 ft., and of only about 0.3 ft. at 100 ft. from the transit (p. 105). As it makes little difference whether a contour-point is one or two feet one way or the other, and as limb-readings can be estimated easily to less than ten minutes, it follows that, as a rule, *it is not necessary to read the vernier in locating contour-points—simply estimate the reading from the limb.*

404. Methods of Vertical Control. (*a*) *Primary stations.* The first step is to ascertain the elevations of the principal stations, or else to establish bench-marks near them, so that a party can start from any station to locate contours, or several parties can start simultaneously from different stations. When the principal stations are located by triangulation, it involves very little additional field work to ascertain their elevations by *trigonometric* leveling,—simply measure the vertical angles. (See *Triangulation and Trigonometric Leveling*, p. 200.) Another method is that of spirit-leveling, and still another, for rough work, is barometric leveling (pp. 245 and 265). Sometimes all three methods are used.

(*b*) *Elevations of traverse stations* or nearby bench-marks are usually determined by spirit-leveling, the leveling party following close behind the traverse party. For compass traverses the aneroid barometer will sometimes give close enough results (p. 267). Permanent bench-marks should be placed near stations and in other places where they will be of use in contour work.

(*c*) *Elevations of contour-points.* When each contour is run in separately (Method (*a*), p. 340), points at the elevation of the contour are usually determined by spirit-leveling, either an ordinary level or a hand-level being used. The spirit-level may be used also for finding the elevations of points taken at random (Method (*b*), p. 340), but when such points are on steep slopes not many of them can be taken without moving the level—a decided disadvantage. It is better in such cases to use the stadia-vertical-angle method (explained on p. 308), so that points up and down the slope can be taken from one set-up. A third method used in rough work is to find the elevation of points by the aneroid barometer.

(*d*) *Reading vertical angles to contour-points.* The directions for measuring horizontal angles (Art. 403 (*h*)) hold good for vertical angles also. For a sight from 100 to 200-ft. long an error of 10 min. causes an error of from 0.3 ft. to 0.6 ft. in elevation; this is no more than the difference in height between the top and bottom of a grass hummock, hence the absurdity of reading vertical angles to the nearest minute in getting heights for contours in ordinary work. Occasionally, on long sights, it may be necessary to consult the vernier, but *most of the angles should be read directly from the vertical arc, estimating the minutes.* (See p. 309.)

405. Choosing Stations for Horizontal Control. When stations for horizontal control are established by triangulation, the accuracy of the work will depend, to a great extent, on the proper proportions of each triangle (p. 192). When traverses are run, the suggestions for choosing stations given in Art. 216, page 149 should be followed. In general, contours are most easily controlled from stations along ridge lines and valley lines. Stations should be located also near prominent topographic features, such as deep gulleys, high cliffs, ponds, lakes, and watercourses. (See Art. 247, p. 192 for other suggestions concerning the choice of primary stations.)

406. Choosing Stations for Vertical Control. In triangulation, stations should be chosen with proper regard for vertical control as well as horizontal control, especially if trigonometric leveling is employed. In traverses, if the suggestions of the preceding paragraph are followed, the stations, as a rule, will be well located for vertical control.

407. Choosing the Controlling Points of Contours. In running in contours by Method (*a*), p. 340, points should be chosen near enough together, so that no appreciable error will exist when the contour-lines are sketched through them. This will depend to a large extent upon (1) the regular or irregular nature of the ground, and (2) upon the scale adopted for the map. The critical points of contours are usually apparent to the eye and these points should always be taken. Thus, points should be near together at sharp curves or abrupt changes in direction,—farthest apart where contours are nearly straight. In uniform slopes the highest and lowest contours having been run in, the others can frequently be interpolated without any field work, or perhaps every fifth contour may be located and the others interpolated.

(*a*) When contours are interpolated from points taken at random (second method, p. 340) the preceding suggestions still hold good, the points in general being taken close together for uneven ground or steep slopes and farther apart for even ground *or gentle slopes.* In this method points on ridge lines, valley lines, tops and bottoms and changes of slope should, as far as possible, be located.

In either method it is a waste of time to take points too near together when the scale of the map is small, while it leads to inaccuracy to take them too far apart when the scale is large.

408. Choosing Contour Intervals. The contour interval depends almost entirely upon the purpose of the survey. Thus, for example, in the survey of a city lot, a two-foot interval or even a one-foot interval may be necessary; for the preliminary survey in railway location a five-foot interval is often adopted; for more extended surveys, such as the United States Geological Survey, 20-ft., 50-ft., and even 200-ft. intervals are used.

409. The Choice of Methods and of Instruments in topographic surveying depends upon the nature of the ground and the purpose for which the survey is made, especially upon the related questions of speed and accuracy. Practically every method and every instrument may be used to advantage at one time or another in topographic surveying.* Even in the same survey various parts of the work may call for the use of different methods and different instruments. Thus, for example, in an extensive survey less accurate methods and instruments are used for running secondary traverses than for establishing the primary stations or skeleton upon which the survey is hung, while the contours and topographic details may be filled in by the use of still less accurate methods and instruments.

The instruments and methods chosen for vertical control should correspond in accuracy to those chosen for horizontal control. For example, if rough instruments are used for locating points, rough instruments are used for finding their elevations. There are, of course, exceptions to this rule. Consistent accuracy should be maintained between linear and angular measurement. For example, it is necessary to measure the base-line of the triangulation with great accuracy, but compass-lines may be measured roughly or even paced in some cases.

The number of possible combinations of instruments and of methods is too large to be given in detail, but some of the most common combinations are suggested below.

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* Even the camera may be used to advantage in topographic surveys extending over large areas, and the science of photographic surveying constitutes a branch in itself. See Wilson's *Topographic Surveying*, p. 292, and *An Elementary Treatise on Photographic Methods and Instruments* by J. A. Flemer (John Wiley and Sons).

COMBINATIONS OF INSTRUMENTS AND METHODS

Points Located	Horizontal Control		Vertical Control	
	Instruments	Methods	Instruments	Method of Leveling
Triangulation Stations	Transit	Triangulation	Transit or level	Trigonometric or spirit
Primary Traverse	Transit and tape	Traversing	Level	Spirit
Secondary "	Transit, tape or stadia	Traversing—Azimuths	Level	Spirit
" "	Compass and chain	Traversing—Bearings	Level or barometer	Spirit or barometric
" "	Plane table, tape or stadia	Traversing	Level or barometer	Spirit or barometric
Contours........	Transit, tape or stadia	Azimuths or squares	Transit or level	Vertical angle or spirit
"	Compass, chain or pacing	Bearings or squares	Level or barometer	Spirit or barometric
"	Plane table, tape or stadia	Radiation, intersections	Level or vertical arc	Spirit or vertical angle
"	Traverse board, chain, pacing	Radiation, intersections	Hand level or barometer	Spirit or barometric

410. Field Notes. There are two general methods of keeping field notes in topographic surveying: (1) Keeping them according to one of the methods of Chapter XV, making no attempt to plot to scale. (2) Plotting the notes to scale and carefully sketching the contour-lines in the field. The latter method requires more time in the field, but saves time in the office. The first method is generally used for small areas, the second for large areas.

First Method. The combination method (p. 166) is generally preferred. The points on the sketch on the right-hand page should be carefully numbered to conform to some well-understood system. Thus, for example, all points on the 100-ft. contour may be numbered 100 regardless of location, all points on the 105-ft. contour may be numbered 105 and so on, the location of the points being given in the tabulated form on the left-hand page. If points are taken at random and contours afterwards interpolated, it is better to give each point a sepa- number, and to note its elevation in a column set apart for that purpose. Notes for stations and details are kept by the usual methods.

Second Method. For large areas the results, as a rule, will be far more satisfactory if the topography is plotted in the field. There are two ways of doing this: (1) By the use of the plane-table and (2) by the use of ordinary plotting methods. The plane-table is especially useful for filling in details from stations which have been established previously by more accurate instruments. Sometimes the entire survey can be made with it. (See Chapter XXVI.) When ordinary plotting methods are used angles and distances are plotted as fast as measured, and contours sketched in. In this way errors and omissions are likely to be detected and doubtful points settled. Thus, with the land actually before him, the topographer can secure a more truthful representation of it than he could otherwise. Various devices are used to facilitate the plotting, such as sketching-boards, various forms of protractors, and especially designed cross-section paper.

Sketching. In order to sketch topography successfully in the field, one must have a faculty of selecting the most important topographic features and transmitting them to paper in their proper relations. This sense of topography comes only from long experience. In ordinary class work the beginner first plots points of known elevation, as fast as they are determined, and, guided by these points, he sketches in each contour; his imagination aided, perhaps, by the conception of a water surface. (See suggestion p. 337.) Successive points on the same contour should be joined as soon as they have been plotted, by an irregular line which corresponds as nearly as possible to the actual contour which it represents. If contours are interpolated, they should be carried along in a similar manner. It answers the purpose in many cases simply to interpolate contour points by eye; for more accurate methods of interpolation see page 497. Each contour should be marked in one or more places with its elevation.

(*a*) *Practical suggestions.* (1) Even when the combination method is used, it is best to sketch the contours as correctly as possible on the right-hand page of the note-book. A very good book for this purpose has the page divided by radiating lines and concentric circles, so that a point can be plotted quickly by azimuth and distance without the use of instruments. (2) A common mistake is to record the azimuth to one point and the distance to another point, as if they were taken to the same point. (3) When notes are plotted in the field, the drawing-board may be mounted on a tripod and set up near the transit. Instead of using a protractor it is sometimes more convenient to plot the notes on paper or Bristol-board which has printed upon it a full circle graduated to quarter degrees. This paper comes in sheets and may be obtained from dealers in such supplies. (4) Keep track of points as fast as they are plotted by marking each point with its elevation. This is especially important when points are taken at random and contours are interpolated, otherwise confusion will result. (5) A disadvantage common to all plotting operations in the field, including the plane-table, is the change in the paper due to moisture. This is partially overcome by using mounted paper, and by dampening it slightly before stretching it on the board. Let the paper extend over the edges of the board and fasten it on the under side by thumb-tacks close together. Sometimes mucilage is used. In very damp places, where water is likely to drip on the board, celluloid sheets have been used in place of paper. (6) The suggestions for choosing a scale for plane-table work (p. 328) hold good for topographic work also.

411. Field Routine. On page 346 several combinations of instruments and of methods are suggested for topographic surveying. In this article the field routine corresponding to each of several combinations is outlined. It is assumed in each illustration that the stations are chosen in accordance with Art. 405, p. 344, that the contour interval has been adopted to suit the purpose of the survey (p. 345), that all details, other than contours, are located by the usual methods (Chapter XIII), and that the controlling points of contours are chosen according to Art. 407, p. 344. The main points to be brought out are: (1) How controlling points are located *horizontally*, and (2) how their *elevations* are determined. (3) How to start at any station. (4) How the levelman, the transitman, and other members of the party may work together to economize time.

Each illustration begins with a statement of the extent of the survey, the accuracy and speed required, the instruments chosen, and the methods used. It is understood that the work pertaining to horizontal control and that pertaining to vertical control are carried on simultaneously, as far as possible.

Remark. The methods of procedure given in the following illustrations may be modified in many ways to suit conditions. Thus, for example, it is often advantageous to use the direct-angle method in place of the azimuth method, especially if stations cannot be occupied in regular order (p. 129). The illustrations suggest how to proceed for different combinations of instruments and of methods—this is the first thing to learn; then the student should begin a study of the advantages and disadvantages of the different combinations, a full knowledge of which can be gained only by extended experience in the field. Special attention is called to the last illustration on p. 352, since it combines in one survey nearly all the methods of topographic surveying and, hence, serves as a summary of this chapter.

(*a*) First Illustration. *Conditions:* comparatively small area, fair degree of accuracy, medium speed, 5-ft. contour interval. *Instruments:* transit, level, and leveling-rod (used also as a stadia-rod). *Methods:* azimuth method (p. 156) and stadia method (p. 300) for horizontal control; spirit-leveling for vertical control; method of running in contours by ascertaining points through which they pass (Art. 402 (*a*), p. 340). *Party:* transitman, note-keeper or draftsman, levelman, and rodman.

Method of procedure for the levelman. (1) Set up where a backsight can be taken on some bench near by, and, at the same time, near enough to the transit station so that any point which can be seen from the transit will be visible from the level also. (It is assumed that benches have already been established near stations.) (2) Backsight, find H.I., figure the grade-rod for one of the contours. (See p. 243 for grade-rod.) It is well to begin with the highest contour that can be run in from the first set-up of the level and work down hill, or else begin with the lowest and work up hill. (3) When as much of the first contour has been run in as can be located conveniently from the transit station, run in two more in the same way, moving the target 5 ft. for each contour. By this time it will be necessary to move the level, hence establish a turning-point as far down hill or up hill as practicable, set up the level as far down hill or up hill as the turning-point will permit, find the new H.I. and the new grade-rod for the next contour, and proceed as for the first set-up. (4) Continue the work, moving the level as often as may be necessary until all the contour-points that can be located advantageously from the transit station have been taken. If the work has been carried on down hill from the station, it may be necessary to start again at the bench-mark and work up hill, or vice versa. When benches have not been established near the stations, it may be necessary to take extra turning-points in carrying the work from one station to the next; otherwise it is better to start anew at each station by backsighting on the bench-mark near it.

Method of procedure for the rodman. (1) Check the grade-rod for the first contour as figured by the levelman, set the target, find a point on the contour by moving the rod up hill or down hill until the levelman signals that the line of sight strikes the target, and then hold the rod for the transitman to take a stadia reading. (The levelman should not split hairs in sighting. Often it will be close enough if the line of sight strikes anywhere on the target.) (2) Repeat for other points, selecting only critical points along the contour. (See p. 344.) (3) Check the grade-rod for the next contour, set the target, and run in this contour by selecting critical points as before. Repeat for all the contours, establishing turning points whenever it becomes necessary to move the level up hill or down hill.

Method of procedure for the transitman. (1) Set up and orient the transit (p. 125). Assuming that transit stations have not been established, it will be necessary at the first station to assume the meridian of the survey. (See p. 121.) At other stations the azimuths of lines must be with reference to this meridian. (See *precaution*, 403 (*f*), p. 342.) (2) Take the azimuths to contour-points as fast as these points are established by the level party, reading the corresponding stadia distance and vertical angles. (It is not necessary to read the vernier for vertical angles in ordinary work—estimate the minutes.) (See Art. 152 (*d*), p. 109.) Vertical angles less than from 3° to 5° may be ignored. (See p. 307.) (3) Observe the method of procedure for the azimuth method (given on page 156), in running transit lines, and read the stadia distance between stations twice, once from each end of the line. Check the azimuths of all transit lines by the needle, and check at least every fifth side shot in the same way.

Method of procedure for the draftsman. (It is assumed that the notes are plotted in the field as fast as they are taken.) (1) Set up the plotting-board near the transit. If it is the first station, draw a long line near the center of the board to represent the meridian. Azimuths may be plotted directly from this line or from parallel lines through the stations, as explained in the chapter on *Plotting Transit Lines*, Part II. Use a protractor for ordinary work or else some form of plotting-paper. (See p. 347.) (2) Plot the azimuth to each point as soon as it is taken, and then the stadia distance, reducing the latter to a horizontal distance when the vertical angle is greater than, say, 3° (p. 480). Keep track of points by marking near each one the elevation of the contour which passes through it, and sketch in each contour-line as fast as it is determined by the plotted points.

try to reproduce each contour as it actually appears on the ground. When the points have been taken too far apart and the sketch shows the need of more points, indicate to the level party where the additional points are needed.

When the notes are not plotted in the field, use the combination method and the form of notes shown on page 311, but make the sketches on the right-hand pages represent the actual contours as nearly as possible.

(*b*) SECOND ILLUSTRATION. *Conditions:* Same as in the preceding illustration, except that points are taken at random and contours are interpolated. *Methods:* The only change in routine is in the leveling and in the plotting. The rodman selects critical points where the slope changes (p. 344), and the elevation of each point as determined by the level party is given to the draftsman. The latter plots these points as in the preceding illustration, and marks the elevation of each point; but instead of sketching contour-lines through the points, he interpolates them as fast as the plotted points permit him to do so.

(*c*) THIRD ILLUSTRATION. *Conditions:* an area of considerable extent, medium accuracy, rather high speed. *Instruments:* transit and stadia-rod or leveling-rod, board mounted on a tripod, with accessories for plotting, including protractor, scale, and triangles. *Methods:* azimuth method and stadia method for horizontal control, vertical angle and stadia for vertical control (p. 308); points chosen at random for interpolation of contours, (p. 340). *Party:* transitman, rodman, a computer, and a draftsman.

Method of procedure for the transitman. (1) Set up and orient the transit as in the first illustration. (2) Co-operate with the rodman in setting the target in order to determine elevations as explained on page 264. Take the azimuth, the stadia reading, and the vertical angle to each point where the rod is held. (3) Unless the lines between stations have already been established, locate them as usual. (See first illustration.) Usually in this work it is better to establish bench-marks near stations by spirit-leveling, but if this has not been done it will be necessary for the transitman to determine elevations from station to station by vertical angles.

Method of procedure for the rodman. (1) Set the target (or the rubber band on the rod) from the nearest bench, or in some cases from the transit station (pp. 244 and 264). (2) Hold the rod on ridge lines, valley lines, wherever the slope changes, and at other critical points (p. 344). Co-operate with the draftsman in determining what points are needed.

Method of procedure for the computer. Compute the horizontal distance from the station to a point as soon as the transitman has given the stadia reading and the vertical angle; while the draftsman is plotting the point, find the corresponding vertical distance, and from that ascertain the elevation of the point with respect to the datum used for the survey. See that the elevation of each point is marked near its plotted position. Use the table in the back of the book for reducing stadia readings.

Method of procedure for the draftsman. Plot the azimuth of the line to each point as soon as the transitman reads it (see first illustration); plot the distance as soon as the computer has ascertained it; mark the elevation near each point as soon as the computer has calculated it. Be careful to keep track of the azimuth, distance, and elevation of each point until there is no chance of using any one of these three data in connection with some other point. Sketch in the contours as fast as they can be interpolated from the points plotted, always with due regard to the ground as it appears to the eye; when in doubt call for other points to be taken near the doubtful places.

(*d*) FOURTH ILLUSTRATION. *Conditions:* small area, high degree of accuracy, low speed. *Instruments:* transit, tape, leveling-rod, and level (or the transit may be used as a level). *Method:* area subdivided into squares or rectangles for horizontal control, elevations of the corners of the squares or rectangles taken for vertical control, contours interpolated (p. 341).

Routine of the field work. Decide how to lay out the squares in order to cover the ground to the best advantage, what size to make them, and in what direction to run the lines (p. 341). Stake out the squares with the transit and tape. (Two or three set-ups on the outside corners will usually suffice, the interior stakes being lined in by eye.) Backsight on a bench near by, or establish one if none exists, find the elevation of the ground at each stake and in between if there is a sudden change in elevation, and thus determine points from which the contours can be interpolated. The level notes should include sketches on the right-hand pages, carefully lettered and numbered according to some well-understood system (p. 341), and showing all dimensions and measurements necessary for plotting.

Note. This same method may be modified for very rough work by using the compass for running out the lines, and a clinometer or some other rough instrument for ascertaining the elevations. Where the contour interval is large, the barometer may be used for vertical control, readings being taken from the altitude scale. If bench-marks have been established here and there, and the barometer is reset to correspond to the elevation of a bench-mark whenever there is an opportunity, contours may be sketched in a fairly satisfactory manner by this method. It could be used, for example, in combination with more accurate methods for securing a rough topographic map of a large wooded area.

(*e*) FIFTH ILLUSTRATION. *Conditions:* the survey progresses in one general direction as in railroad work; the main transit lines have been established, and the elevations of points at frequent intervals, say at every 100-ft. station, have been found by the method of profile-leveling; fair degree of accuracy, fair speed, and a 5-ft. contour interval. *Instruments:* two hand-levels, two contour-rods, a sketch-board, and plotting-paper. *Methods:* running in contour-lines by ascertaining points through which they pass; offsets from the transit line for horizontal control; hand-levels for vertical control. *Party:* two men, who do practically the same work, except that only one of them plots the contours.

Routine of the field work. Leveler No. 1, resting his hand-level on the top of a contour-rod, backsights on a station of known elevation to get the line of sight at the height of some even 5-ft. contour. (See illustration, p. 233.) He then sends Leveler No. 2 out at right angles to the transit line from a 100-ft. station to a point where the line of sight cuts the ground (working up hill). This point must be on a contour, and the offset distance to it may be measured with a tape, or in rough work it may be paced. Leveler No. 2 then rests his level on a contour-rod at this point, and sends Leveler No. 1 up hill to a point where his line of sight strikes the ground, Leveler No. 1 either carrying one end of the tape as he goes or pacing the distance. Thus the men alternately establish contour-points on the offset line as far up hill as desired, then they work from the station down hill in the same way. (How will the work going down hill differ slightly from the method just explained?) All the points on one offset line having been plotted, the party repeats the work at the next 100-ft. station. Thus the work is carried on, contour-lines being sketched by joining points of the same elevation. Offset lines are taken between stations where irregularities in the surface of the ground render it necessary.

When the elevations of stations have not been established previously it will be necessary to take turning-points as in ordinary leveling in going from one station to the next.

(*f*) SIXTH ILLUSTRATION. A plane-table is intended primarily for horizontal control, but if a vertical arc is attached to the telescope, it can be used also for vertical control. In that case the entire work in the third illustration (p. 350) could be done with the plane-table, no other instrument being required. It could be used for horizontal control in the first and second illustrations (pp. 349 and 350), taking the place of the transit and of the plotting-board, the contour-points being located and plotted simultaneously as fast as they are determined by the level party. In the same way, it could be used in connection with the hand-level in the fifth illustration; in fact, the combination of the traverse-board (a rough plane-table) and the hand-level is a very common one.

When the horizontal distances and the elevations of points are reduced from stadia readings and vertical angles, the man who runs the plane-table may be assisted by a computer, whose duties are those described in the second illustration.

(*g*) SEVENTH ILLUSTRATION, SUMMARY OF METHODS. The following outline indicates the work done by a class in making a topographic survey of a rather extended area. It was desired to illustrate as many methods as possible, and to furnish work for a large number of parties, each party going from one kind of work to another until it had been given practice in all the different methods. This illustration will serve to summarize the methods of this chapter and to show how different methods may be combined in the same survey.

(1) Made a reconnoissance with the class, with the following objects in view: to get an idea of the drainage of the country; to ascertain what areas were wooded and what were comparatively open; to find favorable locations for triangulation stations (p. 192), and to plan the triangulation net (p. 191); to select a place for a base-line (p. 193), and decide upon its approximate length (p. 194); to decide tentatively where to run traverses and what instruments and methods to use.

(2) Secured from a railroad company the elevation of the nearest bench, used this elevation in adopting a datum for leveling, and started parties from this bench to run lines of levels to the two ends of the base-line, where the first benches of the survey were to be established.

(3) Built signals at points selected for triangulation stations (p. 198).

(4) Set a hub for each end of the base-line (p. 57), referenced each hub (p. 203), set parties at work measuring the base-line (p. 57).

(5) Set parties at work measuring the angles of the triangulation net (p. 194), and the vertical angles to triangulation stations required for trigonometric leveling (p. 200).

(6) Established a true meridian through one end of the base-line by observation on Polaris (Chapter XXVIII), found the true bearing of the base-line by measuring the angle between it and the meridian, checked this bearing by observations on Polaris at the other end of the base-line.

(7) Started transit parties to establish secondary stations by running traverses (direct-angle method, angles doubled (p. 154)), each party to begin at one primary (triangulation) station and to close on the same or some other primary station, all secondary stations to be established with due regard to their future usefulness (p. 150).

(8) Started level parties from the benches near the base-line to establish benches near secondary stations, checking whenever possible on primary stations, the elevations of which were found by trigonometric leveling in (5).

(9) Set plane-table parties at work in open country, some of the parties to start from primary or secondary stations, and, establishing their own tertiary stations by traversing (p. 322), to close on some primary or secondary station. Other parties are to set up in some commanding position from which three stations previously plotted are visible, and orient by

the three-point problem (p. 329). In either case contours are to be sketched in by stadia and vertical-angle method as in the third illustration (p. 350).

(10) Set transit parties at work in wooded areas unfavorable for triangulation, each party to start from a primary or secondary station, establishing tertiary stations if necessary, and to close on a primary or secondary station. Method of procedure exactly the same as in the third illustration (p. 350).

(11) Set transit parties at work to run a preliminary survey for constructing a new road, tying this survey whenever possible to the main survey and running in contours with the level and hand-level as in the fourth illustration (p. 350).

(12) Detailed compass parties to locate high cliffs and deep gorges, differences in elevations between top and bottom lines or other controling lines to be taken with an aneroid barometer, and less steep slopes with the clinometer, intervening contours to be interpolated. Compass lines are to be tied to primary or secondary stations, and aneroid readings to be compared with elevations at benches whenever possible.

CHAPTER XXVIII

DETERMINATION OF A MERIDIAN

BY OBSERVATIONS ON POLARIS

THE simplest and most direct method of determining a true meridian is by observations on Polaris. This method is easily understood, and easily carried out in the field. It involves almost no computations, and the few astronomical data required are easily obtained.*

412. Why Polaris is Chosen for Observations. The two points in which the axis of the earth, indefinitely produced, pierces the celestial sphere are the *celestial poles.* One of these points will be referred to throughout this chapter as the North Pole or simply Pole. All heavenly bodies *appear* to revolve once a day about the earth, describing circles on the celestial sphere. If the distance of a star from the Pole is less than the altitude of the pole, the circle which the star apparently describes will not meet the horizon at all, and the star will, therefore, neither rise nor set, but it will appear to revolve about the pole as a center. If there were any star exactly at the Pole it would seem to remain stationary, and to determine a true meridian it would be necessary simply to bring the cross-hairs of the telescope upon it; but, unfortunately for the purposes of the surveyor, there is no such star. Contrary to a general impression, Polaris (North Star) is not exactly at the Pole, but like all the other stars, it appears to revolve around it. The radius of its diurnal orbit is so small, however, that of all the stars, Polaris is the most convenient for determining a meridian. Any other star near the pole can be used, provided that certain data concerning it are known.

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* Other methods most used by surveyors for determining a true meridian depend upon observations on the sun. They are somewhat difficult to understand and to carry out in the field, more or less computation is involved, and, for accurate results, astronomical data are required which are not always at hand. These methods may be studied to better advantage in connection with a course in field astronomy, but for purposes of *reference* they are outlined in Appendix I.

413. Facts Concerning Polaris. Before describing the method of taking observations, certain facts concerning Polaris will be given.

1. A simple way of finding Polaris is to find first the "Great Dipper" (Ursa Major), and then to follow the line through the two "Pointers," i.e. the two stars farthest from the handle; this line passes very near to the pole. (See Fig. 413 (*a*).)

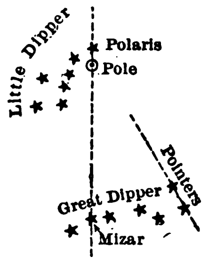

FIG. 413 (*a*).

2. Polaris is the end star in the handle of the "Little Dipper" (Ursa Minor). About 15° from the pole is another star about as bright as Polaris, and between these two is a curved row of three faint stars forming the handle of the "Little Dipper."

3. As Polaris, revolving counter clockwise about the Pole, reaches its [highest point, *U.C.* (Fig. 413 (*b*)), it is said to be at its *upper culmination*, when it reaches its lowest point, *L.C.*, it is at its *lower culmination*. In either case *it is in the true meridian.*

4. When Polaris reaches the most easterly point of its orbit it is at its *eastern elongation*, *E.E.*; likewise at the most westerly point it is at its *western elongation*, *W.E.*

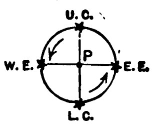

FIG. 413 (*b*).

5. The earth turns on its axis once in about 23 hrs. 56.1 m. (*the sidereal day*), hence Polaris reaches any point in its orbit, as, for example, its culmination or elongation, about 4 min. earlier each day than it did the preceding day.

6. The radius of the circle in which Polaris appears to revolve changes from year to year. This radius, called the *polar distance*, when first recorded was about 12°; it is now (1907) about 1° 11'.37. It will continue to grow less at the rate of 0.31' per year (approximately) until the star is about 30' from the pole, when it will begin to increase. In 1910 it will be 1° 10'.44, in 1915, 1° 08'.89, in 1920, 1° 07'.35. The *mean polar distance* or 90° minus the *mean declination* of Polaris is given for the *beginning* of each year up to 1920 in a table in Appendix I.

7. If an observer at the equator points the telescope of his transit at Polaris when at its *eastern* elongation, the line of sight will be *east* of the true north by an amount equal to the polar distance; at the *western* elongation the line of sight will be the same distance *west* of the true north. But if the observer is in a latitude north of the equator, and points his telescope at Polaris when at its elongation, the line of sight will make an angle with a N. and S. line *greater* than the polar distance, and the farther north the observer the greater this angle. Thus, for example, at the equator in 1907 this angle is 1° 11'.37, but at a latitude of 40° this angle would be 1° 33'.2, or 22' greater than the polar distance. This *horizontal* angle between the north point and the line of sight to Polaris at elongation is called the *azimuth at elongation* and is not to be confused with *polar distance* since the two are not the same except at the equator. (This meaning of azimuth is the same as that given on p. 114.) *Azimuth* varies with the position of the observer, but *polar distance* does not.* The azimuth

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* A line produced indefinitely from the center of the earth through any point on its surface pierces the celestial sphere in the *zenith* for that point, and hence observers at different points will have different zeniths. The spherical triangle *PZS* having for its vertices the Pole (*P*), the zenith (*Z*), and a heavenly body *S* is an *astronomical triangle*. (See Appendix I.)

The *azimuth* of any star *at elongation* is equal to the angle *PZS*, but since *Z* changes for different points of the earth's surface the *azimuth PZS* must change also, but the *polar distance PS* at any given time is, of course, the same, for it is in no way connected with the position of the observer.

If the observer is at the equator his zenith must be 90° from the pole, for any given latitude and date can be found from a table in the back of this book.

8. The *vertical* angle which a line of sight to Polaris makes with the horizontal is equal, *approximately*, to the latitude of the point where the transit stands. If Polaris were exactly at the north pole the vertical angle would be exactly equal to the latitude. (See Appendix I.)

9. When Polaris is at its upper or lower culmination a line of sight directed to it from any point on the earth's surface will be a true N. and S. line. This occurs twice every twenty-four hours.

10. For the year 1905 Polaris reached its culmination 4 min. 21 sec. after it was in the same vertical plane with *Mizar*, the second star from the end of the handle of the Great Dipper. (See Fig. 413(*a*).) For each year after 1905 add 21 seconds. Thus in 1910 the interval will be 6 min. 6 sec.

11. The *azimuth* of Polaris at any given time can be calculated by spherical trigonometry. At elongation it is:

$$\text{Sine of azimuth at elongation} = \frac{\text{sine polar distance}}{\text{cosine latitude}}.$$

414. Three Methods of Observing Polaris.

(1) At either elongation. (2) At either culmination. (3) At any time. The three methods will be outlined first, and then practical suggestions given for carrying on the work in detail.

I. AT ELONGATION. (1) Before the night of the observation look up in the table given in the back of this book the time of elongation of Polaris in 1905 for latitude 40° and longitude 90°, and make proper corrections (see Suggestion (1)). It is convenient, but not essential, to know the time of elongation within a minute or two. (2) Set up the transit ten or fifteen minutes before the time of elongation, and keeping the vertical cross-hair exactly on Polaris follow the star until it reaches its elongation (see Suggestions (2) to (8)). (3) Depress the telescope and set a point 300 or 400 ft. away (see Suggestion (9)). The line from the transit to the point thus established makes an angle with a true meridian equal to the azimuth of Polaris at elongation. The remainder of the work may be done more conveniently in the daytime. (4) Before continuing the work the next day, look up in the table in the back of the book the azimuth of Polaris at elongation for the given year and latitude. (5) Set up over the point occupied the night before, backsight at the point established by the observation on Polaris, and turn off the azimuth found in (4), clockwise if the

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i.e., $PZ = 90°$, and if the star is at its elongation ZS must equal 90° also, hence the angle PZS must equal the arc PS, i.e., the azimuth equals the polar distance. As the observer goes north, however, his zenith moves towards P, i.e., PZ becomes less than 90°, *for PZ must always equal 90° minus the latitude of the observer*. The farther north the observer goes the shorter become the two sides PZ and SZ of the triangle PZS, but since the third side PS remains the same the angle PZS, opposite to PS, must *increase*, i.e., the greater the latitude of the observer the greater the *azimuth of the star*.

observation was taken at the western elongation, counterclockwise if taken at the eastern elongation. The line thus established is in the true meridian.

(*a*) *Method modified.* While the above method of procedure will often answer the purpose there are two objections to it: (1) There is no check on the work. (2) If the transit is out of adjustment so that the line of sight does not revolve in a vertical plane, the resulting error is likely to be large since the vertical angle through which the telescope is depressed after sighting at Polaris is large. Both objections may be overcome by taking a double observation. As Polaris does not move east or west more than about 5 sec. of arc for a period of ten minutes before and after elongation, there is ample time for two observations, the first with the telescope normal and the second with it inverted. If the two points thus established coincide, the instrument is in adjustment, otherwise the error may be eliminated by taking a point half-way between. (See the third adjustment of the transit in Part III.)

(*b*) *Practical suggestions.* (1) The time of elongation as taken from the table should be corrected for the year, and for the day of the month, but the small corrections for latitude and longitude may usually be ignored. This corrected quantity is the local time of elongation, but in the United States the observer's watch is usually set to "standard time" and unless he is on a standard meridian (75°, 90°, 105°, or 120° West Longitude) he must allow for the difference between watch time and local time. To do this, find the difference in degrees and minutes between the longitude of the place of observation and the nearest standard meridian (or the one corresponding to his watch time), multiply by four and call the product of the degrees, minutes, and of the minutes, seconds of time. For example, if the observer is at 70° 20′ West Longitude then: $75^\circ - 70^\circ 20' = 4^\circ 40'$; $(4^\circ 40') \times 4 = 16$ min. 160 sec. $= 18$ min. 40 sec. Hence elongation will take place 18 min. 40 sec. earlier if timed by the observer's watch. If the observation is at any time within four minutes of elongation the observed azimuth will differ from the azimuth at elongation by less than one second of arc, hence it is far more important to eliminate instrumental errors by two or more observations as suggested above than to observe Polaris at the exact time of its elongation. (2) It is well to test the transit for adjustment on the day of the observation, to make sure that the line of sight revolves in a vertical plane. For the same reason great pains should be taken in "leveling up" the transit at the time of the observations. (3) The point over which the transit stands during the observations should be chosen so that the point established after sighting at Polaris may be at least 200 or 300 ft. distant. It is well to select some regular station of a survey, or else, before nightfall, to set a point for this purpose on one of the main traverse lines, so that the bearings of all the traverse lines may be calculated easily. (See Sta. Y, p. 390.) (4) A quick way of bringing Polaris into the field of the telescope is to clamp the latter inclined at an angle equal to the latitude of the observer, and to turn the transit on its spindle until, looking along the top of the telescope, Polaris is in line. (5) The telescope must be focused for distant objects. As this is sometimes difficult to do at night it is well to mark the correct position, once for all, on the object-glass slide. (6) To illuminate the cross-hairs, hold a lantern within three or four inches of the object-glass so that its light will enter the telescope. If the transit is not provided with a reflector, tear a hole about an inch in diameter in a white sheet of paper and by means of a rubber band attach the paper to the telescope in such a position that the star can be seen through the hole while the rest of the paper reflects light into the telescope. A little experimenting should make the cross-hairs perfectly visible—if not try focusing the eyepiece while sighting directly at the lantern. (7) As no angle is to be measured, it is unnecessary to set the vernier on the limb, and either the upper or lower clamp may be used. (8) Bring the vertical hair on Polaris about 15 minutes before elongation and keep it on the moving star by means of the upper or lower tangent-screw. At eastern elongation Polaris apparently ceases to move east, but appears to climb up the vertical hair. It will soon move off the hair westward, showing that it has passed its elongation. In like manner at the western elongation for some minutes the star appears to stay on the vertical hair, but moves downward, and finally eastward. It is well to take two observations at elongation, one with the telescope normal and one with it inverted, as explained above. (9) Before depressing the telescope to establish the point on the ground make sure that all clamps are tight. To set the point bring a lantern into line by sighting across the top of the telescope and signaling with another lantern held at the transit. Set a stake in line a few inches in front of the lantern, driving it until the top is a little above the ground. Hold a pencil on the stake and a handkerchief back of the pencil but in front of the lantern. The handkerchief not only makes the pencil stand out with surprising distinctness, but it renders the cross-hairs plainly visible, and the pencil is easily brought into line and a tack set in the stake. Sometimes it is more convenient to hold a piece of white paper against the side of a fence or building, and then direct a strong light on a pencil held against the paper. A box, open on two sides, with a thin sheet of paper stretched across one of the openings, and a light behind the paper, is sometimes used. (10) Check observations may be taken at any time within an hour of elongation, by correcting the azimuth by the formula $c = kt^2$ (see p. 359).

II. AT CULMINATION. (1) The time may be taken from the table in the back of the book. It may also be known by observing when Polaris and Mizar are in the same vertical plane, and following Polaris for the proper interval afterward (4 m. 21 sec. in 1905, see (10), p. 356). Notice that in this second method, it matters not if the observer's watch is slow or fast. (2) Having sighted on Polaris at the moment of its culmination, depress the telescope and set a point as in Method I. The line from the transit to the point thus established is a true N. and S. line.

(*c*) *Note.* Since Polaris and Mizar cannot be seen within the field of the telescope, it is necessary to alternately sight at Polaris and then at Mizar to determine the moment when they are both in the same vertical plane. This is likely to be the source of a small error.

When Mizar cannot be used, as in the spring months, the star δ Cassiopeiæ is sometimes used instead. Culmination occurs about 5 m. 5. sec. (in 1905) after this star is in the same vertical plane with Polaris, the interval increasing at the rate of about 20 seconds per year.

The time from either elongation to upper culmination is about 5 hr. 55 m., while from either elongation to lower culmination it is about 6 hr. 3 m.

III. AT ANY TIME. (1) Set up the transit and sight at Polaris, noting the time of observation. The nearer to the time of elongation the more slowly the star is moving and the better will be the results. (2) Set a point several hundred feet away, as in the two preceding methods. (3) The azimuth of this line for the year, hour, and latitude may be ascertained at leisure (see suggestions below), and the angle turned off the following day to establish a true meridian, as in Method I.

(*d*) *Practical suggestions.* In this method it is necessary to know the local time of the observation to the nearest minute, and to ascertain the *azimuth of* Polaris for that time and for the latitude of the observer. This *azimuth may be found* from tables given in the "Manual of Surveying

Instruction," issued by the General Land Office,* or it may be computed. If the observation is made within, say, an hour before or after elongation the following approximate correction will give the azimuth to within a small fraction of a minute. $c=kt^2$, where c = correction *in seconds* of arc to be applied to the azimuth at elongation, t = time of observation before or after elongation *in minutes*, and k a coefficient depending on the latitude. For 30° $k=0.052$, for 40° $k=0.058$, and for 50° $k=0.069$.†

The true value of the azimuth of Polaris at *any* time may be computed by the methods explained in books on field or geodetic astronomy.

415. Methods Compared. (*a*) *At elongation.* The proper time for sighting on the star is known without depending upon the watch. There is time to make two or more observations while Polaris is moving apparently neither east nor west. Little or no calculation is involved. With the same care the error is likely to be smaller than those made in the other two methods. The only disadvantage is that both elongations may occur at inconvenient times for observations, very late at night, or in the daytime. (See April 15, in table for time of elongations.) If instrumental errors are eliminated either by careful adjustment and leveling or by repetition (modified method, p. 357), the error should never exceed one minute of arc, and usually it should be less than 30″.

(*b*) *At culmination.* The star appears to be moving rapidly, and an error in time of two minutes is equivalent to about one minute of arc, hence the observation must be taken more quickly and at the proper moment. It is more difficult to eliminate errors of adjustment by two observations. If the time of culmination is found by sighting at Mizar, it is somewhat difficult to ascertain when this star is in the same vertical plane with Polaris, as the two are not in the field of the telescope at the same time. With reasonable care, however, the error of observation at culmination should not exceed one minute of arc.

(*c*) *At any time.* The method has all the disadvantages of the preceding method, and in addition it involves considerable calculation, correct data for which are not always at hand. The method is used only when the other two are impracticable.

416. Rough Determination of the True Meridian. If a transit is not at hand, any of the preceding methods, especially the first two, may be carried out roughly as follows: Suspend a long plumb-line, and on a level support place the rear-sight of a compass or anything having a small hole in it to look through. Shift the sight to the right or left to keep it in line with the star and the plumb-line until the time of elongation or culmination, as the case may be. The line of sight thus established may be marked by stakes, but of course it will be shorter and much less accurate than if established with a transit. With care, however, a meridian can be established in this way with sufficient accuracy for the purposes of compass surveying. For example, if the plumb-line is 50 ft. from the peep-sight and an error of, say, 0.2 ft. is made in sighting and setting the stake, the corresponding angular error will be about 14 minutes of arc. (See p. 105.)

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* See Johnson's *Theory and Practice of Surveying*, p. 569.

† Ibid., p. 567.

417. To Determine the Magnetic Declination. (1) Determine a true N. and S. meridian by one of the methods of Art. 414. (2) With vernier at zero, backsight along the meridian, and measure the angle between this line and a line found by turning the upper plate until the needle points to the zero of the compass-box. This angle is the magnetic declination.

Practical suggestions. (1) If there is a declination arc, be sure that it is set at zero. (2) The chief difficulty in determining magnetic declination lies in the lack of sensitiveness of the needle. It will be found that when the needle is almost at zero, the upper plate can often be turned several minutes by using the tangent screw, with little if any effect on the needle. One way to overcome this is to observe the reading of the vernier when the needle points to N., then unclamp the upper plate, turn it several degrees in any direction, the needle having been raised off its pivot, lower the needle, quickly set the vernier at its original reading and see if the needle settles at N. If not, repeat the process, changing the setting of the vernier a little each time until the needle does settle at zero. Even then it is often difficult to determine the declination within three or four minutes. As this, however, is closer than one can read the compass, this error is usually of little importance. (3) If the *azimuth* of Polaris has been determined, it is not necessary of course, to actually stake out a N. and S. line. The declination may be measured from the line fixed by the observation and allowance made for azimuth. For example, a line is established by observation at eastern elongation, and the azimuth of this line for the date of observation is 1° 34′. Backsighting along this line with vernier at zero, and then turning the upper plate until the needle points exactly at zero, the vernier reading is, say, 12° 10′. If the declination is *west* it must be 12° 10′ − 1° 34′ = 10° 36′. If the declination is *east* it must be 12° 10′ + 1° 34′ = 13° 44′.

PART II

OFFICE WORK

INTRODUCTORY

OFFICE WORK may be divided into two parts, viz.: (1) *Computing* and (2) *Drawing*.

418. Computations are of two kinds: (1) Computations required for purposes of plotting, which include such work as *Calculation of Bearings* (Chapter XXX), *Latitudes and Departures* (Chapter XXXI), *Omitted Measurements* (Chapter XXXII), and other miscellaneous computations preparatory to plotting maps and profiles.

(2) Computations made for the purpose of obtaining data to be used in laying out work in the field, or for ascertaining desired information concerning the tract surveyed. Under this head may be classed such work as *Calculation of Areas* (Chapter XXXIII), *Parting off Land* (Chapter XXXIV), *Earthwork Computations* (Chapter XXXV), and miscellaneous work in computing dimensions and elevations.

For the most part office computations involve only the simplest principles of plane trigonometry and of geometry. Accuracy and facility in the use of logarithms is essential.

419. Drawing consists in *plotting* and in *finishing maps and profiles*, and, occasionally, in the graphic solution of problems.

(1) *Plotting* consists mainly in reversing on paper operations in the field, drawing instruments being used in place of field instruments, and measurements being laid off to a greatly reduced scale. The nature of this work is suggested by the following chapters: *Methods of Plotting Angles* (Chapter XXXVIII), *Methods of Plotting Traverses* (Chapter XXXIX), *Plotting Details* (Chapter XLI), and *Plotting Profiles* (Chapter XLIII). Many of the methods are analogous to field methods, the scale being used in place of the tape, and the protractor in place of the transit or compass.

(2) *Finishing Maps and Profiles* (Chapter XLII) consists in inking the pencil drawing, in lettering the map, in putting on conventional signs, and in miscellaneous work such as drawing border-lines and meridian needles.

CHAPTER XXIX

OFFICE COMPUTATIONS

GENERAL DIRECTIONS AND SUGGESTIONS

IN this chapter are given general directions and suggestions which apply to all kinds of office computations. The aim is to outline methods of avoiding mistakes and of economizing time.

420. Checking Computations is recognized in practice as a very essential part of office work. No important result is allowed to stand until it has been checked. Each student, therefore, should learn to check his own work as a matter of course; otherwise he will find himself severely handicapped when he enters upon actual practice. Once he understands a method, he should find his own mistakes, and make sure of his own results, before submitting work to an instructor.

Remark. Frequently students may work together with profit *when studying methods*, but the habit of comparing results at every opportunity is a real weakness which is likely to cling to one long after graduation. It is true that in practice two men often work on the same problem, but unless they work independently any agreement in results is of little value as a check.

There are several general methods of checking computations, among which are: (1) Checking arithmetical work step by step, as, for example, checking multiplication by casting out nines (p. 366). (2) Checking results by known conditions, as, for example, checking the three computed values of the interior angles of a triangle by comparing their sum with 180°. (3) Checking a result by using a different method of computation, as, for example, finding the length of the hypothenuse of a right-angled triangle by trigonometry and then checking it by squaring the other two sides. (4) Checking a result by using different data, as, for example, checking the length of a diagonal of a quadrilateral by solving each of the two triangles of *which it is* a common side. Usually one or more of these methods of checking can be used, no matter what the nature of the computations may be.

(*a*) *Approximate checks* are of great value in making sure that no large mistake has been made. Computed areas may be checked approximately by means of the planimeter (p. 408). Latitudes and departures of a traverse which does not close may be checked roughly by plotting the traverse (p. 463). Computed lengths may be checked by scaling the map. Accurate computations may be checked roughly by the use of the slide-rule. Thus many approximate methods are constantly used to check results which have been obtained by more accurate methods.

421. Systematic Methods. The main object of systematic methods in office work is to economize time. Checks are used to detect mistakes, systematic methods to avoid them. The remarks in Art. 15, p. 8, are especially true of office computations.

Remark. It is sometimes difficult to convince students that systematic methods actually *shorten* work. In the long run it will take less time to avoid mistakes than to correct them. Why not take a little trouble to avoid much trouble?

The following suggestions are offered for systematizing office computations:

(1) *A "separate-leaf" note-book* with pages about 8″×10½″ is recommended for class-work. The computations for each exercise can be handed in, checked, returned, arranged in logical order in the note-book, and preserved for future reference. Avoid the use of loose scraps of paper for computations.

(2) *Arrangement.* Adopt some economical and systematic arrangement. If the same kind of work is always arranged in the same way, blunders and omissions often become self-evident. It is usually advantageous to arrange work in columns. (See pp. 393 and 425.) The head of each column, each final result, and all other important quantities should be labeled so that every step in the computations may be followed by simply glancing through the work. At the head of the page the problems should be concisely stated, and the data given.

(3) *Order of procedure.* Having brought together all the necessary data, *check them* before beginning computations. Too frequently mistakes are made in copying data from field notes. Aim to perform at one time all operations that are alike. (See

p. 374.) Adhere to the same order of procedure for similar problems, so that much of the work will become automatic, and the chances of error will be diminished.

422. Speed in computing may be gained in several ways, viz.: (1) By adopting economic methods of procedure; (2) by using short cuts in arithmetical work; (3) by eliminating unnecessary steps and worthless refinements; (4) by the use of tables; (5) by the use of graphic methods; (6) by the use of mechanical methods.

423. Economic Methods of Procedure consist largely in grouping like operations. Frequently the entire work in a problem can be blocked out on paper in some convenient form before any numerical work is begun, so that afterward the attention may be concentrated upon the mechanical work of computation (see p. 399). This is especially true of logarithmic computations. The less the mind is compelled to turn from one thing to another the more quickly and accurately it works.

424. Short Cuts in Arithmetical Work. There are a few simple methods of shortening arithmetical work in common use that are of great value in computing. If the student is not already familiar with those given in this article, he can easily learn them, and, once he becomes accustomed to their use, he will be surprised at the amount of time they save.

(*a*) MENTAL MULTIPLICATION. The product of composite numbers may be obtained mentally by resolving them into factors and finding the product of these factors.

(1) 32×28 = 16×8×7 = 896. (2) 32 cu. yds. ×27 = 16×6×9 = 864 cu. ft.

When the multiplier is a factor of 100 multiply by 100 and divide by the proper quantity:

(3) 68×25 = 6800÷4 = 1700 (4) 98×50 = 9800÷2 = 4900

(5) 76×75 = 3800+1900 = 5700 (6) 41×150 = 4100+2050 = 6150.

Mental division is the reverse of the above.

(7) 864 cu. ft. ÷ 27 = 864 divided by 9 and again by 3 = 32 cu. yds.

(*b*) LONG MULTIPLICATION can be shortened in many cases by a similar method. It is surprising how many times two adjacent figures in the multiplier form a number which is an even multiple of another figure in the multiplier.

```
   (1)                         (2)
  2875             Circum. = 2468
   273                     0.3183
 ------                   -------
  8625                 7404
 77625                   44424
 ------                     7404
784875                   -------
                 Diam. = 785.5644
```

Illustration. In the first illustration 8625 is multiplied by 9, since 27 = 9×3. In the second illustration multiply first by the left-hand 3, and then 7404 by 6, since 18 = 6×3.

(3) Multiply 205894×497328, multiplying by only 4 digits instead of 6. Suggestion: 32 = 4×8 and 49 = 7×7.

(*c*) LONG DIVISION. There is little gained in applying the corresponding process in division except when the divisor can be factored into two divisors, each of which does not exceed 12.

Illustration. 48926 ÷ 63. The work as shown requires 16 figures, while long division by the regular method would require at least 29.

```
7|48926.
 ------
9|6989.4
 ------
  776.6
```

(*d*) ABRIDGED MULTIPLICATION may be used when the product is required only to a given number of places of figures. The method of procedure is as follows: (1) Reverse the order of the figures of the multiplier, writing them directly below those of the multiplicand. (2) Multiply in the usual way, except that in multiplying by any figure of the multiplier begin with the one in the multiplicand directly above it, mentally adding anything which should be carried from the multiplication of the preceding figure of the multiplicand by this figure of the multiplier; the result is the first right-hand figure of the corresponding partial product. The partial products are written so that their right-hand figures are in the same vertical column. The decimal point is determined by inspection.

Illustration. Let it be required to multiply 140.66 by 52.13 to one decimal place. The long method of multiplication and the short method are given side by side.

```
    140.66
     52.13
    ------
     42198
    14066
   28132
  70330
 ---------
 7332.6058
```

```
140.66
 31.25
------
 70330
  2813
   141
    42
------
7332.6
```

Notice that in multiplying by 1 the result was put down as 141 instead of 140. This was because the amount to carry (0.6) was nearer 1 than 0.

(*e*) *Note.* If a result is required to *n* places of figures, it is usually best to write the first or right-hand figure of the multiplier under the $n+1$ place of the multiplicand, i.e., carry the computation one more place than required. This is illustrated below, where the result was required to four places of figures: (Or two decimal places.)

```
      48.6678
       1.0623
      -------
      1460034
      973356
    2920068
   486678
 -----------
 51.69980394
```

```
48.6678
3260.1
-------
48668
 2920
   97
   14
-------
51.699
```

```
48.6678
3260.1
-------
4867
 292
  10
   1
-------
51.70
```

The right-hand multiplication is accurate enough for most purposes, but by carrying the work to five places, as in the middle example, the result is exact to four places. (In this case it happens to be exact to five places.)

(*f*) *Note.* Always begin by multiplying that figure in the multiplicand which stands in the first place to the right of the multiplier. Before carrying over the result, change it to the single digit which most nearly expresses it in the next left-hand place. For example, if the product is less than 5 there is nothing to carry; if it is 5 or more but less than 15 there is one to carry; if it is 15 or more but less than 25 there is two to carry, and so on. Thus in the middle example shown above when multiplying by 6 the process is $6\times 6=36$ or nearly 4 and $(6\times 6)+4=40$, giving a 0 for the first figure. In the right-hand example in multiplying by 2 the process is $2\times 8=16$ or nearly 2 and $(2\times 4)+2=10$.

(*g*) ABRIDGED DIVISION consists in deleting a figure of the divisor in each partial division after the first, instead of bringing down the next figure in the dividend.

```
3.1416)41.689348(13.27        3.1416)41.689348(13.27
       31416                         31416
       -----                         ------
       10273                         102733
        9425                          94248
       -----                         ------
         848                          84854
         628                          62832
         ---                         ------
         220                         220228
         220                         219912
         ---                         ------
                                        316
```

Illustration. Notice that in multiplying the divisor after deleting, the figure deleted is multiplied mentally and the nearest digit carried over; thus in multiplying by 3 begin with 6, the figure deleted, then $3\times 6=18=2$ (nearly) to carry, and $(3\times 1)+2=5$, the first digit in 9425. Likewise, in multiplying by 7, $7\times 4=28=3$ (nearly) and $(7\times 1)+3=0$, the first digit in 220.

Remark. If a quotient is desired to n places of figures, it is better in some cases to obtain $n\div 2$ or one half of the required figures before beginning to delete.

(*h*) RULE FOR CHECKING MULTIPLICATION. Cast out nines from (1) the multiplicand, (2) the multiplier, and (3) the product. The result in (3) should equal the product of the results of (1) and (2).

Methods of casting out nines. Casting out nines from any number is equivalent to subtracting from that number the nearest multiple of nine. This may be accomplished by subtracting from the *sum* of all the digits in the number the nearest multiple of nine, but it is somewhat easier to cross out first all 9's and all figures whose sum is 9 or a multiple of 9, and then to add the remaining figures. For example, let 4 867 428 be the given number; first cross out 4, 8, and 6, also 2 and 7, thus leaving 4 and 8; then $4+8=12$ and $12-9=3$ (or what amounts to the same thing, $1+2=3$). The same result is obtained by adding the digits, thus: $4+8+6+7+4+2+8=39$ and $39-36=3$ (or $3+9=12$, and $1+2=3$). Another method is to begin at one end to add the digits, but to keep casting out nines as the addition progresses, thus: $4+8+6=0$, and $0+7+4+2=13=1+3=4$, and $4+8=12=1+2=3$. Having once become accustomed to casting out nines the method affords a very simple and rapid check for multiplication, as illustrated below. (See explanation top of next page.)

$$\begin{array}{rlrl}
\cancel{2}8\cancel{7}5 & = 4 & 2\cancel{4}\cancel{6}\cancel{8} & = 2 \\
\cancel{2}\cancel{7}3 & = \underline{3} & 3\cancel{1}\cancel{8}3 & = \underline{6} \\
\hline
\cancel{7}\cancel{8}\cancel{4}\cancel{8}75 & = 12 = 12 & 78\cancel{5}\cancel{5}6\cancel{4}\cancel{4} & = 21 = 12 \\
 & & & 3 = 3 \\
 & & & \\
14066 & = 8 & \cancel{4}\cancel{8}\cancel{6}678 & = 3 \\
\cancel{5}2\cancel{1}\cancel{3} & = \underline{2} & 10\cancel{6}2\cancel{3} & = \underline{3} \\
\hline
\cancel{7}3\cancel{3}\cancel{2}\cancel{6}058 & = 16 = 16 & \cancel{5}\cancel{1}\cancel{6}\cancel{9}\cancel{9}\cancel{8}0\cancel{3}\cancel{9}\cancel{4} & = 0 = 9
\end{array}$$

Illustration. Taking the multiplicands, multipliers, and products from pages 364 and 365, the nines may be cast out as indicated above. Notice that 7848 = 27 = 0 in the first illustration. In the number 5 169 980 394, the three 9's were first crossed out, then 6 and 3, then 8 and 1, leaving 5 and 4 equal to 0. In this way by crossing out from inspection two or more figures whose sum is 9 or 18 or 27, the work becomes very simple.

Remark. Of course this method is not an absolute check, for it will happen occasionally that two mistakes are made which offset each other For example, one figure in the product may be 1 too large and another 1 too small; the sum would check though the product would be wrong.

If the remainder in either factor is 0, it is necessary, not to cast out nines from the other factor, but simply to note whether the remainder of the product is 0. Why?

(*i*) Rule for checking division. Cast out 9's from (1) the dividend, (2) the divisor, (3) the quotient, and (4) the remainder if there is one. Multiply the result in (2) by the result in (3) and add the result in (4). This final result should equal the result in (1). This process is, of course, the reverse of the preceding.

Illustration. Take the dividend, divisor, quotient, and remainder from p. 366:

Dividend, 41.689348 = 7
Divisor, 3.1416 = 6
Quotient, 13.27 = 4

Remainder, 316 = 1 + 24 = 7 (check)

(*j*) To square a small number. (1) Add to or subtract from the given number a quantity such that the result will be a multiple of ten. (2) Subtract from or add to the number the same quantity. (3) Multiply the result in (1) by the result in (2), and add the square of the quantity subtracted or added in (1) and (2).

$84^2 = 80 \times 88 + 16 = 7056$ $\qquad 48^2 = 50 \times 46 + 4 = 2304$

$(8\tfrac{1}{2})^2 = 8 \times 9 + (\tfrac{1}{2})^2 = 72\tfrac{1}{4}$ $\qquad (8\tfrac{1}{4})^2 = 8 \times 8\tfrac{1}{2} + (\tfrac{1}{4})^2 = 68\tfrac{1}{16}$

The above rule is particularly easy of application in squaring multiples of 5.

$(25)^2 = 20 \times 30 + 25 = 625$ $\qquad (45)^2 = 40 \times 50 + 25 = 2025$

(*k*) SQUARE ROOT OF SMALL NUMBERS. The following approximate method usually gives results which are accurate enough for most purposes: Divide the given number by that number whose square is nearest the given number and find the arithmetical mean of the quotient and the divisor. Expressed algebraically, let N = the given number, S = the nearest square, and $r=\sqrt{S}$.

$$\sqrt{N}=\left(\frac{N}{r}+r\right)\div 2$$

$$\sqrt{84}=\left(\frac{84}{9}+9\right)\div 2=(9.33+9)\div 2=9.17 \text{ (9.1652 correct)}$$

$$\sqrt{47}=\left(\frac{47}{7}+7\right)\div 2=(6.71+7)\div 2=6.86 \text{ (6.8557 correct)}$$

(*l*) SQUARE ROOT OF LARGE NUMBERS may be found in the same way provided a table of squares is also used. The above formula may be written in a more convenient form when applied to large numbers.

$$\sqrt{N}=r+[(N-S)\div(2\times r)]$$

$$\sqrt{32200}=179+[(32200-32041)+(2\times 179)]=$$
$$179.444 \text{ (179.44358 correct)}$$

$$\sqrt{175440}=419+[(175440-175561)\div(2\times 419)]=$$
$$418.856 \text{ (418.85558 correct)}$$

The value of S, the nearest square, is taken from a table of squares. Not only is this method very much shorter than the ordinary process of extracting a square root, but results are usually correct to three and often to four decimal places. Deleting the divisor (see p. 366) shortens the process still more.

(*m*) CHECKING BY SECOND DIFFERENCES. This method is useful in a variety of problems involving equations of the second degree. It is especially useful in checking ordinates, taken at *equal* intervals, to horizontal or vertical curves. In this case the method consists simply in finding the "first differences" between successive ordinates, and then finding the "second differences" between the "first differences"; the "second differences" should be equal. Due regard should be paid to algebraic signs.

Illustration. Let it be required to check the elevation given on page 292 for a vertical curve. The work below shows that the "second differences" e *equal*, and hence the elevations are probably correct.

Elevations	First Differences	Second Differences
98.6000		
	+.4125	
98.1875		+1750
	+.2375	
97.9500		+1750
	+.0625	
97.8875		+1750
	−.1125	
98.0000		+1750
	−.2875	
98.2875		+1750
	−.4625	
98.7500		+1750
	−.6375	
99.3875		+1750
	−.8125	
100.2000		

(*n*) ADDITIONAL ARITHMETICAL CHECKS. *Addition.* The usual check is to add both up and down the columns. Another check is to go over the work a second time, adding in the carried figures after the others, instead of before, or *vice versa.*

Subtraction. Add the subtrahend and the remainder, but cover up the minuend while doing so.

Multiplication. In place of the check afforded by casting out 9's a similar check, more laborious but more reliable, is that by excess of 11's. Thus in the examples on p. 367:

2875÷11=	261+	remainder of	4	
273÷11=	24+	“ “	9	
784875÷11=	71352+	“ “	3	
9×4=36 and 36÷11=3+		“ “	3	(check)
2468÷11=	224+	remainder of	4	
3183÷11=	289+	“ “	4	
7855644÷11=714149+		“ “	5	
4×4=16 and 16÷11=1+		“ “	5	(check)

When a product is not carried to the full number of places, as in abridged multiplication, neither the check by excess of 9's nor that by excess of 11's can be used. In such a case a good check is to reverse the position of the multiplier and multiplicand.

Division. Instead of casting out 9's a check by excess of 11's may be used. When the work has not been carried to the full number of places, as in abridged division, about the only check is to multiply divisor and quotient together and add in the remainder.

425. Eliminating Worthless Refinements consists chiefly in avoiding the use of unnecessary figures. Data may be given to more places of figures than there is any need of, or time may be wasted in trying to attain a higher degree of precision than the data will warrant. The aim in this article is to show that in all calculations it is desirable to make a critical inspection of the numbers which enter into every computation for the purpose of rejecting figures which cannot affect the accuracy of the final results. Not only is it a waste of time to use an unnecessary number of figures in computations, but the results thus obtained are misleading, for they appear to be more accurate than they really are. Worthless figures may be avoided by an intelligent use of the following facts: (1) Numbers expressing measurements cannot be exact, for the true value of a single measurement is never known. (2) A result cannot be more accurate than the data from which it is obtained, no matter how many places of figures are used in each step of the calculation. (3) In dealing with inexact numbers *consistent* accuracy should be the aim instead of *absolute* accuracy, and consistent accuracy is not gained by carrying all numbers to the same number of decimal places regardless of how many figures each number contains.

(*a*) *Illustration.* The length of one line measured to the nearest *foot* is 3216 ft.; a second line measured to the nearest *tenth* is 321.6 ft.; a third line measured to the nearest *hundredth* is 32.16 ft.; a fourth line measured to the nearest *thousandth* is 3.216 ft. The last figure in each number is *uncertain*, while the first three are *certain*, and the *relative precision* of each measurement is $\frac{5}{32160} = \frac{1}{6432}$. The position of the decimal point has nothing to do with the *relative precision* of a measurement, and the mistaken idea that the number of decimal places to which a result is carried indicates the accuracy of that result is responsible for a great deal of stupid and unnecessary labor.

(*b*) *The accuracy to be attained* in any computation is determined by the *number* of certain figures in the data. In order to determine the number of certain figures in any measurement some approximate expression of its accuracy must be obtained. For example, 5600 ft. may contain one, two, or three certain figures according to whether the measurement was made to the nearest 100 ft., to the nearest 10 ft., or to the nearest foot.

(*c*) *Note.* In office computations it frequently happens that one man works from field notes taken by others. In this case, unless it is evident that a measurement has been given to too many places, it may be assumed, as a rule, that each number contains but one doubtful figure. In the case of a decimal fraction the zeros used merely to indicate the position of the decimal point are not counted as certain figures. Thus, for example, 0.00043 contains only one certain figure; 0.000430 may contain two, since the cipher at the end indicates that the measurement was carried to one more place. A final zero used for this purpose should never be omitted, for it may be the only doubtful figure.

(*d*) *Remark.* The number of *significant figures* in any number *representing a measurement* is always one greater than the number of *certain figures; the last significant* figure is the first uncertain figure. It must not be assumed that because a figure is uncertain it has no significance. Its uncertainty may be very small, hence it has significance, but there can be no advantage gained in writing a number beyond the *second* doubtful figure.

(*e*) *The effect of uncertain figures* in the data upon the results of computations may be illustrated as follows: Four measurements are: 6843.2 ft., 253 ft., 282.42 ft., and 1200 ft. Each may be assumed to contain only one uncertain figure except the distance 1200 ft., which was measured to the nearest hundred feet and therefore contains three uncertain figures. It is required to add the four numbers, and also to find the product of the first two. Let the uncertain figures be indicated throughout the work by smaller type.

6843.2	6843.2	684	6843.2
253.	253	25	352
282.42	205296	28	1369
1200	342160	120	342
8578.62	136864	8570	20
	1731329.6		1731000

Since the uncertain figures in the data render so many figures worthless in the sum and product, the results 8570 and 1 731 000, obtained by abridged methods, are better than 8578.62 and 1 731 329.6, since the latter appear to be much more accurate than they really are. It is seldom necessary to give a final result beyond the first doubtful figure except when it is desired to show that the uncertainty of that figure is small. Thus, ordinarily, the preceding results may be written 8600 ft. and 1 730 000 sq. ft.

Not only is it misleading to retain more than two uncertain figures in any result, but it is a reflection upon the intelligence of the computer.

The preceding examples illustrate two general propositions:

I. *In addition or subtraction the result cannot be accurate beyond the first doubtful place in any of the numbers added or subtracted.*

II. *In multiplication or division the relative accuracy of the product or quotient cannot exceed that of the factor whose relative accuracy is least.*

(*f*) *Note.* The first of these propositions is self-evident and the second is easily proved. Let a and b be the errors of two numbers A and B. Then $\frac{a}{A}$ is the relative accuracy of A and $\frac{b}{B}$ of B. $(A + a) \times (B + b) = AB + Ba +$

$Ab+ab$. Dropping the very small quantity ab the error of this product is $Ba+Ab$ and the relative accuracy of the product is $\frac{Ba+Ab}{AB}=\frac{a}{A}+\frac{b}{B}$. Hence the relative accuracy of a product is practically equal to the sum of the relative (or fractional) accuracies of the factors, and hence less than either.

It is unwise to adopt hard and fast rules in shortening computations, but by following intelligently the suggestions given below a great deal of time may be saved without sacrificing accuracy in the results.

(*g*) SUGGESTIONS FOR SHORTENING COMPUTATIONS. *Multiplication.* (1) Before beginning a multiplication ascertain by inspection which of the factors contains the smallest number of certain figures. Call this the *limiting* factor. (2) Use the abridged method. Reverse the limiting factor for a multiplier, count off from left to right one more place in the multiplicand than there are certain places in the multiplier, and begin under this place to write the multiplier backward from right to left. *The number of certain figures in the product can never exceed the number of certain figures in the limiting number by more than one; frequently the number of certain figures in both will be the same.*

Remark. This method gives the result exactly to one uncertain place, and the error in the second uncertain place, if there is any, will rarely exceed one unit. See Art. 424 (*e*), p. 365. By observing what figures are affected by the uncertain figure in the multiplier the number of uncertain figures in the product will become evident. (See abridged multiplication, p. 371.)

Addition and subtraction. (1) The limiting number will be the one whose first uncertain figure is farthest to the left. (2) In writing down the quantities omit all figures in each number beyond the second doubtful place of the limiting number. (See abridged addition, p. 371.)

Division. Use the abridged method on p. 366, and determine by inspection the figures rendered uncertain in the divisor.

General suggestions. (1) Retain throughout the different steps of a calculation enough places of figures to correspond to two uncertain figures in the result in order that the first uncertain figure may be free from accumulated rejection errors. (2) In dropping figures, add 1 to the last figure retained when the first left-hand rejected figure is greater than 5; when it is less than 5 leave the last figure unchanged. If the rejected figure is 5, add 1 to the last figure retained if this will make it even, otherwise leave it unchanged.

(*h*) APPLICATION OF RULES. We usually know before beginning a calculation what degree of precision is required in a result, or else we desire to compute a result as accurately as the data will permit. In either case we may be guided by the rules just given. In the following illustrations of the applications of the rules it is understood, unless otherwise stated, that the last digit in a number is the only uncertain figure.

Illustration I. The lengths of two sides of a rectangular lot are given as 626.8 ft. and 73.2 ft. What is its area? There cannot be more than two certain figures in the result, since there are only two in 73.2. Hence reverse 732, and writing the 7 under the *third* figure in 626.8 obtain the result by abridged multiplication = 45 880; since the last three figures are uncertain this result may be written 45 900 sq. ft. By direct multiplication 626.8 × 73.2 = 45 881.76, the last five figures of which are uncertain, hence the result is no more accurate than before.

Illustration II. The lengths in the above illustration are given as 626.834 ft. and 73.253 ft. The data will bear inspection. Was allowance made in chaining for temperature and other sources of error? (p. 43). If not, there cannot be more than three certain figures in either measurement. Assuming this to be true, 626.8 × 73.25 = 45 912 (by abridged multiplication), or 45 910 sq. ft. Taking the measurements as given, by direct multiplication 626.834 × 73.253 = 45 917.471002; only four figures of this result can be certain, hence it may be written 45 917 sq. ft., with the last figure uncertain. This is a good illustration of the folly of carrying computations to an excessive number of places.

Illustration III. The radius of a circle is given as 21.84, what is the circumference? There can be only three certain figures in the answer, hence use 2π to four places. By abridged multiplication 6.283 × 21.84 = 137.22 ft., with the last two figures uncertain. Using π to one more place, 2 × 3.1416 × 21.84 = 137.225088 (by direct multiplication), a result with only three certain figures and no more accurate than before.

Illustration IV. The base of a right-angled triangle is given in the field notes as 428.63 ft. and the adjacent angle is 44° 24′, measured to the nearest minute. What is the length of the perpendicular side? The natural tangent for 44° 24′ is 0.97927 and for 44° 25′ it is 0.97984. Since the maximum error in the angle may be as large as 30″ the natural tangent may vary nearly 3 in the fourth place, and there can be only three certain figures. Hence drop the last figure in 428.63 and by abridged multiplication 428.6 × 0.9793 = 419.72, or since there are but three certain figures the result is 419.7. If the length is taken as it stands and a five-place tangent is used, the result by direct multiplication is 428.63 × 0.97927 = 419.7445001 ft., only three figures of which are certain, hence the result is 419.7, the same as before.

Illustration V. Suppose in the above illustration that the angle was measured to the nearest 10″. The variation in the tangent would be only $\frac{1}{6}$ × .00057 or .00009, hence the first four figures are certain and five places may be retained in the length and in the tangent or, by abridged multiplication, 428.63 × .97927 = 419.744, four figures of which are certain, hence the result is 419.74.

Illustration VI. Suppose in the above illustration that the base, measured to the nearest foot, is 428 ft. There can be only two certain figures in the result, hence use only three figures in the tangent. By abridged multiplication 428 × .979 = 419.0 ft.

(*i*) TRIGONOMETRIC TABLES. The following general rules may be given for simple computations such as ordinarily arise in office work. They hold good for logarithmic tables as well as for tables of natural functions.

For two certain figures in the result use three-place tables, for three certain figures use four-place tables, for four certain figures use five-place tables. In computations involving several steps it may be necessary to use tables to one more place in each case.

If angles are given to the nearest minute use four-place tables; if angles are known to be in error less than 30″ use five-place tables.

(j) *Remark.* Six-place tables are seldom needed in office computations except for measurements of the highest precision, such as those which are made in accurate triangulation. In direct logarithmic multiplication and division with 4-, 5-, and 6-place tables the work is about as 2 : 3 : 4, i.e., the work is doubled by using six-place tables instead of four.

426. Logarithmic Computations. The following suggestions are intended to supplement the directions commonly given for the use of logarithmic tables.

(1) A large proportion of the time required for the use of tables is spent in merely turning leaves, a fact not always appreciated. Make it a rule, therefore, when the tables are open at a given page to look up all that is to be taken from that page before turning to another. In order to do this it will be necessary, at the beginning of a calculation, to prepare a form or schedule in which a place is left for each logarithm, and for the result of each step throughout the computation. The undivided attention can then be given to looking up the logarithms, thus saving time and diminishing the chances of error.

Illustration. On page 393 the bearing, the length, and the letters of each given line were entered in the first column, and this column was entirely completed before the logarithmic tables were used at all. (See also the form for triangulation computations, p. 399.)

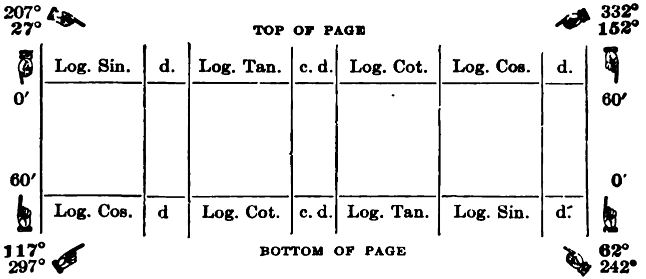
(2) Mistakes are less likely to occur if like operations are always carried out in the same way. For example, in *interpolation*, it is well to work always from the next smaller towards the next larger minute, regardless of whether this takes one up or down the page. Thus if the sine of 42° 20′ 20″ is desired, add $\frac{1}{3}$ the difference in the table to the sine of 42° 20′ rather than subtract $\frac{2}{3}$ the difference from the sine of 42° 21′, though the latter method would be correct. Likewise if the cosine of 64° 10′ 15″ is desired, subtract $\frac{1}{4}$ the difference in the tables from the cosine of 64° 10′ rather than add $\frac{3}{4}$ the difference to the cosine of 64° 11′. Of course one must always be careful to *observe whether* the tabular values of functions are increasing

or decreasing, as one glances from the next lower to the next higher minute, for this will determine whether the proportional difference is to be added or subtracted.

Remark. Form the habit of roughly checking a quantity obtained by interpolation. Thus, for example, if $\frac{1}{4}$ the tabular difference has been added to one tabular value, see if the result, as nearly as can be estimated, is $\frac{3}{4}$ the tabular difference from the next tabular value. A quick inspection of this sort is likely to reveal any serious blunder that has been made in interpolating.

Some tables give proportional parts, but if they are not given, aim to perform interpolation *mentally* without setting down figures.

(3) When an angle is more than 90° there is danger not only of ignoring the algebraic sign of a function, but of using the wrong heading or the wrong column of minutes. In some tables the degrees are indicated from 0° to 180° by giving four different values on each page, one in each corner. In that case follow the simple rule *to use the line of headings and the column of minutes which are nearest to the degree number used.*



It will pay to go through a new set of tables and to draw indexes or arrows on each page, as shown above. This simple device will often save mistakes by reminding one of the proper minute column and line of headings to use. If many calculations are to be made with angles greater than 180°, additional numbers may be marked on each page of the tables, e.g., the numbers 207°, 332°, 242°, and 297°, shown above.

(4) Another useful device is to letter a narrow strip of cardboard with headings at the top and bottom, as shown above, to fit the tables used, and to slide this strip up or down in taking quantities from the tables.

(5) When, as is often the case in trigonometric computations, any one of several methods may be used, the following suggestions will help to decide which to choose.

(*a*) *An angle is best determined from a function which changes rapidly,* i.e., *with large tabular differences.*

(*b*) *A quantity to be found from a given angle is best determined by means of a function which changes slowly.*

(*c*) *Tangents and cotangents vary more rapidly than sines and cosines, hence use the former to determine angles and the latter to determine lengths.*

Remark. The more poorly proportioned a triangle the more important these rules become.

Illustration. Let the three sides of a right-angled triangle be 20.00 ft., 140.00 ft., and 141.42 ft. respectively. Required the angles A and B. Each can be found in four ways:

$$\tan A = \cot B = \frac{20}{140} = 0.14286 = 8^\circ\ 7\frac{24'}{29} = 8^\circ\ 7'.82+$$

$$\tan B = \cot A = \frac{140}{20} = 7.00000 = 81^\circ\ 52\frac{282'}{1456} = 81^\circ\ 52'.19+$$

$$\sin A = \cos B = \frac{20}{141.42} = 0.14142 = 8^\circ\ 7\frac{23'}{29} = 8^\circ\ 7'.79+$$

$$\sin B = \cos A = \frac{140}{141.42} = 0.98996 = 81^\circ\ 52\frac{2'}{4} = 81^\circ\ 52'.5$$

The denominator in each fraction of a minute is the tabular difference for the corresponding function. This is largest for *tan B* or *cot A*, i.e., the function is changing about 0.01456 per minute of arc, and hence this is the most accurate determination of A or B. The least accurate result, and one of comparatively little value, is that obtained from sin B or cos A, the tabular difference being only 4 for one minute of arc. Of course so poorly proportioned a triangle should be avoided in practice when great accuracy is desired.

(6) The student will find it a great help to memorize a few logarithms, especially those of the whole numbers up to ten, and of those constants most frequently used. Thus, for example, he can learn, in a few minutes, the logarithms of 2, 3, 4, 5, 6, 7, 8, and 9, then he will know the logarithms of their squares, and of simple multiples, and hence the logarithms of at least a third of the numbers from 1 to 100 will be at his command.

(7) *Finally,* the student is urged to use logarithms in preference to arithmetical calculations whenever the work involves many figures. He must overcome the antipathy to the use of logarithms which seems prevalent among students, and recognize that in long multiplications, in long divisions, in squaring or cubing numbers, and in extracting square or cube roots a table of logarithms is a great labor-saving device.

427. Useful Tables. Among the tables most useful in office computations in addition to the tables of natural functions and the logarithmic tables are the following: Stadia reduction tables, tables of latitudes and departures (see p. 388), multiplication tables, such as those of Crelle, which give the products of three figures by three figures to 999×999, tables of squares and cubes, decimals of a foot for each $\frac{1}{16}$ of an inch, earthwork tables, table of chords, and tables giving squares of feet and inches to each $\frac{1}{16}$ of an inch up to 100 feet.

Many simple tables may be prepared for special use. For instance, any arithmetical calculations which occur again and again may be performed once for all and the results tabulated. Thus, for example, such constants as 43 560 (sq. ft. per acre), 27 (cu. ft. per cu. yd.), and $\pi = 3.1416$ may be multiplied by each digit from 2 to 9 once for all, thus saving time and avoiding errors in subsequent multiplications or divisions in which these quantities are involved.

428. Graphic Methods often give satisfactory results in less time than they could be obtained by computation. For example, horizontal curves may be plotted and scaled (see p. 208), gradients and vertical curves may be determined in the same way (p. 289), areas may be checked by scaling distances and by means of ruled squares (p. 408), contour points may be interpolated graphically (p. 433 and p. 498).

Diagrams may be constructed to be used in place of tables, such, for example, as those for reducing stadia readings (p. 482), for determining latitudes and departures (p. 388), and for estimating quantities in earthwork.

Remark. Excellent devices for shortening office work appear in the engineering papers from time to time, either in the form of diagrams or tables. When engaged in a special line of work, be on the alert to devise similar diagrams or tables. By exercising a little ingenuity a great deal of drudgery may be saved.

429. Mechanical Methods. Among the most common mechanical devices for supplementing office computations are the slide-rule, the planimeter (p. 408), the pantograph (p. 443), the protractor, and the computing machine. There are several forms of slide-rules designed for special purposes, such, for example, as the stadia slide-rule for reducing stadia readings. There are also various forms of planimeters, pantographs, and computing machines.

CHAPTER XXX

CALCULATION OF BEARINGS

If the bearing of any line of a survey is known and the angle which that line makes with a second line is also known, the bearing of the second line can be calculated. The method of calculation is given in this chapter.

430. Algebraic Signs of Bearings and Angles.

For convenience in computation all angles measured to the *right*, or *clockwise*, are plus; to the *left*, or *counter-clockwise*, minus. This applies not only to angles measured in the field *but also to angles of bearing*. Thus N.E. and S.W. bearings are *plus* and N.W. and S.E. bearings are *minus*.

431. The Angular Distance.

If through one end of any line a north and south line is established, four angles are formed, two of which are measured from N. (one clockwise and one counter-clockwise) and the other two from S. (one clockwise and one counter-clockwise). Each of these angles may be called the *angular distance* of the given line. In giving the angular distance of any line, it is necessary, therefore, to state whether it is *plus* or *minus* and whether it is measured from N. or from S.

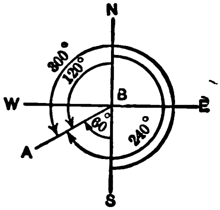


FIG. 431.

Illustration. The bearing of a line, *BA*, is S. 60° W. The four angular distances are:
(1) 240° *plus* from N.
(2) 120° *minus* from N.
(3) 60° *plus* from S.
(4) 300° *minus* from S.

Note. Notice that when *any* angular distance is 90° or *less*, it is identical with *bearing*.

432. To Convert Angular Distance into Bearing.

(1) Notice whether the angular distance is measured from N. or from S. and whether clockwise or counter-clockwise. (2) Imagine the angle turned off from N. or from S. as the case may be and from the number of quadrants passed over, determine the *quadrant* of bearing (i.e., whether it is N.E., S.E., S.W., or N.W.), and then the *angle* of bearing.

(*a*) *Illustration.* The angular distance of a line from N. is 160° *plus.* Since this is more than 90° clockwise from N., but less than 180°, the line must be in the S.E. quadrant, 20° from S., i.e., the bearing is S. 20° E.

(*b*) *Illustration.* The angular distance of a line from S. is 220° *minus.* Since this is more than 180° counter-clockwise from S., but less than 270°, the line must be in the N.W. quadrant, 40° from N., i.e., the bearing is N. 40° W.

(*c*) *Illustration.* The angular distance of a line from N. is 280° *plus.* This is more than 270° (or three quadrants) clockwise from N., hence the line is in the N.W. quadrant, i.e., bearing is N. 80° W.

Note. It is important that the student should be able to convert angular distance into bearing or *vice versa* without puzzling over it, and without relying upon rules. Form a clear mental picture of the four quadrants of bearing, the starting-point N. or S. as the case may be, the direction plus or minus, the number of quadrants passed over, and the final location of the line. The mental process is a simple one, and once the idea is grasped, it ceases to be a matter of memory. Practice on the following illustration, checking the columns of bearings until sure of the method.

ANG. DIST.	BEARING	ANG. DIST.	BEARING
N. 60°+	N. 60° E.	S. 20°+	S. 20° W.
N. 170°+	S. 10° E.	S. 130°+	N. 50° W.
N. 80°−	N. 80° W.	S. 280°−	S. 80° W.
N. 260°+	S. 80° W.	N. 310°+	N. 50° W.
N. 300°−	N. 60° E.	S. 260°+	N. 80° E.

(*d*) *To convert azimuth into bearing.* This is simply a special case of the method just explained, for the *azimuth* of a line is the same as its *angular* distance measured *plus* from N.

433. Calculating Bearings. The bearing of one line is known, the angle which that line makes with a second line is also known, required the bearing of the second line.

(1) *Set down the given BEARING followed by the corresponding sign* (Art. 430).

Note. Every line has two bearings (Art. 155, p. 112). Be sure to use the one corresponding to that end of the line at which the given angle was measured,—change the given letters if necessary. See illustration at the end of this article.

(2) *Set down the given ANGLE followed by the corresponding sign* (Art. 430).

(3) *Add* (1) *and* (2) *ALGEBRAICALLY; put the first letter of the given bearing in front of the result and the proper algebraic sign after it.*

Note. The result in (3) is an *angular distance;* the letter in front tells whether it is measured from N. or from S., and the sign gives the direction,—clockwise if plus, counter-clockwise if minus.

(4) *Convert the angular distance in* (3) *into bearing* (Art. 432).

(*a*) *Illustration.* The bearing of a line *BA* is N. 60° E.; angle *ABC* = 80° clockwise; required the bearing of *BC*.

BA = N. 60° E. +
ABC = 80° +
Ang. Dist. of *BC* = N. 140° +
Bearing of *BC* = S. 40° E.

The student should draw a figure showing the four compass-points and the lines *AB* and *BC*.

(*b*) *Illustration.* The bearing of a line *BA* is N. 60° W.; angle *ABC* = 80° clockwise; required the bearing of *BC*.

BA = N. 60° W. −
ABC = 80° +
Ang. Dist. of *BC* = N. 20° +
Bearing of *BC* = N. 20° E.

Since one sign is plus and the other minus, the algebraic sum is really one angle subtracted from the other. Show by a figure that this is correct.

In each of the two preceding illustrations the given angle was measured clockwise. The method holds good for angles measured counter-clockwise, or to the left.

(*c*) *Illustration.* The bearing of a line *BC* = S. 40° E.; angle *CBA* = 80° counter-clockwise; required the bearing of *AB*.

BC = S. 40° E. −
CBA = 80° −
Ang. Dist. of *BA* = S. 120° −
Bearing of *BA* = N. 60° E.

By the same figure used in connection with the first illustration the student can see that the process here indicated is correct.

(*d*) *An important distinction.* The bearing taken at one end of a line should never be used in connection with an angle taken at the other end without first changing the letters. The importance of this distinction is shown in the following illustration. The bearing *AB* is given as S.E., but the given angle was measured at *B*, hence the bearing of *BA* must be used, and this is N.W., not S.E. (See note at the top of p. 113.)

INCORRECT	CORRECT
AB = S. 60° E. −	*BA* = N. 60° W. −
ABC = 105° +	*ABC* = 105° +
Ang. Dist. of *BC* = S. 45° +	Ang. Dist. of *BC* = N. 45° +
Bearing of *BC* = S. 45° W. Incorrect	Bearing of *BC* = N. 45° E. Correct

It is seen that if the incorrect bearing S.E. is used to start with, the result makes the point *C* appear to be S.W. of *B* when it really is N.E. of *B*.

(*e*) *Check.* By arranging the last two letters of any angle directly under the two letters of the preceding line, as shown above, the following check may be used: *When the middle letter of an angle is not the same as the letter directly above it the bearing of the preceding line must be changed, otherwise it may be left as it is.* (Why?) Thus, in the first column, *B*, the middle letter of *ABC*, is under *A*, hence *AB* (S.E.) is changed to *BA* (N.W.) in the second column. (See also the note, Art. 434 (*a*), p. 382.)

(*f*) *Remark.* The above method holds good no matter in which quadrant of bearing the given line may lie, or whether the given angle is measured to the left or to the right. The method is a very simple one, and the somewhat extended explanation on the preceding pages is justified only by the fact that for some reason many students find difficulty in calculating bearings, especially in the field. At first, in order to be sure of the correctness of the method in any given case, it may be necessary to make a sketch, drawing the given line in its proper quadrant of bearing, and then roughly plotting the given angle. It is far better, however, to do all this mentally. Until the student can *see* in a mental picture the reason for each step of the method as he takes it, he cannot consider himself proficient in calculating bearings. To this end, practice on the examples given below, checking the bearings in the last column. The calculation *is purposely* omitted in some cases. Notice in each case whether the *given bearing is forward* bearing or *back* bearing.

Examples in Calculating Bearings.
R = Angle to the right; *L* = Angle to the left.

Given Bearings	Angle	Bearing Used	Calculation	Angular Distance	Required Bearing
AB: N 30° E	*ABC*: 50° *R*	*BA*: S 30° W	+30 + 50	S 80° +	*BC*: S 80° W
AB: N 30° W	*ABC*: 50° *R*	*BA*: S 30° E	−30 + 50	S 20° +	*BC*: S 20° W
BC: S 50° E	*BCD*: 30° *R*	*CB*: N 50° W	−50 + 30	N 20° −	*CD*: N 20° W
BA: N 30° W	*ABC*: 50° *R*	*BA*: N 30° W	−30 + 50	N 20° +	*BC*: N 20° E
AB: N 30° E	*ABC*: 120° *R*	*BA*: S 30° W	—	S 150° +	*BC*: N 30° W
CD: N 30° E	*DCB*: 200° *R*	—	—	N 230° +	*CB*: S 50° W
CD: N 30° E	*CDB*: 200° *R*	*DC*: S 30° W	—	S 230° +	*DB*: N 50° E
BC: S 80° W	*BCD*: 300° *R*	*CB*: N 80° E	—	N 380° +	*CD*: N 20° E
BC: S 80° E	*BCD*: 300° *R*	*CB*: N 80° W	—	N 220° +	*CD*: S 40° W
AB: N 50° W	*ABC*: 320° *R*	—	—	—	*BC*: N 90° E
BC: N 80° W	*BCD*: 40° *R*	—	—	—	*CD*: S 40° E
BA: N 60° E	*ABC*: 150° *R*	—	—	—	*BC*: S 30° W
AB: N 30° E	*ABC*: 50° *L*	*BA*: S 30° W	+30 − 50	S 20° −	*BC*: S 20° E
AB: N 30° W	*ABC*: 50° *L*	*BA*: S 30° E	−30 − 50	S 80° −	*BC*: S 80° E
AB: N 60° E	*ABC*: 200° *L*	*BA*: S 60° W	+60 − 200	S 140° −	*BC*: N 40° E
AB: S 70° E	*ABC*: 300° *L*	—	—	—	*BC*: N 10° W
BC: S 60° W	*BCD*: 90° *L*	—	—	—	*CD*: N 30° W
BA: N 80° W	*ABC*: 200° *R*	—	—	—	*BC*: S 60° E
AB: S 70° W	*ABC*: 200° *L*	—	—	—	*BC*: S 50° W
AB: S 70° W	*ABC*: 200° *R*	—	—	—	*BC*: S 90° W

434. Application of the Method to a System of Transit Lines. When in any survey the bearing of each transit line is calculated from the bearing of the preceding transit line, the *back* bearing of the preceding line must be used, for reasons given in the preceding article.

This is illustrated by Fig. 434 and the corresponding computations given below. Thus if the bearing of *AB* is S. 30° E. this is changed to N. 30° W. in order to find the bearing of *BC* (N. 60° E.). This bearing of *BC* must then be changed to S. 60° W. in order to find the bearing of *CD*, and so on. In the computation these changes are indicated by letters in parenthesis, and *the letter placed before the angular distance will be the first letter in the preceding parenthesis.* In this way the computations can be kept in a continuous form as shown here, and as still better illustrated on p. 391.

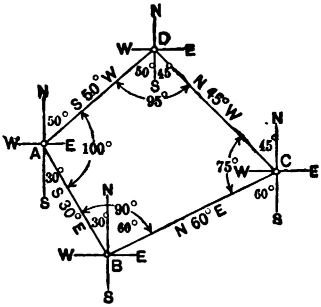


Fig. 434.

It is seen from these two sets of computations that it is immaterial in which direction angles are taken provided proper account is kept of algebraic signs.

ANGLE TO RIGHT	ANGLE TO LEFT
$AB=$ S. 30° E. $-$(N.W.)	$AB=$ S. 30° E. $-$
$ABC=$ 90° $+$	$BAD=$ 100° $-$
$BC=$ N. 60° E. $+$(S.W.)	$AD=$ S. 130° $-$
$BCD=$ 75° $+$	$AD=$ N. 50° E. $+$(S.W.)
$CD=$ S. 135° $+$	$ADC=$ 95° $-$
$CD=$ N. 45° W. $-$(S.E.)	$DC=$ S. 45° E. $-$(N.W.)
$CDA=$ 95° $+$	$DCB=$ 75° $-$
$DA=$ S. 50° W. $+$(N.E.)	$CB=$ N. 120° $-$
$DAB=$ 100° $+$	$CB=$ S. 60° W. $+$(N.E.)
$AB=$ N. 150° $+$	$CBA=$ 90° $-$
$AB=$ S. 30° E. (check)	$BA=$ N 30° W. $-$ (check)

(*a*) *Note.* Apply, by inspection, the check of Art. 433 (*e*), p. 380. In the second column A in the angle BAD is *like* the letter directly above it, and hence the bearing of AB is left unchanged, i.e., S.E. On the other hand the middle letter of every other angle is *unlike* the letter above it, and hence the bearing preceding each of the other angles is changed as indicated by the letters in the parentheses.

(*b*) *Calculating bearings from deflection angles.* Precisely the same method holds good; except that the *forward* bearing of the preceding line is used each time instead of the *back* bearing. (See Art. 189 (*a*), p. 127, for an illustration.) Prove that this method is correct.

(*c*) *To change the bearings of all lines of a traverse by the same amount.* This is also a special case. Give the proper sign to the angle by which each bearing is to be changed, according to whether the lines are to be swung clockwise or counter-clockwise. Add this angle algebraically to each bearing.

Example. (1) In Fig. 434 change the bearing of AB to S. 50° E. and find the corresponding bearings of the other three lines, changing each by the same amount. (2) Change BC to N. 30° E. and find the corresponding bearings of the other three lines.

435. To Calculate the Angle between Two Lines when their Bearings are Known.

This is simply the reverse of the process already given.

Method. (1) Make sure that the two bearings are both from the same station, i.e., if necessary reverse the letters.

(2) Choose either line as a backsight and convert the bearing of the other line into *angular distance*, measured from the

same point (N. or S.) from which the bearing of the backsight is measured.

(3) Give to the bearing of the backsight and to the angular distance of the other line each its proper algebraic sign, and subtract ALGEBRAICALLY the former from the latter, i.e., the bearing from the angular distance.

(4) The result is the required angle measured clockwise or counter-clockwise *from the assumed backsight* according to whether the sign is plus or minus.

If the student thoroughly understands the method given on the preceding pages he will have no difficulty in proving the correctness of the method given above.

Illustration. Calculations of the angles in Fig. 434 from the given bearings. Given: $AD = \text{N. } 50° \text{ E.}$ and $AB = \text{S. } 30° \text{ E.}$ to find DAB. This can be done four different ways,—two assuming AD as a backsight, and two assuming AB as a backsight.

AD assumed as a backsight.

$$\begin{array}{rllcrll}
AB = & \text{N. } 150° & + & \text{or} & AB = & \text{N. } 210° & - \\
AD = & \text{N. } 50° \text{ E.} & + & \text{or} & AD = & \text{N. } 50° \text{ E.} & + \\
\cline{2-2} \cline{6-6}
DAB = & 100° & + & \text{or} & DAB = & 260° & -
\end{array}$$

AB assumed as a backsight.

$$\begin{array}{rllcrll}
AD = & \text{S. } 130° & - & \text{or} & AD = & \text{S. } 230° & + \\
AB = & \text{S. } 30° \text{ E.} & - & \text{or} & AB = & \text{S. } 30° \text{ E.} & - \\
\cline{2-2} \cline{6-6}
BAD = & 100° & - & \text{or} & BAD = & 260° & +
\end{array}$$

In the above illustration the algebraic sign of each result indicates in which direction, clockwise or counter-clockwise, the angle is measured from the assumed backsight. Find the angles at B, C, and D in a similar manner.

CHAPTER XXXI

LATITUDES AND DEPARTURES

THE theory of latitudes and departures explained in this chapter is as simple as it is important. Latitudes and departures are used in plotting maps (see Chapter XLI); also in calculating omitted measurements (Chapter XXXII) and areas (Chapter XXXIII).

436. Latitude and Departure are terms for two co-ordinate distances, one (*the latitude*) measured parallel to a *north and south* line, the other (*the departure*) measured parallel to an *east and west* line.

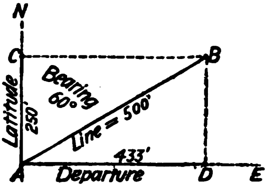


FIG. 436.

Illustration. In Fig. 436 a line AB is in the N.E. quadrant. $AC=DB$ is its *latitude* and $AD=CB$ is its *departure.* The angle CAB is the *bearing* of the line.

Latitude difference and *longitude difference.* Terms preferred by some writers to the old and established terms latitude and departure.

437. Latitude and Departure is Found from the Bearing. It is evident from Fig. 436 that

Length of the Line × Cosine Bearing = Latitude of the Line.

Length of the Line × Sine Bearing = Departure of the Line.

Illustration. A line AB in Fig. 436 is 500 feet long. Its bearing is N. 60° E. The natural cosine of 60° = 0.500; the natural sine of 60° = 0.866.

Length × Cos. Bear. = Latitude.	Length × Sin. Bear. = Departure.
500 ft. × 0.500 = 250 ft.	500 ft. × 0.866 = 433 ft.

Note. For other trigonometric relations, see p. 400.

438. Algebraic Signs of Latitudes and Departures are determined by the quadrants of bearing. *Latitudes* are *plus* when the bearing is *north, minus* when the bearing is *south.* Departures are *plus* when the bearing is *east, minus* when the bearing is *west.* Thus the first letter of a bearing (N. or S.) determines the sign of the latitude, and the second letter of the bearing (E. or W.) determines the sign of the departure.

Illustration. In Fig. 438 a line has been assumed in each of the four quadrants of bearing, thus illustrating all possible cases. The algebraic signs are shown on the figure and indicated also in the following table:

Line	Bearing	Latitude N.(+)	Latitude S.(−)	Departure E.(+)	Departure W.(−)
AB	N.E.	*AC*		*AD*	
FG	S.E.		*FI*	*FH*	
JK	S.W.		*JM*		*JL*
OP	N.W.	*OQ*			*OR*

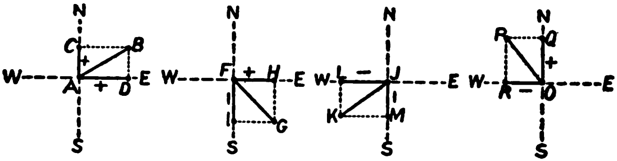


FIG. 438.

Remark. These algebraic signs correspond to those in common use for ordinate and abscissa, i.e., *above* the axis of reference *plus*; *below,—minus*; to the *right* of the axis *plus*; to the *left,—minus*.

Illustration. The calculation of latitudes and departures according to the method of Art. 438 is indicated below.

Line × Cos. Bearing = Latitude.	N.(+)	S.(−)	Line × Sin. Bearing = Departure.	E.(+)	W.(−)
AB × Cos. *BAN*.	*AC*		*AB* × Sin. *BAN*.	*AD*	
FG × Cos. *GFS*.		*FI*	*FG* × Sin. *GFS*.	*FH*	
JK × Cos. *KJS*.		*JM*	*JK* × Sin. *KJS*.		*JL*
OP × Cos. *PON*.	*OQ*		*OP* × Sin. *PON*.		*OR*

Remark. It is evident that the *departure* of any *north and south* line is zero, and that the *latitude* of any *east and west* line is zero.

439. The Latitudes and Departures of a System of Transit Lines may be used for determining the position of any station with respect to any other station. The *algebraic* sum of the latitudes of all the lines between any two stations gives the distance that one of those stations is north or south of the other; and the *algebraic* sum of the departures gives the distance one station is east or west of the other.

Illustration. In Fig. 439 let it be required to find the position of *J* with respect to *F*. Suppose that the latitudes and departures have been found by calculation to be the quantities indicated on the figure. Tabulate the quantities thus:

Line	Bear.	Latitude +(N.)	Latitude −(S.)	Departure +(E.)	Departure −(W.)
FG	S.E.		65	80	
GH	N.E.	40		97	
HI	N W.	54			66
IJ	S.E.		48	127	
		+94	−113	+304	−66
			+ 94	− 66	
			− 19	+238	

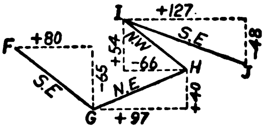


FIG. 439.

It is seen that *J* is 19 ft. south of *F* and 238 ft. east of *F*. It is evident that the position of any point with respect to any other point can be determined in like manner. Thus:

I is $+97-66=31$ ft. east of *G* and $+40+54=94$ ft. north of *G*.

Remark. This principle is made use of in plotting (Chapter XLI) and in the calculation of omitted measurements and of areas (Chapters XXXII and XXXIII); also in parting off land (Chapter XXXIV).

440. In a Closed System of Transit Lines the algebraic sum of the latitudes should be zero and the algebraic sum of the departures should be zero.

Remark. This simply means that if a man walks around a closed system of transit lines returning to the starting-point he will have walked as far north as he walked south, and as far east as he walked west, i.e., *northings* and *southings* are equal and *eastings* and *westings* are equal.

441. The Error of Closure of a Survey (see p. 119) **may be** ascertained by finding the difference between the north latitudes and the south latitudes and the difference between the east departures and the west departures.

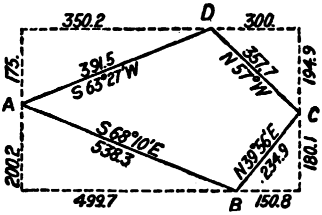


FIG. 441.

Illustration. In Fig. 441 is shown a closed system of transit lines. Suppose the latitude and departure of each line have been found from calculation to be equal in feet to the numbers indicated in the figure. They may be tabulated as follows:

Line	Length	Bearing	N. Lat.(+)	S. Lat.(−)	E. Dept.(+)	W. Dept.(−)
AB	538.3	S.–E.		−200.2′	499.7′	
BC	234.9	N.–E.	180.1′		150.8′	
CD	357.7	N.–W.	194.9′			−300.0′
DA	391.5	S.–W.		−175.0′		−350.2′
	1522.4		+375.0	−375.2′ +375.0′	+650.5	−650.2′ +650.5′
			Error =	−0.2 S.	and	+0.3 E.

$$\text{Error of closure} = \sqrt{(0.2)^2 + (0.3)^2} = \sqrt{0.13} = 0.36 \text{ S.E.}$$

Since the total distance around the polygon is 1522.4, the error of closure is $\frac{0.36}{1522.4}$ or about $\frac{1}{4230}$. This is the *apparent* error of the field work. The *real* error in field work may have been much greater, since the apparent error includes compensating errors. For example, a north latitude which is too large may be offset by a south latitude which is also too large, or a west departure which is too small may be offset by an east departure which is also too small.

The direction of the error is indicated by the algebraic signs. Thus in this example the error in latitude (-0.2) and the error in departure ($+0.3$) indicates error south and east, i.e., 0.36 southeast.

Permissible error of closure. (See p. 160.)

442. Each Line has Two Bearings,—Which is Used? In computing latitudes and departures one may go around the polygon in either direction, but the bearings used must all be *forward* bearings *for the direction chosen.*

Illustration. If in Fig. 441 one goes around the polygon from A to B to C to D to A, the bearings used are those noted on the figure because they are *all* forward bearings for the direction chosen. But if one goes around in the opposite direction (A to D to C to B to A), the bearings should be changed so that they *all* will be *forward* bearings for that direction, i.e., AD = N. 63° 27′ E., DC = S. 57° E., CB = S. 39° 56′ W., and BA = N. 68° 10′ W. The principal thing to avoid, therefore, is the careless use of a back bearing in place of a forward bearing. For example, suppose that in Fig. 441 the bearing of CD had been put down S. 57° E. instead of N. 57° W., the other bearings remaining as they are in the figure. In the tabulated form the latitude of CD would then be entered in the south column and its departure in the east column, and of course the survey would not close.

443. To Compute Latitudes and Departures, Using Azimuths. It is not necessary to convert azimuths into bearings. If the azimuths are measured clockwise from the north point as zero, and the usual trigonometric rules for signs are observed, the signs of the trigonometric functions of any azimuth will agree with the signs of the corresponding latitudes and departures as given on p. 385. Thus the N.E. quadrant will correspond to the first quadrant in trigonometry, the S.E. quadrant the second, S.W. the third, and N.W. the fourth. This is the principal reason for reckoning azimuths from N. instead of from S. (see p. 114, Art. 160 (*b*)).

Illustration. In Fig. 441 the azimuth of DA would be 243° 27′. The cosine and sine of 243° 27′ are both minus, hence the latitude and departure of DA are both minus, or south and west respectively. The student can easily satisfy himself in regard to the signs in the other quadrants in a similar manner.

444. Different Methods of Finding Latitudes and Departures. (*a*) *By computation.* The length of each line may be multiplied by the *natural* cosine and sine of its bearing, but usually it is quicker to work with logarithmic tables. Systematic forms of computation facilitate the work and diminish chances of error. A complete example, showing a form for logarithmic work and a method of tabulating results, is given on pages 393 and 394.

(*b*) *Traverse tables* are simply tables of natural sines and cosines from 0° to 90° multiplied by 1, 2, 3, etc., to 10, and in some tables to 100. In the common forms, values are given for every quarter degree, but such tables are good only for rough compass surveys and work of a similar nature. Tables such as those of Boileau giving values for every minute of arc, are exceedingly useful. In finding the latitude and departure of any length expressed by several figures, the latitude and departure for each digit is read from the table (using the given bearing), and brought to the correct value by moving the decimal point. The several quantities are then added.

Illustration. The following line of values is taken from a traverse table. Only half a line is shown; the other half is usually given on the opposite page.

	1		2		3		4		5	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
30°	0.866	0.500	1.732	1.000	2.598	1.500	3.464	2.000	4.330	2.500

Suppose that a line is N. 30° E. and its length is 4 ft.; then its latitude is 3.464 and its departure is 2.000. If the line is 40 ft. instead of 4 ft., the two values are 34.64 and 20 ft.; if 400 ft. long the values are 346.4 and 200.0 ft., and so on.

Let it be required to find the latitudes and departures of the two lines *AB* and *CD*, whose lengths and bearings are given below.

AB		*CD*	
L'gth = 542.3 ft.,	Bearing = N. 30° W.	L'gth = 503.1 ft.,	Bearing = S. 30° W
Latitude	Departure	Latitude	Departure
433.0	250.0	433.0	250.0
34.64	20.0	00.0	00.0
1.732	1.0	2.598	1.5
.26	.15	.086	.05
469.63 N.	271.15 W.	435.68 S.	251.55 W.

Remark. When a zero occurs in a number, instead of inserting the zeros, as shown above, move the next number over two places. A natural mistake which might be made in the second illustration is that of putting down 0.8 instead of 0.08 and 0.5 instead of 0.05 for the latitudes and departures, respectively, of the digit 1 in the quantity 503.1.

(*c*) *Trigonometer.* A device for finding graphically the latitude and departure of any line. A diagram is constructed on regular cross-section paper, or squares may be ruled to any desired scale. From the lower left-hand corner of the diagram as an origin, distances are numbered along the horizontal and vertical edges. A revolving arm, pivoted at this origin, has a straight-edge, which if produced would pass through the origin. This straight-edge is graduated to correspond to the scale of the cross-section paper. The upper and right-hand edges of the diagram are marked to correspond to degrees and fractions of degrees, with the origin as a center. By means of this angular graduation the revolving straight-edge is set to the bearing of the given line, the length of the line is noted on the straight-edge, and the corresponding latitudes and departures are read from the cross-section scale. If azimuths are used *instead of bearings* the numbering of angles should be continued up to 360°. This instrument is useful in rough determinations, such for example as those needed in plotting to a small scale, but of course the degree of precision which can be attained is far less than that obtained by computation.

445. A Complete Example in the Computation of Latitudes and Departures. The computations on pages 390 to 394 illustrate every step in finding the co-ordinates for plotting by the method of latitudes and departures.

(1) A rough sketch of the transit lines was drawn (p. 390); angles and distances taken from the field notes were copied on this sketch and checked by again comparing with the field notes.

(2) Angles were adjusted so that the sum of the interior angles in each polygon would be correct. (See p. 119.) The error was not distributed equally in this case, as there were certain checks on some of the angles, while on others there were no checks. Where adjusted values differed from the original values the latter were crossed out and the adjusted values written in red ink.

(3) The field notes gave the angle between Polaris at its eastern elongation and the line YV as 63° 25′ (see p. 390), hence the true bearing of YV was 63° 25′ + 1° 37′ = N. 65° 02′ E.

(4) Starting with YV = N. 65° 02′ E. the bearings of all the other lines were calculated *from the adjusted values of the angles.* The form shown on p. 391 is not quite so compact as that given on p. 382, and there are other minor changes, but the method is the same as that explained in the preceding chapter. The student is advised to follow closely the form on p. 382 instead of that on p. 391.

Remark. In many surveys no observations are taken to determine the true meridian. In that case the magnetic bearing of any line which is thought to be correct may be assumed to start with, or this magnetic bearing may be changed to a true bearing if the declination is known. So far as plotting is concerned, it makes no difference what value is taken for the bearing of the first line; an absolutely fictitious bearing may be used, provided the bearings of all other lines are computed from the traverse angles. The *relative* positions of all points on the map will be correct, and this in many cases is all that is required.

(5) Another rough sketch was made of the transit lines (see p. 392), and the length and the calculated bearing marked along each line.

(6) Latitudes and departures were computed by logarithms (see p. 393). The data from the sketch on p. 392 were entered in the first column as shown, then all the log. sines and cosines were looked up, then all the logarithms of the lengths, then the additions were performed and finally the latitudes and departures were ascertained from corresponding logarithms. In this way like operations were grouped to save time. (See p. 374 for other suggestions for the use of logarithms.)

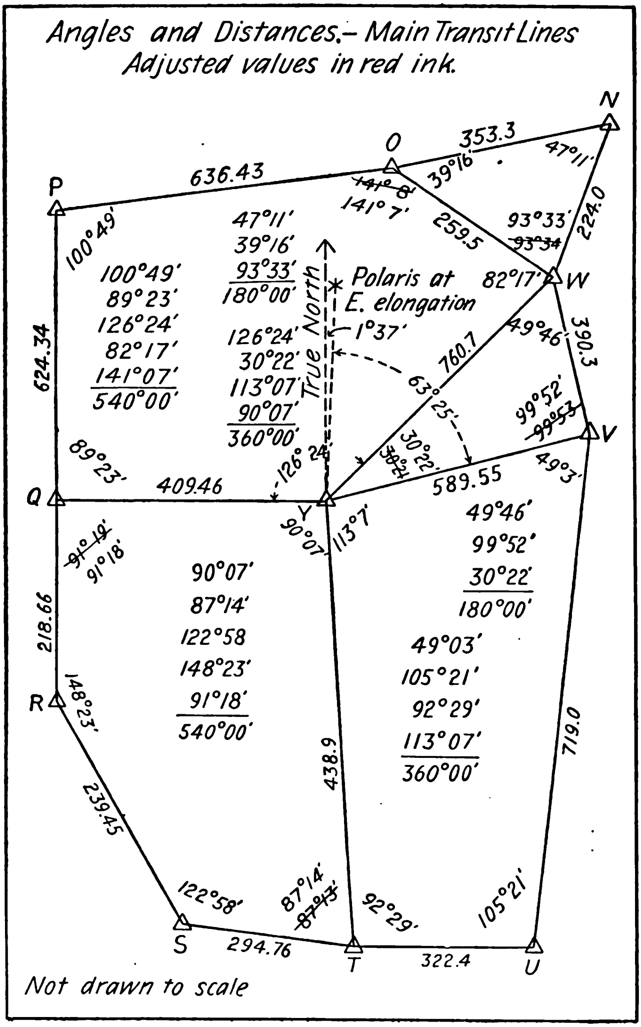
(7) The results obtained in (6) were tabulated in the first seven columns on p. 394. The errors are found to be 0.02 and 0.14 ft. for latitudes and departures respectively.

(8) The last step was to compute and tabulate the last three columns (p. 394) for purposes of plotting as explained on p. 472.

Checks. If the errors in (7) are found to be very large, apply two checks: (1) Compare each latitude and each departure with the first and last letters, respectively, of the bearing of the corresponding line to see if each has been entered in the proper column, N. or S. and E. or W.

(2) See if a latitude and a departure have not been interchanged, the latitude being put in a departure column and *vice versa.* To do this notice if the bearing of each course is greater or less than 45°; if it is greater, the departure should be greater than the latitude, otherwise it should be less. (See an article by Mr. Gould, p. 397.)

(3) If no mistakes are discovered in (1) and (2), make sure that the data were copied correctly and go over the computations again. One can often tell from inspection which latitude and departure is most likely to be incorrect, and therefore which computation to check first.



Angles and Distances.– Main Transit Lines
Adjusted values in red ink.

Calculations for Bearings

Outside Lines

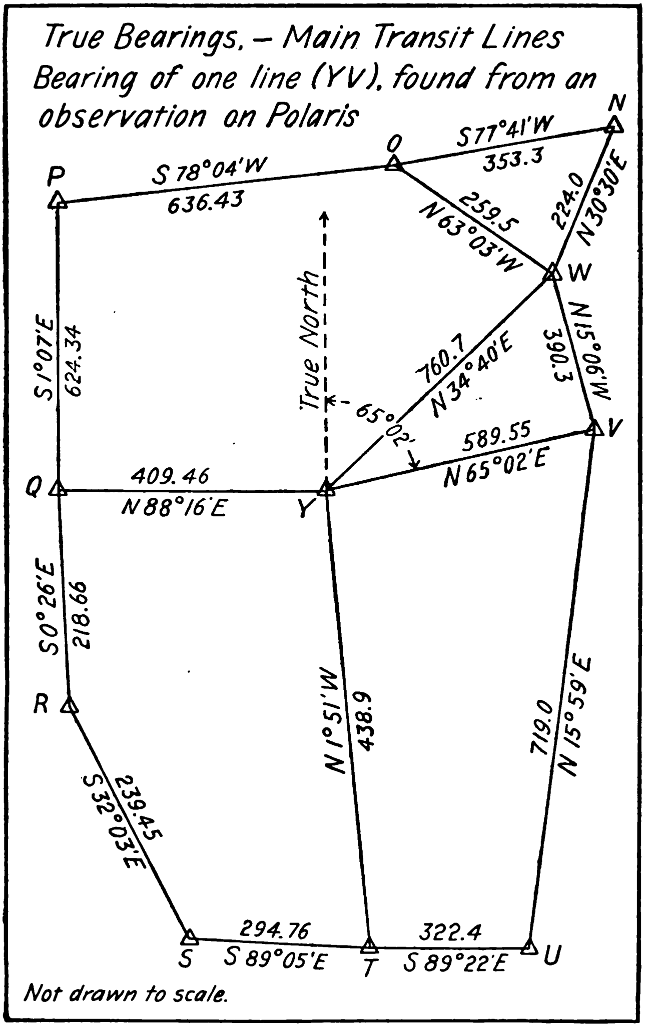
Elongation 1°37'
Polaris 63°25'
YV = N 65°02'E + (S.W.)
YVW = 99°52' +
164°54' +
180° 0' −
VW = N 15°06'W − (S.E.)
VWN = 225°36' +
210°30' +
180° 0' −
WN = N 30°30'E + (S.W.)
WNO = 47°11' +
NO = S 77°41'W + (N.E.)
NOP = 180°23' +
258°04' +
180° 0' −
OP = S 78°04'W + (N.E.)
OPQ = 100°49' +
178°53' +
180° 0' −
PQ = S 1°07'E − (N.W.)
PQR = 180°41' +
179°34' +
180° 0' −
QR = S 0°26'E − (N.W.)
QRS = 148°23' +
147°57' +
180° 0' −
RS = S 32°03'E −

Outside Lines

SR = N 32°03'W −
RST = 122°58' +
90°55' +
180° 0' −
ST = S 89°05'E − (N.W.)
STU = 179°43' +
90°38' +
180° 0' −
TU = S 89°22'E − (N.W.)
TUV = 105°21' +
UV = N 15°59'E + (S.W.)
UVW = 148°55' +
164°54' +
180° 0' −
VW = N 15°06'W −
(Check)

Inside Lines

TS = N 89°05'W −
STY = 87°14' +
TY = N 1°51'W − (S.E.)
TYV = 113°07' −
114°58' −
180° 0' +
YV = N 65°02'E +
VYW = 30°22' −
YW = N 34°40'E +
WYQ = 126°24' −
91°44' −
180° 0' +
YQ = S 88°16'W +


True Bearings. – Main Transit Lines
Bearing of one line (YV), found from an observation on Polaris
N
O
S 77°41'W
353.3
P
S 78°04'W
636.43
259.5
N 63°03'W
224.0
N 30°30'E
W
True North
S 1°07'E
624.34
760.7
N 34°40'E
390.3
N 15°06'W
65°02'
589.55
N 65°02'E
V
Q
409.46
N 88°16'E
Y
S 0°26'E
218.66
R
N 1°51'W
438.9
719.0
N 15°59'E
239.45
S 32°03'E
294.76
S 89°05'E
S
T
322.4
S 89°22'E
U
Not drawn to scale.

Calculation of Latitude and Departures.

Quantity	Departure	Log. Sin	Log. Cos	Latitude
S 1°07′E		8.28977	9.99992	
624.34		2.79542	2.79542	
PQ =	12.17	1.08519	2.79534	624.23
S 0°26′E		7.87870	9.99999	
218.66		2.33977	2.33977	
QR =	1.65	0.21847	2.33976	218.65
S 32°03′E		9.72482	9.92818	
239.45		2.37922	2.37922	
RS =	127.07	2.10404	2.30740	202.95
S 89°05′E		9.99994	8.20407	
294.76		2.46946	2.46946	
ST =	294.72	2.46940	0.67353	4.72
S 89°22′E		9.99997	8.04350	
322.4		2.50840	2.50840	
TU =	322.38	2.50837	0.55190	3.56
N 15°59′E		9.43990	9.98288	
719.00		2.85673	2.85673	
UV =	197.98	2.29663	2.83961	691.22
N 15°06′W		9.41582	9.98474	
390.3		2.59140	2.59140	
VW =	101.68	2.00722	2.57614	376.82
N 30°30′E		9.70547	9.93532	
224.0		2.35025	2.35025	
WN =	113.69	2.05572	2.28557	193.00
S 77°41′W		9.98989	9.32902	
353.3		2.54814	2.54814	
NO =	345.17	2.53803	1.87716	75.36
S 78°04′W		9.99051	9.31549	
636.43		2.80375	2.80375	
OP =	622.67	2.79426	2.11924	131.59

Tabulation of Latitudes and Departures for Plotting

Line	Bearing	Length	Latitude		Departure		Sta.	Total from P	
			North	South	East	West		Latitude	Departure
PQ	S 1°07'E	624.34		624.23	12.17		Q	− 624.2	12.2
QR	S 0°26'E	218.66		218.65	1.65		R	− 842.9	13.8
RS	S 32°03'E	239.45		202.95	127.07		S	−1045.8	140.9
ST	S 89°05'E	294.76		4.72	294.72		T	−1050.5	435.6
TU	S 89°22'E	322.4		3.56	322.38		U	−1054.1	758.0
UV	N 15°59'E	719.0	691.22		197.98		V	− 362.9	956.0
VW	N 15°06'W	390.3	376.82			101.68	W	+ 13.9	854.3
WN	N 30°30'E	224.0	193.00		113.69		N	+ 206.9	968.0
NO	S 77°41'W	353.3		75.36		345.17	O	+ 131.6	622.8
OP	S 78°04'W	636.43		131.59		622.67	P	0.0	0.1
			1261.04	1261.06	1069.66	1069.52			
			Error = 0.02		Error = 0.14				

Line	Bearing	Length	North	South	East	West	Sta.	Latitude	Departure
PQ							Q	−624.2	12.2
QY	N 88°16'E	409.46	12.39		409.3		Y	−611.8	421.5

Note: Plotting co-ordinates are from P, (most westerly station) as the origin.

446. Balancing a Survey means making such corrections that the sum of the north latitudes shall equal the sum of the south latitudes, and the sum of the east departures shall equal the sum of the west departures. This applies, of course, only when the survey forms a closed polygon. In such a case there is practically always some error of closure, but it does not follow that this error must always be corrected. Thus, for example, on page 394, the errors of 0.02 ft. in latitude and 0.14 ft. in departure may be entirely ignored so far as plotting is concerned. This is usually the case, for if the error of closure is large enough to be appreciable in plotting it usually exceeds the permissible error, and measurements have to be corrected in the field. Hence it is seldom necessary to balance a survey for purposes of plotting. When, on the other hand, the data are to be used for other purposes, as, for example, in describing boundaries of land, it may be necessary to distribute the errors so that the polygon described shall be a closed figure.

447. Two Methods of Balancing a Survey. (1) When all measurements have been made with the same degree of precision, the errors may be distributed among the sides of the polygon in proportion to their respective lengths. (2) When some measurements have been made with a greater degree of precision than others, the less accurate measurements should be given the larger corrections.

(*a*) FIRST METHOD. The following proportions are used:

$$\frac{E_l}{P}=\frac{\text{Total error in latitude}}{\text{Total length of perimeter}}=\frac{\text{Error in latitude of one side}}{\text{Length of that side}}=\frac{e_l}{s}.$$

$$\frac{E_d}{P}=\frac{\text{Total error in departure}}{\text{Total length of perimeter}}=\frac{\text{Error in departure of one side}}{\text{Length of that side}}=\frac{e_d}{s}.$$

$$e_l=\frac{E_l}{P}\times s \quad \text{and} \quad e_d=\frac{E_d}{P}\times s.$$

It is not worth while to split hairs—the corrections can usually be made mentally. Thus approximate values are found for $\frac{E_l}{P}$ and $\frac{E_d}{P}$ once for all and these multiplied by the lengths of the different sides in succession. Corrections should be subtracted from numbers in a column where the sum is too large and added to numbers in a column where the sum is too small.

Illustration. Let it be required to distribute the errors of -0.2 in latitude and $+0.3$ in departure on p. 386.

$$\frac{E_l}{P}=\frac{0.2}{1522.4}=\frac{1}{7500} \qquad \frac{Ed}{P}=\frac{0.3}{1522.4}=\frac{1}{5000}$$

AB	$538\times\frac{1}{7500}=0.07$	$538\times\frac{1}{5000}=0.11$
BC	234× " =0.03	234× " =0.04
CD	357× " =0.05	357× " =0.07
DA	391× " =0.05	391× " =0.08
	0.20	0.30

The sums of the corrections equal 0.2 and 0.3 respectively. In this example (p. 386) the sum of the south latitudes exceeds the sum of the north latitudes, hence subtract 0.07 and 0.05 from 200.2 and 175 respectively, but add 0.03 and 0.05 to 180.1 and 194.9 respectively. The west departures exceed the east departures, hence subtract 0.11 and 0.04 from 499.7 and 150.8 respectively, but add 0.07 and 0.08 to 300.0 and 350.2 respectively. The corrected latitudes and departures may be written in the table in red ink directly over the original corresponding values.

Note. Correcting the latitude and departure of a line is equivalent to changing the two short legs of a right triangle, hence, to be strictly consistent, the line itself (or hypothenuse) should be changed to correspond, i.e., a new bearing and a new length should be found which correspond to the corrected latitude and departure. This is often a needless refinement, but when it is necessary it can be done easily by simply reversing the trigonometric method used in finding latitudes and departures. (See also p. 400.)

(*b*) *Another method* is given by Mr. Llano (*Engineering News*, Nov. 23, 1899, and Jan. 18, 1900). Let S = the corrected length of any side, L = uncorrected latitude, D = uncorrected departure, and s, E_l, Ed, and P same as on p. 395.

$$S=s-\frac{E_l}{P}L-\frac{E_d}{P}D \text{ (formula for correcting lengths).}$$

Let B = the uncorrected bearing of any side and Cb the correction in minutes to be applied to this bearing; then

$$C_b=3400\left(\frac{E_l}{P}\sin B-\frac{E_d}{P}\cos B\right) \text{ (formula for correcting bearings).}$$

The quantities $\frac{E_l}{P}$ and $\frac{E_d}{P}$ having already been worked out, the extra work involved is small. Use natural functions in the last formula.

(*c*) SECOND METHOD OF BALANCING A SURVEY. This consists in distributing the errors where it is most likely that errors in field work were made. On the whole this method is often preferable to the preceding method. In using it the following points should be kept in mind.

(1) Measurements made under the most unfavorable conditions may be given the greatest corrections. Thus, for example, linear measurements on steep slopes, or over other ground unfavorable for chaining, are more likely to be too long than if made under favorable conditions.

(2) All linear measurements are likely to be too long pro- *d the length* of the tape is not longer than it should be,

hence in making corrections it is better, as a rule, to decrease distances than to increase them, i.e., it is better to subtract from the latitudes and departures in the columns whose sums are the greater than to add to latitudes and departures in columns whose sums are the smaller.

(3) A slight change in the bearing of a long line will affect the latitudes and departures more than the same change in a short line.

(4) If a line is nearly north and south, a slight change in its bearing will change its departure without greatly affecting its latitude, while a slight change in its length will change its latitude without greatly affecting its departure. Likewise, if a line is nearly east and west a change in its bearing affects its latitude, while a change in its length affects its departure.

(*d*) In an article by Mr. E. Sherman Gould (*Engineering News*, Jan. 4, 1900) some excellent suggestions are given as follows:

> The error of closure must be due either to a mistake in calculating the traverse or to an error in field work.
>
> To detect it, the traverse must be first examined. We assume that the courses have been called back so that there is no error in copying, also that the taking out of sines and cosines and multiplications have been checked, so that there is no error there. We also assume that before the columns were footed up care was taken to see that all entries were made in the proper columns. In this connection it may also be stated that it will be found conducive to accuracy to head the columns of Latitudes "N" and "S", and those of Departures "E" and "W", rather than + and −. It is then very easy to see if any mistakes have been made in putting the latitudes and departures in the wrong column. To see if they have not been interchanged, that is, latitude placed in the departure column, or *vice versa*, observe if the bearing of each course has more or less than 45° of departure. If it has more the departure is greater than the latitude; if it has less the latitude is the greater. Thus, suppose the bearing to be N. 36° 24′ E. and the distance 463 ft. The sine and cosine of this angle are, respectively, 0.59342 and 0.80489. Multiplying both by the distance gives 274.753 and 372.664. A glance shows that the latter must be the latitude and the former the departure.
>
> If all this is correct, then without a shadow of a doubt the error, if a considerable one, is in the chaining. If numbered stakes have been driven every hundred feet, and leveled over as in the case of a railroad line, there will be no mistake in the full stations, because the leveling party would detect and report it. The chief source of error is in the pluses, and these errors are commonly made, if at all, by reading the chain backward, for instance, taking 64 ft. for 36. The distances used in the traverse must be then carefully examined to see if such a change, or any change in fact, in one or more distances would reduce the error to limits of normal inaccuracy. By diligent scrutiny such an error is sure to be detected by an expert eye, and then a couple of trustworthy chainmen are sent out to verify the suspected measurement, not informing them, of course, of what the error is suspected to be. In almost all cases a resurvey can be avoided by an intelligent guess at the point where the mistake was probably made. Undoubtedly a great deal of unnecessary work is frequently gone through with in such cases merely for the lack of a little reflection.
>
> All actual blunders or "busts" being thus eliminated, there will still remain some discrepancies between the Northings and the Southings and Eastings and Westings, unless by pure chance they should happen to

balance. . . . The practice of the old land surveyors, when given a survey to locate from a description, was to first work out a traverse for it, which as a matter of course never closed, and then retain all the courses and distances intact till the last one was reached. A closing course and distance was then calculated and the area of the survey determined accordingly. This, I believe, is the legal way of treating a survey which does not close, unless some metes and bounds are cited which conflict. It is, however, awkward when the last course is a course of an adjoining tract, particularly if of an earlier date.

I believe the best way to balance such a survey as we have been considering, that is, one in which there can be found no absolute mistake, and only a slight discrepancy remains to be eliminated, is to go very carefully over the traverse and see to what changes of distance it will respond most sensitively. For instance, supposing that in a large survey of several hundred acres the North latitudes exceeded the South by a few feet, and the East departures were less than the West by a somewhat smaller discrepancy. In the first place we would look to see if this difference was about in the right proportion; that is to say, if the total latitudes were greater than the total departures, then the difference between the Northings and Southings should be greater than between Eastings and Westings, and the fact of their being so would give us additional confidence that the errors were simply due to ordinary small inaccuracies, although this evidence would not be conclusive either way. We would then see what courses, if any, ran nearly North and South. It is clear that by lengthening and shortening these by a small amount we would make a great impression on the latitudes. And the same in reference to the departures. On the other hand, the change of a single minute in the bearing of a long course running nearly North and South will make a great change in the departure. In a word, by an intelligent study of the traverse all minor discrepancies can be eliminated with a small number of changes when placed where they do the most good, and this method will prove the most satisfactory in practice.

CHAPTER XXXII

CALCULATION OF OMITTED MEASUREMENTS

When all the angles and sides of a polygon are known except two, those two can be found by calculation. In that case, however, it is necessary to assume the measurements actually taken in the field as correct,—there can be no check on the field work, nor can the survey be balanced. Hence the importance of taking closing angles and distances notwithstanding the fact that they can be calculated if omitted. There are times, however, when because of some oversight or because of obstacles to field work it becomes necessary to compute omitted measurements. Methods for doing this are outlined in this chapter.

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448. Omitted Parts of a Triangle. The most general case is that of the oblique triangle. If any three parts, including at least one side, are known the remaining three may be found by trigonometrical methods, which need not be explained here. The case, however, in which one side and the two adjacent angles are known arises so often in triangulation that suggestions are given below for the order and arrangement of the work.

(*a*) *Computations for triangulation.* (1) Find the most probable value of the base-line and adjust the angles in each triangle of the triangulation net so that the sum will be 180°. (2) Make out a form similar to one of the two given below, leaving spaces for logarithms and results. (3) When the form is completed, and not before, look up the logarithms. (4) Using the computed length of a side as a base, solve each of the triangles of the triangulation net in a similar manner.

(*b*) *Illustration.* Given $a = 416.32$ ft., $A = 64° 18'$, $B = 58° 40'$, and $C = 57° 02'$.

To find b	To find c	Check
$b = \frac{a \sin B}{\sin A}$	$c = \frac{a \sin C}{\sin A}$	$b = \frac{c \sin B}{\sin C}$
$\log a =$	$\log a =$	$\log c =$
$\log \sin B =$	$\log \sin C =$	$\log \sin B =$
colog $\sin A =$	colog $\sin A =$	colog $\sin C =$
$\log b =$	$\log c =$	$\log b =$
$b =$	$c =$	

A check which is longer but more sure than that given above is to find b from the formula $\frac{b-a}{b+a}=\frac{\tan\frac{1}{2}(B-A)}{\tan\frac{1}{2}(B+A)}$. This check is quite certain to reveal mistakes if any have been made.

(*c*) *Remark.* In triangulation, one side of the triangle farthest from the base may be measured, and an agreement between its measured and its calculated length is a check on the calculated sides of all the triangles in between.

(*d*) The following form is more compact than that given on page 399.

(1) $\log \sin C =$
(2) $\log a =$
(3) $\operatorname{colog} \sin A =$
(4) $\log \sin B =$

Add the logarithms in the order indicated to get results (5) and (7).

(5) $\log c =$ (1) + (2) + (3)
(6) $\operatorname{colog} \sin C =$

(7) $\log b =$ (2) + (3) + (4) = (4) + (5) + (6) (check).

(*e*) *Remark.* Various formulas for the solution of oblique triangles are given in the back of this book. While the use of these formulas usually shortens the work of computation, attention is called to the fact that any oblique triangle can be divided into two right-angled triangles by dropping a perpendicular from one vertex to the opposite side and the required parts found by solving these two right-angled triangles. Whatever solution is used, prepare a form before looking up the logarithms.

449. Trigonometric Relations of Bearing, Length, Latitude, and Departure. The *latitude* and *departure* of any line form two sides of a right-angled triangle in which the *line itself* is the hypothenuse. One angle in this triangle is the bearing of the line, i.e., the angle between the hypothenuse and the latitude. Any two parts of this triangle, including at least one side, being known the other two can be found by trigonometry.

	GIVEN	REQUIRED	FORMULA
(*a*)	*Bearing and Length*	*Latitude*	= *Length* × *Cos. Bearing*
(*b*)	*Bearing and Length*	*Departure*	= *Length* × *Sin. Bearing*
(*c*)	*Latitude and Dep.*	*Tan. Bear.*	= *Depart.* ÷ *Latitude*
(*d*)	*Latitude and Bearing*	*Length*	= *Lat.* ÷ *Cos. Bearing*
(*e*)	*Departure and Bear.*	*Length*	= *Dep.* ÷ *Sin. Bearing*
(*f*)	*Latitude and Length*	*Cos. Bear.*	= *Lat.* ÷ *Length*
(*g*)	*Depart. and Length*	*Sin. Bear.*	= *Dep.* ÷ *Length*
(*h*)	*Lat. and Departure*	*Length* = $\sqrt{(\textit{Latitude})^2+(\textit{Departure})^2}$	

450. Omitted Measurements. (*a*) There are no omitted measurements in any polygon when the bearing and length of each side is known. When the exterior or interior angles are given in place of the bearings, and the bearing of one side is known (or assumed for purposes of calculation) the bearings of all the other sides can easily be found by calculation (Chapter XXX). When not more than two bearings, or two lengths or one length and one bearing, are omitted the missing parts may be calculated. This applies of course only to closed polygons. *When there are three* or more missing parts they cannot be calculated. Missing parts are understood to mean bearings or lengths or both.

There are four general cases, as follows:

I. (*a*) When the *bearing* of *one* side is omitted.

(*b*) When the *length* of *one* side is omitted.

(*c*) When the *bearing* and the *length* of *one* side are omitted.

II. When the *bearing* of *one* side and the *length* of *another* side are omitted.

III. When the *lengths* of *two* sides are omitted.

IV. When the *bearings* of *two* sides are omitted.

Remark. When two unknown sides are parallel, Case III is indeterminate unless the area is known. Cases II and IV are also indeterminate unless it is known which of two angles that correspond to a given sine or cosine is the right one to use.

Notice that in Case I only one side of the polygon is affected. This is the simplest and most common case, and the solution of each of the other three cases involves the general method used in Case I. This method consists simply in computing and tabulating the latitudes and departures by the method of the preceding chapter just as if no side were missing. Of course, the survey will not balance, the error of closure (p. 386) being very large, i.e., equal to the required side. Hence the discrepancy between the sums of the north and south latitudes will be the latitude of the missing side, and the discrepancy between the sums of the east and west departures will be its departure. Two sides of a right triangle being known its hypothenuse (the required side) is easily found.

451. Methods of Calculation for each of the four cases are outlined in succeeding articles.

Caution. Before beginning calculations make sure that the given bearings are either all forward bearings or all back bearings.

(*a*) CASE I

WHEN THE BEARING OF ONE SIDE OR THE LENGTH OF ONE SIDE IS OMITTED, OR WHEN BOTH ARE OMITTED

(1) Calculate and tabulate the latitudes and departures of the given sides (Chapter XXXI).

(2) Find the difference of the sums of the north and south latitude, and the difference of the sums of the east and west departures; these two differences are the latitude and departure respectively of the omitted side.

(3) From the latitude and departure thus found calculate the omitted bearing or the omitted length, or both.

Remark. The choice of several trigonometric formulas may be had. Thus, formulas (*c*) and (*d*) or (*c*) and (*e*) (p. 400) may be used first, and then formula (*h*) furnishes a check.

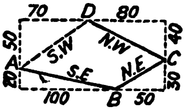


FIG. 451 (*b*).

Question. What determines the *quadrant* of the omitted bearing thus found, i.e., N.E., S.E., S.W., or N.W.?

(*b*) *Illustration.* In Fig. 451 (*b*) suppose that the bearing and the length of *AD* are omitted in the field notes and that the latitudes and departures of the other three sides have been calculated and tabulated as follows:

LINE	N. LAT.	S. LAT.	E. DEP.	W. DEP.
AB		20	100	
BC	30		50	
CD	40			80
	70	20	150	80

$$\text{Tan Bearing of } AD = \frac{\text{Dep.}}{\text{Lat.}} = \frac{70}{50} = 1.4 = \text{Tan } 54° \; 30', \text{ hence } DA = \text{S. } 54° 30' \text{ W.}$$

$$\text{Length of } AD \quad = \frac{\text{Lat.}}{\text{Cos } B} = \frac{50}{0.5807} = 86.0 \text{ ft. or} \sqrt{50^2 + 70^2} = 86.0 \text{ ft. (check).}$$

(*c*) *Illustration.* In Fig. 451 (*c*), let the broken lines represent boundary lines, the lengths and bearings of which were not taken in the field. The transit lines *A–B–C–D* were measured, the interior angles were taken, and each corner of the boundary-line was located by the angle and distance. No bearings were recorded. Required to find the lengths and bearings of the boundaries (broken lines). This problem simply involves four applications of the method of Case I. Assume a bearing for any line as *AB* and calculate in the usual way the bearings of the eight lines shown by full lines. Then each boundary-line is an omitted side of a four-sided polygon, and its length and bearing can be calculated. The bearings of the four boundary-lines having been calculated, the angles *EFG*, *FGH*, *GHE*, and *HEF* are easily found. (Chapter XXX.) If the lengths of the boundary-lines had been measured in the field they would have furnished an important check on the calculation of the bearings of the boundary-lines.

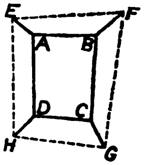


FIG. 451 (*c*).

Remark. This problem could be done by the method explained on p. 406, and one method could be used to check the other.

(*d*) GENERAL METHOD FOR CASES II, III, AND IV

When two sides are affected by omitted measurements. Let the sides *AB* and *BC* be affected by omitted measurements. Leaving these two sides out altogether, the polygon may be completed by a *closing line*, *AC*, the bearing and length of which can be calculated by the method of Case I. This having been done, enough parts of the triangle *ABC* will be known to give the omitted parts by the usual trigonometric methods. Cases II, III, and IV can all be solved in this way. It is immaterial whether the deficient sides adjoin or not. *This will become* evident in any given case if in omitting the two

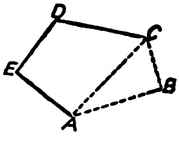


FIG. 451 (*d*).

deficient sides the other sides are shifted, each parallel to itself until they form a connected series of lines.

Illustration. Let the sides C and E in Fig. 451 (*e*) be affected by omitted measurements. In Fig. 451 (*f*) D has been moved parallel to itself to the end of B. Omitting C and E the closing line is L. The length and bearings of the three sides A, B, and D remain unchanged, and hence the line L must form a triangle with C and E as indicated by broken lines in Fig. 451 (*f*).

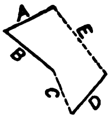


FIG. 451 (*e*).

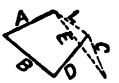


FIG. 451 (*f*).

(*e*) CASE II

WHEN THE BEARING OF ONE SIDE AND THE LENGTH OF ANOTHER SIDE ARE OMITTED

(1) Find the length and bearing of a closing line. (Case I.)

(2) Find the angle between the closing line and the side of omitted length but known bearing, using the bearing found in (1) and the known bearing. (Art. 435, p. 382.)

(3) Given two sides of a triangle and an angle opposite one of them to find a side and either of two angles. (Formula in the back of the book.)

(4) From the angles found in (3) calculate the omitted bearing.

Remark. Two answers are possible in this case. (See Remark, p. 401.) It is usually evident in any given case, however, which result corresponds to the line of the survey.

(*f*) *Another method for Case II* is to assume for purposes of calculation the side of unknown length to be due north and south, the bearing of the other sides being changed to correspond. This eliminates one unknown quantity, i.e., the departure of the side assumed as north and south becomes zero; the other unknown quantities can then be found by latitudes and departures. It is immaterial whether the deficient sides adjoin or not.

(1) Assume the side of unknown length to be a north and south line, and calculate the bearings of the other side.

If the bearings of the sides are already known, the bearing of the side of unknown length can be changed temporarily by an amount (clockwise or counter-clockwise) required to make it N. and S. The bearings of the other sides should then be changed by the same amount and in the same direction (clockwise or counter-clockwise). (See Art. 434 (*c*), p. 382.)

(2) Compute and tabulate the latitudes and departures of all sides except the two affected by omitted measurements. Of these two the side of unknown length has no departure since it was assumed N. and S., hence the departure of the other side

must be equal to the difference of the sums of the E. and W. departures of the known sides.

(3) From the departure of the unknown side (found in (2)) and the length of that side, calculate its bearing (Art. 449 (*g*)) and its latitude (Art. 449 (*a*) or Art. 449 (*h*)).

(4) Insert in the table of (2) the latitude found in (3) and find a *new* difference between the sums of the N. and S. latitudes. The difference is the latitude of the side assumed as N. and S., and since its departure is zero, the length and latitude of this side are identical.

(*g*) CASE III

WHEN THE LENGTHS OF TWO SIDES ARE OMITTED

(1) Find the length and bearing of a closing line. (Case I.)

(2) Find the angle between the closing line and each of the deficient sides, using the bearing found in (1) and each of the two known bearings. (Art. 435, p. 382.)

(3) Given one side and two adjacent angles of a triangle to find the other two sides and the remaining angle. (Formula in the back of the book.)

Remark. When the two unknown sides are parallel this case is indeterminate unless the area is known.

(*h*) *Another method for Case III* is to assume either side of unknown length to be a north and south line and then to proceed as in the second method of Case II. (Art. 450 (*f*).) The only difference is that in (3) the *length* is required and the *bearing* is known instead of *vice versa.*

(*i*) CASE IV

WHEN THE BEARINGS OF TWO SIDES ARE UNKNOWN

(1) Find the length and bearing of a closing line. (Case I.)

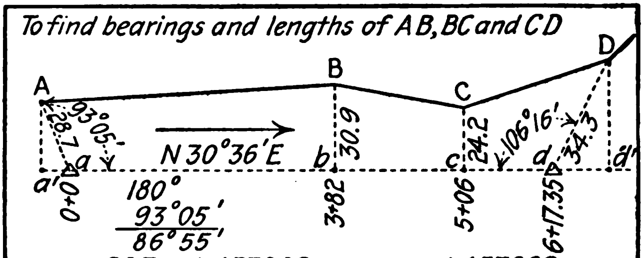
(2) Given the lengths of three sides of a triangle to find the three angles. (Formula in the back of the book.)

(3) From the angles found in (2) and the bearing of the closing line find the two omitted bearings. (Chapter XXX.)

Remark. As in Case II there will be two answers possible.

452. Simple Trigonometrical Methods of computing omitted measurements are frequently used instead of those already explained. An example of this sort is shown on the opposite page, and explained in detail on p. 406.

To find bearings and lengths of AB, BC and CD



180°
93°05'
86°55'

28.7 = 1.457882
Cos 86°55' = 8.730688
+1.54 = 0.188570
382.00
AB = 383.54
−0°20'
N 30°36'E
AB = N 30°16'E

1.457882
Sin 9.999371
28.66 = 1.457253
30.90 Offset at A
2.24 = 0.350248
383.54 = 2.583810
−0°20' = 7.766438 (Tan)

(AB = Lat. 331.26 Dep. = 193.30)

6.7 = 0.826075
124.0 = 2.093422
+3°05'34" = 8.732653
N 30°36'00E (Tan)
N 33°41'34"E = BC = 124.18

= 0.826075
= 8.732028 (Sin)
2.094047

(BC = Lat. 103.31 Dep. = 68.88)

34.3 = 1.535294
Cos 73°44' = 9.447326
+9.60 = 0.982620
111.35
120.95
8.73 = 0.941014
120.95 = 2.082606
−4°08' = 8.858408
N 30°36'E (Tan)
N 26°28'E = CD = 121.27

= 1.535294
Sin = 9.982257
32.93 = 1.517551
24.20 Offset at D
8.73
= 0.941014
= 8.857280 (Sin)
2.083734

(CD = Lat. 108.54 Dep. 54.04)

FIG. 452.

(*a*) *Illustration.* Given the measurements shown in the sketch to find the lengths and bearings of AB, BC, and CD.

(1) $Aa \times \cos Aaa' = aa'$ and $Aa \times \sin Aaa' = Aa'$.
$28.7 \times \cos 86° 55' = 1.54$ ∴ $28.7 \times \sin 86° 55' = 28.66$.
$a'a + ab = a'b$ ∴ $Bb - Aa =$
$1.54 + 382 = 383.54$ $30.9 - 28.66 = 2.24$.
$(Bb - Aa) \div a'b =$ tan of angle which AB makes with $a'b$
$2.24 \div 383.54 =$ tan of $0° 20'$.
N. 30° 36′ E. − 0° 20′ = N. 30° 16′ E. = bearing of AB.
Length of AB is practically the same as length of $a'b$ in this case.
Traverse tables give for AB: Lat. = 331.26, Dep. = 193.30.

The computations for BC and CD are similar except that the length of each is found by dividing the difference in the offsets at each end by the sine of the angle which the line makes with $a'd'$, thus:

$BC = (Bb - Cc) \div$ sine of angle which BC makes with $a'd'$.
$124.18 = 6.7 \div \text{sine } 3° 05' 34''$.

After the computations were completed the latitudes and departures of Dd, da, aA were looked up in a traverse table as a check.

	N.	S.	E.	W.
AB	331.26		193.30	
BC	103.31		68.88	
CD	108.54		54.04	
Dd		25.03	23.45	
da		531.38		314.26
aA	13.26			25.45
	556.37	556.41	339.67	339.71
	+.04		+.04	

It is seen that the polygon closes within a reasonable limit of error.

Question. Assuming that angles are measured to minutes and distances to the nearest tenth, does the use of six-place tables in this example give better results than could be obtained by five-place tables? Can the bearing of BC be obtained to the nearest second as implied in the answer 33° 41′ 34″? (See p. 373.)

CHAPTER XXXIII

CALCULATION OF AREAS

In this chapter are explained the methods of obtaining areas from field notes and from maps, including the geometrical and trigonometrical methods, the method of latitudes and departures, the method of coördinates and the mechanical (planimeter) method.

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453. The Area of Land in plane surveying means the area as projected on a *horizontal* plane. (See Art. 5, p. 2.)

454. Units of Area. Areas in surveying are usually given in square feet or in acres: 1 acre = 43 560 sq. ft. = 10 sq. chains = 10×66^2 sq. ft. (See p. 3.)

Sq. Miles	Acres	Sq. Rods	Sq. Yards	Sq. Feet
1	640	102 400	3 097 600	27 878 400
	1	160	4 840	43 560
		1	30.25	272.25
			1	9

1 Sq. Ft. = 0.0929 Sq. Meters   1 Sq. Meter = 10.7639 Sq. Ft.
1 Sq. Yd. = 0.8361 Sq. Meters   1 Sq. Meter = 1.1960 Sq. Yds.
1 Acre = 0.4047 Hectares   1 Hectare = 2.4710 Acres
1 Sq. Mile = 2.5899 Sq. Kilometers   1 Sq. Kilometer = 0.3861 Sq. Miles

In Spanish measure 1 acre = 5650 sq. varas, more or less. (See page 3.)

455. Geometrical and Trigonometrical Principles. The following formulas are for areas:

GEOMETRICAL

1. *Triangle* = base × half perpendicular height.
2. *Parallelogram* = base × perpendicular height.
3. *Trapezoid* = half the sum of the parallel sides × perpendicular height.
4. *Trapezium*, found by dividing into two triangles.
5. *Regular polygon* = length of perimeter × half perpendicular distance from center to sides.
6. *Circle* = diameter squared × 0.7854 = diameter squared × $\frac{1}{4}\pi$.
7. *Sector of circle* = length of arc × half radius.
8. *Segment of circle* = area of sector minus area of triangle.
9. *Ellipse* = major axis × minor axis × 0.7854.
10. *Parabola* = base × $\frac{2}{3}$ height or $\frac{2}{3}$ area of circumscribed rectangle.

TRIGONOMETRICAL

11. *Right triangles.*

Given	Area
Hypothenuse and acute angle	$(c^2 \sin 2B) \div 4$
Angle and opposite side	$(b^2 \cot B) \div 2$
Angle and adjacent side	$(b^2 \tan A) \div 2$
Hypothenuse and another side	$[a\sqrt{(c+a)(c-a)}] \div 2$
Two sides about right angle	$ab \div 2$

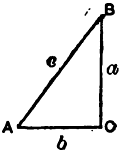

FIG. 455.

12. *Oblique triangle* (three sides given). Area $=\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the sides and $s=(a+b+c)\div 2$.

13. *Oblique triangle* (two sides and included angle given). Area $=\frac{1}{2}ab \sin C$.

14. *Area of segment of a circle* $=\frac{R}{2}(L-C\times\cos\frac{1}{2}I)$, where R = radius of curve, C = length of chord, and I = intersection angle (see p. 204), and L = length of arc.

Approximate formulas for area of segments: $A=\frac{2}{3}MC=\frac{C^3}{12R}$, where M = middle ordinate $=\frac{C^2}{8R}$ (approximately).

15. *Area between tangents and curve* $=R\left(T-\frac{L}{2}\right)$, where R = radius, L = length of arc, and T = length of tangents (see p. 204).

456. General Methods of Finding Areas. (1) From a map by scaling and computing, or by the use of the planimeter. (2) By computations made directly from the field notes without the use of a map. The first method is generally the quicker, the second method the more accurate.

457. Obtaining the Area from a Map. (*a*) The area is subdivided into geometrical figures, such as triangles, rectangles, squares, trapezoids, etc., and the quantities necessary for computing the areas of these different figures, such as lengths of bases and altitudes, are scaled from the map.

(*b*) Two sets of lines drawn at right angles to each other across the map divide it into squares of any desired size, say 100 ft. squares to the scale of the map. The number of squares which fall completely within the boundaries gives part of the area at once. The fractional areas inside of the boundaries, formed by the boundary lines cutting through squares, are usually triangles and trapezoids, the areas of which are found as in (*a*) and added to the area found by counting the number of whole squares. (See trapezoidal rule, p. 411.)

Remark. It is usually more convenient to keep pieces of tracing-cloth upon which squares have been drawn to the scales commonly used in plotting, and to lay the cloth over the map instead of drawing parallel lines. Another method is to plot the boundaries on cross-section paper.

Both of these methods are extensively used as rough checks on computed areas.

(*c*) THE PLANIMETER is a mechanical integrator and is used by engineers for ascertaining the area of any figure which has been plotted. The following directions for the use of the *polar* planimeter may be modified to apply to other forms also.

1. Remove the instrument from the case; if it has settings for different units, as sq. in., sq. ft., sq. cm., make this adjustment first by sliding the bar to the unit wanted, then fasten securely by the set-screw.

Place the tracing-point anywhere in the perimeter and press it into the paper hard enough to make a *slight indentation* to show definitely the starting-point.

3. Set the pole, or fixed needle-point, in such a position with reference to the area to be measured that the tracing-point can reach the whole boundary without difficulty.

4. Read the graduated wheel when the tracing-point is in the initial position, making use of the vernier, and set down the number.

5. Guide the tracing-point by hand around the area to be measured in a *clockwise* direction, and stop exactly at the starting-point.

6. Read the graduated wheel again and set down the result over the figures found at the beginning.

7. Subtract the initial reading from the final reading and the result is the area desired. (For exception see step 8.)

8. If the fixed point (see direction 3) was placed *within* the boundary, which is necessary only in large areas, then to the reading by the wheel as found under step 7 must be added the *constant* engraved on the back of the instrument, directly opposite the mark showing the units. Then the sum of this constant and the reading found by the wheel is the true area.

(*d*) *Practical suggestions. Step 1.* Some instruments read only in one unit: square inches; then no adjustment is required.

Step 3. Some planimeters have a *metal block* resting on the paper, into the top of which the fixed point is set. Then the block is held by its own weight, and no needle-mark is made in the paper. Before starting to measure the area it is well to make a rough preliminary excursion of the tracing-point over the area to see that all points of the boundary can be reached.

Step 4. It is often convenient to set the wheel at zero before starting. This can be done conveniently by shifting the pole or needle-point until the reading is zero, and thus step 4 is avoided.

Step 5. In case the area has a loop, or negative portion, start in a clockwise direction over the main area, then the loop or negative part will be traced in a counter-clockwise direction and the instrument gives the difference or net area.

Step 6. If the final reading is less than the initial, it usually indicates that the perimeter has been traced in the negative (counter-clockwise) direction.

Step 8. If the reading given by the wheel is backward when the tracing-point moves clockwise, call this reading negative and *subtract* it from the constant instead of adding.

General direction. Try the instrument first by measuring a circle, a hexagon, or a square of known area, to test its accuracy. By making several such tests of different known areas, a correction constant per unit of area may be determined. If there has been any shrinkage or expansion of the paper since the map was plotted the trial square or circle should be drawn to agree with the slightly changed scale of the map.

458. Computing Areas Directly from Field Measurements. (*a*) The simplest cases are those in which the area required is of some plane figure which can be divided into simple figures of known dimensions.

(*a*) *Illustration.* In the chain survey shown in Fig. 458 and described on p. 56, the lengths of the four fence-lines and of the four broken lines were measured. Hence the three sides of each triangle are known, and the area of each can be found from Formula 12, p. 408.

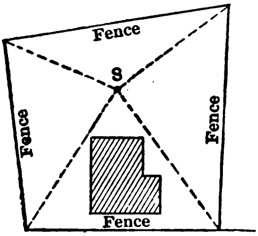

FIG. 458.

(*b*) *Illustration.* Suppose that in the above survey a transit was set up at *S*, the angles between the broken lines taken, and the length of each broken line measured. Then in each triangle two sides and the included angle would be known and the area could be found from Formula 13, p. 408.

(*c*) *Illustration.* Suppose that in the above survey the lengths of the fence-lines and the angle at each corner had been measured but not the broken lines, the method of calculation would be:

(1) Divide the polygon into *two* triangles by a diagonal.

(2) Find the length of this diagonal from the two sides and included angle of one triangle and check it by solving the other triangle.

(3) The three sides of each triangle are now known and the area of each can be found from Formula 12, p. 408.

Check. Divide the polygon into two different triangles by the other diagonal and find the area of each of these triangles as before.

Remark. Usually the transit lines do not coincide with the fence-lines. This case will be discussed on p. 418.

(*d*) *Another method.* Instead of solving the oblique triangles the polygon may be divided into right triangles. Draw one diagonal as *AC*, Fig. 458 (*d*), and from each end drop a perpendicular to one side. These perpendiculars, *Aa* and *Cc*, are easily calculated from the right triangles, and each is the altitude of one of the two triangles into which the quadrilateral is divided. Thus:

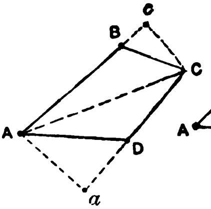

FIG. 458 (*d*).

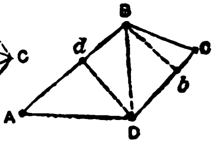

FIG. 458 (*e*).

$$AD \sin ADa = Aa \quad \text{and} \quad \tfrac{1}{2}Aa \times DC = \text{area of } ADC.$$
$$CB \sin CBc = Cc \quad \text{and} \quad \tfrac{1}{2}Cc \times AB = \text{area of } ABC.$$

The work may be checked by drawing the other diagonal *BD* Fig. 458 (*e*), dropping the perpendiculars *Bb* and *Dd* and proceeding as before.

(*e*) *Curved boundary.* Suppose that in Fig. 458 the lower left-hand corner were rounded by an arc of a circle and the points of tangency were known. A chord drawn between these points of tangency would form with the other sides an irregular polygon, the area of which could be found as in the preceding illustration. To this area must be added the area of the segment between the chord and the arc. If the length of the chord and the middle ordinate are known, the approximate area is $\frac{2}{3}MC$ (see p. 408). If the length of the arc and its radius is known, use Formula 8, p. 407. If the intersection angle, etc., is known, use Formula 14, p. 408.

459. Areas from Offsets. There are two cases: (1) when the offsets are taken at regular intervals, and (2) when they are taken at irregular intervals. It is assumed that in both cases offsets are measured from a straight base-line.

(*a*) *Offsets at regular intervals.* There are several formulas designed to shorten the work of calculating each trapezoid separately. In Fig. 459 (*a*) the first offset is numbered 0 and the other offsets 1, 2, 3, etc., . . . 6. Let *a* and *b* be the end offsets at 0 and 6 respectively, and y_1, y_2, y_3, . . . y_5 the intermediate offsets, *d* the common distance between offsets and *S* the sum of the intermediate offsets y_1, y_2, y_3, etc., . . . y_5.

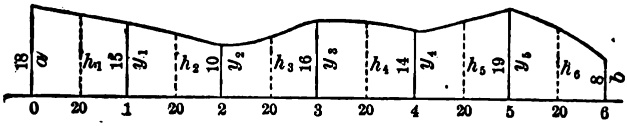

FIG. 459 (*a*).

Trapezoidal Rule: $\text{Area}=\frac{d}{2}(a+2S+b)$. I

$$1740 \text{ sq. ft.} =\frac{20}{2}(18+148+8).$$

This rule obviously is based on the assumption that the figures are trapezoids. It holds good for any number of offsets, and gives results accurate enough for ordinary purposes.

Another form of the trapezoidal rule is

$$\text{Area}=d\Sigma h, \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{II}$$

in which Σh is the sum of the middle ordinates h_1, h_2, h_3, etc., . . . h_n.

$$1740 \text{ sq. ft.}=20\times(16.5+12.5+13+15+16.5+13.5).$$

Formulas I and II are used not only in computing areas from field measurements, but also in obtaining areas directly from the map, the offsets or the middle ordinates, whichever are used, being scaled at regular intervals.

(*b*) *Formulas for curved boundaries.* When the boundary-line between two offsets is curved the assumption that the figure is a trapezoid may give an area too large or too small, according to whether the boundary-line curves away from or towards the base-line. If the curve departs very much from a straight line more accurate results may be obtained by either of the following formulas than by the trapezoidal rule.

Poncelet's Rule. Let h_n = the last middle ordinate.

$$\text{Area} = d\Sigma h + \frac{d}{12}(a - h_1 + b - h_n). \quad \ldots\ldots \quad \text{III}$$

$$1733 \text{ sq. ft.} = 20 \times 87 + \frac{20}{12}(18 - 16.5 + 8 - 13.5).$$

Francke's Rule:

$$\text{Area} = d\Sigma h + \frac{d}{72}(8a + h_2 - 9h_1 + 8b + h_{n-1} - 9h_n). \quad \ldots\ldots \quad \text{IV}$$

$$1730.8 \text{ sq. ft.} = 20 \times 87 + \frac{20}{72}(8 \times 18 + 12.5 - 9 \times 16.5 + 8 \times 8 + 16.5 - 9 \times 13.5).$$

Remark. Formula IV is more accurate than III; either is more convenient to use and generally gives better results than *Simpson's Rule*,* which is sometimes used for the same purpose.

(*c*) *Offsets at irregular intervals.* The most obvious method is to find the area of each trapezoid separately by multiplying its base by its middle ordinate. Thus in Fig. 459 (*c*) the total area is:

$$8530 \text{ sq. ft.} = 50 \times \tfrac{1}{2}(40 + 30) + 40 \times \tfrac{1}{2}(30 + 36) + 60 \times \tfrac{1}{2}(36 + 44) + 30 \times \tfrac{1}{2}(44 + 28) + 60 \times \tfrac{1}{2}(28 + 38).$$

A second method is a special case of the coördinate method given on p. 418.

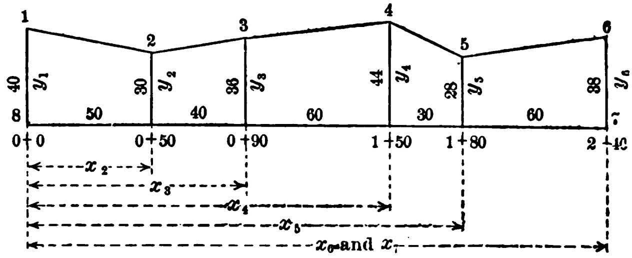

FIG. 459 (*c*).

Formula II, on page 418, may be arranged for convenience as shown below. The area is equal to the sum of the products of the quantities joined by full lines minus the sum of the products of the quantities joined by broken lines. From Fig. 459 (*c*) it is evident that x_1, x_8, y_8, and y_7 are all zero and that $x_7 = x_8$, hence the last two terms do not affect the result.

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* *Derived* on p. 690, Johnson's book on Surveying.

$$\frac{1}{2}\left(\frac{x_1}{y_1}\times\frac{x_2}{y_2}\times\frac{x_3}{y_3}\times\frac{x_4}{y_4}\times\frac{x_5}{y_5}\times\frac{x_6}{y_6}\times\frac{x_7}{y_7}\times\frac{x_8}{y_8}\times\frac{x_1}{y_1}\right)$$

$$\frac{1}{2}\left(\frac{0}{40}\times\frac{50}{30}\times\frac{90}{36}\times\frac{150}{44}\times\frac{180}{28}\times\frac{240}{38}\times\frac{240}{0}\right)$$

$$\tfrac{1}{2}(2000+2700+5400+7920+6720+9120-1800-3960-4200-6840)$$

$$\text{Area} = \tfrac{1}{2}(17060) = 8530.$$

Remark. If at any point the irregular line turns back upon itself, as, for example, if x_5 were less than x_4, the method still holds good, the algebraic signs indicating the additional area to be subtracted.

Another convenient form is to place the ordinates of the corners in determinant array. Rule: *Multiply diagonally from left to right, and consider a downward diagonal plus, upward minus, or vice versa. Divide the algebraic sum of all the products by two.*

Illustration. Taking the example on p. 417 for illustration, and assuming *D* as an origin, the coördinates of the corners and the computations are as follows:

Station	y	x	Downward (+)	Upward (−)
D	0	0		
E	+10	+14	0×14 = 0	10×0 = 0
A	−14	+24	10×24 = +240	−(−14×14) = +196
B	+16	+36	−14×36 = −504	−(16×24) = −384
C	+22	+16	16×16 = +256	−(22×36) = −792
D	0	0	22×0 = 0	0×16 = 0
			−8	−980

[−8 + (−980)] ÷ 2 = −494 sq. ft. Answer.

Remark. The minus sign in the above answer signifies that the area was obtained by taking the corners around the field clockwise; if taken counterclockwise the area would be the same, but with a plus sign.

When total latitudes and departures have been found for plotting, this method is very convenient. It is equally applicable, of course, to an example like that of Fig. 459 (*c*).

460. Computation of Areas from Latitudes and Double Longitudes. This method is the one most often used for computing the area of a closed traverse. When the latitudes and departures of the traverse lines have been computed for purposes of plotting, very little additional labor is required in finding the area. The method is a simple one, though considerable space is required to explain it clearly in detail. Several definitions will be given first.

(*a*) Reference meridian. The north and south line through the most westerly station of a survey.

Remark. This is the sense in which the term is usually understood in the calculation of areas. It is not essential, however, to assume the reference meridian through the *most westerly* station; it may pass through any station provided proper account is taken of algebraic signs. It is usually assumed through the most westerly station in order to simplify matters and avoid mistakes. (See p. 472, for method of determining which is the most westerly station.)

(*b*) The first and last courses of every polygon are the two courses which meet at the most westerly station. The *first* course is that one for which the given bearing is *easterly*, the last course is that for which the bearing is *westerly*, it being understood that the given bearings *are all forward* bearings. (See p. 112.) To determine the most westerly station (see p. 472).

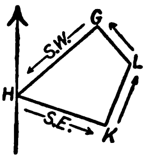

Fig. 460 (*b*).

Illustration. In Fig. 460 (*b*), let H be the most westerly station, then HK is the *first* course and GH the *last* course, provided the bearings indicated on the figure were those used in calculating the latitudes and departures. If, however, the *opposite* bearings were used KH = N.W. and HG = N.E., then HG would be the first course and KH the last course. *Note that the two courses can never both be easterly, or both westerly.*

(*c*) The longitude of a course is the distance from the *middle* point of the course to the reference meridian, measured at right angles to the meridian.

Illustration. In Fig. 460 (*c*) L_1 is the longitude of AB, L_2 of BC, L_3 of CD, and L_4 of DA.

The longitude of any course is equal to the longitude of the preceding course plus half the departure of the preceding course plus half the departure of the course itself.

For the *first* course there is no preceding course, so that the first two terms are zero and *the longitude of the first course is therefore half its departure.* In applying the rule, east departures must be given *plus* signs and west departures *minus* signs.

Illustration. The correctness of the rule just given will become evident from inspection of Fig. 460 (*c*).

Longitude of Course	=	Long. of Preced.	+	½ Dep. of Preced.	+	½ Dep. of Course itself
L_1 of AB	=	0 of 0	+	0 of 0	+	a of AB
L_2 of BC	=	L_1 of AB	+	a of AB	+	b of BC
L_3 of CD	=	L_2 of BC	+	b of BC	+	$-c$ of CD
L_4 of DE	=	L_3 of CD	+	$-c$ of CD	+	$-d$ of DE

Note that c and d both have minus signs.

(*d*) GENERAL METHOD OF CALCULATING AREAS BY LATITUDES AND LONGITUDES. If in any survey an east and west line is drawn from each station to the reference meridian, triangles and trapezoids will be formed. *One side* of each triangle (or trapezoid) will be some *course* of the survey, the *base* of the triangle (or trapezoid) will be the *latitude* of that course, and the mean *length* or *altitude* of the triangle (or trapezoid) will be the *longitude* of that line. The *area* of each triangle (or trapezoid), therefore, will be the *latitude of some course multiplied by the longitude of the same course.* (Thus the area of $FBCG$ is $l_2 \times L_2$ = Latitude of $BC \times$ Longitude of BC.) The areas of all the trapezoids and triangles thus found in any survey are *not* to be added together to find the total area, for each trapezoid, as $FBCG$, includes area which is *not* within the boundaries of the survey, while the area of each triangle may be wholly outside of those boundaries. If, however, all areas found from *north* latitudes (called north products) are added, and all areas found from south latitudes (south products) are added, the difference of these two sums will be the required area.

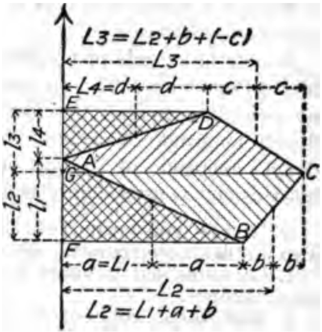

FIG. 460 (*c*).

Illustration. **The area of *ABCD* in Fig. 460 (*c*) may be found as follows:**

NORTH PRODUCT

Area = Latitude × Longitude
$GFBC = l_2$ of $BC \times L_2$ of BC
$GCDE = l_3$ of $CD \times L_3$ of CD

SOUTH PRODUCT

Area = Latitude × Longitude
$FAB = l_1$ of $AB \times L_1$ of AB
$EDA = l_4$ of $DA \times L_4$ of DA

It is evident that the Area of $ABCD$ = (Area of $GFBC$ + Area of $GCDE$) − (Area of FAB + Area of EDA), or equals the difference of the sums of the north and south products.

(*e*) DOUBLE LONGITUDES. If the longitude of any course is found from the longitude of the preceding course by the method of Art. 460 (*c*), it is necessary to use *half* departures. As *whole* departures and not *half* departures are usually given in the tabulated form for latitudes and departures (see p. 474), it is more convenient to multiply each latitude by twice the longitude (double longitude) and divide the final result by two.

To find a double longitude of any course C from the longitude of any preceding course P.

Double Longitude of $C =$ Double Longitude of $P +$ Departure of $P +$ Departure of C. In the case of the first course there is no preceding course; the first two terms disappear and the double longitude of the *first* course equals its departure. The double longitude of the *last* course also equals its departure, but this fact is used simply as a check.

(*f*) RULES FOR COMPUTING AREAS BY METHOD OF LATITUDES AND DOUBLE LONGITUDES. (SUMMARY.)

1. Compute and tabulate the latitudes and departures in the usual manner.
2. Add three columns to the table, headed respectively *Double* Longitudes, North Products, South Products.
3. Ascertain which station is *farthest west,* and which of the two lines from this station is the *first line.* (See pp. 414 and 472.)
4. Complete the double longitude of each course by adding to the double longitude of the preceding course the departure of the preceding course plus the departure of the course itself. West departures are *minus.* The double longitude of the *first* course equals the departure of the first course.
5. Check the double longitudes by the fact that the double longitude of the *last* course is equal to the departure of the last course.
6. Multiply the latitude of each course by the corresponding double longitude; if the latitude is *north* put the result in the column headed North Products, but if the latitude is south put the result in the column headed South Products.
7. Add up the north products, add up the south products, subtract the smaller sum from the larger and *divide the result by two.*

If the latitudes and departures are in feet, as is usually the case, the result will be in square feet. To reduce to acres divide by 43 560, or multiply by 0.0000229568.

8. Having computed an area by the method of latitudes and double longitudes, always check the result roughly by means of the planimeter or by the graphic method (see p. 408). If a more accurate check is desired, repeat the work with the most easterly station as an origin, thus using a new set of coördinates.

Remark. In steps 6 and 7 reject all figures beyond the second doubtful place. (See Art. 425, p. 369.) In step (6) use abridged multiplication or logarithms. In converting square feet to acres it is fully as easy to multiply by 0.000022957, using the abridged method, as it is to divide by 43 560. A table in which these constants are multiplied once for all by every digit from 2 to 9 is useful. (See p. 377.)

Sources of error: Starting with the wrong line for a *first line;* adding departures instead of subtracting when they are *west;* entering north products in the column for south products or *vice versa; forgetting to divide the final product by two.*

Illustration. Let it be required to find the area of the polygon *A-B-C-D-E-A* in Fig. 460 (*f*). Assume that the latitudes and departures have been calculated and tabulated in the usual manner, the next thing to do is to add a column of double longitudes. It is found that *D* is the most westerly station (Art. 507 (*a*), p. 472), and that therefore *DE* is the first line (Art. 460 (*b*), p. 414).

It is not necessary to tabulate the work done in finding double longitudes,—it is tabulated below for sake of clearness.

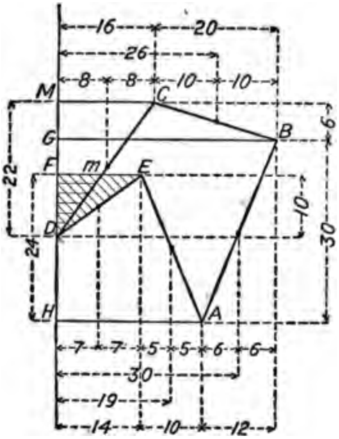

FIG. 460 (*f*).

Double Long.	of	Preceding Course	+	Departure	of	Preceding Course	+	Departure	of	Course itself	=	Double Long.	of	Required Course
0	of	0	+	0	of	0	+	14	of	*DE*	=	14	of	*DE*
14	of	*DE*	+	14	of	*DE*	+	10	of	*EA*	=	38	of	*EA*
38	of	*EA*	+	10	of	*EA*	+	12	of	*AB*	=	60	of	*AB*
60	of	*AB*	+	12	of	*AB*	+	−20	of	*BC*	=	52	of	*BC*
52	of	*BC*	+	−20	of	*BC*	+	−16	of	*CD*	=	16	of	*CD*

Notice that the double longitude of *CD* comes out equal to its departure —a check on the work. In surveys which do not exactly close, the double longitude of the last line will differ from the departure of that line by twice the *total* error in departures (unless errors are balanced). Inserting the double longitude found above and multiplying each latitude by its corresponding double longitude, the tabulated form becomes that given below:

LINE	BEARING	LENGTH	LATITUDE N.	LATITUDE S.	DEPARTURE E.	DEPARTURE W.	DOUBLE LONGITUDE	PRODUCTS N.	PRODUCTS S.
AB	N.-E.		30		12		60	1800	
BC	N.-W.		6			20	52	312	
CD	S. W.			22		16	16		352
DE	N. E.		10		14		14	140	
EA	S. E.			24	10		38		912
			46	46	36	36		2252	1264
								1264	
								2)988	

Area = 494 sq. ft.

Question. In the illustration the work practically amounts to subtracting the sum of the areas *MCD* and *FEAH* from the sum of the areas *MCBG*, *GBAH*, and *FED*. Why, from inspection of the figure, should *FED* be included in the latter sum?

Note. For more practical examples in areas, see pp. 425 and 427.

461. To Compute Areas from Coördinates. This is simply a more general case of the method of computing areas from latitudes and double longitudes. In Fig. 461 let 1–2–3–4 be any irregular polygon, the coördinates of the corners with respect to two coördinate axes being known and represented by x_1, x_2, x_3, x_4, and y_1, y_2, y_3, and y_4.

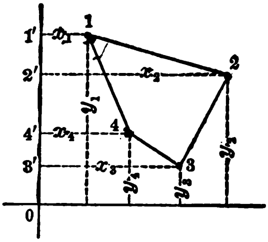

FIG. 461.

The area of the polygon is equal to the sum of the areas of two trapezoids minus the sum of the areas of two others, i.e.,

$$A = (\text{Area } 1'\text{–}1\text{–}2\text{–}2') + (\text{Area } 2'\text{–}2\text{–}3\text{–}3') - (\text{Area } 3'\text{–}3\text{–}4\text{–}4') - (\text{Area } 4'\text{–}4\text{–}1\text{–}1');$$

$$A = \tfrac{1}{2}[(y_1 - y_2)(x_1 + x_2) + (y_2 - y_3)(x_2 + x_3) - (y_4 - y_3)(x_4 + x_3) - (y_1 - y_4)(x_1 + x_4)];$$

$$A = \tfrac{1}{2}[y_1(x_2 - x_4) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_1 - x_3)]; \quad . \; . \; . \quad \text{I}$$

$$A = \tfrac{1}{2}[y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1 - (y_1x_4 + y_2x_1 + y_3x_2 + y_4x_3)]. \quad . \; . \quad \text{II}$$

These two equations may be expressed as follows:

Rule I. *Multiply the ordinate of each corner by the difference between the abscissas of the two adjacent corners, always subtracting the following abscissa from the preceding abscissa. The area is one half the sum of the products.*

Rule II. *Multiply the ordinate of each corner by the abscissa of the next corner and sum the products. Multiply the ordinate of each corner by the abscissa of the preceding corner and sum the products. One half the difference of the two sums is the area.*

Remark. To avoid mistakes in algebraic signs it is better to choose the axes so that the whole polygon lies in the first quadrant. In the above rules it is assumed that the corners are numbered *consecutively* in either direction around the polygon. Of course the rules hold good if the words abscissa and ordinate are interchanged wherever they occur.

Rule I generally involves less work than Rule II. When Rule II is used it is convenient to arrange the ordinates and abscissas as shown on p. 413.

462. Summary of the Different Methods of Computing Irregular Areas. The most general case likely to arise in *practice* may be stated as follows: Boundary-lines are irregular; *traverse lines do not* coincide with boundary-lines; boundaries located from the traverse lines by offsets taken at every point of abrupt change and near enough together so that the boundary between two successive offsets may be considered straight.

The lines bounding the area to be computed form a polygon; in the perimeter are included not only any straight boundary-lines which may exist, but also the series of straight lines which lie between offsets and which follow approximately irregular boundaries such as winding roads and streams. To distinguish this polygon from that formed by traverse lines, it will be referred to as the boundary polygon.

(*a*) *First method.* (1) Compute the length and bearing of every line in the boundary polygon. (See practical illustration, p. 405.) (2) Compute and tabulate the latitudes and departures for all the lines in the boundary polygon (p. 389). If the error of closure is small the work in (1) is probably correct. (3) Find the area within the boundary polygon by the method of latitudes and double longitudes.

Remark. The length and bearing of each side in step (1) may be found in two ways: (1) If the boundary lines are irregular the method explained on p. 405 is preferable. (2) If, however, the boundary line is straight and corresponds to the omitted side of a polygon the method of Case I, p. 402, may be used although the first applies equally well.

(*b*) *Second method.* (1) Compute the area within the polygon formed by the traverse lines by latitudes and double longitudes (p. 416). (2) Compute the areas of the triangles, trapezoids, etc., which lie between the traverse lines and the boundary-lines. (3) Add to the area found in (1) any area found in (2) which lies within the boundary polygon and subtract any area found in (2) which lies outside.

(*c*) *Remark.* In step (2) the area between any traverse line and an *irregular* boundary may often be computed by the trapezoid rule (p. 411) or by one of the rules on p. 412. In the case, however, of the area between a traverse line and a *straight* boundary-line the method explained in the following illustration is frequently used.

(*d*) *Illustration.* Let it be required to compute the area of the piece of land shown in Illustration III, insert sheet, opposite page 190. (1) The area *ABCD* is found by the method of latitudes and double longitudes. (2) The area of the polygon 9–C–D–11 is equal to the area of the trapezoid 9–x–y–11 minus the areas of the two shaded triangles. Since the distances C–9 and D–11 were measured in the field, and angles taken to 9 and 11, the sides 9–x, Cx, Dy, and 11–y are easily calculated. (3) The areas of the polygons D–A–7–11 and A–B–8–7 are found in a similar manner. (4) To find the area between *CB* and the brook, drop perpendiculars to *CB* at *C* and *B* and find these approximate lengths. (It would have been well to have measured them in the field.) The area between these two perpendiculars from *B* to *C* may be found by the method on p. 412, and to this area the areas of the small triangle at each end must be added.

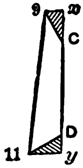

Fig. 461 (d).

(*e*) *The two methods compared.* The chief advantage of the first method is the excellent check in step (2). There is no such check in the second method. The first method would seem to involve more labor, but, in most cases, the work in step (1) of calculating lengths and bearings of boundary-lines has to be done anyway, so that the extra labor involved is not as much as would at first appear. Whichever method is used it is well to check the computed area roughly by the area obtained from the plotted boundary-lines (by scaling and computing, p. 408, Arts. 457 (*a*) and 457 (*b*), or by means of the planimeter, p. 408).

CHAPTER XXXIV

PARTING OFF LAND

MANY of the problems which arise in parting off or dividing up land involve only simple geometrical principles which the student can readily discover for himself. Of the more complicated problems, two are of such frequent occurrence that methods of solving them are explained in this chapter.*

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463. General Problem *Given a line L intersected by two lines making angles with L of M and N degrees respectively. Required the distance x from L of a line C parallel to L, which shall form with L and the two inclined lines a trapezoid of given area A. Also required the length of C and of the non-parallel sides of the trapezoid.*

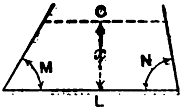

FIG. 463.

$$C = L - x(\cot M + \cot N)$$

$$A = (L + C)\frac{x}{2} = Lx - \frac{x^2}{2}(\cot M + \cot N)$$

$$-2A = x^2(\cot M + \cot N) - 2Lx$$

$$x^2 - \frac{1}{\cot M + \cot N}\cdot 2Lx = -\frac{1}{\cot M + \cot N}\cdot 2A$$

$$\text{Let } t = \frac{1}{\cot M + \cot N}$$

$$x^2 - 2Ltx = -2tA$$

$$x = Lt \pm \sqrt{L^2t^2 - 2tA} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

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* For a great variety of problems in parting off land see Gillespie's *Surveying*, Vol. I, pp. 263–299.

From this value of x and the angles M and N, the other unknown distances may be found by trigonometry, for x is the value of the perpendicular side of a right triangle one angle of which is M, and also of another right triangle one angle of which is N. Compute the two bases of these triangles and subtract their sum from L to get C. The lengths of the non-parallel sides of the trapezoid are equal respectively to the hypothenuses of the triangles.

464. To Part Off a Required Area from a Given Irregular Polygon. The two general cases which arise most frequently in practice are: (1) When the *direction* of the dividing line is known, and it is required to find the two points where this line meets the perimeter of the polygon. (2) When a *starting-point* on the perimeter is known, and it is required to find the direction of the dividing line. The different steps in the calculation will be outlined and then illustrated by a practical example for each case.

(*a*) CASE I. *To divide an irregular polygon into two parts, one of which shall have a specified area, by a line having a given direction.*

If no bearings are given the dividing line may be assumed as north and south, and the corresponding bearings of the sides of the polygon calculated by the methods of Chapter XXX, beginning with the side which makes a known angle with the dividing line. If the bearings are given it is better to use them as they stand.

(1) Compute the latitude and departure of each side of the given polygon to see if it is a closed figure. (If it does not close within a reasonable limit of error nothing further can be done with the problem.) If it does close compute its area by the method of double longitudes (p. 416).

(2) Draw the polygon accurately to as large a scale as can be conveniently used, and check the area obtained in (1) by some approximate method. (See p. 408.)

(3) Find by trial the approximate position of the dividing line. Usually this may be done very roughly by a method similar to that used in (2), since the object is simply to find which vertex of the polygon is nearest to the dividing line.

(4) Through the nearest vertex found in (3), assume an *auxiliary line* having the same direction as the dividing line. *A trapezoid is then* formed having for its two parallel bases this

auxiliary line and the dividing line. If the altitude of this trapezoid can be found the position of the dividing line is determined.

(5) Compute the area on one side of the auxiliary line by the method of latitudes and double longitudes. To do this it will be necessary first to calculate the missing parts of the polygon formed by the assumed line and that part of the original perimeter bounding the area to be computed; one of these missing parts is the length of the auxiliary line. (Chapter XXXII.)

(6) Find the difference between the area found in (5) and the area to be parted off on the corresponding side of the dividing line. This difference is the area of the trapezoid referred to in (4).

(7) Substitute this difference for A in Formula I, p. 421, and solve for x, the altitude of the trapezoid. Find also by trigonometry the length of the non-parallel sides of the trapezoid, thus giving the two points where the dividing line meets the perimeter of the polygon. Since the dividing line is a base of the trapezoid, its length may easily be found.

(8) As a final check, compute the area on each side of the dividing line by latitudes and double longitudes, to see if the area which was to be parted off has been obtained, and also if the sum of the two areas is equal to the total area of the original polygon as found in (1). (This is a double check, but see *Caution*, p. 424.) It is well to check the area of the trapezoid from the computed lengths of the two parallel bases and its altitude x, before applying the double check just given.

(*b*) *Illustration.* Let it be required to subdivide the tract of land shown in Fig. 464 (*b*) by a line *de* whose direction shall be N. 55° 10′ E., in such a way that the area *dEABed* shall be 8 acres. The solution of this problem is shown on p. 425. The different steps were as follows:

(1) Calculated the latitudes and departures and found that the figure closed. Found the area by method of double longitudes (p. 416) to be 478 810 sq. ft. or 10.992 acres.

(2) Drew the polygon to scale, and using the approximate method on p. 408 checked the area found in (1).

(3) Found approximately where *de* would have to be in order to make the area between it and the highway equal to 8 acres. This was done *very roughly* by the approximate method used in (2), the sole object being to find which vertex in the perimeter is nearest *de*. This vertex is evidently *E*.

(4) Assumed a line through *E* parallel to *de* (N. 55° 10′ E.) intersecting the opposite side of the polygon at *b*. In the polygon *ABbEA* there are two missing parts, the lengths *bE* and *Bb*. By the method given on p. 404 these lengths were found to be $bE = 1087.7$ ft. and $Bb = 15.6$ ft. (The computations made in getting these results are not shown in the work on p. 425.)

(5) Found the area of *ABbEA* by latitudes and double longitudes to be 84 690 sq. ft.

(6) Found the area of the trapezoid *dEbe* by subtracting 84 690 from 348 480 sq. ft. (or 8 acres) to be 263 790 sq. ft. Substituting in Formula I, p. 421, found $x = 299$ ft. Found by trigonometry $eb = 338.0$ ft. and $Ed = 391.8$ (work not shown).

(7) Checked the area of the trapezoid as indicated. Also checked the area between *de* and the highway by double longitudes. Found it to be correct within 60 sq. ft. If worth while, this error could be corrected by moving the line *de*, which is 676.8 ft. long, an amount equal to $60 \div 676.8$. As this is less than 0.1 ft. it was allowed to stand in this case. As a final check the area *DdeCD* was calculated by double longitudes (work not shown), and found equal to 2.993 acres. Adding the areas on either side of the dividing line *de*, $7.999 + 2.993 = 10.992$ = the area of the entire tract (check).

(*c*) ***Suggestions.*** (1) It is well to keep all areas in square feet until the problem is finished. (2) This method can be used for subdividing a polygon into any number of parallel strips, each of any required area. (3) In thec ase of very irregular boundary-lines it may be impossible to find a vertex from which an auxiliary line can be drawn and a trapezoid formed. In that case the auxiliary line should be assumed as near to the position of the required line as can be estimated and the area computed as in (5). This area will probably be incorrect, but it will indicate about how much to move the auxiliary line for another trial. Two or more trials may be necessary. This trial method makes no use of Formula I, and is frequently used in place of the trapezoid method. (4) It is not essential even in the trapezoid method to have the auxiliary line pass through a vertex of the polygon; it may start at any point on the perimeter, but it usually saves work to have it start from a vertex.

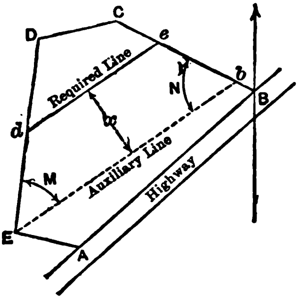


FIG. 464 (*b*).

(*d*) *Caution.* It often happens in subdivisions of areas that several of the same D.L's and N. or S. latitudes are used in two or more area computations (as in the total area and in one or more of the component areas). In such cases an error in the N. or S. products, if they are not multiplied independently each time, might extend through the whole computation, yet would not be revealed by the checks in step (8). (See preceding illustration.) In such cases it is often best to use the most easterly meridian as a starting-point in computing the last set of D.L's.

(*e*) CASE II. *To divide an irregular polygon into two parts, one of which shall have a specified area, by a line starting from a given point on the perimeter.*

If no bearings are given, assume some line of the polygon as north and south, and calculate the corresponding bearings of the remaining sides, otherwise use the bearings given. Steps (1), (2), (3), (5), and (8) are practically identical with the corresponding steps of the preceding case.

(1) and (2) Compute the latitudes and departures, find the area and draw the polygon to scale, as directed in steps (1) and (2) of the preceding case.

(3) *Find* by trial the approximate position of the dividing *line, in order to* ascertain which vertex of the polygon is nearest

	Line	Bearing	Length	N	S	E	W	D.L	N.Prod.	S.Prod.
Area of ABCDEA	AB	N.47-48 E.	1000.6	672.2		741.2		1072.2	720 730	
	BC	N.62-38 W.	573.3	263.5			509.1	1304.3	343 680	
	CD	S. 76-40 W.	330.3		76.2		321.4	473.8		36 100
	DE	S. 5-26 W.	805.0		801.4		76.2	76.2		61 070
	EA	S. 70-39 E.	175.4		58.1	165.5		165.5		9 620
				935.7	935.7	906.7	906.7		1064410	106 790
	Total Area of ABCDEA = (1064410 − 106 790) ÷ 2 = 478 810 Sq.ft. = 10.992 Acres.									
Area of ABbEA	AB			672.2		741.2		1072.2	720 730	
	Bb	N.62-38 W.	(15.6)	(7.2)			(13.9)	1799.5	12 960	
	bE	S.55-10 W.	(1087.7)		(621.3)		(892.8)	892.8		554 700
	EA				58.1	165.5		165.5		9 610
	Missing Parts in parenthesis			679.4	679.4	906.7	906.7		733690	564 310
	Area of ABbE = (733690 − 564310) ÷ 2 = 84690 Sq.ft. = 1.944 Acres									
To find x, eb, etc.	8 Acres = 348 480 M = 49°44′ cot = 0.84706 t = 0.7277 $x = Lt \pm \sqrt{L^2t^2 - 2tA}$									
	Area of ABbE = 84 690 N = 62°12′ cot = 0.52724 Lt = 791.5 $x = 791.5 - \sqrt{626472 - 383920}$									
	" " dEbe = 263 790 cot M + cot N = 1.37430 $L^2t^2 = 626472$ x = 299.0									
	ANSWERS: x = 299.0 eb = 338.0 eB = 338.0 + 15.6 = 353.6 Ed = 391.8 **de** = 1087.7 − 410.9 = 676.8									
Check Areas	Area dEbe = $\frac{1}{2}$(de + Eb) x = $\frac{1}{2}$(676.8 + 1087.7) · 299 = 263 790 = (check)									
	dE	S. 5-26 W	391.8		390.0		37.1	37.1		14470
	EA	S.70-39 E	175.4		58.1	165.5		165.5		9620
	AB	N.47-48 E	1000.6	672.2		741.2		1072.2	720 730	
	Be	N.62-38 W	353.6	162.5			314.1	1499.3	243 640	
	ed	S.55-10 W	676.8		386.6		555.5	629.7		243440
				834.7	834.7	906.7	906.7		964 370	267 530
	Area of dEABed = (964370 − 267530) ÷ 2 = 348 420 Sq.ft. = (Check within 60 Sq.ft.)									

to the *unknown* end of the dividing line. (See step (3) of the preceding case.)

(4) From the vertex found in (3) draw an auxiliary line to the *known* end of the dividing line, and find its length and bearing (p. 401).

(5) This auxiliary line is the base of a triangle having two unknown sides, one the required dividing line, the other a portion of one side of the polygon. (See *dBe*, Fig. 464 (*f*).) The solution of the problem depends upon finding the area of this triangle.

(6) Calculate the area on one side of the auxiliary line by the method of latitudes and double longitudes.

(7) Find the difference between the area found in (6) and the area to be parted off on the corresponding side of the dividing line. This difference is the area of the triangle referred to in (5).

(8) The base of the triangle being known from (4) and its area from (7), its altitude follows at once. Then the lengths of the two unknown sides may be computed by trigonometry.

(9) As a final check, calculate the area on each side of the dividing line as explained in step (8) of the preceding case.

(*f*) *Illustration.* Required to subdivide the tract of land shown in Fig. 464 (*f*) by a line of unknown direction, starting from a given point *d* and parting off an area *deBAEd* = 7.999 acres.

(1) and (2) Found the area of the given polygon to be 478 810 sq. ft. (See preceding illustration.) Checked it roughly by drawing the polygon to scale.

(3) Found approximately the direction of *de* such as to make the area between it and the highway equal to 7.999 acres. Found *B* to be the vertex nearest the unknown point *e*.

(4) Drew the auxiliary line *Bd* and found its bearing and length by the method for missing measurements to be, respectively, S. 75° 33′ W. and 898.0 ft. (See opposite page.)

(5) Found the area of *dEABd* to be 242 565 sq. ft.

(6) Found the area of the triangle *deB* by subtracting 242 565 from 348 436 sq. ft. (or 7.999 acres) to be 105 871 sq. ft.

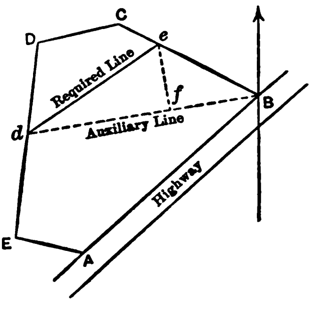


FIG. 464 (*f*).

(7) Found the altitude *fe* of the triangle to be 235.8 ft., and the two unknown sides by trigonometry are: *Be* = 353.6 ft. and *ed* = 676.8. The angle *edB* = 20° 23′, from which the bearing of *ed* = S. 55° 10′ W.

(8) Checked the position of *ed* by calculating the area on each side as in step (7) of the preceding illustration. (Work for one area shown on p. 425.) (*See Caution*, p. 424.)

Suggestions. See suggestions at the end of the preceding case.

Area of dEABd

Line	Bearing	Length	N	S	E	W	D.L.	N.Prod.	S.Prod.
dE	S. 5-26 W	391.8		390.0		37.1	37.1		14 470
EA	S.70-39 E	175.4		58.1	165.5		165.5		9 620
AB	N 47-48 E	1000.6	672.2		741.2		1072.2	720 730	
(Bd)	(S 75-33 W)	(898.0)		(224.1)		(869.6)	943.8		211 510
			672.2	672.2	906.7	906.7		720 730	235 600

235 600

Area of dEABd in sq.ft. = 485 130 ÷ 2 = 242 565

Length and Bearing of Bd. Altitude ef

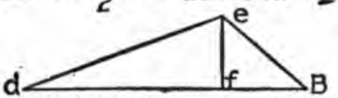
log. 869.6 = 2.93932
" 224.1 = 2.35044
" tan. Bear. = 0.58888
Bearing Bd = S. 75° 33′ W
log 869.6 = 2.93932
" sin 75-33 = 9.98604
" Bd = 2.95328
Length Bd = 898.0

7.999 Acres = 348436 sq.ft.
dEABd = 242565 " "
Area of △ deB = 105871 " "
Base of △ = Bd = 898.0 ft.
Altitude of △ deB = 105871 ÷ $\frac{898.0}{2}$ = 235.8 ft. = ef

Length and Bearing of de (dividing line)

Bearing Bd = S 75-33 W = N 104-27 –
" Be = Bearing BC = N 62 38 –
" Angle dBe = 41° 49′

log 235.8 = 2.37254 2.37254
" sin 41-49 = 9.82396 tan 9.95164
" Be = 2.54858 fB 2.42090
Be = 353.6 fB = 263.6
Bd = 898.0
fB = 263.6
df = 634.4



2.37254
log. 634.4 = 2.80236
" tan. edB = 9.57018
edB = 20°23′
Bearing Bd = S 75-33 W
ed B = 20-23
Bearing ed = S 55-10 W

2.37254
Sin. = 9.54208
log. ed = 283046
ed = 676.8

ANSWERS

CHAPTER XXXV

EARTHWORK COMPUTATIONS

In this chapter are given simple methods of computing earthwork, such, for example, as those most used in estimating cut and fill for ordinary grading. For a more comprehensive treatment of earthwork computations the student is referred to standard books on railroad engineering.

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465. Grade Elevations. The *grade* or *gradient* between two points of fixed elevations is equal to the difference in elevation divided by the distance between the points. This grade is generally given in *per cent*, as 3%, 5%, etc. (See 310 (*a*), p. 243.) The per cent being known, the elevations of intermediate points are easily obtained. The usual method is to find the elevation of any point by adding to the elevation of the preceding point, if lower, a quantity equal to the distance from the preceding point multiplied by the grade. If the intermediate points are equal distances apart, the quantity added each time is of course a constant.

(*a*) *Illustration.* In Fig. 465 (*a*) let the elevation of two points 200 ft. apart be respectively 52 and 60 ft., and let the finished grade be a straight line between the two points. Let the existing surface be represented by the full line. The grade is $\frac{60-52}{200}=0.04$ or 4%. For 50-ft. stations the constant change of grade is $0.04\times 50=2$ ft. The elevation of A is $52+2=54$; of B, $54+2=56$; of C, $56+2=58$. As a check $58+2=60$, the original elevation. This check should never be omitted.

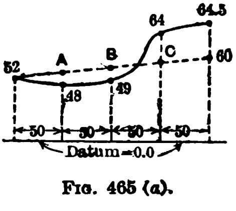

FIG. 465 (*a*).

The grade elevation of any other point between 50-ft. stations may be found easily. Thus, for example, at a point 24 ft. from B toward C the grade elevation is $56+(24\times 0.04)=56.96$.

Note. For a more practical illustration, see computations for sewer-grade points, p. 286, and also *Illustration* (*b*), p. 281.

(*b*) *Cut and fill.* The cut and fill at any point is the difference between the grade elevation at that point and the elevation of the existing surface. If the former is always subtracted from the latter *algebraically*, a *minus* result indicates *fill* (earth to be added), a *plus* result *cut* (earth to be taken away). Thus in Fig. 465 (*a*) the fill at A is $48-54=-6$ ft., and at B $49-56=-7$ ft. The cut at C is $64-58=+6$ ft., and at the last point $64.5-60=+4.5$ ft.

466. Geometrical Definitions and Principles. A *polyhedron* is a geometrical solid bounded by planes. A *prism* is a polyhedron two of whose faces (bases) are equal polygons lying in parallel planes and having their homologous sides parallel, the other faces being parallelograms formed by planes passed through the homologous sides of the equal polygons. A *truncated prism* is one in which the two bases are not parallel. A *right* truncated prism is one in which one of the bases is perpendicular to the lateral edges.

The volume of any truncated prism is equal to the area of a *right section* multiplied by the length of a line joining the centers of mass of the two bases. The volume of any *truncated triangular* prism is equal to the product of its right section by ⅓ the sum of its lateral edges. Any truncated prism whose base is a symmetrical polygon can be divided into equal truncated triangular prisms, and hence the volume of such a prism is equal to the area of a right section multiplied by a length equal to the mean of all the lateral edges.

467. Calculating Earthwork by the Method of Unit Areas. (*a*) In Fig. 467 (*a*) let *abcd* be a rectangle which has been staked out on the ground, and suppose that the four corners are at different elevations, but lie in the same plane. The surface bounded by the rectangle is therefore an *inclined* plane. Suppose that it is desired to grade down to a level surface a certain distance below the lowest corner. The earth to be moved is a *right* truncated prism, with vertical edges at a, b, c, and d. Suppose, on the other hand, that the finished grade is also an inclined plane (not necessarily parallel to the original surface), then the earth to be moved is a truncated prism whose *right section* is the rectangle *abcd*. In either case the volume is equal

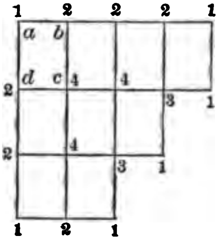

FIG. 467 (*a*).

to the area of the rectangle *abcd* multiplied by one fourth of the sum of the vertical edges through *a*, *b*, *c*, and *d*, or

$$V\text{ (in cu. ft.)} = A\frac{a+b+c+d}{4} \text{ or } V\text{ (in cu. yds.)} = \frac{A(a+b+c+d)}{4\times 27}. \quad \text{(I)}$$

This formula is, of course, equally applicable to fills, i.e., when the grading is carried up to a plane wholly above the original surface.

(*b*) When any surface has been divided into a number of *unequal* rectangles and the surface within each rectangle is a plane, there will be as many prisms as there are rectangles, and the total volume will be the sum of the volumes of all of the prisms. By making the rectangles equal, however, the work of computation is simplified. Thus in Fig. 467 (*a*) each corner-height will be used as many times as there are rectangles joining at the corner (indicated on the figure by numbers). The total volume is therefore

$$V\text{ (in cu. yds.)} = \frac{A(\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + 4\Sigma h_4)}{4\times 27}. \quad . \quad \text{(II)}$$

A = area of one rectangle in sq. ft.; Σ = sum of; h = corner-height; subscripts indicate number of times a corner-height is used.

(*c*) *Remark.* Both formulas (I) and (II) are based on the assumption that the original surface within a unit rectangle is a *plane*. In practice the unit area is assumed small enough so that this assumption is approximately true, but it is practically never exactly true, and the volume obtained from either formula will be slightly different from the true volume.

(*d*) *If the finished surface is inclined* it may slope in one direction (Case II, p. 288), or it may slope in two directions (Case III, p. 288). In either case the elevations of the finished surface at the different corners must be obtained either by calculation or graphically before the corner-heights can be found. The method of ascertaining these grade elevations will depend upon how the unit areas were laid out in the field. The directions for setting grade-stakes (pp. 288–289) will suggest ways in which the method of Art. 465, p. 428, may be used to find grade elevations at points in any given row.

(*e*) *Method of procedure.* (1) Draw a diagram of the rectangles as laid out in the field, not necessarily to scale but large enough so that the shortest side of a rectangle is at least an inch. (2) At each corner mark the elevation of the original surface as obtained from the field notes. (3) Calculate the elevation of the graded surface at each corner and put the result on the diagram directly under the original elevation. (4) Subtract on the diagram the original elevations from the grade elevations (or *vice versa* if it is fill instead of cut), thus getting the corner-heights. (5) Sum up all the corner-heights in the perimeter used *once;* sum up all those used *twice* and multiply by 2; sum up all those used *three* times and multiply by 3; this will take into account all points in the perimeter; sum up all *interior* corner-heights and multiply by 4. (6) Sum up all the results obtained in (5) and multiply by $\frac{A}{108}$, where A = area in square feet of unit rectangle—the result is the total volume in cubic yards.

When the outline of the entire area is not rectangular this method of procedure must be modified. (See Fig. 467 (*g*).)

(*f*) *Suggestions.* Any systematic method of procedure may be followed in place of that given, provided that corner-heights are correctly calculated and that there is no confusion as to the number of times each corner-height is used. There is always danger of omitting a corner-height altogether, hence in step (5) arrange corner-heights in four columns according to whether they are used once, twice, thrice, or four times, taking interior corners in order, vertically or horizontally. Before adding, see if each of the quantities in the four columns agrees with the corresponding quantity on the diagram; if it does put a check-mark near the latter, and finally inspect the diagram to see if there are any corner-heights without check-marks.

(*g*) *When the outline of an area is not rectangular*, as in Fig. 467 (*g*), it cannot be subdivided into squares or rectangles, but as much as possible of the calculation is done by the method of the preceding article, and the rest by finding the volumes of the irregular solids around the edges. Thus in Fig. 467 (*g*) the unshaded portion was divided into 50-ft. squares and the corresponding volume computed by the preceding method. Extending the lines to the boundaries, a number of irregular areas are formed (indicated by cross-lines). From each area and the corresponding corner-heights the volume of the corresponding prism may be found, and the sum of these volumes added to the volume of the unshaded portion. In the majority of cases the irregular areas will be trapezoids; the middle height

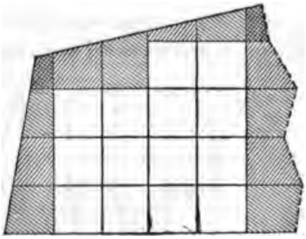

FIG. 467 (*g*).

of each trapezoid (computed or scaled) multiplied by the base gives the area, and this area multiplied by the mean of the corresponding corner-heights gives the volume. In Fig. 467 (*g*) all the shaded areas are trapezoids except the two in the upper corners (double-cross-lined), and each of these may be divided into a trapezoid and a triangle.

When a cut or fill "runs out to zero" the boundary-line is usually irregular, as shown by the dotted line. (See Fig. 468.) In a perfectly general case of this kind all four sides of the figure would be bounded by irregular lines, but the method would still hold good.

(*h*) *Suggestions.* Good judgment in choosing the size of the unit square or rectangle, and in planning the general arrangement, will save work both in the field and in the office. The surface within a unit area should be approximately a plane, and the aim should be to have the smallest number of corners consistent with this rule. (See Remark, p. 430.) Frequently large unit areas can be used for the greater portion of the work where the ground is even, smaller unit areas being adopted for places where the ground is uneven.

In very uneven ground the assumption that the four corners of a unit area lie in the same plane is seldom true, but any three points will lie in a plane, hence better results may be obtained by subdividing each unit square or rectangle into two triangles by drawing that one of the two diagonals which more nearly fits the ground. Each corner-height will then be used as many times as there are triangles meeting at that corner. (Why cannot this number exceed eight?) Formula II, p. 430, then becomes

$$V \text{ (in cu. yds.)} = \frac{A(\Sigma h_1 + 2\Sigma h_2 + 3\Sigma h_3 + \text{etc.}, \ldots 8\Sigma h_8)}{6 \times 27}, \quad \text{(III)}$$

in which A is the area of a *unit rectangle* or *square*, and the subscripts denote the number of triangles meeting at a corner.

468. Estimates of Cut and Fill. In Figs. 467 (*a*) and 467 (*g*) it was assumed that the graded or finished surface would be entirely below or above the original surface, i.e., that the grading would be all cut or all fill. More often in practice the problem is one in both cut and fill. Suppose that in Fig. 468 the elevations of the ground at all corners of the squares have been ascertained, and it is desired to grade down to an elevation of 20 ft. above datum. Let the outside broken line represent the boundary of the area to be graded. Suppose that the elevations of

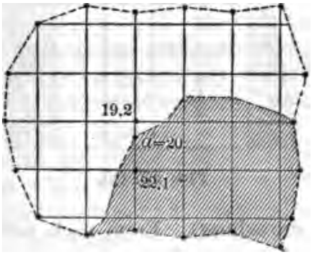

FIG. 468.

all the corners within the shaded area are *above* 20 ft. and those in the unshaded area are *below* 20 ft. Then there must be a line somewhere between (represented by the inside broken line) which passes through points of exactly 20 ft. elevation, i.e., a line at grade, corresponding to a 20-ft. contour. Points through which this line passes may be determined by interpolation as explained below. The volume of cut (shaded area) and the volume of fill (unshaded area) can each be calculated by the method of the preceding article, and the difference between the two is the amount of earth which must be "borrowed" or taken away, according to whether the fill or the cut is in excess, assuming that no allowance is made for shrinkage or expansion.

Remark. In the above illustration the finished surface is level, hence the grade elevation at any point, as, for example, at any corner of any square or at any point between corners, is 20 ft. The interpolation for any point, as *a*, is very simple, since its elevation must be 20 ft. (See Fig. 469 for method.) When the finished surface is inclined the grade elevations at the corners of the squares will differ, but once the cut or fill at each corner is known the work of interpolation is no more difficult than for a level grade.

Sometimes portions of the outside boundary-line are determined by the points where the grade "runs out," i.e., where there is neither cut nor fill. In such a case these points may also be found by interpolation.

469. Interpolation. (*a*) Let the elevations of the surface of the ground at A and B, two corners of a square, be 19.2 and 22.1 respectively. Suppose that the graded or finished surface is level and its elevation is 20 ft. It is required to find a point on the line AB where the existing surface is intersected by the graded surface, assuming the former to slope uniformly from A to B. There are several methods, arithmetical, graphic, and mechanical, based on the properties of similar triangles. One of the simplest graphic methods is shown in the figure. At A $19.2-20=-0.8$ (fill), and at B $22.1-20=+2.1$ (cut), hence lay off to any scale 8 units in one direction and 21 units in the opposite direction, lay down a straight-edge between the points thus obtained (indicated by the broken line), and mark the required point C. Since the 8 units and the 21 units may be laid off on any two parallel lines through A and B, it saves time to use the sides of the square, and points can be interpolated in a case like that of Fig. 469 without drawing any lines whatever. This method is equivalent to plotting a vertical section through A and B in

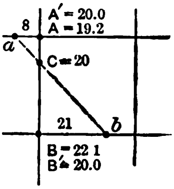

FIG. 469.

which $A'B'$ is a level line of 20-ft. elevation, a a point 0.8 ft. below this line, and b a point 2.1 ft. above it.

In other words the fill (−0.8) is laid off in one direction and the cut (+2.1) is laid off in the other. It is well, therefore, before beginning to interpolate to go over the plot and mark the amount of cut or fill at each corner.

(*b*) *When the grade is not level.* (1) Find the grade elevation at each corner. (2) Find the cut or fill at each corner. (3) Proceed as above, i.e., lay off the fill in one direction, the cut in the other on parallel lines, and lay a straight-edge between the points thus obtained; it will cross the side of the square in the required point. (Draw a figure and prove this.)

Note. For other methods of interpolation see page 497.

470. Prismoids. *Definition.* "A prismoid is a solid having for its two ends any *dissimilar* parallel plane figures of the *same* number of sides, and all the sides of the solid plane figures also." Any prismoid may be resolved into prisms, pyramids, and wedges, the common altitude of which is the perpendicular distances between the two parallel end planes.

Many excavations and embankments, especially those which occur in the construction of roads and railroads, are more nearly prismoids than prisms. There are several methods of computing the volume of a prismoid. Two of the most common are indicated by the following formulas, in which A_1 and A_2 represent the areas of the two parallel end faces, M the area of the middle section parallel to the end planes, and l the length or perpendicular distance between the end planes:

End-area Formula: $V \text{ (in cu. yds.)} = \frac{1}{2}(A_1 + A_2) \times \frac{l}{27}$. . (IV)

Prismoidal Formula: $V \text{ (in cu. yds.)} = \frac{1}{6}(A_1 + 4M + A_2) \times \frac{l}{27}$. (V)

The "end-area formula" gives approximate results, close enough for rough estimates, and is in general use. The "prismoidal formula" is of wide application, since it gives the correct volume of any solid bounded by two parallel plane faces, and defined by the surfaces generated by a straight line moving upon the perimeters of these faces as directrices. It is used as a basis for earthwork computations in railroad work, and it would be used more extensively than it is were it not for the extra labor involved in finding M. The area M is *not* the mean *of A_1 and A_2, but* each of its linear dimensions is the average of the corresponding dimensions of A_1 and A_2, and from these linear dimensions the area M is found. The middle section is sometimes measured on the ground; or in a series of such sections, equally spaced, each alternate section may be taken as M, and the length of the prismoid as twice the distance between cross-sections. For a continuous line of earthwork the prismoidal formula then becomes:

$$V=\frac{l}{3\times 27}(A_1+4A_2+2A_3+4A_4+2A_5+4A_6\ldots+A_n)\ldots \quad \text{VI}$$

In ordinary cross-sectioning it is customary to take elevations of the surface of the ground at three points (middle point and two side points), on a line at right angles to the center-line, at every 100-ft. station, unless a sudden change in the ground requires an intermediate cross-section. For "three-level" sections the work of computation is simplified by finding the volume first by the inexact method of "end areas" and then applying a "prismoidal correction". The work is further shortened by the use of diagrams.*

471. Estimating Cut and Fill from a Contour Map. There are several methods in which approximate estimates for grading can be made from contour maps.

(*a*) *First Method.* In Fig. 471 (*a*) the contours of the existing surface are represented by full lines. The figures around the outside of the bounding lines give the grade elevations at those points, and hence the straight lines represent contours of the finished or graded surface at one-foot intervals. At every point where a straight line intersects a contour of the existing surface the cut or fill can be found by simply subtracting the elevation of the straight line from the elevation of the contour. By joining points of equal cut or fill, a

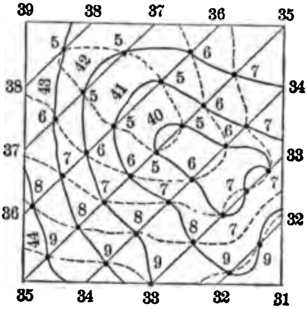

FIG. 471 (*a*).

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* For prismoidal-correction formulas and for complete discussions of the whole subject of earthwork calculation consult books on railroad surveying such as Webb's "Railroad Construction" (John Wiley & Sons), or Allen's "Railroad Curves and Earthwork" (Spon & Chamberlain, New York).

set of lines is obtained (represented by broken lines in Fig. 471 (*a*)) which are the horizontal projections of lines cut from the existing surface by planes parallel to the finished surface. In this case these planes are one foot apart, measured vertically. The layer of earth between any two of these parallel planes is a solid having for its two parallel bases the irregular areas enclosed by the two corresponding broken lines, and for its altitude the vertical distance of one foot. The volume of this solid may be obtained by the end-area method or by the prismoidal formula, the irregular areas being found from the map by scaling and computing or by the use of the planimeter. The sum of the volumes of all the layers is the total volume required.

(*b*) *Illustration.* The line passing through the points marked 5 in Fig. 471 (*a*) is the horizontal projection of a line on the existing surface which encloses an imaginary irregular surface that is parallel to and 5 ft. above the finished surface. Likewise the line through points marked 6 represents the boundary of an imaginary surface parallel to and 6 ft. above the finished surface. The area enclosed by the 5-ft. line and the area between the 6-ft. line and the top and left bounding line are ascertained, and the volume between these two irregular surfaces computed as explained above. The volumes of the other layers may be found in a similar manner.

(*c*) *Second Method.* (1) Divide the map into unit squares or unit rectangles by ruled lines. (2) Find by interpolation the elevation of the existing surface at the corners of each unit square. (This is done by a method the reverse of that explained in Art. 469, p. 433.) (3) Ascertain the grade elevation at each corner and then the corner-height. (4) Calculate the volume by the method of Art. 467, p. 429.

(*d*) *Remark.* The two preceding methods are used most in estimates where the finished or graded surface is a plane—level or inclined. For warped or undulating surfaces, or for structures like dams or reservoirs and terraces having regular outlines, another method (given below) is often used.

(*e*) *Third Method.* (1) Make an accurate map on which the existing surface of the ground is shown by contours drawn in black ink. (2) On this map lay out contours in red ink which *correspond in elevation* to those in black ink, but which represent the finished or graded surface. If this final surface is to be curving or undulating, the red-ink contours will be curving lines, but if it is a surface composed of *plane* faces, like the inside slopes of a rectangular reservoir, the red-ink coutours will be a series of straight lines.* (3) The area enclosed by a red-ink contour and a black-ink contour of the same elevation is the horizontal area of the cut or fill at that elevation. (4) The layer of earth between two of these successive areas is approximately a prismoid, the altitude of which is the contour interval. The volumes of successive layers may be obtained by the end-area method or by the prismoidal formula, but if the latter is used it is better to consider every other area as a middle area, and the height of the prismoid as *twice* the contour interval. Areas are usually found from the map by means of the planimeter.

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* See Raymond's "Plane Surveying" (American Book Co., New York), *p. 285, or Johnson's* "Plane Surveying" (John Wiley & Sons), p. 446, *for an illustration of this* method applied to a reservoir.

CHAPTER XXXVI

THE USE OF THE DRAWING INSTRUMENTS *

In this chapter are given such directions and suggestions for the use of drawing instruments as are of special importance in mapping.

472. Use of the Pencil. (1) Use a 6-H pencil for ordinary map-drawing, but for penciling directly on tracing-cloth a 3-H pencil (or softer) is better. Sharpen as follows: Cut the wood back a little more than an inch at *each* end of the pencil, *leaving at least $\frac{3}{8}$ in. of lead exposed.* Sharpen the lead at one end to a *long thin wedge* by rubbing the flat sides on fine sandpaper alternately as in sharpening a knife, until they taper gradually from the wood to a knife-edge. Sharpen the lead at the other end to a *long* cone-shaped point by rolling the lead as it is rubbed.

Remark. The above directions are made explicit because so many students neglect to sharpen their pencils properly. It is practically impossible to do accurate work in plotting with a lead that is dull or soft—clear hair-lines are absolutely essential.

(2) Draw lines so close to the ruling edge that they are scarcely visible until the edge is moved away. To insure this, it may be necessary to incline the top of the pencil slightly outward, thus bringing the flat side of the lead in contact with the *lower* corner of the ruling edge. The pencil should be held nearly vertical, and it should be *drawn,* not *pushed.* Do not use the cone-shaped lead for ruling lines—it will wear off too fast; keep it sharp for marking points.

473. Use of the T-square. (1) Make sure that the upper edge of the blade is straight,† and do not use the lower edge unless it is known to be truly parallel to the upper edge.

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* This chapter is taken largely from a similar chapter in the author's "An Introductory Course in Mechanical Drawing" (American Book Co.). The student is referred to that book for a more complete treatise on the use of drawing instruments.

† To test an edge for straightness draw a fine hair-line in exact contact with the edge, and then turning the T-square end for end, see if the same edge coincides with the line throughout its length.

(2) Use the T-square against the left-hand edge of the board for horizontal lines, and acquire the habit of *feeling* the head in perfect contact with the edge before starting to draw a line.

(3) Do not use the T-square with its head against the right-hand edge of the board unless you draw with your left hand.

(4) Do not draw vertical lines by placing the head of the T-square against the top or bottom of the board except in rough work, for adjacent edges of the ordinary drawing-board are seldom exactly at right angles to each other.

(5) In drawing long oblique lines it is sometimes necessary to turn the T-square over in order that it may lie flat. In that case use the edge of the blade which is known to be straight and take extra pains to keep the blade from slipping.

474. Use of the Triangles.* (1) In drawing a line through a point the corner of the triangle should never be at the point. For example, do not erect a perpendicular to an oblique line by fitting one edge of a triangle against the line and drawing the perpendicular along another edge without moving the triangle, but fit one edge to the line, bring the edge of a second triangle or the edge of a T-square into contact, and slide the first triangle until the perpendicular passes through the point. Another method is to fit the hypothenuse of a triangle to the line, and bringing a straight edge into contact with a second side, turn the triangle until its third side is in contact with the stationary edge, thus bringing the hypothenuse perpendicular to the given line. In either method, exercise great care to keep the stationary straight edge from slipping.

(2) By placing the 45° triangle against the horizontal edge of a T-square and the 30° or 60° triangle against the edge of the 45° triangle, 15° and 75° lines can be drawn.

(3) In drawing a line parallel to a given line, slide the triangle along the edge of the T-square if the lines are too far apart to use the usual combination of two triangles.

(4) It is difficult to make the edge of a triangle pass through a point or coincide with a line if that edge casts a shadow.

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* A new triangle should always be tested. The right angle may be tested by placing one edge against the T-square and drawing a vertical line along the other edge. Turn the triangle over, placing the same edge against the T-square, and if the triangle is not right-angled the vertical edge will not coincide with the vertical line. A 60° angle may be tested by constructing an equilateral triangle wholly with the triangle and T-square. Similar tests will suggest themselves for 30° or 45°.

Avoid shadows by turning the drawing-board or by working from different sides of the board.

475. Use of the Scale. The scale commonly used in mapping is the *decimal* scale. The form recommended is the triangular scale in which the smallest divisions on the six sides are respectively $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{50}$, and $\frac{1}{60}$ of an inch. In using the scale for plotting do *not* reduce measurements by arithmetic, but read distances directly from the scale itself *by changing in the mind the value of each division on the scale.*

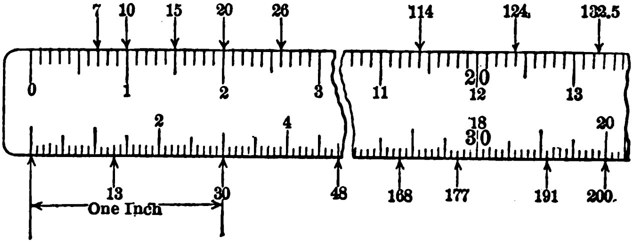

Fig. 475.

Illustration. The smallest division on the upper edge of the scale, Fig. 475, is $\frac{1}{20}$ of an inch. When plotting 20 ft. to the inch consider each of these divisions as representing one foot. The portion to the right of the break is at the center of a 12-inch scale, and the "20" marked in large numerals indicates the number of divisions to the inch. On the lower scale the "30" indicates that each inch is divided into thirty parts. The other four edges of the scale not shown in the figure are marked at the center 10, 40, 50, and 60 respectively.

In reading the scale simply add a cipher to the nearest number preceding the point. The distance in *feet* that each arrow in Fig. 475 is from the zero-point is indicated by the corresponding number.

To plot 100 ft. to the inch use the 50 side and double the value of each division, i.e., $\frac{1}{50}$ in. = 2 ft. To plot 200 ft. to the inch, let each $\frac{1}{50}$ in. = 4 ft.; 300 ft. to the inch, $\frac{1}{60}$ in. = 5 ft., or $\frac{1}{30}$ in. = 10 ft., and so on.

For additional suggestions for using the scale see p. 448.

476. Use of the Ruling-pen. (1) Never begin to ink a drawing until the ruling-pen can be relied upon to produce a clear clean-cut line. If good ink is used and the pen is properly held, a ragged line indicates that the nibs are not exactly of the same length or that they are not sharp. A nib which is too short or too dull will leave *its* side of the line more or less broken. The best of pens become dull with use, and the draftsman should keep his pen sharp.

(2) Hold the pen almost perpendicularly, *thumb-screw out,* both nibs pressing evenly on the paper, and with the top *inclined slightly in* the direction in which it is moved. (The tend-

ency is to incline it too much.) Acquire a certain "touch" which brings the pen *lightly* but *firmly* in contact with the ruling edge. Endeavor to get into the easiest position in which to ink a line, even though it is necessary to walk around the drawing. In this way the ruling edge may be kept between the line and the body, so that in moving the pen the tendency is to draw it *against* the ruling edge rather than away from it. Thus the pen will cling to the edge without that excessive pressure which pushes the nibs together and causes the width of line to vary. Steady the hand by sliding on the end of the little finger. The pen should be moved from left to right and should be *drawn*, not *pushed*. See that there are no small particles of dust or lint in its path.

(3) Adjust the nibs of the pen to the width of line most often used in map-drawing, and make a fine scratch across the milled head of the adjusting-screw, so that by noting the position of this scratch the nibs can be set quickly to this width of line or to one either twice as wide or half as wide. When the pen is once set at the proper width avoid changing its nibs until all lines of that width are inked.

(4) Tack a cloth to the drawing-desk and clean the pen frequently when in use by forcing the cloth out through the nibs without disturbing the adjusting-screw. Keep the pen bright and clean when not in use, as ink left to dry in the pen will almost surely corrode the points.

(*a*) *To sharpen a pen* use a thin oil-stone. Screw the nibs of the pen close together, and draw the pen across the stone a few times precisely as if drawing lines, but changing the inclination of the pen from one side of the vertical to the other, so as to keep a rounded point. This will make the nibs of equal length but *dull*. Unscrew the nibs and sharpen each of them separately by rubbing its *outer* side on the stone with a rotary motion, as if sharpening a knife, taking care to hold the pen at a small angle with the horizontal—almost flat. The burr on the inside of the nibs may be removed by a stroke or two on the stone, or with a flat file.

477. Use of Dividers. Dividers are used chiefly for dividing a line into equal parts and for transferring distances from one map to another, or from one part of a map to another part of the same map. *They should not be used for transferring distances from scale to paper*—a method both slow and inaccurate. Whenever possible use the scale in preference to the dividers.

Proportional dividers are used to transfer measurements to an enlarged or reduced scale. Thus if the measurements are to be reduced one third, the dividers are adjusted so that any distance included between one pair of points is always one third of the distance included between the pair of points at the other end of the dividers.

478. Use of the Compasses. (1) The leads that come with compasses are often too soft for accurate work. Cut off two inches or so of a 6-H pencil, take out the lead, sharpen it to a fine cone-shaped point, and keep it sharp by rubbing it frequently on a file or sand-paper.

(2) Adjust the pivot-needle so that it will be right for the pen and do not disturb it thereafter, but adjust the lead to the pivot-needle. The shoulder of the needle-point will then be at the end of the pen-point or the pencil-point when the compass-legs are together, and the pencil alone will need readjustment from time to time as it wears off.

(3) Bend the legs of the compasses until each is perpendicular to the paper when the arc is drawn, especially in inking, for in no other way can the nibs of the pen be made to bear evenly on the paper.

(4) Guide the pivot-needle to place with a finger of the left hand, be sure it is set before starting the arc, lean the compasses a little in the direction of revolution, let the handle roll between the thumb and forefinger, ink the circle with one continuous motion, and stop the pen at the end of one revolution lest the line be made uneven by further inking.

(5) Use bow-compasses for very small circles, the lengthening bar for large circles, and beam-compasses for circles too large to be drawn with ordinary compasses.

(6) Note that the directions for filling, cleaning, and sharpening the ruling-pen apply to the compass-pen also.

479. Use of the Curve-ruler. (1) It is assumed that points have been plotted close enough together to determine the curve completely. First draw the curve in pencil, using a hair-line and noting which portions of the curve-ruler are used. Ink in small portions of the curve, keeping the blades of the ruling-pen tangent to the curve by turning the handle; as the curve-ruler will coincide with the pencil outline for a short distance only, great care is necessary to make the different sections join well. To insure a smooth curve it is well to let the edge of the ruler coincide with a small portion of the curve beyond the points where any particular section is begun or stopped.

In inking narrow ellipses, parabolas, and similar curves with *a sharp turn, the* common difficulty is to avoid a slight angular break in the curve at the turn. The part of the ruler used for one side of the bend should be such that if a line were continued it would nearly coincide with the curve for a short distance the other side of the bend, so that when the ruler is reversed for the curve on the other half of the turn the angular break referred to should not occur. A *small* portion of a narrow turn can usually be drawn better with the bow-compasses than with the curve-ruler.

Flexible curves which can be adjusted to any curved line, and hard-rubber splines held in place by weights, are useful for inking curves, especially irregular lines and those of gentle curvature. These curves are sold by dealers in drawing instruments.

480. The Use of the Pantograph. The pantograph is an instrument for copying drawings to the same, a larger, or a smaller scale. The theory is based on the following proposition: If any parallelogram is jointed at the corners, as *A*, *B*, *C*, and *E* (Fig. 480), and the four sides (produced if necessary) are cut by a straight line, the four points thus obtained will continue to lie in a straight line no matter how much the angles of the parallelogram may be changed. Moreover, the distances between these points will remain in a constant ratio to each other.

In the pantograph, Fig. 480, two of the sides are cut by the straight line at their intersection *A*, while the other two are cut at *F* and *B* respectively. Hence if *F* is fixed and *A* is moved to any point as *A′*, the point *B* will move to a point *B′* in a straight line with *F* and *A′*. Moreover *FA*:*FB*::*FA′*:*FB′*. Hence *AA′*:*BB′*::*FA*:*FB*.

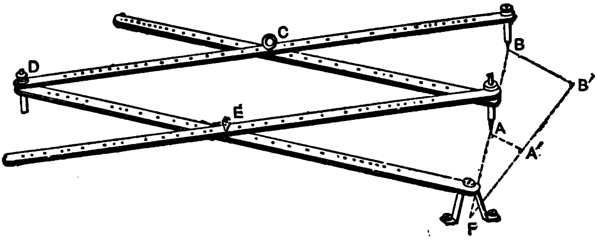

FIG. 480.

(a) *Illustration.* Suppose the parallelogram is adjusted at the points C and E so that $FB = 2FA$. Then if a point is moved along any line AA' the pencil at B will move parallel to it along a line BB' twice the length of AA'. It matters not whether the line AA is straight, curved, or irregular, and hence if the point A is moved over any outline a pencil at B will reproduce a similar outline twice as large. Conversely, if a point at B is moved over any outline a pencil at A will reproduce a similar outline half as large. If A is made the fixed point and either F or B is moved over any outline the remaining point will reproduce a similar outline of the same size, but in an inverted position.

(b) *To set the pantograph.* (1) If the original drawing is to be enlarged, put the pencil at B and the tracing-point at A, and, *vice versa*, if the drawing is to be reduced, put the pencil at A and the tracing-point at B. (2) Whatever the setting, remember that the portion $ACDE$ must form a parallelogram and that F, A, and B must lie in the same straight line. (3) Various pantographs have different systems of marking the arms for purposes of setting. The best test, however, is to measure the distances FA and FB. Thus to enlarge three times, FB must equal $3FA$; to enlarge $1\frac{1}{2}$ times, $FB = 1\frac{1}{2}FA$; to reduce $1\frac{3}{4}$ times, $FB = 1\frac{3}{4}FA$, and so on. (4) To reproduce exact size, make A the fixed point, and $FA = AB$. The pencil may then be put at either F or B.

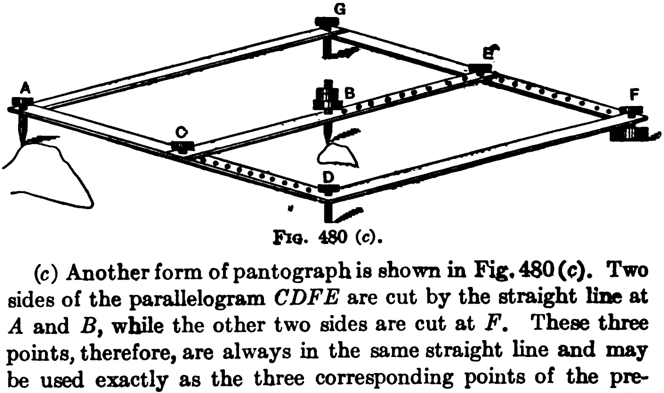

FIG. 480 (c).

(c) Another form of pantograph is shown in Fig. 480 (c). Two sides of the parallelogram $CDFE$ are cut by the straight line at A and B, while the other two sides are cut at F. These three points, therefore, are always in the same straight line and may be used exactly as the three corresponding points of the preceding form are used. The portions EG and GA serve simply to stiffen the arm CA.

(d) *Remark.* In both of the forms shown one of the three working points is at the intersection of two sides of the parallelogram. This is by no means essential so long as the three working points lie in a straight line at the intersection of at least three sides of the jointed parallelogram. It is evident therefore that there may be an indefinite number of different forms of the pantograph. The two shown are inexpensive and are not intended for accurate work. To attain any considerable degree of accuracy one of the expensive metal pantographs must be used.

CHAPTER XXXVII

DRAFTING-ROOM METHODS *

In this chapter general directions and suggestions are given for penciling and inking, for mixing inks, for laying off measurements to scale, for section-lining, for tinting, for tracing, and for making blue-prints.

481. To Fasten the Paper to the Board. Insert a tack in one corner, make the paper square with the board by means of the T-square, stretch it *diagonally* across to the other corner and insert a tack; stretch the paper diagonally in the other direction and insert tacks. It is not essential that the paper should be square with the board, but take pains to stretch it flat and smooth. In large sheets it will probably be necessary to put additional tacks along the edges. Press the tacks down until the heads are flush with the paper, so that the T-square can slide over them; if one should interfere with the work at any time, it may be temporarily removed.

(a) *Note.* In certain kinds of work the best results can be obtained only by stretching the paper while damp. Moisten the whole sheet with the exception of a half-inch border; secure this dry border to the board with mucilage, beginning at the centers of opposite edges and leaving the corners until last; use a sponge in smoothing the paper, working from the center out, and see that the mucilage is set before the body of the paper dries, so that the latter may be stretched uniformly by its own contraction. Cut the paper from the board when the map is finished, following the edges around successively in order.

If possible keep the paper in flat sheets. If kept in rolls it is more difficult to make it lie smoothly on the board.

482. Precautions to Insure Neatness. (1) Wipe the triangles and T-square free from dust before beginning work—occasionally the triangles should be washed. (2) Keep the paper clean by frequently applying a brush or a cloth—after the eraser has been used never proceed with the work until the leavings have been brushed from the surface of the paper. (3) After sharpening a pencil on sandpaper wipe the lead clean with a cloth—keep the sandpaper away from the drawing. (4) Never use a pencil soft enough to rub or blur.

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* This chapter is taken largely from a similar chapter in the author's "An Introductory Course in Mechanical Drawing" (American Book Co.).

483. Penciling. (1) *Use a hard pencil and draw hair-lines.* (2) Draw accurately in pencil, for accuracy is seldom, if ever, increased in the inking in. (3) Do not try to stop pencil lines at exactly the right place, for just as much care is required to make the ink lines stop where they should, whether the pencil lines do or not. It saves time and usually makes intersections and corners more definite to let the pencil lines overrun *slightly*, the projecting ends being erased *after the whole drawing has been inked.* Likewise draw a pencil line a little longer than the distance which is to be marked off on it. (4) It is often better not to draw a construction line throughout its entire length. For example, to find where two lines intersect, draw an inch or so of one line near the estimated point of intersection, and simply cross it with the other. In drawing lines to plot corners of houses or other details the approximate positions of points may be estimated and only so much of each line drawn as is really necessary. Many similar cases occur where time would be wasted and unnecessary erasing caused by drawing entire lines.

484. Inking. (1) As a rule, do not begin to ink until the map is practically completed in pencil. (2) Make the lines wide enough to be clear and distinct—the tendency with beginners is to set the nibs of the pen too close together, making the lines too fine and often ragged. Keep a *separate* piece of paper or tracing-cloth at hand for testing the pen to see if lines are of the right width. (3) Hold the pen nearly vertical, both nibs bearing evenly on the paper. (See p. 440.) (4) Keep the pen sharp and in good condition, so that it will begin to ink a line the instant it touches the paper; no pressure should be required. (5) Do not ink too close to the edge of the T-square or triangle, but place the ruling edge so that it does not quite coincide with the pencil line. When the line is inked, remove the T-square or triangle by drawing it *away* from the line to avoid blotting or blurring. (6) Get into an easy position for inking each line, keeping the ruling edge between the line and the body. (7) Never push a pen backward over a line. It is *difficult to patch* up a ragged line, and it should not be necessary if the pen is in proper condition; occasionally it may be necessary to go over a line a second time, but the pen should be moved *in the same direction as before*, great care being taken not to increase the width of line. (8) Ink in circles and arcs of circles first, as it is easier to draw straight lines tangent to circles than to draw circles tangent to straight lines. (9) Ink in *top* or *left-hand* lines first and work from the top down or from left to right to avoid waiting for lines to dry. In inking in small details, place one triangle so that the lines to be inked lie within the open space in the center; a second triangle can then be laid across the first and used as a ruler in any direction without blurring wet lines. (10) When several lines meet in a point, if possible ink *from* and not *towards* the point, otherwise too much ink will gather at the intersection of the lines. Allow one line to dry before inking another. The same precautions are necessary in inking acute angles. (11) In inking wide lines, as, for example, a heavy border, draw several parallel lines of medium width about their own thickness apart and fill in between with another set of parallel lines after the first are dry. This is better than drawing two outside lines and filling in between all at one time—a method that often leaves a large quantity of ink which warps the paper and takes a long time to dry. (12) Finally, while inking keep the surface of the paper clean, watch for particles of dust or lint in the path of the pen, and clean the pen frequently.

(*a*) *To mix India-ink.* Many draftsmen prefer to mix their own ink from Chinese stick-ink when very fine and sharp lines are required. This is done as follows: Place as little water in the saucer as will suffice for the amount of ink required. Rub the end of the stick on the bottom of the saucer with a rotary motion until the mixture is of the desired density. The liquid will appear black before it really is; to get it just right test it in the inking-pen—it should flow readily and dry quickly; the resulting line should be jet-black, and not easily erased or blurred when dry. If the line fades as it dries add more ink to the mixture from the stick. If it lacks the other desired properties try another make of ink. Chinese inks vary in quality, but it does not follow that the expensive kinds are the best. Try different sticks until one is found which is satisfactory. Wipe this dry after using to prevent it from crumbling, and it will last for a long time. Observe the following precautions: Clean the saucer before using; avoid dust or undissolved particles of ink in the mixture—they clog the pen; keep the mixture covered when not in use.

485. Erasing. (1) Avoid erasing pencil lines until the map is inked. If the eraser must be used, as, for example, to correct the pencil drawing, brush the surface of the paper *clean* before doing anything else. (2) If the eraser does not of itself wear off fast enough to keep clean, rub it occasionally on the drawing-board. (3) Avoid erasing *across* ink lines, rub in *between.* Much erasing with the pencil-rubber will deaden any ink line, destroying the sharp black effect. (4) In erasing ink lines use an ink-eraser, but do not press too heavily on the paper, as it injures the surface for further inking. For the same reason a knife should be used sparingly, though it is useful for taking out short portions of a line to make it "dashed" when it has been inked in full by mistake, or for removing the surface of a blot before applying the eraser. (5) If the surface of the paper has been injured in erasing, rub briskly with a clean hard substance until the paper becomes "polished" and ready for the ink. (6) To erase part of a drawing and leave adjacent portions untouched, cover the part not to be erased with a piece of stiff paper or a triangle. A flat piece of celluloid or cardboard with a narrow slit in it makes an excellent erasing-shield for portions of lines, the rubber being applied through the slit. (7) A sponge rubber or crumbs of wheat bread can be used to clean the drawing when it is finished. (8) For erasing and cleaning tracings, see (12), p. 452.

486. Laying Off Measurements to Scale. (1) Use a needle for pricking off distances. A handle can be easily made of wood or sealing-wax. Make prick-marks small, and draw free-hand circles around them, if necessary, to keep track of the points. (2) Get the eye opposite the mark on the scale before pricking the point. (3) Do not lay off distances with the compasses or dividers when the scale can be used. (4) Avoid arithmetical work; note what the smallest division on the scale represents, and read directly from the scale. (See p. 440.) (5) In laying off several successive distances along a line, set the scale once for all instead of moving the zero-point up each time to the end of a space in order to lay off the next space. In the latter method an error in a single space affects all succeeding spaces, and thus errors will accumulate. For this reason it is better to adopt the system of plus distances in field work. (See note, p. 139.) (6) Ordinarily it is not necessary to plot a point nearer than $\frac{1}{100}$ in. Hence if the scale is 100 ft. to the inch, plot to the nearest half foot; 200 ft. to the inch, plot to the nearest foot, and so on. If the scale is 20 or 30 ft. to the inch, by using the "40" or "60" side the quarter of a foot may be more easily estimated, and thus distances plotted to the nearest *eighth of a foot.* (7) It is more accurate to mark off the radius of a circle on a straight line each side of a point, and open the compasses until the circumference of the circle passes through the extreme points, than to take the radius directly from the scale. (8) Form the habit of frequently checking measurements by the eye—large errors may be detected usually by simple inspection. For small errors apply the scale a second time where there is the least doubt of accuracy. Other ways of applying checks should suggest themselves to one familiar with similar checks used in field work. (See p. 494.) (9) A common mistake is to use one side of a triangular scale when intending to use another. A "scale-guard", costing but a few cents, will keep the scale right side up and help to avoid this common error.

(*a*) *Method of laying off plus stations.* No subtraction is necessary. Suppose a station already plotted is 87+20 and it is desired to lay off the distance to Sta. 90+40, using a scale of 1 in. = 100 ft. Let the zero of the scale represent Sta. 87, and set the mark on the scale which represents 20 ft. from zero at Sta. 87+20 on the paper; then Sta. 88 is at the first inch, Sta. 89 at the second inch, Sta. 90 at the third inch, and 40 ft. beyond the third inch is Sta. 90+40.

A similar method may be used for any scale, although the 100-ft. stations will not fall on the inch-marks to any other scale. Thus if 1 in. = 200 ft., the 100-ft. stations fall at the half-inch marks. If 1 in. = 30 ft., Sta. 88 in the above illustration would fall at the point on the 30 scale marked 10 (see p. 440), Sta. 89 at 20, and Sta. 90 at 30.

487. Section-lining. (1) As a rule draw the section-lines at 45° and from $\frac{1}{16}$ in. to $\frac{1}{8}$ in. apart—the larger the area covered the farther apart the lines. (2) Make the section-lines finer than ordinary lines, usually about as fine as the pen will draw. Ink slightly thinned best serves the purpose. The common tendency is to make section-lines too heavy and too close together. (3) Do not draw the lines in pencil. A draftsman should be able to draw the lines in ink, making the distances apart equal by the eye alone, but the use of one of the standard section-liners lessens the strain on the eyes and is a surer method of securing a uniform appearance.

488. Tinting. (1) Water-colors are used in tinting maps. To mix the tint, place as much water in the saucer as will suffice for the amount of tint required. Transfer tint from the cake to the water with a brush, or rub the cake on the finger-end held in the water until the mixture is of the required density.

Note. Use a camel-hair or sable brush similar to Keuffel and Esser's No. 3121–14. A nest of saucers for mixing different tints will prove convenient.

(2) The surface to be tinted must be clean. Avoid as far as possible all pencil lines within the area to be tinted: if drawn and not erased they can be seen through the tint; if they are erased the surface of the paper is often injured and a smooth, even wash cannot be laid on. It is well to tint before inking; if the drawing is inked first, water-proof ink must be used. The paper, of course, should be stretched flat and tight on the board—if necessary the edges may be glued (see p. 445).

(3) Raise the rear of the board slightly so that the surface will be sloping—if the top of the drawing-table cannot be inclined, block up the board. Fill the brush from the *surface* of the tint to avoid specks in the bottom of the saucer. Small particles of coloring-matter transferred to the paper will dissolve and leave small blots. Apply the brush moderately full of tint to the upper left-hand corner of the area and draw towards the right. This should leave a "pool" of tint extending across the area. This pool should contain enough tint so that it will almost but not quite run down the paper of itself. The secret of success is to *coax* this pool down the paper, keeping it moving constantly by applying the brush back and forth from side to side, not allowing the pool to stand in one place longer than in another. The pool must be kept full by adding tint from time to time with the brush. The tint *must not be painted on;* it is not even necessary to touch the brush to the paper. The whole operation must be done rapidly; it is better to let the brush overrun a little *outside* the outline than to spend so much time on the edges that tint on the interior soaks in or dries in streaks. The tint outside the boundary-line can be removed *when thoroughly dry* by covering the portion not to be erased with a shield and using a pencil-eraser. In laying on the wash do not go over the same place twice to "touch it up"; it will usually make it worse. When the pool reaches the bottom of the area, squeeze the brush comparatively dry, and use it like a sponge for taking up the surplus tint, working rapidly and drawing the brush frequently between the finger and thumb to *discharge the tint.* The whole surface should now have a

uniform flat tint, free from streaks. (4) Better results may often be obtained by first going over the surface with clean water exactly as in tinting. When the surface, held towards the light, ceases to glisten, begin to lay on the tint. The work can be done a little more leisurely and carefully than if the surface were dry, since the tint is not absorbed so quickly by the paper and therefore dries more slowly and evenly. If, on the other hand, the paper is too wet the tint will run. (5) If a deep or very dark tint is desired, it is better to put on several successive layers of a lighter shade, waiting each time for the preceding one to dry, than to attempt a single dark tint at first. The tendency is to make the colors on maps too deep and striking.

489. Tracing. (1) Draftsmen differ as to whether the tracing should be made on the rough or smooth side of the cloth. If much penciling is to be done directly on the cloth, the rough side is preferable. It is easier to draw clean-cut lines on the smooth side and it is also easier to erase them if necessary, though erasures are more noticeable than on the dull side. Tracings will not curl or roll up to such an extent if ink lines are on the dull side, but blue-prints will be slightly clearer if the lines are on the smooth side. When different portions of the map are to be colored with crayon pencils, the ink lines should be on the glazed side and the crayon rubbed on the back or unglazed side. (See p. 507.) (2) Tear off a strip about half an inch wide from each of the two finished edges of the tracing-cloth and then pin it down, stretched tight and smooth as possible. (3) Sprinkle the surface with powdered chalk, and rub it off with a cloth. This is necessary to make the tracing-cloth take the ink, especially if working on the glazed side. Specially prepared chalk in a box with perforated cover may be had from dealers, but dust scraped from ordinary school crayon answers the purpose. (4) Make sure that the ruling-pen is *sharp*—it is more important than ever that the pen should be in good condition. (See p. 441.) (5) Lines, letters, and figures should be made slightly heavier on tracing-cloth than on drawing-paper, in order that the blue-print may be clear. It is not unusual to find a beginner carefully putting on almost microscopic figures with a crow-quill pen. Such fine work practically disappears on the blue-print. *Figures above everything else should be bold and clear*, since they cannot be guessed at. (6) *Erasures* should be made very carefully, using a *sharp* knife or hard-rubber ink-eraser to start with and finishing with a soft-rubber pencil-eraser; too much erasing wears a hole through the cloth. After erasing, the smooth surface should be restored before inking by rubbing with soapstone, otherwise the ink is likely to spread or blot. Many expert draftsmen prefer pumice-powder for making erasures on the glazed side of the cloth. Dip the finger into the powder and rub over parts of the drawing which are to be erased, replacing the discolored powder at frequent intervals until a perfectly clean surface is obtained. Of course a rubber may be used in place of the finger. The advantages of this method are: It is quick, erasures are less noticeable, and the surface is left in good condition for inking. Many draftsmen claim that pumice-powder if vigorously applied is better than prepared chalk for preparing the surface of tracing-cloth to take sharp unbroken lines, and that in addition the gloss is so effectually removed that pencil lines for alterations may be readily drawn on the glazed side. (7) If a considerable area of a tracing needs to be altered, it is possible to cut out a portion of the cloth and insert a new piece in its place, gluing the corners or edges. This can be done very neatly. (8) Tracing-cloth is very susceptible to moisture, and if left on the board for more than a day is liable to become ruffled and uneven from expansion. All important lines, therefore, should be traced the same day, if possible. (9) If tracing-cloth is of poor quality, the ink is liable to strike through and spread; such cloth is useless for tracing. (10) Red ink and colored inks in general do not give pure-white lines on blue-prints, but they are used for dimension lines and other portions of a drawing which it is desirable to render less prominent. Never use anything but black ink for dimensions themselves or for other important data on a map. (11) Tracing-paper is often used, and is very satisfactory though less durable than cloth. Water-proof ink is best for drawing on either. (12) *To clean a tracing*, rub the surface with a cloth saturated with naphtha or gasoline (benzine). This will remove pencil marks and dirt without injuring the cloth or affecting ink lines if the ink is water-proof. It is an excellent method also of freshening the color of old and much-handled tracings.

Remark. Do not allow water to touch the tracing—a single drop will make a spot which will show in the blue-print. Ink lines may be entirely removed from spoiled or useless tracings, however, by soaking or boiling the cloth in water, leaving a clean white linen which is useful for many purposes in the office.

490. Blue-printing. (1) Trim as many sheets of blue-print paper as there are prints required, each sheet slightly larger than the tracing, but not too large to go inside the frame. Keep the sheets away from the light. (2) Turning the frame until the glass side is down, remove the back boards and the pads. (3) Put the tracing in the frame *with the ink lines next to the glass.* (4) Place the blue-print paper *with the yellow or sensitized side next to the tracing.* (5) Insert the pad, taking pains to make the tracing lie flat, and insert the back boards. (6) Turn the frame over; make sure that the tracing is free from wrinkles, and that the ink lines are next to the glass—if they are not words and numbers will read backward. (7) Expose squarely to the sun, making sure that no shadows fall across the glass. (8) After exposure, remove the paper and wash thoroughly in water until the greenish solution entirely disappears. (9) Allow the paper to soak for at least ten minutes—twenty is better—and then hang it up to dry.

(*a*) *Practical suggestions.* (1) The length of time required for exposure varies for different kinds of blue-print paper. In a bright sun with quick paper from 20 to 30 seconds will suffice; for medium paper one to three minutes; for slow paper from three to six minutes. The safest way is to test a new roll by exposing small strips until the correct time is ascertained. The same test should be made when it is cloudy, or when the paper is not exposed to the direct sunlight, for of course the weaker the light the longer the exposure. In an under-exposed print the blue will be too light; in an over-exposed print the blue will be too dark and the lines will appear bluish instead of pure white. (2) To get a good print it is important to keep hairs and specks from being reproduced. The stray hairs from the felt should be carefully dusted from the glass. The paper and the tracing should be in perfect contact, pressed smooth and flat against the glass. If wrinkles appear when the frame is turned over, take the back boards out and fix the tracing-cloth. Sometimes the trouble is caused by too weak or too short springs on the back boards or by too thin a pad between the back boards and the paper. (3) Blue-print paper may be exposed to the light in an ordinary room for several minutes without being affected, but it should not be subjected to strong light through a window. It is safer, especially with very sensitive papers, to pull the window-shade down and work in subdued light while placing the paper in the frame. (4) In washing the print make sure that it is completely immersed, and move it back and forth or splash water over it so that the chemicals will be removed from all parts of the paper as quickly as possible. (5) Keep the tracing where no water can spatter on it, and do not handle it with moist fingers. A drop of moisture on the cloth will make a spot which cannot be taken out, and which will show in every subsequent print. (6) Keep blue-print paper in a dark dry place until wanted for use; it must be fresh to get the best results; with old paper the lines will be gray instead of white. (7) Various white-ink mixtures and bleaching solutions are used for making corrections on blue-prints; a solution of quicklime and water, well shaken, is fairly satisfactory. White lines may also be obtained by using Chinese white water-color, and red lines by using scarlet-vermilion water-color (see 537 (a), p. 507). To obliterate white lines or figures go over them with a blue pencil.

Note. Print-frames can be purchased from dealers in engineers' supplies. Spring clips like Keuffel and Esser's No. 249–3 are useful for hanging up prints.

(*b*) *To prepare blue-print paper.* Prepared paper for blue-prints can be purchased in rolls of different widths at such a low price and of such excellent quality that it hardly pays to prepare it for oneself, unless a large quantity is required. If it is desirable, however, to make it, the chemicals may be used in the following proportions:

1 oz. (weight) prussiate of potash in 5 oz. water;
1 oz. (weight) citrate of iron and ammonia in 5 oz. water.

These two solutions may be prepared separately in the light, but they should be mixed (in equal parts) in subdued light or in a darkened room. Apply an even coating to one side of hard white paper with a soft sponge or a broad thin brush and hang the paper up to dry. The proportions of the chemicals in the blue-print mixture may be varied considerably without any effect except to change the time required for exposure; the larger the proportion of citrate of iron and ammonia the quicker the paper.

Blue-print cloth may be prepared in the same way as blue-print paper.

491. *Brown solar paper* is a process paper for making positive prints, i.e., dark lines on a white background. The process is similar to blue-printing, except that the tracing is used to print a negative on the solar paper, and this negative is used to make positive prints on either solar or blue-print paper.

(*a*) *To make the negative* place the tracing in the frame with the ink lines *against* the process-paper instead of against the glass. Expose until the paper which projects from underneath the tracing is a dark tan color; this will take from 2 to 5 minutes in bright sunlight in summer, nearly twice as long in winter. Wash for at least 5 minutes in clear water and then immerse in a fixing-bath composed of 2 oz. of hyposulphite of soda to one gallon of water. The negative should be left in the bath just long enough for the lines to become white and the background a dark brown; if left too long it may become mottled. Some prefer to apply the fixing solution with a brush, especially if only a few prints are desired. After the negative is fixed it must be washed thoroughly to avoid "hypo" stains; let the print soak in clean water for at least 20 minutes—running water is desirable though by no means essential. Hang the negative up, and when it is practically dry, roll it tightly on a small round stick (such as comes in a roll of tracing-cloth), so that the paper negative will be smooth for printing purposes. Any number of prints can now be made from this negative, while the tracing can be filed away and kept in good condition, one advantage of this process. If a large number of positive prints are required more than one negative can be made, so that several positives can be printed at the same time, another advantage.

(*b*) *To make positive prints* place the negative in the frame with the brown side next to the sensitized side of the printing-paper, not against the glass. To make blue-prints proceed exactly as outlined in Art. 490, except that the negative is used in place of the tracing, the result being blue lines on a white background. By using solar paper instead of blue-print paper, brown lines on a white background may be obtained, but these prints must be treated with a fixing solution as explained above. In either case corrections in pencil or ink can be made on the white background, another advantage as compared with ordinary blue-process paper. Corrections on the negative itself may be made by blocking out unnecessary lines with black ink or by inserting white lines by means of a bleaching solution furnished by the maker.

CHAPTER XXXVIII

METHODS OF PLOTTING ANGLES

Five methods of plotting angles are given in this chapter. The use of these methods in plotting maps is explained more fully in the next chapter.

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492. Five Methods of Plotting Angles. (1) By the *protractor.* (2) By the *tangent* of the angle. (3) By the *cosine and sine* of the angle. (4) By the *chord* of the angle. (5) By *latitudes* and *departures* or *coördinates.*

493. The Protractor Method. There are various forms of protractors in common use. With the ordinary form, angles are laid off as follows: (1) Place the center of the protractor exactly at the point which is to be the apex of the required angle, and at the same time make the zero-point fall on the given line so that the line from the center of the protractor to the zero-point will coincide with one side of the required angle. (2) To get the other side, follow around the circular edge of the protractor the required number of degrees and minutes from zero, and make a fine dot. (3) Draw a line through this dot to the apex. If there are several angles to be laid off from the same line, make all the dots, if possible, before removing the protractor to draw the lines.

(*a*) *Practical Hints.* (1) Make sure of the *exact center* of the protractor once for all, by means of the following test. With a triangle and T-square or with hair dividers construct on paper two intersecting lines exactly perpendicular to each other. Place the protractor so that the 0° and the 180° are on one line and the 90° on the other. Observe carefully what point of the protractor coincides with the intersection of the two lines. This test should always be made when using any protractor for the first time. In some of the semicircular protractors the center is marked by a nick, the bottom of which is often mistaken for the center. The correct and incorrect points are shown in Fig. 493 (*a*).

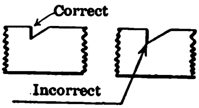

Fig. 493 (*a*).

(2) Accuracy depends largely on the correct centering of the protractor. Thus if in Fig. 493 (*b*) an angle is to be laid off at *B* from a line *AB*, produce *AB* so that the 180° mark can be used in centering the protractor.

(3) On most protractors there are two rows of numbers, the second increasing from 0° to 180° counter-clockwise for use in laying off angles to the left.

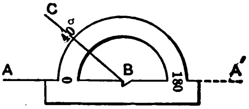

Fig. 493 (*b*).

(4) There are three methods of laying off an angle greater than 180°, viz.: 1. Subtract 180° from the angle, lay off the result in the usual manner, and produce the line thus obtained through the center. For example, to plot 220°, mark off 40° clockwise, as shown in Fig. 493 (*b*), and produce the line thus obtained through *B*. 2. Subtract 180° from the angle, place the protractor on the opposite side of the line, and, considering 180° of the angle to have been plotted by simply producing the given line, plot the remainder with the protractor. 3. Invert the protractor as in the preceding method, but plot 360° minus the angle backward. This last method involves subtracting degrees *and minutes*, and hence mistakes are more likely to occur than if either of the first two methods are used.

(*b*) *Remark.* The ordinary protractor is seldom divided more finely than quarter degrees, and hence anything less than 15 minutes must be estimated. Some of the expensive vernier protractors read to minutes, but the difficulty in plotting to such a degree of precision is in properly centering the protractor. Moreover, unless the circular arc on the protractor is large, errors are likely to occur which render it nearly impossible to plot to minutes with any certainty.

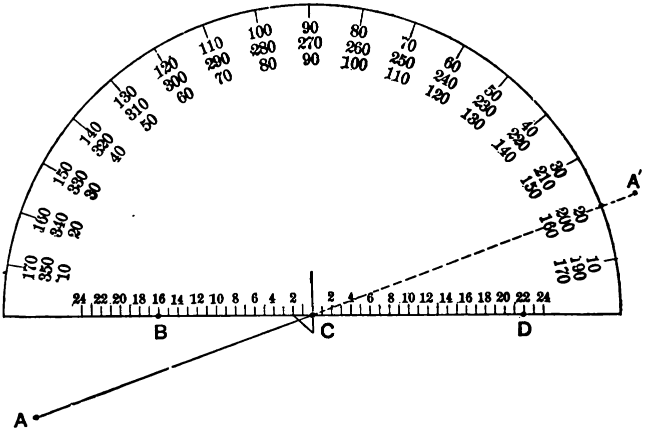

FIG. 493 (*c*).

(*c*) ANOTHER FORM OF PROTRACTOR is shown, much reduced in size, in Fig. 493 (*c*). (On an actual protractor the circle is subdivided to degrees and quarter degrees, but, on account of the small size of the cut, these subdivisions are not shown in the figure.) There are similar protractors on the market, but many draftsmen prefer to make their own as follows: (1) Purchase a *full circle* Bristol board protractor about 14″ in diameter. Rule lines between 0° and 180°, and between 90° and 270°, thus getting the exact center. (2) Along the first of these lines construct a scale 1″ = 50 ft., or 1″ = 40 ft., or whatever scale is to be used in plotting. The zero of the scale is at *the center of the* protractor, the numbers increasing outward in either direction. (3) Number the ten-degree marks as shown in the figure. (4) Cut the circular protractor in two parts along the line from 0° to 180°, leaving, however, a small V-shaped point about $\frac{1}{4}''$ deep as shown in the figure. Better results may be obtained by cutting clear across with a machine such as all bookbinders have; then cut a small depression in the board, under the center, half-way through the back, and gum a thin piece of celluloid into it to form the V. The celluloid may be notched slightly to receive the needle about which the protractor revolves.

(*d*) *To use the paper protractor.* Let it be required to draw a line *CB*, 160 ft. long, and making an angle of 20° clockwise with *CA*. Produce *AC* to *A′*, stick a very *fine* needle at the point *C*, bring the corner of the protractor (center) against the needle, turn the protractor until the 20° mark (outermost row) is on the line *CA′*, and mark *B* opposite 160.

For an angle over 180° use the middle row. Thus the same setting as above gives *ACD* 200° *clockwise*, and a point *D* any distance, say 220 ft., from *C*.

If angles *counter-clockwise* up to 180° are desired, use the innermost row. Thus the same setting gives *ACD* *counter-clockwise* = 160°.

(*e*) *Remark.* This form of protractor is exceedingly useful in plotting details where a large number of angles and distances are laid off from the same point, for it serves both as protractor and scale. It is necessary, however, to construct a separate protractor for each different scale, but this is not a serious drawback, since in ordinary work less than half a dozen scales are commonly used.

Notice that the required side of the angle is always found along the straight edge of the protractor. The fact that this straight edge is used as a scale offsets the disadvantage of resetting the protractor for each angle.

(*f*) *Practical suggestions.* (1) Fine hair lines should be ruled for the scale. Do not cut the Bristol board in two until these lines have been penciled and inked, if it is desired to ink them. Make the lengths of marks correspond to those on a regular scale. (See p. 440.) (2) If only clockwise angles are to be plotted, it is better to omit the innermost row of numbers, otherwise print them in red ink to avoid confusion with the other two rows which may be printed in black. If counter-clockwise angles larger than 180° are to be plotted, a fourth row of numbers may be added. (3) Instead of sticking a needle into the paper it is better to construct a goose-neck device like that shown in Fig. 493 (*f*), the space between the needle-point and the weight being large enough to permit the protractor to revolve. The needle-point just touches the paper, and can be moved to any desired position by simply sliding the goose-neck. The device is easily made of brass tubing, filled with lead just over the center to increase its stability, and with a wooden plug to hold the needle.

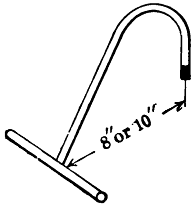

FIG. 493 (*f*).

494. The Tangent Method is a trigonometric method based upon the fact that in any right-angle triangle the *base* × *tangent* = *perpendicular*. The steps are as follows: (1) Mark off on the given side of the angle, beginning where the apex is to be, a base of any convenient length, say 8″ or 10″ or 12″, and at the end of the base erect a perpendicular extending toward the required side of the angle. (2) In a table of natural tangents look up the tangent of the required angle and multiply this tangent by the length of the base, 8″ or 10″ or 12″ as the case may be. (When there is room the base is usually made ten inches so that the multiplication can be performed by changing the decimal point one place to the right.) (3) Mark off the number of inches obtained from (2) on the perpendicular erected in (1) and join the point thus found with the apex.

(*a*) When an angle is much over 45° the tangent becomes too long and the complement of the angle is plotted instead of the angle itself. If the angle is a little over 90°, plot 90° of it by erecting a perpendicular and the balance by the tangent method, *using the perpendicular as a base.* In a similar manner any large angle can be plotted by laying off a portion of it either by erecting a perpendicular to the given line or by producing that line, so that the part of the angle remaining to be plotted by the tangent method is less than 45°.

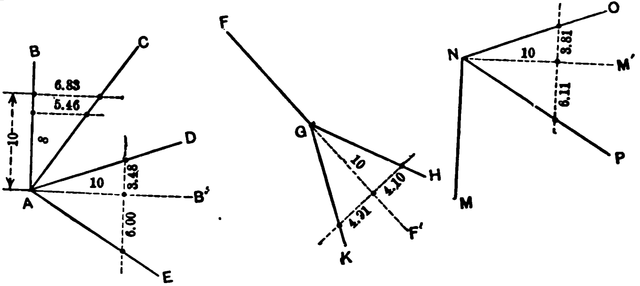

FIG. 494.

Required Angle	Angle Plotted	Tangent
$BAC = 34^\circ\ 20'$	$BAC = 34^\circ\ 20'$	0.683
$BAD = 70^\circ\ 49'$	$B'AD = 90^\circ - 70^\circ\ 49' = 19^\circ\ 11'$	0.348
$BAE = 120^\circ\ 58'$	$B'AE = 120^\circ\ 58' - 90^\circ = 30^\circ\ 58'$	0.600
$FGH = 157^\circ\ 42'$	$F'GH = 180^\circ - 157^\circ\ 42' = 22^\circ\ 18'$	0.410
$FGK = 206^\circ\ 09'$	$F'GK = 206^\circ\ 09' - 180^\circ = 26^\circ\ 09'$	0.491
$MNO = 249^\circ\ 08'$	$M'NO = 270^\circ - 249^\circ\ 08' = 20^\circ\ 52'$	0.381
$MNP = 301^\circ\ 26'$	$M'NP = 301^\circ\ 26' - 270^\circ = 31^\circ\ 26'$	0.611

The above examples indicate how the method is used. Notice that in every case except the first the angle actually plotted was not the required angle, but the *smallest* angle that could be found with the required side either by producing the given side or by erecting a line at right angles to it.

(*a*) *Practical suggestions.* (1) The accuracy of this method depends very largely upon the correct laying off of perpendiculars. If a triangle is used it should be tested for its right angle. (See p. 439.) In using the triangle to erect a perpendicular it is better to resort to one of the sliding methods (p. 439) than to depend on fitting two sides of one triangle to a 90° angle. A steel square, however, may be used in that way, and is preferable to a triangle, but it also should be tested.

(2) The inches laid off for the base and perpendiculars are of course *real* inches and not to any scale. As scales are not usually divided to hundredths of an inch, use the side with fiftieths. Thus in the first illustration 83 hundredths equals 41½ fiftieths.

(3) If there is not room for a 10″ base or its corresponding tangent, use a shorter base, say 5″ or 8″, *but be sure to use the corresponding length* for the perpendicular. Within reasonable limits, however, the longer the base the more accurate will be the angle, hence a 20″ base is sometimes used.

(4) After plotting an angle by the tangent method always check it roughly with the protractor.

(5) If an angle is known to be wrong and no mistake can be found in the work, plot the angle over again, using a *new* length of base and its corresponding length of perpendicular. In addition it may be well to plot from some other direction, i.e., plot the complement or supplement of the angle if possible, thus using an entirely different base line. This is usually equivalent to checking by measuring the cotangent.

(6) This same method can be used in the field for laying off angles with the tape. Measurements are then in feet instead of in inches.

495. The Cosine and Sine Method is practically a modification of the tangent method. The cosine of an angle is laid off as a base, and the sine as the perpendicular. The hypothenuse is then unity—a useful check.

Illustration. To plot 34° 20′. Cosine = 0.826, sine = 0.564. Lay off 8.26 inches as a base and 5.64 inches as a perpendicular. Check by seeing if the hypothenuse is exactly 10 inches.

Note. When an angle is much larger than 45° its complement or its supplement may be plotted as in the tangent method. Most of the practical suggestions given for the tangent method apply to this method also.

496. The Chord Method. In this method the angle is constructed by two intersecting arcs. The first arc is of any convenient radius, say 5″ or 8″ or 10″, and is described with the *vertex* of the required angle as a center. The length of the radius of the second arc must be equal to that chord of the given angle which corresponds to the radius of the first arc.

(*a*) *Illustration. At A to plot an angle of* 47° 8′ *clockwise from AB.* With *A* as a center describe an arc *CD* of any convenient radius, say 10″. The chord for 47° 8′ for a unit radius is 0.7996 (found by one of the methods given below). For 10″ the chord is 7.996″, or for practical purposes 8″ long. With *C* as a center and with a radius of 8″, describe a second arc *FG* intersecting the first at *D*, a point which determines the angle.

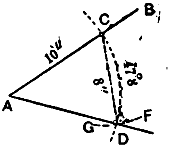

FIG. 496 (a).

(*b*) *To find the chord length.* Take from a table of chords the chord for the required angle for a *unit length* and multiply by the length of the radius of the first arc. If a table of chords is not at hand look up in a table of natural sines the sine of HALF the required angle and multiply by TWICE the length of the radius of the first arc. (For the chord of any arc is equal to the radius of that arc multiplied by twice the sine of half the subtended angle, or $chord = radius \times (2 \times sine\ \frac{1}{2}\ angle) = 2 \times radius \times sine\ \frac{1}{2}\ angle$. Draw a figure showing this to be true.)

Illustration. In Fig. 496 (*a*) one half the required angle is 47° 8′ ÷ 2 = 23° 34′ natural sine of 23° 34′ = 0.39982, and 20 × .39982 = 7.9964 = length of chord. (8″ is near enough.)

(*c*) *Practical suggestions.* (1) The accuracy of the work depends upon the precision with which the arcs can be drawn. An arc of 10″ radius can be drawn with ordinary compasses if the lengthening bar is used, but it takes time to set the compasses at *exactly* 10″, and it is sometimes difficult to keep them there until the whole arc is completed. A better way is to keep beam compasses set at *exactly* 10″, and to use them only for drawing the *first* arc of each angle. Then, instead of describing a second arc, find the chord by trial with a scale; place the zero of the scale where one end of the chord must be, and knowing the length of the chord, fit the scale in place and thus find the point on the first arc where the other end of chord must be. If beam compasses are not at hand, use a straight stick with a needle at one end for a pivot, and a *very hard* lead at the other, sharpened to a *fine* point. Be sure that the distance between the needle and the point of the lead (which should be *exactly* 10″) cannot change while describing an arc. By exercising a little ingenuity beam compasses of this kind can be made with a device for adjusting the lead slightly to keep it exactly 10″ from the needle-point.

(2) If a radius of 5″ is used the natural sine of half the angle is multiplied by 2 × 5 = 10, which simply changes the decimal point. The objection to this, however, is that for accurate work the first arc should have a radius greater than 5″. If 8″ or 9″ or 12″ is used it is more convenient to multiply the sine by two and then by the radius 8″ or 9″ or 12″, as the case may be. When there is room, however, use a radius of 10″, multiply the sine by 2, and change the decimal point one place. If the fiftieths on the scale are used it is not necessary to multiply the sine of half the angle by two; simply change the decimal point *three* places, call the result fiftieths, and read directly from the scale, adding of course a cipher to the number printed on the scale.

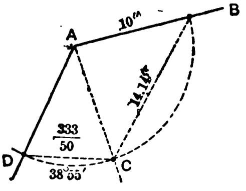

FIG. 496 (*c*).

Illustration. At A to plot an angle of 128° 55′ *clockwise from AB.* From *A* as a center describe an arc of 10″ radius. Plot 90° of the required angle by erecting the perpendicular *AC*. Half of the remaining angle is (128° 55′ − 90°) ÷ 2 = 19° 27.5′. The sine of 19° 27.5′ is 0.33312. Hence with the zero of the scale at *C* find the point *D* where the 333 mark (fiftieth) comes, i.e., the required chord is $0.333 \times 2 \times 10 = \frac{666}{100} = \frac{333}{50}$. (This is really 6.66″.)

(*d*) *Large angles.* The largest angle that can be laid off, *using* the ordinary scale, is 73° 46′, for the sine of half that *angle is: sine of* 36° 53′ = 0.60019, and hence the chord equals

600 fiftieths or the limit of a 12″ scale. For any angle between 73° 46′ and 90°, erect a perpendicular and plot the complement of the required angle. In a similar manner any angle, no matter how large, can be plotted by either erecting a perpendicular if the angle is nearest 90° or 270°, or by producing the original line if the angle is nearest 180°. See Fig. 494, p. 458; for parallel cases.

(*e*) *Erecting perpendiculars.* If two sides of a right triangle are each 10″ long, the hypothenuse is 14.142″ long, i.e., the chord of 90° is 1.4142. Hence, if desired, perpendiculars can be erected by the chord method, as indicated in Fig. 496 (*c*). Keep a straight edge with two fine scratches 14.14″ apart especially for that purpose. It is convenient to have these marks on the beam compass or on the straight stick used in place of the beam compass.

497. The Latitude and Departure Method consists in plotting points by coördinate distances corresponding to latitude and departure. While it is seldom used for laying off a single angle, it is of great value in plotting a series of connected lines, especially if they form a polygon, as, for example, the transit lines of a survey which closes. This method will be explained in detail in the next chapter.

498. Methods Compared. (1) *The ordinary protractor* should not be used for plotting lines much longer than the radius of the protractor, if more than medium accuracy is required. With the expensive vernier protractors angles can be laid off to the nearest minute, but ordinarily fifteen minutes is as close as the protractor can be read. The protractor is used for plotting short lines to relatively unimportant points, as, for example, in plotting the angles at a transit station to the corners of buildings, fences, and other points a short distance away. For the transit lines some other method is usually used unless a vernier protractor is at hand.

(2) *The tangent method,* (3) *the cosine and sine method,* and (4) *the chord method* are all good methods, and each has its advantages over the others. In the first two methods the difficulty lies in quickly and accurately erecting perpendiculars; in the chord method it lies in drawing arcs correctly and making their intersection perfectly definite. All three methods require the use of trigonometric tables, with the attending danger of blunders in looking up the values of functions.

(5) *Latitude and departure method.* See p. 476 for advantages and disadvantages.

499. The Three-point Problem. Given three points, A, B, and C, whose relative positions are determined by the two distances n and o and the *interior* angle M. Let N and O represent the values of the angles at P, a point to be plotted.

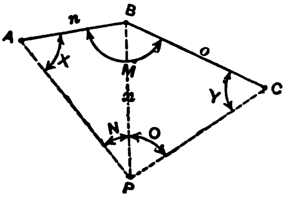

FIG. 499.

(*a*) *Graphic method.* (1) Plot the two angles N and O on tracing-cloth, making the lengths of the lines PA, PB, and PC indefinite. (2) Shift the tracing-cloth over the drawing until each of the three lines passes through the plotted position of the corresponding point. (3) Prick through the point P.

A three-arm protractor should be used in place of the tracing-cloth if there are many points to be located. Each arm is set to correspond to a line radiating from P.

(*b*) *Plane-table methods.* (See pp. 329 to 336.)

(*c*) *Algebraic method.* Let $R = X + Y = 360° - (M + N + O)$
Then

$$x = \frac{n \sin X}{\sin N} = \frac{o \sin Y}{\sin O},$$

$$Y = R - X,$$

$$\sin Y = \sin R \cos X - \cos R \sin X,$$

$$\cot X = \cot R \left(\frac{n \sin O}{o \sin N \cos R} + 1 \right).$$

The angle Y may be found in a similar manner. Plotting these two angles, the lines AP and CP intersecting determine P.

(*d*) *Note.* The problem is indeterminate when A, B, C, and P are on the circumference of a circle, i.e., when $M + N + O = 180°$. When $N + O = M$ or nearly, the solution is impossible.

CHAPTER XXXIX

METHODS OF PLOTTING TRAVERSES

For every method of running traverses explained in Chapter XII, p. 116, there is a corresponding method of plotting. Distances between stations are laid off to scale and angles or bearings are plotted by one of the methods given in the preceding chapter. The different steps for each method are outlined in this chapter, without entering upon the practical details of mapping. Such questions as the choice of scale, the general arrangement of the map, the accuracy required, and the best methods of plotting details are discussed in the next chapter.

500. Different Methods of Plotting Traverses, corresponding to the methods of running transit lines given on p. 119, are:

I. 1. *Plotting each traverse line by a* DIRECT ANGLE,
2. *Plotting each traverse line by a* DEFLECTION ANGLE.

II. 3. *Plotting each traverse line by its* AZIMUTH,
4. *Plotting each traverse line by its* BEARING.

In addition there is a fifth method:

5. *Plotting traverses by* COÖRDINATES OF LATITUDES AND DEPARTURES.

In each of the first four methods the angle at each station may be plotted by means of the *protractor*, or by the *tangent method*, or by the *chord method*, or by the *cosine and sine method*.

501. Importance of Traverse Lines. Since the traverse lines form the basis of the whole map, they must be plotted with the greatest care, using fine hair lines. On a small scale map, say 1″ = 200 ft., a line drawn with a soft or poorly sharpened pencil may easily be from one to two feet wide measured to scale, hence use a 6-H pencil sharpened to a *fine* wedge, and a needle to prick the points.

Where a high degree of precision is required *an ordinary protractor should not be used for plotting traverse lines.* It is inconsistent to measure angles in the field to the nearest minute, and then to plot them with a protractor which reads only to half degrees, especially if the lines between stations on the map are more than, say, five inches long. Of course there are many cases where the protractor will give results consistent with the accuracy required, but its chief use is in plotting details, and in roughly checking angles plotted by some other method. Even with vernier protractors it is difficult to attain a high degree of precision, though angles can be plotted very rapidly with it, and much more accurately than with an ordinary protractor.

502. The Use of the Protractor in plotting direct angles, deflection angles, bearings, and azimuths requires no explanation in addition to that given on p. 455. The line from which the angle is plotted should always be extended beyond the station to aid in centering the protractor (see p. 455), and occasionally a line at right angles is drawn in order that the 90° mark on the protractor may be used for centering in addition to the 0° and 180° mark.

Remark. In studying the remainder of the chapter keep in mind the fact that wherever the tangent method or the chord method is used for plotting an angle, the protractor can be used also, provided a high degree of precision is not required. Likewise the cosine and sine method can be used if desired in place of the tangent method, but the two are so nearly alike that the former is not explained in this chapter.

503. Plotting Each Transit Line from the Preceding Transit Line. This method consists simply in plotting the angle at each station by the protractor method, by the tangent method, by the chord method, or by the cosine and sine method, the distance between stations being laid off with a scale. When the angle is a large one it may be necessary to plot its complement or supplement as explained on page 458.

In order to start the map some traverse line must be drawn from an assumed point, and its length laid off to scale. The direction in which it is drawn and the position of the assumed point depend upon the general arrangement of the entire map. *(See p. 490 for* directions for starting the map.)

(a) TANGENT METHOD

TABLE I

Angle	Value	Line	Length	Angle Plotted	Tangent
FAB	56° 44′	*AB*	404	90° − 56° 44′ = 33° 16′	0.6560
ABC	258° 30′	*BC*	312	270° − 258° 30′ = 11° 30′	0.2034
BCD	31° 56′	*CD*	550	31° 56′	0.6232
CDE	150° 36′	*DE*	386	180° − 150° 36′ = 29° 24′	0.5635
DEF	67° 48′	*EF*	407.8	90° − 67° 48′ = 22° 12′	0.4081
EFA	154° 26′	*FA*	454.2		

Illustration. Fig. 503 (*a*). Before beginning to plot, Table I was prepared; the first four columns were taken from the field notes, the fifth column gives the angles to be actually plotted, and the last column gives the corresponding *natural* tangents. Thus the draftsman has before him all the data necessary to plot the lines. In preparing the fifth column the fact was kept in mind that the tangent method is best adapted to plotting angles of 45° or less. (See p. 458.) Hence at *A* and *D complements* of the angles taken in the field were used, at *D* the *supplement*, at *B* 270 *minus the angle*, but at *C* the original angle was used, since it was less than 45°. In the original drawing from which the figure was reduced the work was as follows: Since no bearings were given, the line *AF* was assumed in any convenient direction and its length laid off to scale. A perpendicular at *A* was erected and the complement of *FAB* = 90° − 56° 44′ = 33° 16′ was plotted. Ten *real* inches were used as a base and from Table I 6.56″ was the length of the tangent. Thus a point on *AB* was found, *AB* drawn, and its length laid off to scale. The angle at *B* is nearly 270°, hence a perpendicular was erected at *B*. 270° − 258° 30′ = 11° 30′, and since 258° 30′ is *less* than 270° the angle 11° 30′ must be plotted backwards or counter-clockwise from the perpendicular. The dotted lines indicate how the tangent method was used for plotting the remaining angles. The plot closes if the last line passes through *F*, *and the distance EF scales correctly*. If another check is desired the angle at *F* may be tested by producing *EF* and measuring the perpendicular at a point 10″ from *F*. In this case the distance *EF* came out one foot too long to a scale of 1″ = 50′ 0″. The tangent measured 4.78″, which corresponds to 25° 32′, to 25° 33′, and to 25° 34′. The correct angle is 180° − 154° 26′ = 25° 34′.

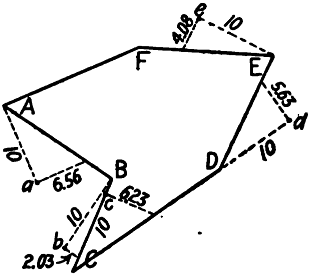

FIG. 503 (*a*).

Remark. As pointed out on p. 458, although any base may be used, 10″ is the most convenient length.

(b) CHORD METHOD

TABLE II

Angle	Value	Line	Length	Angle Plotted	Angle ÷ 2	Sine	Chord
FAB	56° 44′	*AB*	404	56° 44′	28° 22′	.4751	9.50
ABC	258° 30′	*BC*	312	11° 30′	5° 45′	.1001	2.00
BCD	31° 56′	*CD*	550	31° 56′	15° 58′	.2750	5.50
CDE	150° 36′	*DE*	386	29° 24′	14° 42′	.2538	5.08
DEF	67° 48′	*EF*	407.8	67° 48′	33° 54′	.5578	11.16
EFA	154° 26′	*FA*	454.2	25° 34′	12° 47′	.2213	4.43

Illustration. Before beginning to plot, Table II was prepared; the first four columns were taken from the field notes and the last four were added for purposes of plotting. In preparing the fifth column the fact was kept in mind that in the chord method it is more convenient to use angles less than 73° (p. 460). Thus at D the *supplement* of the angle taken in the field was used, while at B 270° *minus the angle* was the most convenient angle to plot. At A, C, and E the original angles are less than 73°. If a table of chords is at hand the sine column is unnecessary.

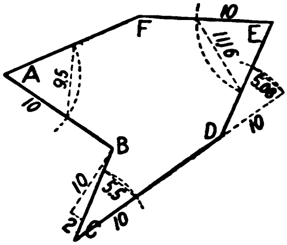

FIG. 503 (*b*).

The line AF was assumed to start with. An arc of 10″ radius described from A as a center, and the corresponding chord 9.5″ (from Table II), gave a point on AB. The distance to B was laid off to scale, a perpendicular at B was erected, the angle 11° 30′ (270° − 258° 30′, see Table II) was plotted, and the direction of BC was thus obtained. The dotted lines in the figure indicate how the arcs and chords were used in a similar way to plot the remaining angles. If the last line passes through F and *the distance* EF is correct, the plot closes. The angle at F can be measured if an additional check is desired. The distance EF came out a little less than 407 instead of 407.8 to a scale of 1″ = 50′ 0″. The chord measured 4.43, showing the angle to be correct.

Remark. Any radius may be used in the chord method, but 10″ is the most convenient length, especially when the fiftieths on the scale are used as suggested on page 460.

(*c*) *Checks.* Never lay off an angle by the tangent or chord method without checking it roughly with the protractor. When a traverse is plotted by direct angles an excellent check is to compute the bearings of all the lines, and at every fourth or fifth station measure the angle of bearing of the line that starts from that station. If the survey is a closed traverse, it is only necessary to check this angle roughly with the protractor, otherwise check it carefully by the tangent or chord method. (See p. 494 for additional directions for checking traverses.)

Illustration. Let the bearing of AB = S 55° 0′ E. The calculated bearing of DE would be N. 26° 02′ E. Draw a meridian at A making 55° 0′ with AB. Through D draw a parallel meridian, and see if the angle which it makes with DE is 26° 02′

Remark. A check like the above is more necessary in a survey having a large number of stations.

504. Plotting by Deflection Angles. This method is a modification of the preceding method. It corresponds to the method of running transit lines given on page 120, and is used for lines which progress more or less in one direction, as in railway work. The deflection angle at each station is plotted by the protractor method, by the tangent method, or by the chord method, preferably the latter. A mistake to be avoided is that of plotting the angle on the wrong side of the transit line produced. As some angles are measured to the right and *some to the left,* this mistake is likely to occur.

(a) CHORD METHOD

TABLE III

Back-		Deflection Angle							
sight.	Sta.	Right	Left	Line	Length	Ang. Plot.	Ang. + 2	Sine	Chord
				AB	210.1				
A	*B*	34° 18′		*BC*	255.6	34° 18′	17° 09′	.2948	5.90
B	*C*		64° 20′	*CD*	321.1	64° 20′	32° 10′	.5324	10.65
C	*D*	76° 48′		*DE*	342.8	13° 12′	6° 36′	.1149	2.30

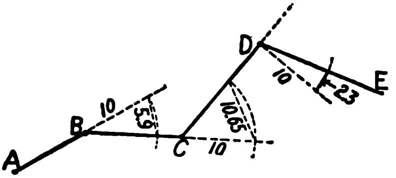

FIG. 504 (*a*).

Illustration. The first six columns in Table III are the field notes. The last four were added for purposes of plotting. Fig. 504 (*a*) studied in connection with these last four columns will illustrate the method of plotting when chords are used.

(*b*) *Checks* are the same as those given for the preceding method. As this method is used most often for surveys that do not close, lines should be checked very carefully by calculated bearings at, say, every fourth station.

505. Plotting by Bearings. This method consists in plotting each line by laying off its angle of bearing from a line assumed as a north-and-south meridian. The protractor, the chord method, the tangent method, or the cosine and sine method may be used for laying off the angle of bearing.

Note. Except in compass surveying *the bearings laid off are always calculated bearings*, found by the method explained in Chapter XXX. Sometimes these bearings are calculated from the *true* bearing of some traverse line, sometimes from the *magnetic* bearing. When no bearings are given, the bearing of some traverse line is assumed, and the corresponding bearings of the others calculated. These bearings, though fictitious, will give the true *relative* positions of the different stations.

Any adjustments of angles which may be necessary (see p. 478) should be made before the bearings are calculated, and the adjusted values used in the calculations.

(1) There are two general methods of procedure. In the first, all bearings are plotted from the same line and afterwards transferred. (2) In the second, lines parallel to the assumed meridian are drawn through all the stations, and the bearing of each traverse line is plotted from the meridian at the station from which the traverse line starts. In either method it is convenient to assume the meridian at right angles to the T-square, i.e., a vertical line, with the north point at the top of the map.

Sometimes, however, it is desired to assume the meridian in some other position, thus making the east-and-west line oblique to the T-square. In such cases it is convenient to use a T-square with an adjustable head so that the edge of the blade may be made to concide with an east-and-west line when the head is in contact with the left-hand edge of the board. (See *Caution*, p. 473.)

The first method (plotting lines from the same meridian) will be explained in detail, both for the tangent and the chord method of plotting bearings.

(*a*) TANGENT METHOD OF PLOTTING BEARINGS

Construct near the center of the paper a 20″ square and subdivide it into four 10″ squares. This must be done with accuracy, and the large and small squares tested by measuring the diagonals. There is no use in proceeding farther until the diagonals of the 20″ square are equal, the sides each 20″ long, and the diagonals of all the 10″ squares equal to each other. Assuming the vertical line through the center of the 20″ square as a north-and-south line, each of the 10″ squares will correspond to one of the four quadrants of bearing, N.E., S.E., S.W., and N.W. Plot the bearing of each line of the survey by measuring ten times the natural tangent of the bearing (or if the bearing is over 45°, the complement of the bearing) along a side of that 10″ square which corresponds to the quadrant of bearing. The *direction* of each line of the survey thus obtained may be transferred to its proper place in the plot by any of the accurate methods of drawing one line parallel to another (p. 439).

Remark. **Some other length than 20″ could be used for the side of the large square, but, as pointed out on page 458, 10″ is the most convenient base by which to multiply the natural tangent.**

Illustration. In Fig. 505 (*a*) the transit lines of page 465 are plotted from bearings using the tangent method. The bearing of *AB* if not known may be assumed as any convenient bearing, and the bearings of the other lines calculated from the transit angles (Chap. XXX); but in that case while the outline will be correct, none of the transit lines will be in its really correct position with respect to a north-and-south line. In the illustration, however, the bearing of *AB* was known to be S. 55° E. The following table was prepared for plotting.

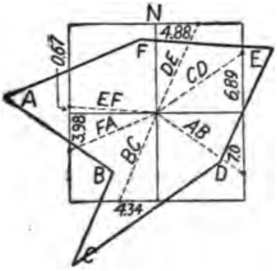

Fig. 505 (a).

TABLE IV

Line	Bearing	Length	Angle Plotted	Tangent
AB	S. 55° 0′ E.	404	E. 35° 0′ S.	.7002
BC	S. 23° 30′ W.	312	S. 23° 30′ W.	.4348
CD	N. 55° 26′ E.	550	E. 34° 34′ N.	.6890
DE	N. 26° 02′ E.	386	N. 26° 02′ E.	.4884
EF	N. 86° 10′ W.	407.8	W. 3° 50′ N.	.0670
FA	S. 68° 16′ W.	454.2	W. 21° 44′ S.	.3986

The letters in the fourth column tell how to lay off the tangent. Thus E. 34° 34′ N. means to measure the corresponding tangent (0.689) from E. towards N.

Each dotted line in Fig. 505 (*a*) was determined by the natural tangent of the corresponding angle in the fourth column of the table. Thus *AB* is E. 35° S., hence measure down 7.0″ along the right-hand side of the 20″ square, beginning at the middle point, which represents the east. *BC* is S. 23° 30′ W., hence from the middle of the lower side measure 4.34 (or better 4.35) westward. In this manner the *directions* of all the lines in the survey are ascertained.

Each line in the perimeter of the polygon may now be drawn parallel to the corresponding dotted line. A starting-point *A* having been assumed (see p. 491), *AB* was drawn parallel to the corresponding line in the S.E. quadrant, and 404 feet to scale gave *B*. Through *B*, *BC* was drawn parallel to the corresponding line in the S.W. quadrant, and 312 feet to scale gave *C*. The final position of each transit line was determined in a similar manner.

(*b*) *A modified method of plotting bearings by natural tangents* is to draw near the center of the paper a 10″ square instead of a 20″ square, using the right-hand side for a north-and-south line when a bearing is N.W. or S.W. and the left-hand side of the square for a north-and-south line when a bearing is N.E. or S.E. Thus one square is made to answer for all four quadrants of bearing.

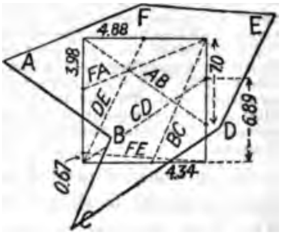

FIG. 505 (*b*).

(*c*) CHORD METHOD OF PLOTTING BEARINGS

Proceed as follows: (1) Near the center of the paper draw a circle exactly 20″ in diameter. (2) Through its center draw a north-and-south line and an east-and-west line. (3) Make sure that these two lines are mutually perpendicular,—the four chords should be exactly equal to each other and each equal to 14.14″. (4) Prepare a table similar to that below. (5) Plot each line in its proper quadrant by using the chord value of its bearing. (6) Transfer each line in order to its proper position in the plot by any of the accurate methods of drawing one line parallel to another.

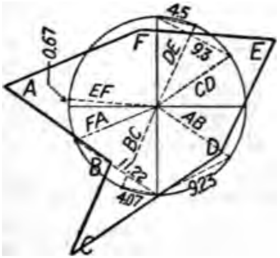

FIG. 505 (*c*).

Remark. A circle of some other diameter could be used, but 10″ is the most convenient radius, especially if the fiftieths on the scale are used as suggested on p. 460.

Illustration. In Fig. 505 (*c*) the same survey as plotted in the preceding examples is shown. If no bearings were taken in the field, one may be assumed for one of the lines and the bearings of the other lines calculated. In that case none of the lines will be in its correct position, but the outline of the figure will be correct, which in many cases is all that is wanted. In the illustration, however, *AB* was known to be S. 55° E. The following table was prepared for plotting.

TABLE V

Line	Bearing	Length	Angle Plotted	Angle ÷ 2	Sine	Chord
AB	S. 55° 0′ E.	404	S. 55° 0′ E.	27° 30′	.4617	9.23
BC	S. 23° 30′ W.	312	S. 23° 30′ W.	11° 45′	.2036	4.07
CD	N. 55° 26′ E.	550	N. 55° 26′ E.	27° 43′	.4651	9.30
DE	N. 26° 02′ E.	386	N. 26° 02′ E.	13° 01′	.2252	4.50
EF	N. 86° 10′ W.	407.8	W. 3° 50′ N.	1° 55′	.0334	0.67
FA	S. 68° 16′ W.	454.2	S. 68° 16′ W.	34° 08′	.5611	11.22

The dotted lines radiating from the center of the circle in Fig. 505 (*c*) were plotted by laying off the chords given in the last column of the table. Thus *AB* is S. 55° 0′ E., hence from the lower end of the vertical diameter lay off a chord easterly of 9.23″. *BC* is S. 23° 30′ W., hence lay off a chord from the same point westerly of 4.07″. In this manner the directions of all the lines in the survey were determined.

Each line in the perimeter of the polygon was then drawn parallel to the corresponding dotted line. A starting-point *A* having been assumed (see p. 491), *AB* was drawn parallel to the corresponding line in the S.E. quadrant, and 404 feet to scale gave *B*. Through *B*, *BC* was drawn parallel to the corresponding line in the S.W. quadrant, and 312 feet to scale gave *C*. The final position of each transit line was obtained in a similar manner.

(*d*) *A modified method of plotting bearings by chords* is shown in Fig. 505 (*d*). The four quadrants occupy the same space, otherwise the method is unchanged. This method may be used when a 20″ circle will not come within the limits of the paper and it is not desirable to use a radius of less than 10″. The dotted lines in the figure correspond to the transit lines in Fig. 505 (*c*).

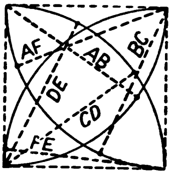

FIG. 505 (*d*).

(*e*) *A second method of plotting bearings by tangents or chords.* In this method instead of plotting the bearings of all the transit lines from *one* north-and-south line, the bearing of each line is plotted from a north-and-south line drawn through the station at the beginning of the line. Otherwise the method is the same *the preceding* one. In this method errors are less likely to occur in drawing parallel lines, but more work is involved in the plotting.

Another method sometimes used for large maps is to draw a rectangle which shall include as much as possible of the map and at the same time be divisible into ten-inch squares. The bearing of a line starting from any station may then be laid off in the square nearest the station and afterward transferred to the station. If the outside rectangle is carefully checked and the squares are laid off entirely by measurement, the errors due to the use of T-square and triangles are avoided. (See *Caution*, p. 473.)

(*f*) *Checks.* The angle at each station should be checked roughly with the protractor, or if the survey is not a closed traverse, more accurately by the tangent or by the chord method. When lines have been plotted by bearings there is no check which, if applied at any station, will check all of the previous courses. This is one objection to this method of plotting, since if each angle must be checked by some accurate method, that method might as well be used to start with. The method of plotting by bearings is best suited, therefore, for closed traverses where it is sufficiently accurate to check angles with the protractor. (See p. 494 for additional directions for checking traverses.)

506. Plotting by Azimuths. The method is practically the same as that given for plotting bearings. The only difference is that the angle plotted at each station is the *azimuth* of the next line instead of the bearing. The checks are similar to those given above.

507. Plotting Transit Lines by Latitudes and Departures. This method consists in selecting one of the stations as a reference-point (origin) and plotting each of the other stations by its total distance *east* or *west*, and its total distance *north* or *south* of the reference-point (origin). These two coördinate distances by which each station is plotted are obtained from the latitudes and departures. For purposes of plotting, it makes no difference which station is chosen for a reference-point; it is well to select the *most westerly* station, especially as this is done when areas are computed from the latitudes and departures.

Note. Except in compass surveys the bearings used in computing latitudes and departures are always *calculated* bearings, based upon a magnetic, a true, or an assumed meridian, and upon the adjusted or final values of traverse angles. (See note, p. 467.)

Latitudes and departures may also be calculated directly from azimuths (see p. 387), and the above holds true for azimuths as well as for bearings.

(*a*) *To determine which is the most westerly station:* This can usually be done by inspection of the bearings. Start with any line and follow around the survey until the station is reached where the bearing changes from W. to E. Thus if *AB* is N.E., *BC* is S.E., *CD* is S.W., *DE* is S.W., *EF* is N.W., and *FA* is N.E., without consulting the figure at all it is seen from the bearings that *F* is the most westerly station, because at *F* the lines stop running west and begin to run east. When, however, there are *two* such points where lines change from west to east, as, for example, if *F* in Fig. 507 (*a*) were at *F′*, it is sometimes necessary to add up the departures each way, starting from some easterly station. Thus, if the sum of the departures of *CB* and *BA* is greater than the sum of the departures of *CD* and *DE* then *A* is the most westerly station, otherwise *E* is farther west than *A*. A still better method, especially when there are a large number of sides to the polygon, is to start with any station as an origin, and find by a preliminary tabulation similar to the last column on page 474, the *total* departure of each station from the assumed origin. The station having the largest *minus* departure is the most westerly station.

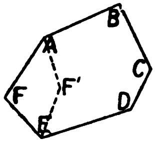

FIG. 507 (*a*).

(*b*) GENERAL METHOD OF PLOTTING BY LATITUDES AND DEPARTURES.

(1) Compute and tabulate the latitudes and departures of all transit lines. (See p. 389.)

(2) Add to the table three columns for plotting, headed respectively *Station, Total Latitude, Total Departure.*

(3) Choose the most westerly station as a point of reference or origin. Its total latitude and its total departure (its two coördinates) are *both* zero.

(4) With this station as an origin, compute the two coördinate distances for each of the other stations in turn as follows: Add *algebraically* the latitudes of all traverse lines passed over in going from the origin to the required point; this will give one coördinate, the *total latitude.* Add algebraically the departures of these same lines; this will give the other coördinate, the *total departure.* Enter the result for each station in the last *two columns.*

(5) Find the size of the rectangle * which just encloses the survey, i.e., the most westerly station will be on the left-hand side of the rectangle, the most easterly station on the right-hand side, the most northerly on the upper side, and the most southerly on the lower side. The largest total *north* latitude in the plotting column, plus the largest total *south* latitude, will be one side of the rectangle; since there is no total *west* departure in the plotting column the largest total departure *east* will be the other side of the rectangle.

(6) Draw the rectangle to the scale chosen for plotting the map. Check it by measuring each side, and by seeing that the diagonals are equal. The accuracy of the whole map depends upon the accuracy of this rectangle,—it is worth while to get it right to start with. (See suggestion (3), p. 475.)

(7) Locate each transit station by its two coördinate distances. *Measure from the two nearest sides of the rectangle,* subtracting if necessary values in the table from the lengths of the corresponding sides of the rectangle.

(8) Connect the transit stations by fine lines and *measure the length of each transit line* to see if it checks with the length used in calculating the latitudes and departures. (See also additional check, p. 475.)

Caution. The object in measuring from the two *nearest* sides in (7) is to reduce the errors likely to arise when the T-square is used for projecting a point any considerable distance across the drawing-board.

In very accurate work it is better not to rely upon the T-square and triangles but, having made sure that the rectangle is correct, to proceed as follows: Find by measurement a point on the east side of the rectangle exactly opposite the station used as the origin, and draw an east-and-west line from the origin to this point for use as a reference-line in measuring latitudes. To plot any station, lay off its total departure twice, once along the north side of the rectangle and once along the south side. On the line between the two points thus obtained, lay off the total latitude of the station, north or south as the case may be, measured from the east and west reference-line.

(*c*) *Illustration.* For the sake of simplicity whole numbers are used for latitudes and departures in the following illustration. Since the values of the bearings and the lengths of the lines do not enter directly into the plotting, they are purposely omitted.

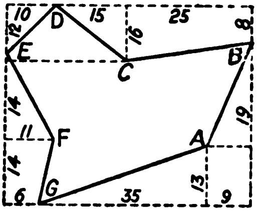

FIG. 507 (*c*).

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* The author prefers to use this rectangle for measuring coördinates in place of two coördinate axes through the most westerly station.

Line	Bearing	Length	Latitude North	Latitude South	Departure East	Departure West	Station	Total Lat.	Total Dep.
			+	−	+	−			
AB	N.-E.		19		9		*B*	+ 4	50
BC	S.-W.			8		25	*C*	− 4	25
CD	N.-W.		16			15	*D*	+12	10
DE	S.-W.			12		10	*E*	0	0
EF	S.-E.			14	11		*F*	−14	11
FG	S.-W			14		5	*G*	−28	6
GA	N.-E		13		35		*A*	−15	41
			48	48	55	55			

(1) The latitudes and departures are first computed and tabulated.

(2) The headings of the last three columns are entered as shown.

(3) Running the eye down the second column it is seen that either E or G must be the most westerly station. Art. 507 (*a*). Adding departures from B to E $(25+15+10)=50$; from B to $G=9+35=44$. Hence E is 6 feet farther west than G. The coördinates of E are 0 and 0.

(4) Coördinates of the other stations from E as the origin are found as follows: Latitudes: $F=-14$. $G=-14+(-14)=-28$. $A=-28+13=-15$. $B=-15+19=+4$. $C=+4+(-8)=-4$. $D=-4+16=+12$. $E=+12+(-12)=0$ (check). The departure coördinates are $F=11$. $G=11+(-5)=6$. $A=6+35=41$. $B=41+9=50$. $C=50+(-25)=25$. $D=25+(-15)=10$. $E=10+(-10)=0$ (check).

(5) In the column of total latitudes the largest north latitude is 12; the largest south latitude is 28, hence one side of the rectangle is $12+28=40$ feet. The other side of the rectangle is equal to the largest total departure or 50 feet. Size of rectangle is 40 feet by 50 feet.

(6) Draw the rectangle to scale, and check it by measuring its sides and diagonals.

(7) Locate the origin E. It is on the western side of the rectangle, *a distance from the top equal to the largest* TOTAL LATITUDE NORTH = 12 feet. In this case this distance happens to be the latitude of ED also. Check E by measuring its distance from the lower side of the rectangle; it should be equal to the largest *total latitude south* = 28 feet. Locate each of the other stations by its coördinates, measuring as far as possible along the sides of the rectangle. Thus to locate C make a point 4 feet below E on the west side of the rectangle, and another point on the north side 25 feet from the corner and draw projecting lines from these two points as indicated. For any station remote from E as A, it may be more convenient *and more accurate* to measure from the two nearest sides.

(8) *Test the length of every transit line by scaling it.* Test also the angle DEF. (Why?) See Art. 507 (*f*).

(*d*) *Illustration.* In Fig. 507 (*d*) the transit lines already plotted by the other methods are plotted by latitudes and departures. The actual measurements used are given on the figure. Station D was located by the departure of $DE=169.4$ and by the latitude of $CD=312.0$, because these two distances were more convenient to use than the two coördinates given in the table. Similar modifications of the method will become apparent from inspection.

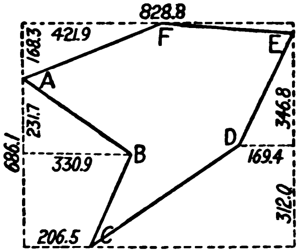

Fig. 507 (*d*).

Line	Bearing	Length	Latitude North	Latitude South	Departure East	Departure West	Station	Total Lat.	Total Depart.
AB	S. 55° 0′ E.	404.0		231.7	330.9		*B*	−231.7	330.9
BC	S. 23° 30′ W.	312.0		286.1		124.4	*C*	−517.8	206.5
CD	N. 55° 26′ E.	550.0	312.0		452.9		*D*	−205.8	659.4
DE	N. 26° 02′ E.	386.0	346.8		169.4		*E*	+141.0	828.8
EF	*N. 86° 10′ W.*	407.8	27.3			406.9	*F*	+168.3	421.9
FA	*S. 68° 16′ W.*	454.2		168.2		421.9	A	0.1	0.0
			686.1	686.0	953.2	953.2			

(*e*) *Suggestions.* (1) On large maps it is often advisable to subdivide the reference rectangle into squares of any convenient size, say 10″, and to use these squares for measuring off the coördinates. If these squares are all drawn at the outset, any shrinkage or expansion which may take place in the paper will not affect the relative positions of stations subsequently plotted, provided the squares are taken as the basis of measurement instead of the original scale. It is sometimes expedient not to draw the entire reference rectangle, but only those supplementary squares needed for plotting, especially if the survey does not close but zigzags in one general direction. (2) It is frequently advantageous to assume the reference rectangle so that no side is horizontal or vertical. (See directions for starting the map, p. 490.) In that case it may be expedient to use a T-square with adjustable head, and to set the blade to coincide with the lower and upper sides of the rectangle. (3) If in testing the reference rectangle it is found that the diagonals are not exactly equal, each right angle may be tested by laying off 10 inches along each side and measuring between the two points thus obtained to see if the distance is 14.142 inches. See Art. 496 (*e*), p. 461.) (4) Sometimes instead of plotting all stations from the same origin, each station is plotted by its latitude and departure from the preceding station. The only advantage in this is that by checking the *total* latitude and departure of the last station from the first station all lines between these two stations are checked. This check is of the most value when the traverse does not form a closed figure. (5) Let a fine dot represent the position of each station, draw a small free-hand circle around the dot and draw no lines of any kind inside this circle. Then there will always be a definite point to measure from, unobscured by lines.

(*f*) *Checks.* In addition to the check afforded by measuring distances between stations, it is well to test with the protractor the angle between two lines when either of these lines is nearly parallel to one of the coördinate axes. This is because a mistake made in measuring perpendicular to that axis does not greatly affect the length of the line. If preferred, the test in such a case may be made by checking the bearing of the line. (See also check in suggestion (4) above.)

508. Methods of Plotting Traverses Compared. The methods of plotting angles by the protractor, by tangents, by cosine and sine, and by chords have already been compared on page 461. In a test carried on with a large number of students the author has found that, apparently, there is little difference between the tangent method and the chord method, either in the time required to plot a closed traverse or in the accuracy attained. If anything the chord method seemed to be the quicker, and when the "home-made" beam compass (p. 460) was used the results were fully as accurate. This is, perhaps, contrary to the general impression.

The chief advantage in plotting *direct angles* or *deflection angles* lies in the fact that the check (p. 466) applied every few stations checks all previous courses, while if lines are plotted by *bearings* or *azimuths* no such single check can be used. On the other hand if a mistake is made in laying off the direction of one course and it is not discovered until a number of succeeding courses have been plotted, the direction of each of these courses must be changed, while if they have been plotted by bearings or azimuths the correction may be made by moving each line parallel to itself. For the same reason, when plotting by direct angles or deflection angles small errors in direction are likely to accumulate, though this may be remedied by plotting every fifth line or so by its bearing (p. 466).

The method of bearings or the method of azimuths is best adapted to plotting surveys which close, for it is difficult to check lines plotted by either of these methods if the traverse does not form a closed figure.

The method of plotting by coördinates (or latitudes and departures) is generally recognized as the best method for plotting the average map. The preliminary computations are somewhat longer than those required for the other methods, but on the other hand they may be used for supplying missing data and for calculating areas. The draftsman knows before he begins to plot whether or not a traverse closes, while in the other methods if it fails to close he is uncertain as to whether the error is in the field work or in the plotting. Since stations are plotted independently errors cannot accumulate, but on the other hand intermediate stations cannot be checked by checking the last station unless suggestion (4), p. 475, is carried out. One of the chief advantages of the method is that the size of the map may be ascertained before plotting is begun, since in the majority of cases it is approximately the size of the reference rectangle.

CHAPTER XL

WORKING UP FIELD NOTES PREPARATORY TO PLOTTING

The details of a survey are usually plotted directly from the field notes with little if any preparatory work. On the other hand more or less preliminary computation is necessary before the traverse lines can be plotted. Moreover the notes pertaining to traverse lines are often scattered through many pages of the note-book. It saves time, therefore, to get together all the data needed for computation and plotting, to make all the preliminary calculations and to arrange results in such a systematic manner that when plotting is begun the undivided attention may be given to the mechanical portion of the work.

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509. The Method of Working Up Notes depends largely upon the method to be used in plotting. In any event it is well to make a rough free-hand sketch of the main lines or skeleton of the survey, and to put on this sketch all lengths and angles which will be needed for computation and plotting. It is better to take the time for this at the beginning of the work than to stop in the midst of a computation to search the field notes for necessary data.

Having thus gathered all the data that in any way affect the main lines, it may be necessary to correct or adjust values. The adjusted values of lengths and angles (or bearings) may be written in red ink on the original sketch, or a new sketch may be made showing only such data as will be needed for computation or plotting. From this sketch the draftsman can see what angles are to be plotted (they are often the complements or supplements of the angles measured in the field), or if the lines are to be plotted by latitudes and departures this sketch will be very useful in the preliminary calculation.

When the protractor is used, no further preparation is necessary, as the lines can be plotted directly from the data on the sketch. When, however, the more accurate methods are employed, additional data will be needed before the work of plotting is begun. The additional data required for each of the different methods of plotting are indicated in Art. 515, p. 483.

Remark. It is well to keep all sketches, computations, and results in a separate-leaf note-book, such as that described on p. 363.

510. Correcting Measurements. Paradoxical as it may seem, the more accurate the field work the more important it may be to correct the field measurements, for in order to obtain results of a high degree of precision, certain refinements are necessary which are entirely omitted in ordinary work. The corrections given below may or may not be necessary, depending upon the nature of the work. They are usually so small that they are not needed for purposes of plotting, but they may be needed for purposes of calculation or for permanent record.

(*a*) *Linear measurements* may be corrected for error in the length of the tape, p. 39, for error due to temperature, p. 40, and occasionally for sag and pull, pp. 41–42. If a distance is chained along a slope instead of in horizontal stretches, the calculated correction is applied, p. 35. In the case of inclined stadia readings, corrections are made as explained on p. 480.

(*b*) *Angular measurements.* Few corrections can be applied to angular measurements. In compass surveying bearings affected by local attraction may be corrected as explained on p. 297.

511. Adjusting Measurements means changing the angles and distances given in the notes so that they will conform more nearly to conditions which are known to exist. Adjustments are usually made for purposes of calculation, not for plotting.

(*a*) *Linear measurements.* If more than one value is given for a distance, the mean may be taken, throwing out any value that is obviously incorrect. When only one value is given it is not changed unless some obvious mistake is detected, such as an even hundred feet in chaining or reading from the wrong end of the tape, as, for example, 54 instead of 46. Sometimes several consecutive measurements are made along a line, as in taking offsets; when their sum does not exactly equal the total measurement, if it is worth while correcting the error it may be distributed among the partial measurements in proportion to their lengths.

(*b*) *Angular measurement.* The most common adjustment of angles is in the case of a closed figure, when the error in the sum of the interior angles (p. 119) may be distributed equally.

When a number of angles are clustered about the same point, the same adjustment may be made in order to make their sum equal 360°. When several angles at the same station are checked by measuring another angle, the error is distributed among all the angles, including the check angle. (See p. 14.) One should not be tied down by rules, however, for it is often better to change some of the angles without changing others. An illustration given below shows how, by merely changing six of the twenty angles of a survey, five conditions were fulfilled.

When several values are given for each angle, the mean value may be used. The mean value may be further modified if necessary in order that the interior angles of a polygon may add up correctly, or that some other condition may be fulfilled. Thus, for example, in class work, where several parties work independently but use the same stations, mean values may be found. these values adjusted, and the survey plotted from the composite values thus obtained.

(*c*) *Remark.* It must be remembered that in many cases nothing is gained by adjusting the measurements given in the field notes. The purpose of the survey and the limit of error will determine whether or not adjustment is necessary.

Sometimes after latitudes and departures have been calculated further adjustment of angles and distances may be made. (See pp. 394–398.)

(*d*) *Illustration.* In the survey on page 390 each distance and each angle was originally obtained by taking the mean of several measurements. No further adjustment of distances was possible. The angles, however, were adjusted on the following assumptions:

(1) In the two five-sided polygons the sum of the interior angles must equal 540°.

(2) In the one four-sided polygon the sum of the interior angles must equal 360°.

(3) In the two triangles the sum of the interior angles must equal 180°.

(4) The sum of the four angles at *Y* must equal 360°.

(5) In the ten-sided polygon *NOPQRSTUVWN*, the sum of the interior angles must equal 1440°.

Since it was not desired to carry any angle to seconds, the adjustment was made by changing six of the twenty angles as indicated in the figure. In no angle did the change exceed one minute.

(*e*) *In work of great precision* adjustment of observed values may be made by the method of least squares; this is a theoretical method of deducing the most probable results of a set of measurements. If some of the measurements are thought to be more accurate than others, they are given weight and treated accordingly. Equations are formed from conditions known to exist, probable errors are calculated, and theoretical values of angles and distances are deduced. The method is seldom used in ordinary plane surveying.

512. Calculating Bearings. If the bearings of lines are needed for plotting and they are not given in the notes, they may be calculated by the method explained in Chapter XXX, provided the bearing of one line is known. For purposes of plotting, however, it is not essential that the bearing of even one line should be given, for the *relative* positions of stations will be correct if the bearing of any line be assumed, and the bearings of the other lines be calculated from that assumed value. In such a case it makes little difference what bearing is assumed to start with.

Sometimes a true meridian is established in a survey (Chapter XXVIII) and the true bearing of some main line found by measuring the angle it makes with the meridian. The true bearings of the other lines can then be calculated. (See p. 391 for an example.)

In calculating all bearings the final adjusted values of angles are used. Make sure that bearings are consistent, i.e., all forward bearings or all back bearings.

In compass surveys the magnetic bearings are usually plotted, for, of course, no calculated bearings are possible.

513. Supplying Missing Data. When the length of a line or the value of an angle is not given in the field notes it can often be calculated by the methods explained in Chapter XXXII. Notice, however, that in such cases there is no check on the field work, and hence, as pointed out in the note at the beginning of Chapter XXXII, closing angles and distances should be measured whenever practicable.

514. Reducing Stadia Notes means finding the horizontal distance, or the vertical distance, or both, for each sight in which the vertical angle a is large enough to be taken into account (ordinarily 3° or larger, see p. 308). The formulas for inclined sights derived on p. 303 are:

$$\begin{aligned} \textit{Horizontal distance} &= R - R\sin^2 a + C\cos a \\ &= R\cos^2 a + C\cos a. \\ \textit{Vertical distance} &= R\,\tfrac{1}{2}\sin 2a + C\sin a. \end{aligned}$$

When there are many readings to be reduced, the work involved in substituting in these formulas would become a serious objection to the entire stadia method. Fortunately the terms $R\cos^2 a$ and $R\,\frac{1}{2}\sin 2a$ are independent of the instrument or the linear unit used, and hence their values for different vertical angles can be computed once for all and tabulated. In ordinary work $C\cos a$ is practically equal to C, and $C\sin a$ is often a negligible quantity.

Three different devices for obtaining the value of $R\cos^2 a$ and $R\,\frac{1}{2}\sin 2a$ for different vertical angles are in common use, viz.: (1) Reduction tables, (2) stadia diagrams, and (3) special forms of the slide-rule.

(*a*) Reduction tables. In some tables the values are computed on the assumption that the actual intercept on the rod (R) is unity, as, for example, one foot. Then any inclined distance, as Fm in Fig. 379, p. 306, may be reduced by multiplying that distance by the tabular value corresponding to the vertical angle. In other tables the values are based on the assumption that $R = 100$, and hence the inclined distance is divided by 100 (two decimal places being pointed off), before multiplying by the tabular value. For horizontal distances the constant C (nearly equal to $C \cos a$) must be added to the result, but often $C \sin a$ may be neglected. In more accurate work $C \cos a$ and $C \sin a$, computed for several of the most common values of C, may be obtained from supplementary tables, usually given at the bottom of the main table.

Since for average stadia shots of 100 ft. or over the measurement is not exact beyond the figure in the third place, i.e., the figure in the fourth place is uncertain (see p. 370), not more than four places of figures should be retained in any result. (See p. 371.) Frequently three places will suffice. Hence an ordinary slide-rule may be used for multiplying tabular values. The next best method is that of abridged multiplication (p. 365), writing the extreme right-hand figure of the reversed multiplier under the fourth place of figures in the tabular value regardless of the position of the decimal point. (See Art. 425 (*g*), p. 371.)

(*b*) *Illustration.* Given a stadia reading = 234, vertical angle = 10° 22′, and an instrumental constant = 1.25:

ACCURATE METHOD		APPROXIMATE METHOD	
Horizontal	Vertical	Horizontal	Vertical
96.76	17.70	96.76	17.70
432	432	432	432
19352	3540	1935	354
2903	531	290	53
387	71	38	7
226.42	41.42	226.3	41.40
1.23	.23	1.23	.23
227.6	41.65	227.6	41.63

Notice that for most purposes the approximate results on the right are accurate enough, that is to say, the extreme right-hand figure of the reversed multiplier may be written under the third place of the multiplicand instead of under the fourth.

(*c*) STADIA DIAGRAMS. It is an easy matter to construct a diagram that will give fairly accurate results for vertical angles up to, say, 8° or 10°, but when the diagram is extended to cover 15° or 20°, either the scale must be reduced at a sacrifice in accuracy, or the diagram made inconveniently large. Nevertheless such diagrams are very useful, especially when two men work together, one with the note-book and the other with the diagram. Results can be obtained much more quickly than by the use of tables, and for most purposes they will be sufficiently accurate.

Usually separate diagrams are prepared, one for horizontal distances and one for differences in elevation.

(*d*) *Diagram for horizontal distances.* The simplest form may be constructed on cross-section paper, on which there are, say, 10 lines to the inch each way. (1) Assume the origin or zero-point in one corner of the paper, say the lower left-hand corner. (2) Along the lower edge, number every tenth (or heavy) vertical line, the numbers running 100, 200, etc., up to the longest sight taken in the field, say 1000 ft. (3) Along the right-hand edge number every tenth (or heavy) horizontal line, the numbers running 10, 20, etc., thus making the vertical scale ten times as large as the horizontal. (4) Find the values of $1000 \sin^2 a$ for different vertical angles 1°, 2°, etc., up to say 15° or 20°, using stadia reduction tables or computing the values by trigonometry. Lay these values off along the ordinate marked 1000, using the vertical scale, and draw radial lines from the origin to the points thus obtained. Mark each radial line with its degree number.

(*e*) *Illustration. Use of the diagram.* Given a stadia reading $R = 410$, vertical angle $= 13°$, instrumental constant $C = 1.2$. Find the vertical line next beyond the one numbered 400, follow up this line to its intersection with the radial line marked 13°, note that the nearest horizontal line is numbered 21, and estimate the reading as 20.7. Hence $410 - 20.7 = 389.3$ is the corrected (horizontal) distance, and adding $C = 1.2$, the final result $=$ 390.5 feet.

(*f*) *Suggestions.* (1) The scale adopted depends upon the accuracy required, the maximum length of sight, and the maximum vertical angle. If, for example, the length of the longest sight taken in the field is less than 500 ft., there is no advantage in extending the diagram to 1000,—it is better to double the scale. (2) In large diagrams, intended for accurate work, intermediate radial lines may be drawn at intervals corresponding to 10 minutes of arc. If desired, an arithmetical table giving values of $C \cos a$ and $C \sin a$ for several values of C at 5° intervals may be printed in one corner of the sheet. (3) The reason that the vertical ordinates are plotted for values of $R \sin^2 a$ instead of $R \cos^2 a$ is that the former is a small quantity which changes rapidly for small angles, while the latter is a large quantity which changes slowly. Notice that a quantity taken from *such a diagram* **must be** *subtracted* from R, and C **added to get the final** *result. (Why?)*

(*g*) *Diagrams for obtaining differences in elevation* may be constructed in a similar manner, the vertical ordinates representing values of $R\ \frac{1}{2} \sin 2a$. The two diagrams may be drawn on the same sheet without conflicting, by laying off the abscissas and ordinates in the second diagram along the two edges of the paper not used in the first.

(*h*) *Remark.* A number of modifications of the diagrams just described have been devised with a view of saving space and increasing accuracy. In one of these, given in Professor I. O. Baker's *Engineers' Surveying Instruments*, p. 202, the values are laid off along the sides of triangles, equilateral triangles being used in the diagram for elevations. A combination diagram for obtaining horizontal distances and elevations is given on an insert-sheet of the *Engineering News*, Feb. 3, 1898, and on p. 85 of the same issue is another diagram small enough to go in the back of an ordinary field book.

(*i*) STADIA SLIDE-RULES,* such as those designed by Mr. William Cox and by Mr. B. H. Colby, may be used in place of tables or diagrams. Accurate results can be obtained with great rapidity after a few minutes' practice in the use of these slide-rules. The few simple directions necessary are furnished with the slide-rules.

515. Getting Together Data for Plotting—Summary.

The first five or six steps are practically the same, regardless of the method of plotting that will be used.

1. Make a rough sketch showing approximately the relative positions of stations.
2. Mark on the sketch the lengths and angles given in the field notes for the main lines. If the stadia was used reduce the inclined readings and put the *horizontal* distances on the sketch.
3. If necessary correct the measurements for errors from different sources (p. 478).
4. If desirable, adjust the values to conform to known conditions, as explained on p. 478. If the stadia was used reduce inclined readings (p. 480).
5. Mark the final values of angles and distances in red ink on the sketch.
6. If bearings are to be used either in the calculations or for plotting, start with the bearing of one line and calculate the bearings of the others (p. 479).. (If the survey was made with the compass, this, of course, is not done.) Use the final adjusted values of angles in calculating bearings. Enter the calculated bearings on the sketch, making sure they are all forward bearings.

The remaining steps will differ somewhat according to the method used in plotting.

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* See *Catalogue of Keuffel and Esser Co.*

(*a*) *Protractor method.* No additional data are required. Work directly from the sketch. Angles, bearings, or azimuths may be plotted.

(*b*) *Tangent method.* (1) It is well to indicate on the sketch the actual angles or bearings to be plotted, as many of them may be the complements or the supplements of those previously marked on the sketch. (2) Prepare a table similar to that on p. 465, or if bearings are to be plotted, similar to that on p. 469.

(*c*) *Chord method.* (1) As in the tangent method, indicate on the sketch the actual angles or bearings to be plotted. (2) Prepare a table similar to that on p. 465, or if bearings are plotted, similar to that on p. 470.

(*d*) *Latitude and departure method.* (1) Compute the latitudes and departures, using, of course, final corrected values of lengths and bearings. (See pp. 393 and 394 for forms.) (2) If desired, adjust the values thus obtained, i.e., balance the survey (p. 395). (3) Tabulate for plotting (p. 474), and find the size of the circumscribed rectangle. (A complete illustration of working up notes for plotting by this method is given on pp. 390 to 394.)

(*e*) *Azimuths.* If azimuths are given they may be plotted by chords or tangents, tables similar to those on pp. 469 and 470 being prepared, or latitudes and departures may be calculated without reducing azimuths to bearings (see p. 387).

CHAPTER XLI

PLOTTING THE MAP

GENERAL METHODS AND SUGGESTIONS

IN plotting the map, something more is needed than a mere knowledge of the use of drawing instruments and of the different methods of plotting traverses. Practical questions arise, such as the choice of scale, the general arrangement of the map, the order of plotting, the precautions necessary to insure accuracy, the methods of checking the drawing, of plotting details, of interpolating contours, and of copying maps. These questions are discussed in this chapter.

516. Equipment. (*a*) *Paper.* (1) If maps are to be preserved a long time or are likely to receive rough usage, the durability of the paper is an important quality, while if distances are to be scaled with accuracy, the paper should contract and expand as little as possible under atmospheric changes. These two qualities are best combined in mounted paper or paper backed with muslin. For penciling, preparatory to tracing, almost any *white* paper will answer, provided fine lines can be drawn and its surface will stand erasing. A tough bond-paper is best for maps which will be folded (creased) and carried in the pocket. For maps which are to be tinted, mounted paper is recommended; if an unmounted paper is used it should be of high grade, with a surface adapted to either lines or wash—a pebbled or egg-shell surface is often preferred. For photo-reproduction Bristol board or a hard white paper with a smooth glossy surface is the best, although excellent reproductions can be made from tracings. Tracing-cloth should be of the best quality, otherwise ink is liable to strike through; tracing-paper may be used when only a few prints are needed and it is not necessary to preserve the tracing for a long period of time. For blue-prints thin tough paper will usually stand rough handling better than the smooth thick paper, but prints are likely to be somewhat inferior. The more sensitive a paper is, the quicker it deteriorates, and the more care is required in handling it in a strong light. "Medium" printing-paper is recommended for most purposes; in ordering paper, be careful to state whether quick-printing, medium, or slow-printing paper is desired. (See p. 453.)

(*b*) *Drawing instruments.* The edges of the straight-edge, the T-square, and the triangles should be straight. This is more essential in map-drawing than in almost any other kind of drafting. (See p. 438 for test.) The angles of a triangle should be true, especially the right angle. (See p. 439 for test.) Straight-edges, T-squares, and triangles made of steel are usually the most accurate. The pencil should be hard (6-H) and properly sharpened. (See p. 438.) The ruling-pen should be of the very best quality, and the points should be kept in good condition. (See p. 441.) The scale should be a *decimal* scale and its graduations should be accurate and definite. The surface of the drawing-board or table should be a smooth plane. These are the most essential requirements for instruments used in mapping.

517. Precautions Necessary in Accurate Plotting. As the edges of a drawing-board are seldom exactly straight, lines drawn along the T-square blade, as it is moved up and down the board, are seldom exactly parallel. A steel straight-edge fastened to the left-hand edge of the board remedies this to some extent, but even then there is apt to be some play in the blade of the T-square, and hence, in very accurate work, it is better not to rely upon this method. As comparatively few lines in maps are vertical or horizontal, the T-square is not as necessary in plotting as it is in other kinds of drawing; a steel straight-edge about three feet long may be used in its place. All important reference lines, such as traverse lines, can then be plotted entirely by measurement without depending upon parallel lines drawn with the T-square or upon perpendiculars erected with the triangles. (See the method given in *Caution*, p. 473.) Such precautions, however, are not necessary in much of the plotting done in ordinary work.

The shrinkage of the drawing-paper may be another source of error. If the map is 4 or 5 feet wide the change in the paper is frequently as much as ¼ inch, and sometimes ½ inch. The more seasoned the paper the less it will shrink, hence it is well to keep paper in the office several months before using it.

In plotting new details on old maps test the map first to see if shrinkage has occurred, and if it has make proper allowance for it in plotting. On the other hand, in maps which have been rolled up, test for expansion, as rolling tends to stretch the paper—a good reason for filing maps flat when possible.

518. General Methods. A large part of the work in mapping is the reverse of field work; distances that were measured with a tape or chain are laid off with a scale, and angles that were measured with a transit or compass are laid off with a protractor. Such work involves no calculation. When, however, the plotting is done by *indirect* measurements, that is by measurements which are not the same as those taken in the field, these indirect measurements must be obtained from the field measurements by calculation, as, for example, when angles are plotted by latitudes and departures or by any of the trigonometric methods.

A common method in field work is to run traverses and locate details simultaneously (p. 132), but in mapping, the traverse lines are almost always plotted first, before work is begun on the details. The plotting is done in pencil, usually on paper, but sometimes directly on tracing-cloth, and after the penciling is completed, the lines are inked. If only a single copy of the map is desired, the pencil lines are inked directly on the paper, but if more than one copy is required the lines are usually traced and process prints made from the tracing.

519. Method of Procedure Outlined. (1) Choose a suitable scale for the map (p. 489), decide upon the method of plotting traverse lines (see p. 492), and work up the field notes, making sketches and arranging data so that no delay in plotting will be caused by searching through field books (Chapter XL). (2) Select a suitable drawing-paper (p. 485) and stretch it on the drawing-board until it lies smooth and flat (p. 445). (3) Sharpen a 6-H pencil to a round point at one end and a wedge-shaped point at the other (p. 438). (4) Wipe the triangles and T-square *clean,* and brush the surface of the paper. Throughout the work, take precautions to insure neatness (p. 445). (5) Follow the directions for starting the map given on page 490. (6) Plot the traverse lines in pencil (Chapter XXXIX), marking each station with its letter or number. Use hair lines and prick the points. Observe correct methods of penciling (p. 446) and of laying off measurements (p. 448).

Apply checks continually as the work progresses (p. 494). (7) Plot details in pencil (p. 496). (8) Ink the map or trace it. Observe the directions on pages 440 and 446 for inking and those on page 451 for tracing. (9) Finish the map (Chapter XLI), making sure that it meets the requirements given in Art. 532, p. 502. (10) If blue-prints or brown solar prints are desired, follow the directions given on pages 453 and 454.

520. Scales. (*a*) The most common scales for ordinary maps are those in which the number of *feet* represented by one *inch* is some *multiple of ten.* In general these scales are as follows:

Nature of the Map	Number of Feet per Inch							
Small territory—many details	10	**20**	30	**40**	50	60	80	**100**
Large territory—few details	**100**	**200**		400	500		800	**1000**

The numbers in black-face type are probably the scales most frequently used. Notice that 70 and 90 are the only two multiples of ten below 100 that are not included. The first six (from 10 to 60 inclusive) are the scales usually found on the six sides of a triangular scale; the scales of 80 and 100 can be obtained by using the 40 and 50 sides respectively, but 70 and 90 have no such corresponding sides.

(*b*) The so-called *natural scale* is one in which a given distance on the map is some round fraction of the distance it represents, such as $\frac{1}{10000}$, $\frac{1}{50000}$, etc. Such scales are used for maps of large territories, e.g., those of the United States Coast and Geodetic Survey and of the United States Geological Survey.* The maps of the former are usually plotted to a scale of $\frac{1}{10000}$, while those of the latter are mostly plotted to a scale of $\frac{1}{30000}$ and then reduced in lithographing to $\frac{1}{62500}$ or $\frac{1}{125000}$ (about 1 inch and 2 inches to the mile respectively).

(*c*) SCALE EQUIVALENTS FOR VARIOUS RATIOS †

Feet to One Inch	Miles to One Inch	Meters to One Inch	Kilometers to One Inch	Ratio (Number Ins.)
100	0.019	30.480	0.030	1 : 1200
400	0.076	121.920	0.122	1 : 4800
500	0.095	152.400	0.153	1 : 6000
800	0.151	243.840	0.244	1 : 9600
833⅓	0.157	254.000	0.254	1 : 10000
1000	0.189	304.801	0.305	1 : 12000
5280	1.000	1609.2	1.609	1 : 63360
10560	2.000	3218.4	3.218	1 : 126720

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* See Gillespie, Vol. I, p. 27, for scales used for surveys in different countries.

† See *Wilson's Topographic Surveying*, p. 447, for a more complete table.

521. The Choice of Scale. Primary considerations in choosing the scale are: (1) the use to which the map will be put, and (2) the extent of territory to be represented. Thus, for example, if distances are to be scaled on the map to the nearest foot, the scale should be at least as large as 1 in. = 100 ft., and, if possible, as large as 1 in. = 40 ft. For a small map with many details, such as that of a city lot and buildings, a scale as large as 1 in. = 20 ft., or even 1 in. = 10 ft., is preferable. For large maps with few details, such, for example, as the average topographic map, scales from 1 in. = 100 ft. to 1 in. = 1000 ft. may be used. In the excellent survey of Baltimore, Md., the scales adopted were 1 in. = 200 ft. for plotting topographic maps, and 1 in. = 40 ft. for property maps. Preliminary surveys for railroads are usually plotted to a scale of from 100 to 500 ft. per inch, while maps of mines and mining claims are more often to a scale of 100 or 200 ft. to the inch; of course maps of railroads and mines made for special purposes are often drawn to larger scale. Architects usually prefer a scale of 1 in. = 64 ft., or 1 in. = 32 ft., or 1 in. = 16 ft., or, occasionally, 1 in. = 4 ft., to correspond to the scale which they use for the plans of buildings.

The scales of 1 in. = 20 ft., 1 in. = 40 ft., and 1 in. = 80 ft., all have the common advantage that distances may be scaled approximately with a carpenter's rule or an architect's scale, since ¼ inch equals 5, 10, and 20 ft. respectively. Similarly in the scales 200, 400, and 800 ft. per inch, ¼ in. equals 50, 100, and 200 ft. respectively.

Frequently the choice is restricted to some scale small enough so that the whole map will fall within a rectangle of given size. The dimensions of this rectangle are sometimes determined by the size of the blue-print frame, or by the size of the drawer in which the map will be filed, or by the size of sheet most convenient for handling, or by the standard size adopted for similar maps. The arrangement of the map must be taken into account, for if it is desired to have the north at the top and the border lines square with the compass points, it may be necessary to adopt a smaller scale than would otherwise be used (see next article). When it is desirable to choose as large a scale as a given rectangle will permit, it is often expedient to plot the traverse lines roughly to any scale in order to determine the shape and extent of the survey.

(*a*) TWO GENERAL RULES. I. *Choose a scale large enough so that in plotting or in scaling distances from the finished map it will not be necessary to read the scale closer than $\frac{1}{100}$ inch.* (Allowance should be made for expansion and contraction due to atmospheric changes.)

II. *Choose as small a scale as is consistent with a clear delineation of the smallest details to be plotted,* due regard being paid to Rule I. In topographic maps, especially, it is an advantage to adopt as small a scale as practicable, for not only is a larger area shown on each sheet, but by bringing contour lines closer together a more vivid impression is given of the real nature of the surface depicted.

It is frequently necessary to draw maps of the same territory to different scales, first one map to small scale showing the region as a whole, then other maps to a much larger scale, each sheet corresponding to a portion of the original small-scale map.

522. Arrangement. It is desirable to arrange a map with the *north* at the *top*, making the top and bottom border-lines *true* east-and-west lines, and the side border-lines *true* north-and-south lines. Such an arrangement, however, is seldom economical, the border-lines forming a larger rectangle than is necessary, hence some other arrangement is followed when it is more desirable to confine the map to a small space than it is to have the border-lines square with the compass points. Moreover it often is best to have the natural approach to a piece of property at the bottom of a map, even though this brings the south at the top. For example, in the case of a house and lot, it is natural to look for the street side and the approach to the house at the bottom of the map, even though this may bring the south end of the lot at the top.

There are many questions of arrangement, such as where to put the title, the meridian-needle, and the explanatory notes, which do not have to be settled until the map is plotted and ready to finish. (See Chapter XLII.)

523. How to Begin a Map. Usually the first step is to plot the traverse to a small scale, using the protractor for laying off angles; if latitudes and departures have been calculated, it is quicker to plot by coördinates laid off on cross-section paper. This preliminary sketch is drawn roughly and quickly merely to give an idea of the general shape of the plot and the space that it will occupy.

(*a*) *Which line to begin with.* Unless the traverse is to be plotted by coördinates, the position and direction of some traverse line must be assumed to start with, and its length laid off to scale. In a closed figure it is well to begin at the first station and work half-way around the polygon in either direction to lessen cumulative errors. For the same reason, in a traverse that does not close it is well to begin near the middle and plot toward either end.

(*b*) *How to assume the first line.* A starting-point and a direction for the first line should be assumed in such a way that when the other lines are plotted they will not run off the paper. In this the preliminary plot referred to above is of the greatest assistance. If a meridian is to be used in plotting, it is convenient to make it a vertical line; in that case the direction of the first line cannot be assumed, but must be found by laying off its bearing or its azimuth. It is by no means essential, however, that the meridian should be a vertical line; on the contrary, there are many cases in which if it were assumed vertical the map would not fall within the limits of the paper. It is then necessary to assume the first line in the proper position as judged by a preliminary sketch, and to lay off its bearing or azimuth backward to establish the direction of the meridian. This will bring the east-and-west line oblique to the T-square blade, but if the head of the T-square is adjustable the blade can usually be made to coincide either with the east-and-west line or with the north-and-south line.

For the reasons just given, it is often expedient to lay off the reference rectangle, used in plotting by coördinates, so that no side is vertical or horizontal. The inclination required in order that the lines may not run off the paper is estimated by means of the preliminary plot referred to above. In this case it is often inexpedient to draw the entire reference rectangle. (See p. .475) Notice that the size of the reference rectangle is known from the computations (p. 473), and that if this rectangle can be drawn to the desired scale without running off the paper there is no need of making the preliminary plot.

(*c*) *Remark.* No matter how the map may be arranged with respect to the edges of the paper, after the plotting is finished border-lines may be drawn in any desired direction and the sheet trimmed accordingly. (See p. 504.)

524. Plotting the Triangulation Net. In extensive surveys where primary stations have been established by triangulation, the first step in plotting is to lay out the triangulation net. The lengths of the sides of the triangles having been computed, the lines may be plotted by any of the methods of plotting traverses. Unless the method of coördinates is used it is well to plot first those stations lying on the outside, i.e., in the perimeter of the survey. Sometimes the triangulation net may be plotted with sufficient accuracy by simply swinging intersecting arcs with radii corresponding to the lengths of the sides of the triangles, establishing each triangle in turn by this method. Occasionally a station may have to be plotted by the three-point method (p. 462).

525. Plotting Traverses. (A summary of Chapter XXXIX.)

1. For every method of running traverse lines in the field there is a corresponding method of plotting. In addition there is the method of latitudes and departures.

2. In accurate work the ordinary protractor should not be used for plotting traverses. Use it for checking angles. Even vernier protractors will not give as accurate results as can be obtained by other methods.

3. The tangent method, the cosine and sine method, and the chord method are used for laying off direct angles, deflection angles, bearings, and azimuths. Latitudes and departures are used for accomplishing the same results indirectly.

4. Transit lines must be plotted with more than ordinary care, since upon them the accuracy of the whole map depends. Use hair lines and prick points.

5. Before beginning to plot, get together all data that will be needed, including the tangents or chords of all angles to be plotted if corresponding methods are employed (see p. 484), or the two columns of coördinates if latitudes and departures are used.

(*a*) FIVE METHODS OF PLOTTING TRANSIT LINES

(1) *Each line plotted from the preceding line.* (See p. 464.) The tangent method, the cosine and sine method, or the chord method is used for laying off the angle (or its complement or supplement). A base of 10″ or a radius of 10″ is convenient to use. *Check.* Test the bearing of the last line, and if there are many lines apply a similar check at every fourth or fifth *station.*

(2) *Each line plotted by a deflection angle.* (See p. 466.) A modification of the preceding method. Used in railway work and other similar surveys. Checks same as in (2).

(3) *Each line plotted from its bearing.* (See p. 467.) (1) Draw a 20″ square, subdivide into four 10″ squares to correspond to the four quadrants of bearing, plot each bearing in its proper square by measuring ten times the tangent along the side of the square. A better method still is to make one 10″ square answer for all four quadrants. (2) Draw a circle 20″ in diameter, divide into quadrants of bearing, and plot each bearing in its proper quadrant by its chord. Four 90° arcs, the four centers of which are corners of a 10″ square, may be used in place of a 20″ circle, but no advantage is gained except in economy of space. *Check.* Test the angle between traverse lines at each station. There is no single check which checks all previous courses.

A modification of this method is to draw parallel meridians through the stations, and to lay off bearings from these meridians. Another method is to draw a series of 10″ squares. (See p. 471.)

(4) *Each line plotted from its azimuth.* (See p. 471.) Practically the same methods can be used as those given for bearings.

(5) *Transit lines plotted from the latitudes and departures.* (See p. 471.) The most westerly station is chosen as an origin, and each station is plotted by its two coördinate distances (1) north or south, (2) east from the origin. The coördinates are computed by adding algebraically first the latitudes, then the departures, starting with 0 and 0 as the coördinates of the most westerly station, and finding the coördinates of each successive station in order. As a convenient method of measuring the coördinates a rectangle is drawn, the sides of the rectangle passing through the extreme east and west, north and south stations. If preferred, two coördinate axes through the most westerly station may be used instead of the rectangle. For very accurate work do not rely upon the T-square and triangles, but observe the caution on p. 473. *Check.* Measure the length of each traverse line, and in addition test the angle between two lines if one line is nearly parallel to a coördinate axis.

(6) Methods of plotting traverses are compared on page 475. The method of latitudes and departures is the best for general work.

526. Checks Used in Plotting. Precautions and checks are used constantly in field work; similar precautions and checks should be used in plotting. A draftsman can accomplish more in the long run if he will check his work as he goes along. No reliance can be placed upon a map which has not been checked.

Some of the methods given in this article are only approximate checks intended to disclose blunders in plotting, but blunders, perhaps, are more common than small errors.

(*a*) *Checks for linear measurements.* (1) Any straight line may be divided into two or more segments, each segment scaled separately, and the sum of the results compared with the length of the line given in the field notes. (2) Several lines may be scaled, the results recorded on paper, and then compared with the corresponding distances in the field notes. This is better than to look up a distance in the field notes and then scale the corresponding line, for in the latter method the mind is prejudiced, and any mistake made originally in laying off a distance may be repeated. (3) To check lines of a traverse, lay a paper straight-edge on the drawing, mark off the lengths of the lines consecutively along the edge of the paper, each line beginning at the end of the preceding line, and scale the total distance to see if it equals the sum of the lengths of all the traverse lines. This method is particularly useful when the plus-station system of numbering (p. 147) is employed, since the station number of each mark as read from the scale may be compared with that given in the notes. (4) When two points are plotted by *indirect* measurements, always scale the distance between them as a check. Thus, for example, when two sides of a triangle have been laid off, scale the third side to see if it agrees with the length given in the field notes. This method, which is often a check on angles as well as distances, is especially useful in checking lines plotted by latitudes and departures.

(*b*) *Checks for angles.* (1) To check an angle which has been laid off with a protractor, measure its complement or some other related angle which requires a shifting of the protractor, instead of checking the angle itself. (2) Two or more adjacent angles may be tested by measuring the sum-angle. (3) Never plot an angle by the chord method or the tangent method without checking it roughly with a protractor. (4) In plotting traverse lines, always compute the azimuths or the bearings of all of the lines. If the lines are plotted by azimuths or bearings, check the angle between the traverse lines at each station, and *vice versa* if the traverse-line angles have been plotted check the azimuths or bearings, as more fully explained in the preceding chapter. The aim should be to check one angle by measuring an entirely different angle to avoid repeating mistakes.

If the traverse lines form a closed figure, the check angles may be measured with the protractor, otherwise measure them by some method as accurate as that used for plotting the original angles.

Question. What check often employed in the field corresponds to the check used for traverses plotted by direct or deflection angles?

(*c*) *Combination checks.* (1) Check a reference rectangle by measuring the opposite sides and also the diagonals to see if they are equal. (2) If a traverse closes, not only should the closing line pass through the first station but the distance from the last station to the first should agree with the distance measured in the field. (3) When an extra distance or angle has been measured in the field as a check, this distance or angle should be tested on the map.

(*d*) *To "run down" an error.* If traverse lines forming a polygon do not close when plotted, the error may often be "run down" without going over all of the work again, by making the following tests. (1) If the error of closure is in a direction *parallel* to some line in the traverse, test the length of that line; if it is *perpendicular* to the line, test the bearing. (2) If a traverse has been plotted by direct angles or by deflection angles, calculate the bearing of the last line and see if the angle between this line and the meridian agrees with the calculated value; if it does the *directions* of all the lines are probably correct. In that case test the lengths as directed on page 494. (3) If by the test in (2) an error in the direction of the last line is found, apply the same test to some preceding line, say three or four stations back. Continue to make the test until a line is found whose direction is correct and then work forward applying the test at each station. (4) When lines have been plotted by bearings or azimuths no single test can be made; it is well to begin at the last station and to work backward, testing the direct angle or the deflection angle at each station. (5) If a point is obviously wrong and the mistake is not easily found, try to arrive at the same point from some other direction. (6) Keep in mind the effect of slight changes in the bearings of long lines, and of lines nearly north and south or east and west. (See (3) and (4), p. 397.)

527. Plotting Details. In Chapter XIII are given different methods of locating details in the field corresponding to the seven methods of locating a point given on page 4. The work of plotting details is simply the reverse of that done in the field, enough points being established to determine the outlines of the details.

(*a*) *Points located by angles and distances.* Usually the distances from the station to the points located are comparatively short, frequently less than 100 ft. Moreover, if a slight error is made in plotting a point, it does not affect the entire survey as does an error in plotting a station. Hence in plotting details the protractor is generally used for laying off angles. One of the most convenient forms is shown on page 456. Usually there are a number of points located from the same station, and as fast as they are plotted they should be numbered in such a way that there will be no difficulty or confusion in drawing the outlines. Frequently, only two or three points of a building or other rectangular object are given, the perpendicular sides being completed with the triangle and scale.

(*b*) *Points located by angles only.* Angles are laid off and the lines numbered so that the required points of intersection may be found without confusion. The method of plotting a point located by the *three-point* method is explained on page 462.

(*c*) *Points located by linear measurements only.* Frequently the right angles for perpendicular offsets may be estimated by eye, especially in plotting indefinite details, such as edges of streams and roads. A point located by two intersecting distances may be plotted by swinging two arcs with the compasses, or better still by swinging one arc and using the scale. This applies especially to all tie-line methods. To test the straightness of a line located by offsets, such for example as a fence line, plot the offsets on another paper (cross-section paper will often answer the purpose), using an exaggerated scale for offsets just as heights are laid off to an exaggerated scale in plotting profiles. This is especially useful in determining boundaries.

(*d*) *General suggestions.* (1) Keep the drawing free from unnecessary lines. Thus in plotting by angle and distance it is not necessary to draw radial lines clear to the station; draw only the portions near the points located. Likewise draw only short portions of intersecting arcs.

(2) Test all check distances or angles. For example, if two houses are plotted independently and the shortest distance between them is given in the notes, always check this distance on the map. Likewise test all the lines given as checks. If an angle is given to a point not for the purpose of plotting but as a check, the field work in measuring the angle goes for nothing unless the angle is tested on the map.

(3) Make it a practice to plot all the important points at a station first, such for example as the main corners of a building or of a street.

(4) Do not waste time in trying to establish some one point which does not come right. After a reasonable effort to locate it leave it and plot the nearby points. Frequently when they are plotted the location of the doubtful point will be evident.

528. Interpolating a Contour means plotting a number of points through which the required contour passes, by interpolating each point between two points already plotted, one higher, the other lower than the contour. Assuming the ground to slope uniformly between any two points used for interpolation, there are a number of methods of finding the position of the required point.

(*a*) *Graphic method.* When the surface of the ground has been divided into squares or rectangles and the height of each corner is given, points may be interpolated, as explained on page 433.

Remark. Any three points on the surface of the ground will lie in some plane, but it is not often that *more* than three points lie in the same plane. Hence if a point is found by interpolating between two given points on the map it cannot be checked by interpolating between two other points. Thus, for example, if a point has been interpolated between two diagonally opposite corners of a square, it is not likely that it will coincide with a point interpolated between the other two corners, for it is not likely that all four corners lie in the same plane. This applies especially to the above method of interpolation, but it should be kept in mind in interpolating by any of the other methods.

(*b*) *Tracing-cloth method.* Let *A* and *B*, Fig. 528 (*b*), be two points on a map whose elevations are 30.3 and 32.7 ft. respectively. It is desired to interpolate two points whose elevations are 31 and 32 ft. A piece of tracing-cloth is ruled with horizontal lines, any convenient distance apart, each space representing 0.1 ft., and the lines representing whole feet being numbered from 0 to 10. Any point on the line representing 0.3 ft. is set at *A* and held there by a prick point. The tracing is then swung on this point as a pivot until the 2.7-ft. line passes through *B*. The points where the 1-ft. line and the 2-ft. line crosses *AB* are pricked through, for they are the required points. Thus the tracing-cloth can be used over and over for interpolating any number of points.

Suggestions. (1) The number and spacing of the parallel lines will depend upon the scale of the map and upon the greatest differences in elevation or the maximum distances apart of any two given points used in interpolation. Thus, for example, if the scale of the map is 1 in. = 20 ft., and elevations are given to tenths at distances not exceeding say 40 or 50 ft., then 40 to 60 lines say $\frac{1}{20}$ in. apart may be ruled. If there are 60 lines and each space represents 0.1 ft. in elevation, differences in elevation may be interpolated up to say 5 ft. This range may be doubled by calling each space 0.2 ft., quadrupled by calling each space 0.4 ft., and so on. Thus a piece of cloth the size of a postal card will often be large enough. (2) Having ruled lines to suit the conditions, number the lines along the edge to correspond. It is well also to rule every tenth line heavy or in red ink to aid the eye in reading. (3) If, occasionally, two points are too close together and of too great a difference in elevation to use the lines as numbered, let each space represent two, four, or even ten times its usual value, as suggested above.

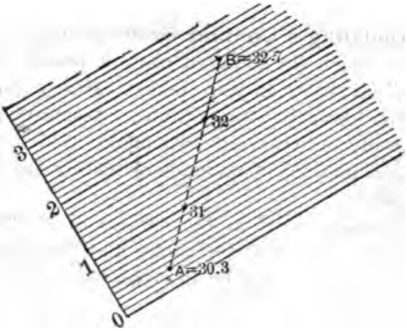

FIG. 528 (*b*).

(*c*) *Home-made device.* Let the parallelogram in Fig. 528 (*c*) represent the inside edges of a jointed frame with parallel threads stretched across, such as that shown in Fig. 528 (*d*). Let it be required to interpolate between two points, *A* and *B*, whose elevations are 52.3 and 58.6 respectively. Set the 2.3 point on one edge at *A* and the 8.6 point on the other edge at *B*. Then thread 5 will cross *AB* at a point on the 55-ft. contour. Likewise if points on the 53-, 54-, 56-, 57-, and 58-ft. contours are desired they will be where the corresponding *threads cross AB.*

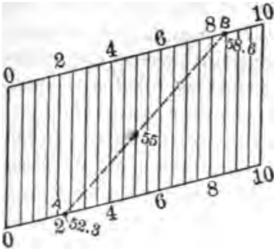

FIG. 528 (*c*).

In making this device the number and spacing of threads will depend upon the conditions named in the suggestions given above for drawing lines on tracing-cloth. The directions for making the particular form shown in Fig. 528 (*d*) were given by Mr. I. T. Farnham in the *Engineering News*, July 30, 1903. This is one of the best of a number of devices in common use, all depending upon the principal of similar triangles.*

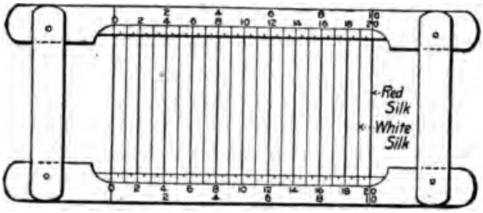

FIG. 528 (*d*).

529. Accuracy Required in Plotting. The same considerations which determine the accuracy required in field work (p. 7) and govern the choice of scale (p. 489) will determine also the accuracy required in plotting. As a rule, distances should be laid off as accurately as the scale adopted will permit. Not all parts of a map, however, should be plotted with the same accuracy. Plot accurately things located accurately in the field—slight things slighted in the field. Thus, for example, traverse lines and boundary lines should be plotted accurately, but indefinite or unimportant details such as river-banks, paths, wood roads, and contour lines may be plotted with much less care.

530. Speed. The suggestions concerning speed in field work (p. 8) hold good for plotting also. Perhaps nothing contributes to speed more than working up the field notes properly, and getting data together before beginning to plot. (Chapter XL.) Free-hand sketches of transit lines with angles and distances marked on the sketches will often save delay in looking up notes. Skill in using the drawing instruments should be supplemented by an economical method of procedure. *Like operations should be grouped.* Thus, for example, all lines that can be drawn with one position of the T-square and triangles should be finished before beginning another set of lines. When the scale is in hand, all measurements which can be made at that time should be laid off. Make sure that the skeleton of the survey is correct by systematic checks (p. 494) before plotting details.

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* See *Engineering News*, 1900, Vol. I, pp. 211, 307, 410, and 1900, Vol. II, pp. 62, 109.

531. Copying and Transferring Maps. Several methods are given below.

(1) Pin the map to be copied on the drawing-board with a clean sheet of paper underneath, and prick through the important points with a sharp fine needle. Work with system so as not to omit any points, and when the upper sheet is removed find first the points which determine the principal lines and then work up the less important details. Several copies can be pricked at once, and the holes in the original map may almost be obliterated by rubbing them on the back of the sheet with a smooth hard substance.

(2) Copy the map with a pantograph (p. 443).

(3) Trace the map and print it on blue or brown process paper (pp. 451 to 454).

(4) Rule squares of any desired size on the original map, draw an equal number of squares on another sheet of paper, and plot each important point by its distances from two sides of the square in which it is located. Unimportant points may be located by eye. If it is desired to reduce or enlarge the map draw the squares on the clean sheet to a smaller or larger scale.

(5) Trace the map on thin tough tracing paper, rub the reverse side of the paper along all of the lines with a soft black pencil, pin the paper over a clean sheet of drawing paper, blackened side downward, and go over the lines with a sharp hard pencil under moderate pressure. This will leave a faint tracing on the clean paper which can be inked afterward. While rubbing the lead on the back of the tracing paper, lay the latter on a hard smooth surface, as a steel triangle, so that the lead will spread evenly.

(6) Maps may be copied by the aid of light from beneath, just as small tracings on thick paper can be made by holding the two sheets against a window pane. The map with the clean paper over it should be laid on a sheet of glass and a strong light reflected through from beneath.* A cloth may be thrown

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* *See Engineering News*, Dec. 29, 1904, for a frame modeled after a

over the head to exclude all light except that which comes through the glass.

(7) Maps may be photographed—a method chiefly used for producing small copies for printed reports by the method of photo-reproduction.

Only methods (2), (4), and (7) can be used for copying to a different scale from that of the original map. Method (3) is the most common, since any number of prints can be made from one tracing. Method (4) is used a good deal for drawing portions of government maps to larger scale. Method (5) is particularly useful in city work, such as transferring plots of buildings from one sheet to another.

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photographer's retouching frame, and also for a rough-and-ready expedient for camp use made from a light placed in a box or barrel, with a piece of glass across the top.

CHAPTER XLII

FINISHING THE MAP

FINISHING the map includes such work as inking, tracing, shading, coloring, lettering, water-lining, putting on conventional signs, designing titles, drawing meridian needles, ruling border lines, and cleaning the drawing. Directions and suggestions for this work are given in this chapter.

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532. Requirements for Maps. Certain requirements common to all good maps are given below, followed by additional requirements to be kept in mind when drawing maps for special purposes.

(*a*) *Specifications for workmanship.* (1) Clean-cut ink lines accurately drawn (p. 446)—plain unmistakable figures (p. 27) —good lettering (p. 507)—symmetrical and orderly arrangement (p. 504)—neat appearance.

(*b*) *What in general should appear on a map.* (1) A neat, explicit title. (2) The scale given in figures and also drawn on the map. (3) A meridian needle showing the direction of the *true* meridian, or two needles showing the direction of the true and of the magnetic meridian, *with the magnetic declination given in figures.* (4) Important data such as the date and the name of the surveyor. (5) A simple border line. (6) Explanatory notes. (7) A key to all symbols used.

(*c*) *Remark.* In some cases, as for example when maps are made for photo-reproduction, the scale is drawn but not given in figures. When a magnetic meridian adds nothing to the value of a map, as is often the case, it is better to draw only the true needle, and to omit the value of the declination. A note, however, should make clear that it is the true and not the magnetic needle which is shown.

(*d*) *Additional requirements for an ordinary map* will depend, of course, upon the use to which the map will be put. Assuming that this consideration governed the field work, it is probable that any data which appear in the field notes (in addition to those required for plotting) should appear also on the map. The most important requirements for an ordinary property map are: (1) Property lines indicated as such. (2) Lengths of all boundary lines and character of boundaries, i.e., kinds of fences, etc. (3) Angles at the intersections of boundary lines. (4) Points of intersection (boundary corners) properly referenced with respect to permanent objects, and all monuments marked as such. (5) Street lines, curb lines, and building lines clearly indicated as such. (6) Property lines of adjoining land which intersect the boundaries of the tract surveyed shown and angles given. (7) At least one boundary line properly referenced (by offsets and bearing if possible, see remark, p. 203). (8) Names of the proprietors of adjoining land. (9) Names of streets, streams, etc.

(*e*) *Remark.* Incomplete maps are sometimes as bad as inaccurate maps, and are frequently the cause of boundary disputes. In some states good laws exist prescribing what shall appear on a map before it will be received for public record. Among such requirements, in addition to those given above, are: The certificate of the surveyor that the plat is a correct one, and that the monuments described in it have been planted as therein described; also the acknowledged signatures of all persons possessing title to any of the land in the tract, and, sometimes those of adjoining owners.

In a paper on *The Ownership of Surveys and what Constitutes a Survey and Map* Professor William G. Raymond discusses the requirements for a good map, and makes some particularly valuable suggestions concerning maps intended for public record. This paper originally appeared in *The Polytechnic*, the student journal of Rensselaer Polytechnic Institute, and is reprinted in Professor Raymond's *Plane Surveying* and also in Professor Johnson's book on surveying.

(*f*) *Requirements for topographic maps* in addition to those already given are: (1) Contours shown and plainly marked with elevations (p. 518). (2) Different topographic features indicated by conventional signs (p. 519). (3) Datum (actual or assumed) given in a note (p. 527). (4) Different portions of the map tinted with different colors (p. 522). This last requirement is for a limited class of maps only.

(*g*) *Remark.* Maps for special purposes must meet special requirements, Thus, for example, on page 211 are given the requirements for a map made for an architect. Entirely different data must be given on maps for use in landscape work, mining, railroad work, and court proceedings. The aim should be to make maps for such purposes explicit and complete like a working drawing, with all necessary dimensions and explanatory notes.

533. The General Method of Procedure in finishing a map after it has been plotted in pencil may be outlined as follows: (1) Decide upon the general arrangement of border lines, lettering, etc., and draw the working-limit line. (See the next article.) (2) Ink in the main lines such as street lines, boundary lines, etc. (p. 506). (3) Ink in details such as buildings, streams, etc. (4) Letter the map, putting on dimensions and the small single-stroke lettering first, but sketching the names of streets, streams, property owners, etc., in pencil if there is any likelihood of the small lettering interfering with or crowding the principal names (p. 506). (5) Finish the conventional signs if there are any. (6) Print the title and explanatory notes (p. 523). (7) Draw the meridian needle and the scale (p. 526). (8) Draw the key or legend, if one is necessary.

Note. The above method of procedure may be followed whether the map is traced or inked directly on the drawing paper. If, however, the map is to be tinted, the tints may be laid on before inking, though many engineers prefer to use waterproof ink and to lay on the tint after inking.

534. Arrangement. (*a*) *Border lines.* Draw the border lines in pencil. If they are to be square with the points of the compass (a question discussed on page 490) one side of the

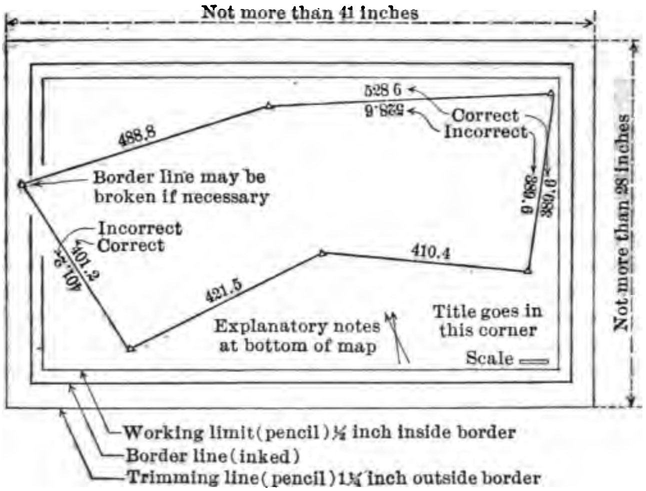
Not more than 41 inches
Not more than 28 inches
528.6
528.6
Correct
Incorrect
488.8
389.6
389.6
Border line may be broken if necessary
Incorrect
Correct
401.2
401.2
410.4
421.5
Explanatory notes at bottom of map
Title goes in this corner
Scale
Working limit (pencil) ½ inch inside border
Border line (inked)
Trimming line (pencil) 1¼ inch outside border

FIG. 534.

rectangle is either drawn parallel to a meridian line or else plotted from a line of known bearing. If the directions of border lines are immaterial, draw the smallest rectangle consistent with a good arrangement of the map as a whole. Space must be provided for a title, a meridian needle, and the necessary explanatory notes. Draw a pencil guide line about ½ in. inside of the border line; no line of the map should extend beyond *this working* limit. Draw another pencil line from 1 to 1¼ in.

outside of the border line which will be the edge of the finished drawing after it is trimmed. Avoid making a map square, or with its vertical edge much longer than its horizontal edge. In Fig. 534 the over-all dimensions are such that the tracing will fit inside of a standard blue-print frame designed for sheets of "double-elephant" size. (See p. 523 for inking border lines.)

(*b*) *Lettering.* Having decided which edge of the map is to be the top (see p. 490), print words and dimensions so that they can be read when either the bottom or the right-hand edge of the map is held next to the body, i.e., print from left to right and from the bottom upward. In only one case should the printing read downward—when the line extends in the general direction of a diagonal from the upper left-hand corner to the lower right-hand corner of the map. (See p. 517.) It is a very common mistake for beginners, working from different edges of the drawing-board, to put on dimensions and words upside down, especially dimensions. (See Fig. 534.)

(*c*) *Titles.* The best place for a title is in the lower right-hand corner, for in searching through a pile of maps it is convenient to raise simply the lower corner of each map without disturbing the pile until the title of the map sought for is exposed to view. Frequently, however, there is no room for a title in the lower right-hand corner, or there is a much larger vacant space elsewhere in which it is desirable to place the title for the sake of a symmetrical appearance. For the same reason the meridian needle and explanatory notes are placed in the most open space, though as a rule the explanatory notes should be near the title, and the meridian needle near the bottom of the map. *Never place the title inside of the property mapped.* (See page 523 for size of title.)

(*d*) *Names of streets, streams, property owners,* etc., are placed in vacant spaces where they will catch the eye, but not where they will obscure any part of the map. If possible place the names of a street in the center of that street, and the name of a property owner parallel to the boundary line between his land and the property mapped.

(*e*) *Final suggestions for arrangement.* (1) Put on necessary dimensions before printing words, and do not put on either until the lines of the map have been inked. (Note an exception in section lining, p. 522.) (2) Ink the border line last—it may have to be changed slightly. (3) When working on the miscellaneous small lettering, plan to leave space for the names which require large letters—it is frequently expedient to sketch the latter roughly in pencil first.

535. Inking. (1) Make sure that the ruling pen is in good working order before beginning to ink (p. 440). (2) Make the lines clear cut and fairly heavy especially on tracings—the general tendency is to make lines too light (p. 451). Widths of lines may vary according to their importance, but use the same width of line for like details. (3) Be careful to follow the pencil lines exactly. Of what avail is it to plot accurately in pencil only to lose accuracy in inking? (4) Follow the general directions for inking, page 446. (5) Ink the lines in the order of their importance, the most important lines first. (6) Edges of streams and similar indefinite lines should not be ruled but should be drawn free-hand zigzagging. Use a ruling pen or a contour pen for free-hand work to secure a uniform width of line. (7) The map will "stand out" better if portions of the outlines of details are drawn heavy, i.e., if outline shading is employed, as in working drawing. (See the next article.) (8) It is not customary to ink the traverse lines, but if they are shown at all they are usually drawn in red ink. If it is desired to show the location of stations this may be done without inking the traverse lines by merely drawing a very small circle in red ink around each station point; small equilateral triangles are used for triangulation stations. In some maps such as those of mines it is important to show all stations. (9) The dashes in broken lines should be short and close together, thus: - - - - - - - -

536. Outline Shading. City blocks, buildings, bridges, outlines of land against water and similar objects should be shaded as *projections,* while streams, lakes, streets, and roadways should be treated as *depressions.* Thus, for example, if the top of the map is north, the *south* and *east* sides of a building are the heavy lines, while in the case of a stream it is the *north* and *west* banks that are shaded. In the free-hand shading of coast lines or roadways, half or quarter strength shade lines at the proper angle are sometimes used.

537. Use of Colored Inks. Lines which represent existing features are almost invariably drawn in black, but proposed *changes,* such as a new layout for a road or a change of grade *for a street,* are usually shown in red. Contour-lines are frequently drawn in sepia or seal-brown, and water-lines in blue (except on tracings). These are the principal colors used unless a map is tinted. On special maps additional colors are used to bring out different features, as, for example, when the workings in the different levels of a mine are shown on the same plan and a different color is used for each level. Dimension lines are usually drawn in red with *black arrow-heads.*

(*a*) Prepared colored inks furnished in bottles are often unsatisfactory and deteriorate rapidly. Test by drawing several lines as wide as will be required on the map. If the ink seems thin and gathers at the end of the stroke, leaving the end of the line a more pronounced color than other portions, it should be rejected. If water-proof qualities are not desired the most satisfactory colors can be obtained by mixing water-colors with just enough water to make the mixture flow readily from the pen. Use scarlet vermilion for red, burnt sienna for brown, and Prussian blue for blue.

538. Tracings. General directions for tracing are given on p. 451. Most of the directions for finishing a map in ink (Art. 535) apply also to tracing. If colored inks are used it should be remembered that colors known as "earths," such as deep yellows and browns, will give the whitest lines on blue-prints, while blues will almost vanish in printing as they offer little resistance to the sun's rays. Many of the prepared red inks will give only faint lines, but a scarlet vermilion water-color (Art. 537 (*a*)) will give fairly white lines.

When the map has been traced on the smooth side of the cloth a nice finish can be given by rubbing colored pencils on the reverse or rough side, using the conventional tints given on page 523. Hold the pencil flat to spread the tint quickly and evenly, then use an old dried-up eraser to rub the color down until it is free from streaks and as uniform as a wash tint. Too soft an eraser will take the tint off instead of rubbing it in. Finish by rubbing with a cloth. The tracing-cloth will only take about so much color. Lighter tints may be obtained by rubbing less color on in the first place. For streams and bodies of water the deepest blue is near the shore-lines and this is gradually worked out to almost no color in the center. Sometimes excellent effects may be obtained by section-lining in color instead of spreading colors evenly. For lettering tracings by shifting letters underneath see page 517.

539. Lettering.[*] Good lettering does not depend to such an extent as is generally supposed upon ability to handle a pen. If letters are properly formed, well proportioned, and well spaced the general effect will be pleasing even though the strokes are made free-hand and are ragged. A knowledge of the essentials of good lettering is all the more important when the ruling-pen and straight-edge are used, for the mere fact that the letters are not drawn free-hand indicates that an extra effort was made to do good work, and hence extra-good results may be expected. Letters in the same word belonging to different styles of alphabets, and letters too wide or too narrow for their heights, are defects which show ignorance, and no mere skill in execution can atone for this. A knowledge of the forms and proportions of the letters in at least one good standard alphabet, such as the Gothic, may be gained in a very short time and will go far toward helping the beginner to do satisfactory lettering.

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[*] The single-stroke system of lettering has already been explained on pages 24 to 29, and hence this article deals mainly with letters which cannot

(*a*) Style of letters. Letters on maps should be simple. Ornate letters indicate extra effort to produce good effects, and when they are poorly executed, as they frequently are, the failure is more pronounced than if only a simple dignified style had been attempted. Poor lettering may ruin the appearance of a map, but it is not necessary to go to the other extreme and to give to lettering an undue prominence. Too often time wasted in ornamental lettering could have been used to advantage in plotting the map more accurately, and occasionally such lettering is used to impress the uninitiated with the excellence of a very poor map.

There are two standard alphabets which form the basis of map lettering, i.e., *Gothic* and *Roman*. Capital letters of the Roman style are simple and very effective. They are perhaps

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be made with single strokes. It is not the intention to treat the subject of lettering in detail, but rather to guide the student in his study of books on lettering, especially those recommended below.

"Lettering," by C. E. Sherman, C.E. (Midland Publishing Co., Columbus, Ohio). Especially good for forms and proportions of letters, for order and sequence of strokes in forming letters, and as an all-around book on map lettering.

"Lettering for Draftsmen, Engineers, and Students," by Charles W. Reinhardt (The D. Van Nostrand Publishing Co., New York). Especially good for single-stroke free-hand lettering.

"Plain Lettering," by Professor Henry S. Jacoby (Engineering News Publishing Co., New York). Gives in detail the exact form and proportions of letters of different styles, and also definite rules for spacing. Especially adapted for exact work with instruments.

"Free-Hand Lettering," by Victor T. Wilson, M.E. (John Wiley & Sons). An admirable presentation of "the development of a letter, a word, *or lines* of words by the sketch method, analogous to that used in other *free-hand drawing*.

the most beautiful of all letters, but the spurs, delicate serifs, and hair-lines are difficult to draw, and if not executed well they are especially noticeable because the eye is so accustomed to the comparatively perfect forms in ordinary type. The student is therefore advised to adopt the Gothic style and to stick to it until he has become skillful enough to undertake the other. One alphabet thoroughly mastered will pave the way to another. Letters may be drawn upright or inclined. It is somewhat easier to secure a uniform appearance if the letters are inclined, since slight variations from the standard slope are less noticeable than equal variations from the vertical.

(*b*) THE GENERAL METHOD OF PROCEDURE is to draw top and bottom guide-lines in pencil, to block out the letters with a sharp pencil, and then to ink them with a writing-pen. Even an expert usually requires a bottom guide-line, and the beginner will need one at the top also.

(*c*) THE SIZE OF LETTERS depends upon the importance of the object described, the scale of the drawing, and the size of the map. The following proportions are approximately those suggested by Professor Charles McMillan:*

Scale	Height of Largest Upright Capitals	Height of Small Letters for Explanatory Notes
$\frac{1}{600}$ or 1 inch= 50 ft.	0.6 inch	0.12 inch
$\frac{1}{2400}$ or 1 inch=200 ft.	0.5 inch	0.10
$\frac{1}{5280}$ or 1 foot=1 mile	0.4 inch	0.06 inch

In the author's judgment the heights given above for capital letters should be reduced 0.1 inch in each case. Gothic capitals 0.5 inch high are large enough for most maps, and often 0.3 inch is a better height.

Frequently it is the size of the map and not the scale which determines the heights of letters. Thus another writer suggests that "As a rule no letter should exceed 0.03 of the length of the shorter border of the drawing on which it appears." It would seem as if this rule places the limit too high for ordinary maps, and that $\frac{1}{60}$ of the length of the shorter border is a better maximum height for letters. The tendency is to make letters too large—they should not "stare one in the face" upon picking up a map.

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* Sherman's "Lettering," p. 13.

(*d*) Proportions of letters. A good unit used in proportioning letters is *one-sixth* the height, and the width of a letter may be expressed in terms of this unit. The widths of Gothic capitals may be given as follows:*

J is	4	units	wide
L is	4¼	"	"
F, **H**, **N**, **P**, and **U** are	4½	"	"
B, **E**, **R**, and **S** are	4¾	"	"
D, **T**, **V**, and **Z** are	5	"	"
A, **C**, **G**, **K**, **O**, **Q**, **X**, and **Y** are	5½	"	"
M is	6	"	"
W is	7½	"	"

The letter **I** is one unit wide, and this is the width of the stems of all of the letters. Although the first step in proportioning a letter is to determine its width, it is also necessary to observe other proportions. These proportions may be studied from a chart in which each letter is outlined upon a surface divided into unit squares. (See pp. 511 to 515.) In ordinary work letters are so small that these squares *are not drawn*, proportions being estimated by eye. Proportions are arbitrary and may be varied, but until the beginner has acquired a sense of proportion, he should be guided by the letters in Plates I to VI, pp. 511 to 515.

(*e*) Penciling the letters. Use a hard pencil with a sharp point. Penciling should be done with care, for if the letters are not well proportioned and well spaced in pencil they are seldom improved in inking. When the Gothic letters are too large for single strokes proceed as follows: (1) Rule top and bottom guide-lines. (2) Place a dot ⅙ of height above bottom guide-line, thus estimating once for all the unit for proportioning and for the thickness of stems. (3) Block out the letters in pencil, using limiting verticals or "width-lines." The distance between the "width-lines" for any letter is given above in units of ⅙ the height; to estimate this distance draw the left-hand width-line and place a dot on one of the guide-lines 6 units to the right, as if the letter were to occupy a square, then cut off or add to this square the necessary space according to whether the letter is narrower or wider. This is done by eye, no measurements being made. It is better to finish one letter before drawing the width-lines for the next in order to space correctly. In drawing any letter there are two things to observe: (*a*) the sequence of strokes and (*b*) the placing of dots to secure proper proportions. Thus in the **H**, Plate I, the first four strokes are vertical strokes and the dot half-way up is made before strokes 5 and 6. In most letters the strokes on or near the width-lines are made first, followed by parallel strokes. In letters like **M**, **N**, **K**, **W**, and **V** the placing of one or more guiding dots is very important.

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* Sherman's "Lettering," p. 8.

Remark. The directions given are for lettering on paper. In lettering on tracing-cloth many devices may be used for shortening the work, such for example as a sheet ruled in squares to slip under the cloth, or one on which inclined lines have been ruled to aid in getting inclined lettering at the proper slope.

(*f*) A STUDY OF THE GOTHIC CAPITALS in detail is given below.* The order of strokes may be varied somewhat, but in practicing it is well to observe that given below for each letter. Consult the letters in the upper row of each plate for proportions, and those on the lower row for methods of construction. Peculiarities of some of the letters can be learned easily at the outset—for example: the horizontal bars of **H**, **E**, **F**, and **B** are more than half-way up, while those of **P** and **R** are not; **O**, **C**, **G**, and **Q** are slightly higher than other letters, extending above the top and below the bottom guide-lines; **B**, **E**, **K**, **S**, **X**, and **Z** are not so wide at the top as at the bottom; strokes 5 and 6 of the **M** do not meet 3 and 4; stroke 5 in **K** does *not* start from the vertical stem.

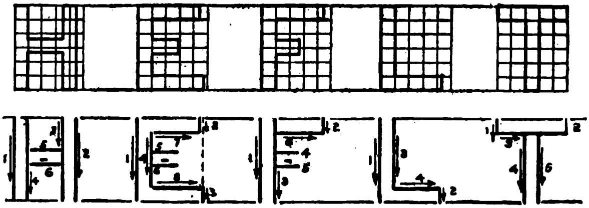

PLATE I

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* With the kind permission of Professor Sherman the drawings in this article have been modeled after similar figures in his book on lettering, and the directions for forming the letters which accompany the cuts on pp. 512–515 are taken verbatim from pages 27 and 28 of that book.

H After cutting off 1½ stems from a square to obtain the circumscribing rectangle, the strokes are as indicated, 1 and 2 coinciding with the left and right sides of the rectangle respectively. Strokes 5 and 6 are correctly placed by first bisecting the height with a dot—the bar to be slightly above mid-height.

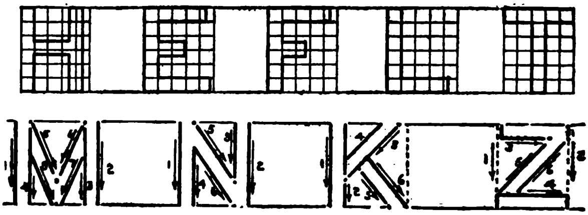
E The middle bar is placed by the use of a dot as in **H** and is half the length of the bottom bar. The top bar is ¼ unit shorter than the bottom one.

F Central bar placed slightly above the middle as above, and is half the length of the top bar.

L Cut off 1¾ units from a square to get its width. If the next succeeding letter in a word is **A**, narrow the **L** to 4 units.

T Bisect stroke 3 with a dot to guide strokes 4 and 5.

PLATE II



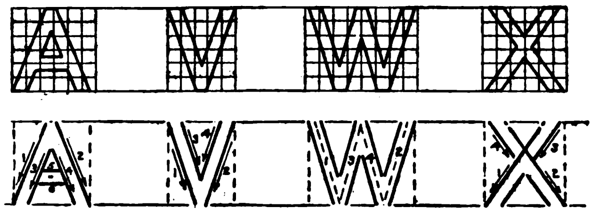
M Is somewhat difficult to draw neatly, the trouble being to keep the junction of the inclined and vertical stems from being too thick without making the middle opening appear too large. After strokes 1, 2, 3, and 4 place a bisecting dot on the lower guide Above this 1¾ units, about, place another dot, and strokes 5 and 6 join this dot with points on the top guide about ⅕ of a space from 3 and 4. Strokes 7 and 8 are parallel to 6 and 5 respectively, making the inclined parts a unit thick. This should make the width of the middle portion on the lower guide less than a unit.

N After strokes 1, 2, 3, and 4 place dots at unit distances from 1 and 2, as shown at top and bottom. Stroke 5 begins at the dot on the top guide and ends on stroke 3 at a trifle less than two units from the bottom. Stroke 6 is parallel to 5 and must end on the lower dot. The inclined stem must not be *over a unit thick.*

K After strokes 1 and 2 place a dot on 2 two units above the bottom. Place a dot on the top guide half a unit from the right side, and 3 joins this point with the dot on 2. On the lower guide place a dot 1¼ units from the right side, and another on 3 at 1¼ units from 2 to guide stroke 5. Strokes 4 and 6 are parallel to and unit distance from 3 and 5 respectively.

Z After strokes 1, 2, 3, and 4 place a dot on 3, 1¼ units from the right side. Stroke 5 joins this dot with the left end of 4. Stroke 6 is parallel to and unit distance from 5, but must end at the intersection of 3 and the right width-line; some shifting of strokes 5 and 6 may be necessary to secure this.

Plate III



A Find its rectangle, bisect the top by a dot which guides 1 and 2. After drawing 3 and 4, place a dot 2 units from the bottom; strokes 5 and 6 are equidistant from this dot.

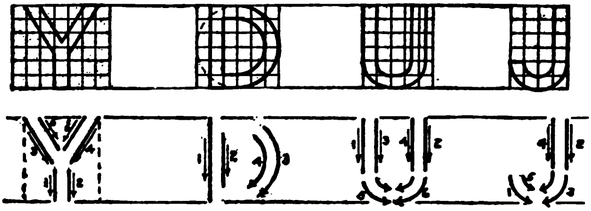
The vertexes of **A** and **V** may be made slightly less than one unit wide, but should not be wider than a unit.

V Bisect the bottom of its rectangle by the dot which guides 1 and 2. Strokes 3 and 4 are simply parallel to 1 and 2 and at unit distance as shown.

W Mark off the proper rectangle by adding 1½ stems to a square, then place dots on the lower guide about 2 units from the width-lines, and a bisecting dot at the top. These dots guide the auxiliary strokes 1, 2, 3, and 4 which guide the outlines.

X Place dots on the upper guide 1½ units from the width-lines. Strokes 1 and 2 join these dots with the lower corners as shown. Strokes 3 and 4 are parallel to 1 and 2 respectively and at unit distances.

Plate IV



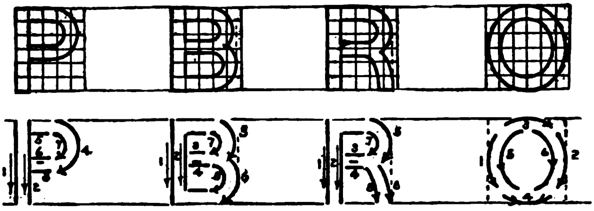
Y Find the center of its rectangle quickly by eye, and place a dot half a unit below; this guides 1 and 2 (a unit apart), and marks their upper extremities. Strokes 3 and 4 join these upper extremities with the upper corners as shown.

D Requires no special instructions except to say that it is helpful to use bisecting dots to guide the curved strokes. Thus, a dot should be placed midheight on the right width-line, and stroke 3 is then made symmetrical with this dot.

U The outside strokes may be made first if desired instead of following the order given.

J Stroke 1 should begin 2½ units above the bottom.

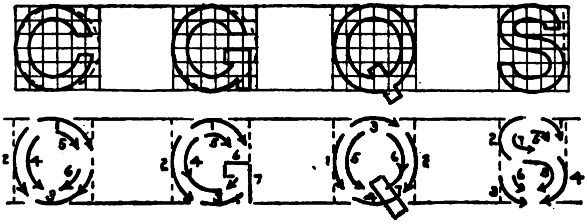
Plate V



P, **B**, and **R** The middle horizontals in these letters are guided by first placing dots at midheight. In **P** the bar is a trifle below the middle, in **B** a trifle above, while in **R** it may be placed in the middle.

O, **C**, **Q** As in the Roman forms, the exteriors of the curved letters are better drawn first, in some such manner as shown *before* the interior strokes are made. Use width-lines and bisecting dots, which help in making the exterior strokes symmetrical.

Plate VI



G Stroke 6 is but ¼ unit below the center.

S One method of making the strokes is given; but methods of drawing this letter vary considerably with different draftsmen. Stroke 3 should begin 2 units above the guide. The middle portion of the letter should be almost horizontal.

All Gothic curved letters, also, should extend a trifle above and below the guide-lines to cure optical defects.

(*g*) Inking the letters. The inking, if free-hand, is done with a writing-pen. Among the pens recommended are Gillott's 303 and 404, and Leonard's ball-pointed pen. The student should try a number of different styles until he finds one that suits him. A pen will work better after it has been "broken in" or slightly worn. A pen-holder with a large grip is the best for lettering.

Begin by inking the pencil outline, taking care to keep the outside edges straight—if the inside edges are ragged it does not matter, provided the space between outlines is to be filled in solid. *Keep inside of the pencil lines.* In filling in, use long strokes. After a letter is dry, corners may be made square and other imperfections touched up, although very little of this should be necessary. Wipe the pen frequently, and do not overload it with ink. Keep a piece of paper tacked to the board to try the pen on and thus start the even flow of ink. Shields are useful in erasing portions of letters (p. 448). *Keep the stopper in the ink-bottle* to exclude dust, to prevent the ink from deteriorating, and to save the drawing in case the bottle is upset.

When the extra time required is warranted, the outlines may be inked with ruling-pen and compasses. Special effort should then be made to secure clean-cut corners and smooth curves.

(*h*) Common defects. (1) The beginner almost always makes Gothic letters too heavy and clumsy. The letters outlined in ink seem correct, but after the outlines have been filled in solid black, the stems are too thick. This is because the thickness of the outlines is added to the space between them. Avoid clumsy letters by making stems slightly narrower in pencil than seems necessary: (2) Another common tendency is to make stems which are curved or inclined thicker than those which are vertical and horizontal, hence such letters as **O**, **U**, **M**, and **W** are apt to be too heavy. (3) Great care is necessary to avoid a too heavy appearance at acute angles, as, for example, at the bottom of the **V**, and at the junctions (top and bottom) of **M**, **N**, and **W**.

(*i*) Spacing.* Aim to space the letters in any word so that the same area of white paper will show between letters. This is the first and most important rule, *but it does not mean that letters should be spaced equal distances apart*, for if they were, letters like **L** and **A** which do not entirely fill the limiting rectangle will often appear separated from other letters. Thus, for example, one of the most difficult words in which to space letters correctly is **LAW**. Whenever the letter **L** is adjacent to a letter like **A** or **W** it is necessary to close up the space. and sometimes to shorten slightly the horizontal stem of the **L**. Likewise when any letter which does not fill its rectangle occurs in a word it may be necessary to close up slightly the space on either side. The closer letters are together, the more this is necessary. When the space between letters is two or three times the width of a single letter the distances between the centers of letters may be made equal without offending the eye.

Ordinarily the space between two words should be at least twice as great as the normal space between two letters When a punctuation-mark is used this space should be measured from the mark, thus increasing slightly the space between words. The space should be still larger if the punctuation-mark is a period.

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* *See Sherman's* "Lettering," p. 10, and Wilson's "Free-hand Lettering," *p. 28.*

In lettering tracings it is often best to sketch the outlines on paper first, and then, by shifting the paper back and forth underneath the tracing, to correct any defects in the spacing of letters or words. This method is especially useful when several lines of unequal length are to be centered with respect to a vertical axis of symmetry, as in printing titles. (See p. 524.)

Remark. Rules for proportioning and spacing should not be followed blindly. They are intended, chiefly, to guide the beginner to a proper sense of harmony in lettering, without which no set of rules is of much avail.

540. General Suggestions for Lettering Maps. (1) The general arrangement of the lettering (Art. 534, p. 504) requires that words be printed from left to right or from the bottom up except when the general direction is that of the line AB in Fig. 540. (2) Aim to make as much of the lettering as possible parallel to the bottom border-line, or, if this is impracticable, parallel to the side border-lines. An exception is the lettering along property lines. (3) In general, words should be in straight lines, although there are many exceptions as, for example, the name of a curved street or of a winding river. Words greatly extended like the names of towns are often in long sweeping curves. When a word is on a curve the axes of the different letters should be radial lines unless the letters are of the inclined type.* (4) When there are two lines of words and it is not desired to center them, it is often better to start the second line so that it will extend beyond the first. This applies especially to descriptive matter in single-stroke lettering which is a little too long for a single line. (5) Names or descriptions should be placed in such a position that there will be no doubt as to where each belongs, either between the boundary-lines of an object, as in the case of a street or a river, or just above, as in the case of a narrow stream, or to just the right of a conventional sign, as in the case of a small building. Avoid a crowded appearance. (6) In printing values of angles *never omit zeroes.* Thus 0°–08′ is correct, but not 08′ or 8′.

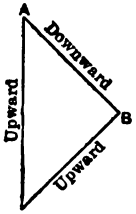

FIG. 540

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* See Sherman's "Lettering," p. 11, and Reinhardt's "Technic of Mechanical Drafting," p. 14.

Likewise in linear measurements 0.8 ft. or 0′.8 is correct, but not .8′. (7) There is danger of reading angular measurements for linear measurements, or *vice versa.* To avoid this always place a dash between degrees and minutes. Thus, for example, 129°–41′ cannot be mistaken for 129′.41 (feet). (8) Since every line has two bearings, in printing the bearing along any line (especially a property line) it is well to draw a short arrow parallel to that line and pointing toward the end which is the *forward* end for the bearing given. (9) Prominence should be given to lettering according to the importance of the object described. Prominence may be gained in several ways, viz.: (1) By increasing the width of letters; (2) by increasing the weight; (3) by increasing the height; (4) by increasing the spacing; (5) by changing the style, and (6) by italicizing.

541. Contour-lines are usually drawn in brown ink or burnt sienna water-color; sometimes red ink or crimson-lake water-color is used, and occasionally contours are represented by broken lines in black ink (short dashes close together). (2) Lines should be of uniform width, just heavy enough to be distinct. They may be drawn with a ruling-pen or a contour pen, but usually the best results are obtained with a well-worn steel pen, such as Gillott's No. 303, applied with little or no pressure to yield a uniform line. (3) Every fifth or tenth contour, counting from the datum plane, should be a little heavier than the others and *also darker in color.* (4) Each accentuated contour should be marked with its elevation above or below datum, and if the map does not contain at least two such contours then every contour-line should be so marked. This contour elevation may be placed just above the line or in a break left for that purpose. The elevations should not be scattered haphazard, but should be placed nearly under each other, or, better still, along a line normal to the contours (usually a winding line). Where the figures are to be inserted in breaks, before inking draw *two* parallel guide-lines in pencil, about ½ in. apart and normal to the contours; when inking the *accentuated* contours stop each one at these guide-lines to give a uniform appearance to the breaks. Elevations may be marked in the color used for the contour-lines, and they should be repeated in different parts of the map wherever necessary, especially where contours change direction abruptly. Aim to *place them where* they will be of the greatest help in ascertaining the elevation of any contour on the map without delay or mental confusion. (5) Contour-lines should not be drawn across buildings, roads, or streams, and they should be turned slightly up hill just before ending at the edges of roads or banks of streams. (If a road is in "fill," the contour lines turn down hill.) (6) Intermediate contours, represented by broken lines, are sometimes inserted to show the outline of some feature of the surface not shown by contours at the regular interval, such, for example, as a summit of a hill or the bottom of a depression. (7) Common mistakes, such as drawing contours of different elevations across each other, or merging contours of the same elevation, may be avoided by remembering the fundamental principles on page 338.

Remark. In small-scale topographic maps hills and mountains are sometimes shown by "hatching" or short disconnected lines. See Reinhardt's "Technic of Mechanical Drafting," p. 37, or Gillespie, Vol. II, p. 350, or Wilson's "Topographic Surveying," p. 462.

542. Conventional Signs. On page 520 are shown some of the conventional signs used on ordinary maps, and on page 521 are some of those used on topographic maps. Most of these conventional signs may be considered standard, though some of them, especially those pertaining to fences, are subject to more or less variation.* A common fault is to make conventional signs too prominent by drawing them too large and heavy. Ordinarily, very light strokes should be used for trees and grass, and ruled lines, such as those for marshes and cultivated land, should be drawn almost as fine as the pen will work. Conventional tints are sometimes used in connection with conventional signs (see p. 523).

(*a*) *Suggestions.* (1) Instead of using conventional signs for fences it is quicker and often better to draw full lines or broken lines as preferred, and then to mark the character of the fence along each line. (2) Small water-courses, where indicated by a single line, should be drawn toward the source so as to end there with as finely tapering stroke as possible. Aim to produce a wavy effect as shown in the right-hand side of Fig. 542 (*b*), avoiding the flat meaningless style of line shown in the left-hand side of the same figure. (3) Outline shading (see page 506) may be used to advantage as shown in the conventional sign for a bridge, page 520. (4) The tufts of grass in "meadow-land," page 521, should begin and end in short light strokes of the pen, the lines radiating from a common center some distance below.

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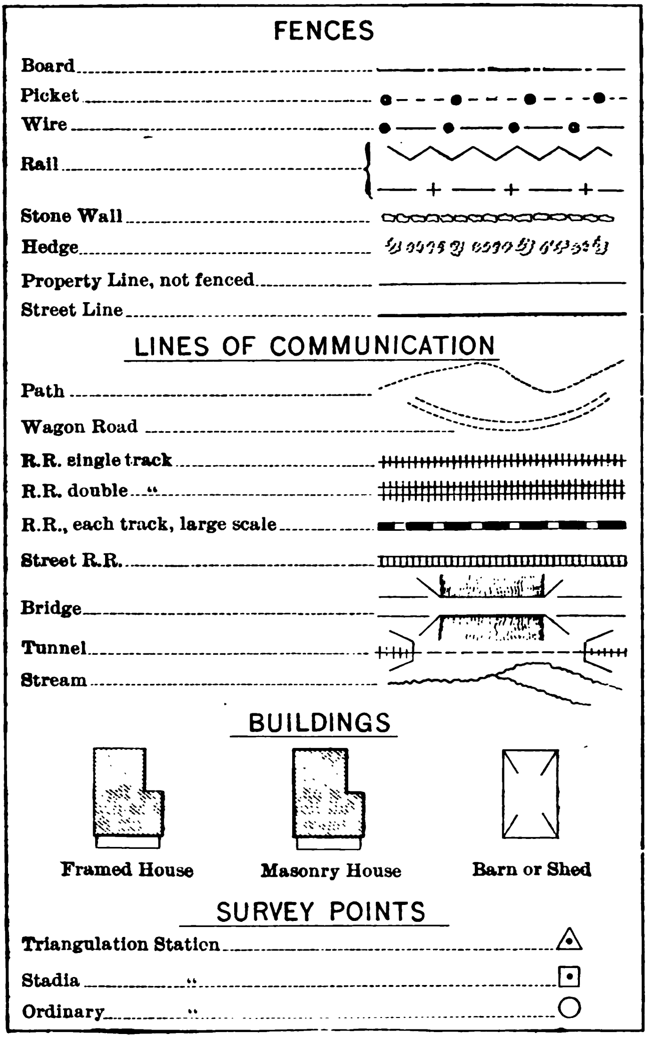
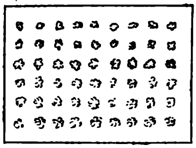
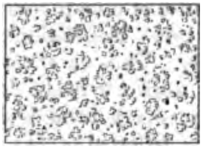
* This variation is evident from the replies to letters of inquiry addressed to city engineers all over the country. For example, many engineers employ a broken line to represent a property line not fenced, although the majority of the replies favored a fine full line.

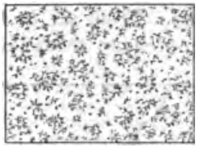

FENCES
Board
Picket
Wire
Rail
Stone Wall
Hedge
Property Line, not fenced
Street Line
LINES OF COMMUNICATION
Path
Wagon Road
R.R. single track
R.R. double "
R.R., each track, large scale
Street R.R.
Bridge
Tunnel
Stream
BUILDINGS
Framed House
Masonry House
Barn or Shed
SURVEY POINTS
Triangulation Station
Stadia "
Ordinary "



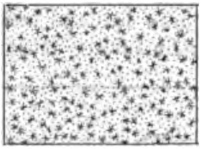
Orchard



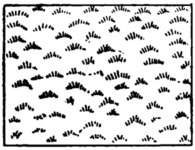
Deciduous Trees



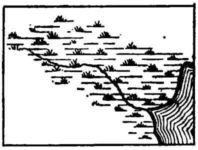
Oak Trees



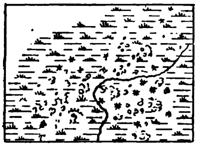
Pine Trees



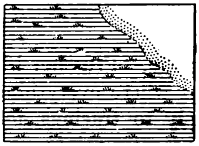
Meadow Land



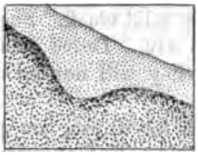
Fresh Marsh and Pond



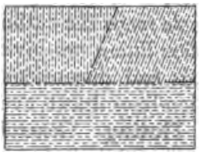
Wooded Marsh



Salt Marsh



Sand-High and Low Water



Cultivated Land

(*b*) *Water-lining* when well done adds much to the general appearance of a map, but when poorly done it is very offensive to the eye. The beginner is inclined to draw water-lines too hurriedly. Thus the effect in the left-hand side of Fig. 542 (*b*) is that of work carelessly done. In the right-hand side, on the contrary, every bend and irregularity of the shore-line was carefully followed. The shore-line is made heavy, and the first water-line drawn as close to it as practicable. The spaces between the water-lines gradually widen, and the little irregularities grow less and less pronounced. From six to ten lines will usually suffice; those farthest from shore should be drawn very lightly. Prussian blue is often used for water-lines, even on maps where everything else is drawn in black. This does not apply to tracings, for blue lines practically disappear in making process prints.

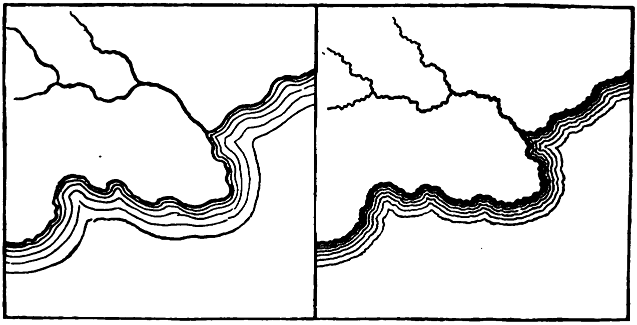


FIG. 542 (*b*).

543. Section-lining. The outline of a building on a map of large scale is frequently emphasized by section-lines, while on small-scale maps the outline is filled in solid black. Suggestions for section-lining are given on page 449. Leave open spaces for words and dimensions, so that they will not be obscured by section-lines drawn over them.

544. Tinting. General directions for tinting are given on page 449. Additional suggestions are: (1) Either tint before inking or use water-proof ink. (Note, p. 504.) (2) The tendency is to use too vivid tints. The beginner will do well to mix his *tint to what seems* a desirable shade, and then to dilute about half. (3) Tinting on ordinary maps is usually confined to filling in outlines of buildings and other structures. Occasionally roads and streams or bodies of water are also tinted.

(*a*) Conventional Tints *

Buildings. If of wood, sepia; for masonry, crimson lake. The outlines of wooden buildings are drawn and shaded with India-ink; outlines of masonry with a very strong tint of crimson lake. To distinguish brick from stone, make the outlines of the latter heavier or else use Payne's Gray for stone and light red for brick.

Roads. The pencil outlines are filled with a flat tint of yellow ochre, the edges being subsequently drawn in India ink.

Bridges. The outline is filled with yellow ochre, and the same distinction as to wood and masonry is observed as in buildings.

Fences are represented with India ink as in pen drawings (p. 520); *stone walls* by their outlines filled with crimson lake.

Water. Prussian blue, a flat tint. If a lower tone is desired for the map use indigo instead of Prussian blue. The surface is also outlined with a strong blue.

Remark. There are a number of conventional tints used on elaborate topographic maps to represent trees, marshes, cultivated land, sand, rocks, etc. For these colors and for a more comprehensive treatise on tinting the student is referred to a chapter on "Topographical Drawing in Color," in Lieut. Reed's book. It takes much practice to do this kind of work, and it is better to leave a map untinted than to produce crude effects in gaudy colors. The student can easily learn, however, to lay on the flat tints given above.

545. Border-lines. (1) Avoid elaborate border-lines. A single heavy line or a heavy line and a light line close together will suffice. Avoid fancy designs at the corners. (2) In drawing the border, keep in mind the size of the blue-print frame and other requirements (p. 504). (3) The heaviest line for a border should rarely exceed $\frac{1}{30}$ inch, and if there are two lines the space between may be about the width of the heavier line. (4) Follow the directions on page 447 for drawing heavy lines. For a width of $\frac{1}{30}$ inch, this means drawing two outside lines and filling in between with a third.

546. Titles.† The title is usually printed after all the other lettering on the map is finished. The best place for it is in the lower right-hand corner unless it is desired to fill up some vacant space elsewhere (p. 505). The size of the rectangle within which the title is inscribed should be *in proportion to the size of the* sheet (see step (4), p. 525), and the size of the largest letters depends to some extent upon the scale used for the map (see p. 509). The general arrangement of the title should conform to the space available. The first consideration in designing a title is SIMPLICITY. *Ornate letters, scrolls, and flourishes are to be avoided.* The subject-matter is usually arranged in several horizontal lines, words which are closely related being put in the same line. The most important words in a title should catch the eye first. Prominence may be gained in several ways (see p. 518). The lines should be centered, i.e., each line made to extend equal distances each side of a common vertical line. Sometimes lines are arranged in curves, but this is to be avoided unless necessary to save space. The spaces in the clear between lines may vary, although often they are made nearly equal. If too much space is left above and below a line of small lettering this line will appear separated from the rest of the title. Avoid making two lines of the same length. The fewer styles of lettering used the neater will be the title. The tendency to use a different style of letter for each line is bad, for it gives the impression that the draftsman is trying to show what he can do. Capital letters are used almost exclusively, lower-case letters seldom appearing in well-designed titles. It is probably better to incline all the letters or to make them all upright than to have inclined letters in one line and upright letters in another. Single-stroke lettering may be used for unimportant words where letters are comparatively small. Very effective titles can be drawn free-hand, and instruments should be used only when the extra time required is warranted. The mere fact that instruments were used in drawing a title leads one to expect superior results, while slight imperfections may be overlooked in a free-hand title. All parts of a title should be finished with equal care—even the smallest lettering should be done as carefully as the largest. It is also inconsistent to take more pains with the letters in a title than with the other letters on the map. It is best not to abbreviate any word in a title.

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* From "Topographical Drawing and Sketching," by Lieut. Henry A. Reed, U. S. Army. (John Wiley & Sons.)

† See Sherman's "Lettering," p. 37, and Wilson's "Free-hand Lettering," p. 59.

Titles for tracings may be penciled on a small sheet of paper *without attempting to center the different lines,* and then by shifting this paper back and forth under the tracing-cloth each line can be centered with respect to a vertical axis of symmetry.

It takes considerable time to execute a good title, and to save time and expense various mechanical devices are sometimes used, such as a stamping outfit or a small printing-press.*

(*a*) *What should appear in a title of a map.* (1) The name or description of the tract or object represented, or its owner's name. (2) The location—town or city, county, state. (3) Scale. (4) Date of survey. (5) Name of surveyor.

If a map is made for a special purpose this should be stated, as, for example, PROPOSED LOCATION OF A HIGHWAY, or PLAN OF BUILDING-LOTS, etc.

(*b*) *Method of procedure in constructing a title.* (1) Write out the title, separating the subject-matter into lines; it is well to give such connectives as "OF THE" and "IN THE" separate lines.

(2) Number the lines in the order of their importance. Usually the name or description of the tract which the map represents is the most prominent. The connectives should be the least prominent.

(3) Choose the style of letters. Heavy Gothic capitals are recommended for the most prominent words, and single-stroke capitals of the same style for the least prominent; the difference in prominence of words between these two extremes may be effected by varying the heights of letters or in other ways (p. 518).

(4) Decide upon the place of the title and upon the approximate size of the limiting rectangle within which it should be drawn. The *maximum* dimensions for this rectangle suggested by Professor Sherman are: Length = ⅓ length of longest border-lines of the map; height = ¼ length of shortest border-lines. Frequently the limiting rectangle should be made smaller than these proportions allow. The tendency is to make titles too large and conspicuous.

(5) Decide upon the height of the largest letters (p. 509) and draw top and bottom guide-lines for the most prominent line of lettering. Draw also a vertical line to serve as an axis of symmetry.

(6) Block out roughly the letters for the most prominent line. If the position of the limiting rectangle for the title is not definitely fixed the vertical axis of symmetry may be drawn through the middle point of this line *after* the letters are completed. When, however, the position of the axis of symmetry is fixed any line of lettering may be centered as follows: Write out the line on another piece of paper, and place over each letter the number of units to allow for its width (p. 510). Estimate roughly the number of units to allow for spaces between letters and between words, keeping in mind that letters are not placed equal distances apart (p. 516). From half the sum of all the units find which letter is nearest the middle point. Beginning with this letter at the proper distance from the axis of symmetry block out the letters in the last half of the line. This gives the distance from the axis of symmetry to the end of the line; begin at an equal distance to the left of the axis and block out the letters in the first half of the line. If the letters are not properly spaced in this first trial usually only a few letters near the center need to be changed,—the chief advantage in lettering one-half the row at a time.

If the title is to be drawn on tracing-cloth the different lines can be centered by shifting the paper as explained above. Conversely sometimes a title can be drawn on a small piece of tracing-paper without *attempting to center the lines*, and then transferred to the paper on which the map is drawn (p. 500), each line being centered by shifting the tracing-paper.

(7) Having penciled the most prominent line, block out the other lines in the same way. Before inking examine the title as a whole to see if it is well balanced. If the inking is done free-hand follow the directions on page 515.

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* See *Engineering News*, Nov. 3, 1904, p. 405.

547. Meridian Needles. Simple arrows like those in Fig. 547 are best. Avoid elaborate needles and compass-points. As with border-lines and titles, simplicity should be the first aim. The true north is usually represented by a full arrow-head and the magnetic north by a half arrow-head. The direction of one of these arrows is found either from the meridian used in plotting the map or from some line of known bearing. If magnetic bearings were used, plot the axis of the magnetic needle and then lay off the declination to find the axis of the other needle, and contrariwise if true bearings were used, find the axis of the true needle first. On many maps nothing is gained by showing the magnetic meridian (see remark p. 502), but if it is shown at all, the magnetic declination should be given in figures marked along the arrow or between the two arrows as in Fig. 547. The needle should be of such length that its direction can be transferred accurately to other parts of the map—usually about 4 or 5 in. long. The arrow-head and the feather-end should both be drawn with care, slim and graceful—the tendency is to make them too clumsy. The needle may be placed near the title or in a vacant space near the top of the map.

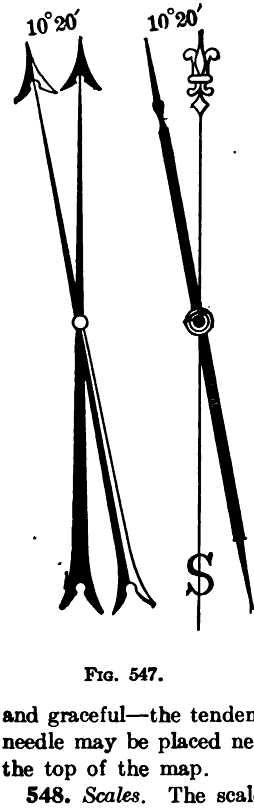

Fig. 547.

548. *Scales.* The scale of a map is usually given in figures as, for example, "Scale: 1 inch = 40 feet" or "Scale: $\frac{1}{10000}$." Frequently in addition a small portion of a scale is drawn as illustrated in Fig. 548. This is most necessary for maps that are to be reproduced by photo-reproduction or by other processes which change the scale, otherwise it would be difficult to scale the reproduction. Such a scale also indicates the shrinkage or expansion in maps and blue-prints, but if designed for this purpose it should be long enough to show the change—

say 6 inches. Distances scaled from tracings and blue-prints are most unreliable no matter what method is used.

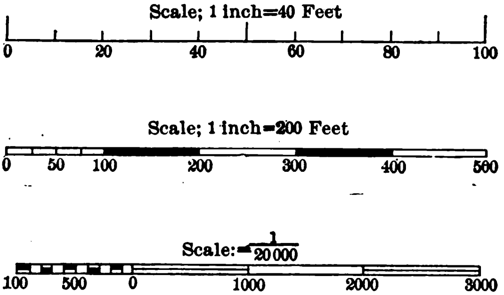

FIG. 548

549. Keys or Legends. Where conventional signs are used it is customary to draw a key near the title explaining the signs. This is especially necessary when the signs are not those commonly employed.

550. Explanatory Notes. Maps, like field notes, should be self-explaining. Anything not clear from the drawing or the dimensions should be made so by explanatory notes. The general directions in Art. 37, page 23, hold good. Notes giving general information concerning the map, such, for example, as the datum used in a topographic survey, may be placed near the title. Occasionally notes should indicate the extent to which measurements may be depended upon by giving the instruments used in the survey, the limits of error adopted, and similar information bearing on the accuracy of the work.

551. Cleaning Maps. Use a soft eraser for removing pencil lines, and a sponge eraser or bread-crumbs for taking off dirt. For tracings use gasoline or benzine. (See p. 452.)

552. Surveyor's Certificate. In filing maps for record the surveyor may be required to furnish a certificate, the exact form of which will of course vary. One form is as follows:

....
(City or Town, State, Date)

I hereby certify that the plot hereon delineated is a correct one and that permanent monuments, consisting of ...., have
(Description of the monuments)
been set at points marked MONUMENT as thereon shown at all angles in the boundaries of the land plotted, and at all intersections of streets and alleys.

................, Surveyor.
(Signature)

Another form sometimes furnished a client but not intended for public record is as follows:

....
(City or Town, State, Date)

This is to certify that I have this date made a survey of ....,
(Description of tract surveyed)
and have plotted the lines thereof on the accompanying map, which to the best of my knowledge and belief is correct.

.... Surveyor.
(Signature)

553. Filing Systems. Although it is not within the province of this book to explain the various systems of filing and indexing drawings and note-books, no discussion of office work would be complete without reference to this important subject. To one who desires to look up some of the systems which have stood the test of every-day use the following references may prove of value:

"Engineering Work in Towns and Small Cities," by Ernest McCullough (Technical Book Agency, Chicago), contains two very complete chapters on "Office Systems" and "City Engineers' Records."

Proceedings of the Connecticut Society of Civil Engineers for 1901. A description of the excellent system used in the city engineer's office of Hartford, Connecticut.

Engineering News, Sept. 5, 1901, Jan. 2, 1902, and April 23, 1903, several articles on different systems in use by railroad and city engineers.

CHAPTER XLIII

PROFILES

In this chapter are given the few directions and suggestions necessary for plotting and finishing profiles.

554. Profile. (*a*) Suppose that in Fig. 554 (*a*) *A, B, C,* and *D,* representing four points in a straight line on the ground, were plotted to a given *horizontal* scale. Let the points *a, b, c,* and *d* be obtained by laying off *A′a, B′b, C′c,* and *D′d* equal, respectively, to the elevations of *A, B, C,* and *D* above a given datum. If the *vertical* scale used in laying off these elevations is the same as the horizontal scale used in plotting *A, B, C,* and *D* the line *abcd* is a *true* profile, bounding a *true* vertical section through the given line, and showing the rise and fall of the ground.

(*b*) As the rise and fall of the ground between two points is usually very small in proportion to the distance between those points, it is customary in plotting profiles to use a much greater scale for laying off elevations than for plotting horizontal distances, in order to make the irregularities of the surface more apparent to the eye, and to render subsequent use of the profile easier and more accurate.

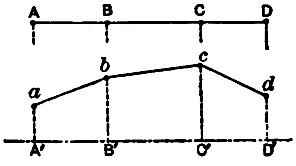

Fig. 554 (*a*)

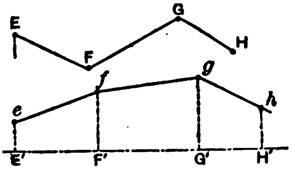

Fig. 554 (*b*)

Suppose that in Fig. 554 (*a*) the vertical scale is ten times as large as the horizontal. The line *abcd* is still called a profile, though it is no longer a *true* profile, and the section which it bounds is no longer a true section.

(*c*) Frequently in practice the line leveled over is not straight, but consists of a series of connected straight lines, as, for example, a traverse line. In Fig. 554 (*b*) *EFGH* is the plan of a traverse line, and *efgh* is the corresponding profile in which the vertical scale is larger than the horizontal. The distances *E'F'*, *F'G'*, and *G'H'* are equal respectively to *EF*, *FG*, and *GH*, but *F'G'* and *H'* cannot be obtained by projecting from *F*, *G*, and *H* respectively. In Fig. 554 (*a*), on the contrary, *A'*, *B'*, *C'*, and *D'* were projected from *A*, *B*, *C*, and *D* respectively. Hence in Fig. 554 (*a*) *abcd* represents a profile cut by *one* vertical plane, while in Fig. 554 (*b*) *efgh* represents a profile cut by a *series* of vertical planes, each of which passes through two stations at consecutive bends of the traverse line.

(*d*) *Remark.* A profile is apt to be very misleading to persons unaccustomed to the methods of plotting. Unlike a map a profile seldom represents points projected on the same plane. Moreover, when the vertical scale is much greater than the horizontal scale, slopes are greatly exaggerated, and the surface of the ground appears to be much more irregular than it really is. Likewise all objects such as manholes, culverts, rivers, roads, etc., which the profile crosses are shown distorted. It is a common experience among engineers to receive protests against a proposed grade because on the profile it appears to be too steep.

555. Profile-papers. Occasionally a profile is plotted on the same sheet with the map, but more often special paper is used upon which horizontal and vertical lines have been printed so that points may be plotted without the use of a scale. There are three standard styles of profile-papers, distinguished by the letters A, B, and C according to the number of lines or spaces to the inch. Vertical lines on A profile-paper are ¼ in. apart, and horizontal lines are $\frac{1}{20}$ in. apart, or 4 by 20 spaces to the inch. B profile-paper has 4 by 30 and C paper 5 by 25 spaces to the inch. These rulings are also furnished on tracing-paper and tracing-cloth. Cross-section paper 10 by 10 to the inch is sometimes used for plotting profiles.

Every tenth vertical line and every fifth horizontal line on profile-papers are usually accentuated for convenience in reading. The rulings are not always exactly correct, but usually the errors are so small that they may be ignored in plotting ordinary profiles, especially as the *relative* elevations are read directly from the paper and not by scaling the drawing.

556. Scale. When a profile is to be used in connection with a map it is often desirable to have the horizontal scale the same as that used for the map; sometimes, however, a larger scale is required. The vertical scale is usually exaggerated to at least five times the horizontal scale and often more, depending upon how close it is desired to read elevations. For city work such as laying out street grades or showing sewers common scales are 40 or 50 ft. to the inch (horizontal) and 6 or 10 ft. to the inch (vertical). When it is desired to show a longer distance on a sheet of given length the horizontal scale may be reduced to 1 in. = 80 or 100 ft., and the vertical scale kept the same (1 in. = 6 or 10 ft.). Common scales for railroad profiles are 1 in. = 400 ft. (horizontal) and 1 in. = 30 ft. (vertical). The following are some of the most convenient combinations of papers and scales.

	Hor. Scale	Vert. Scale	Paper	Smallest Space between V Lines	Smallest Space between H Lines
(*a*)	1 in. = 40 ft.	1 in. = 6 ft.	B	10 ft. distance	0.2 ft. elev.
(*b*)	1 in. = 50 ft.	1 in. = 10 ft.	C	10 ft. "	0.4 ft. "
(*c*)	1 in. = 100 ft.	1 in. = 10 ft.	A	25 ft. "	0.5 ft. "
(*d*)	1 in. = 100 ft.	1 in. = 6 ft.	B	25 ft. "	0.2 ft. "
(*e*)	1 in. = 400 ft.	1 in. = 30 ft.	B	100 ft. "	1.0 ft. "

There are, of course, a number of other combinations. It is desirable to so choose the paper and scale that the smallest space between vertical lines will represent some multiple of 5 ft., while the smallest space between horizontal lines will represent some multiple or decimal part of one foot. It is also an advantage to have each accentuated line stand for a multiple of 5 or 10. Thus in combination (*e*) above the smallest space between vertical lines equals a 100-ft. station, and every tenth station falls on an accentuated vertical line, while the smallest space between horizontal lines represents 1 ft. in elevation, and every fifth foot falls on an accentuated horizontal line.

557. Working Up Notes. When the level notes are brought in from the field the elevations are not usually worked out. In working out elevations the computations should be checked for the elevations on each page as the work progresses by the method explained on page 242. This is not a check, however, on the elevations of *intermediate* stations, and in some cases it may be well to work the notes up by the method of 303 (*e*), page 241 (Form D). The arrangement in Form F, page 256, is very convenient in plotting profiles, since the only two columns needed (El. and Sta.) are side by side.

558. Plotting Profiles—General Method of Procedure. (1) Choose the combination of paper and scale best suited to the purpose (p. 531), and pin the paper to the board; it is slightly more convenient in finishing the profile to have the horizontal lines true with the T square, though this is not of much advantage in plotting. (2) Glance through the level notes to ascertain the elevations of the highest and lowest points. (3) Select one of the accentuated horizontal lines for a base-line or datum line. Its position should be such that the highest and lowest points of the profile will fall on the paper. If there are no points below this line it should still be taken three or four inches from the lower edge of the paper to allow room for lettering. It may be necessary to assume the elevation of this base-line as some whole number of feet above or below datum in order to get the profile on the paper. For example, if the highest elevation is, say, 128 ft. and the lowest is above 100 ft., the elevation of the base-line may be assumed as 100 ft. (4) The profile should begin near the left-hand edge of the paper and extend towards the right. If 100-ft. stations are used, assume Sta. 0 on one of the accentuated vertical lines, or, if the profile begins at some station remote from zero, assume instead a station whose number is some multiple of 10. For example, if the profile begins at 48+21 assume Sta. 50 on an accentuated vertical line near the left-hand edge. Similarly for other horizontal scales aim to have the accentuated vertical lines stand for some multiple of ten. (5) Along the lower edge of the paper number the accentuated vertical lines to correspond to *distances*, and along the left-hand edge number the accentuated horizontal lines to correspond to *elevations*. (6) Plot each point on the profile by finding, first, its position with respect to the nearest vertical line, and then running up this line until the horizontal line is reached which corresponds to its elevation. It is well to connect the points as fast as they are found by straight lines drawn *lightly* in pencil, free-hand. (7) Check points by reading their elevations directly from the profile to see if they agree with those in the level notes. Do not check a point by reading from the level notes first and then looking at the profile, for if a mistake was made in plotting, the mind is liable to repeat it.

(*a*) *Suggestions.* Two men can work to advantage—one reads the notes while the other does the plotting. In checking they change places, the man who did the reading calls off elevations directly from the profile, while the man who did the plotting sees if they agree with the notes. (2) Use a fine pencil-point or needle-point for marking points. (3) When the profile is about to run off the top or bottom of the paper it may be stopped (preferably at some accentuated vertical line), and then continued by beginning some whole number of feet below or above the point where it was broken off. The amount that the line is thus lowered or raised should be some multiple of the interval between two accentuated horizontal lines, and the horizontal lines from the break on should be renumbered accordingly. (4) Turning-points, of course, are not plotted unless they are points of the profile. Benches are not plotted, though they are often described by notes near the points on the profile where they would be found. (5) To avoid the common mistake of plotting the elevations of turning-points and benches, go through the notes, crossing off elevations which should not be plotted. (6) Another common mistake is to plot a plus station on a vertical line as if it were a regular station instead of plotting it between vertical lines by estimating the distance. (7) When very accurate profiles are required a base-line may be laid out on drawing-paper of good quality, and the distances and elevations plotted by scale. (8) When several copies of a profile are required it may be plotted on profile tracing-paper or tracing-cloth and process prints made, or it may be transferred by pricking through, or by carbon paper, or by some similar method

559. Profiles of Several Related Lines. Several related lines are often plotted on the same sheet, one above the other. Thus, for example, the profile of the center line of a street may be plotted about half-way between the top and bottom edges of the paper, the profiles of the right and left gutter-lines being plotted near the bottom and top respectively. In this case a different base-line is used for each profile. Another method is to plot the three profiles from a common base, distinguishing the lines by different colors, as, for example, black for the center-line, red for the right gutter-line, and blue for the left gutter-line.

560. Laying Out Grades. Proposed grades may be laid out on a profile by deciding upon points through which the line must pass, and joining successive points by straight lines. Frequently the vertical angles between these lines must be eliminated by vertical curves which may be determined by one of the methods on pages 289 to 292 and then plotted on the profile.

561. Plotting a Profile from Contours. For any line drawn across a contour map a profile may be plotted, since at each intersection of a contour by the given line an elevation is known, and the distances between these points of intersection may be scaled from the map, or, if the given line is straight, they may be projected from the map to the profile as illustrated in Fig. 398, page 338. Thus, for example, a road may be laid out on a topographic map and a close approximation to its profile may be obtained no matter how crooked or winding the road may be.

562. Finishing Profiles. Many of the directions and suggestions for finishing maps (Chapter XLII) also apply to finishing profiles, especially those that pertain to inking, lettering, notes, and titles. Additional suggestions are: (1) Draw lines representing existing surfaces in black ink, and proposed changes in red ink or broken lines. (2) When the surface represented is smooth the profile may be ruled with a ruling-pen, otherwise it should be drawn free-hand. Thus, ordinarily, the proposed changes are shown by red lines carefully ruled, while the irregularities of the existing surface are represented by a slightly wavering black line.

This irregular line though drawn free-hand should be of even width. When, however, the vertical scale is *very small* the little irregularities between any two plotted points on the profile may disappear, in which case the profile will consist of a series of straight lines each of which is ruled; to draw them free-hand might lead to inaccuracy in the use of the profile, since the surface of the ground would be represented as more irregular than it really is. (3) In inking a pencil profile composed of a series of straight lines the sharp angles may be smoothed out a little unless, as in the case of a sudden and steep incline, such angles exist on the ground. It is not worth while to do this, however, if the vertical scale is greatly exaggerated. (4) Draw the base-line in red ink. At every point of the grade-line where the gradient changes draw a dotted or broken vertical line to the base-line, and mark along this line the grade *elevation* of the point and the "plus" if the point is not a full station, thus: *Sta. 40 + 53 Elev. = 114.25.* When the elevations are those of the existing surface the figures and the broken lines should both be in black ink, but when they indicate points on a proposed surface both should be in red. Among the points where the station number and grade elevation should be marked in this manner are: At the beginning and end of the profile; at each side of an intersecting street; at each end of a bridge; at the P.C. and P.T. of a horizontal curve; at the P.C., the P.T., and the highest or lowest point of a vertical curve, and, if the curve is to be staked out from the profile, at intermediate points where the elevations have been calculated or scaled. (5) At each full station and at such points as those just mentioned mark the cut or fill along a vertical line, if possible in the space between the black line representing the orginal surface and the red line representing the proposed grade; or if this space is too small draw an arrow to one side and put the figures near by, thus: *C. 1.4* or *F. 2.6*. (6) The rate of grade may be printed along the line which represents the corresponding surface, or it may be put some distance below on a horizontal line drawn between vertical lines to the two points where the grade begins and ends. In the latter case the horizontal line should be shown like a dimension-line with an arrow-head at each end. (7) Sometimes, in order to make a profile stand out more prominently, a light flat tint is laid on, extending half an inch or so below the line which represents the original surface, or, in the case of a tracing, colored crayon is rubbed on the back of the cloth instead. Sometimes pencil is rubbed on a proposed profile and erased as fast as the construction is completed, thus showing the progress of the work. Occasionally it is desired to show different materials, as, for example, the different strata determined by borings. Some of the conventional signs are shown in Fig. 562.*

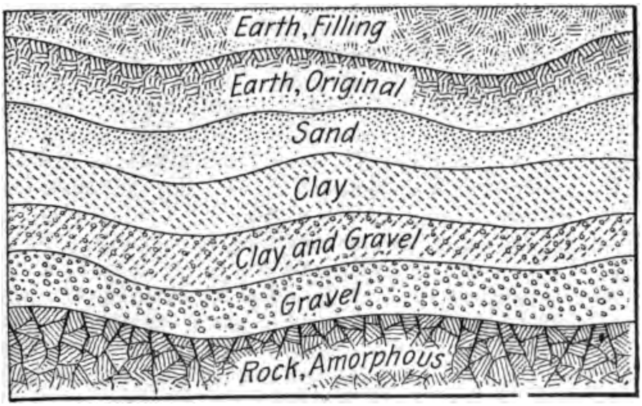

FIG. 562.

563. Lettering the Profile. Names of streets, streams, etc., and descriptions of benches, bridges, culverts, etc., should be printed *along vertical lines* directly above the corresponding points on the profile. Words should read from the bottom up, and all notes should be at least one inch above the profile line except the rate of grade, which may be printed along that line.

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* See Reinhardt's "Technic of Mechanical Drafting" (Engineering News Publishing Co.) for additional conventional signs.

Remark. On long profiles, such as those used in railway work, some engineers prefer to have the vertical lines of lettering read from the top downward so that, in consulting a partly unrolled profile, the zero or left-hand end may be held toward the body as if the eye were looking along the general line represented by the profile.

Notes should be definite but brief, as, for example, "B.M. 36. Elev = 126.192. Tack in root of Oak 30 ft. to left of Sta. 36." The elevations of the accentuated horizontal lines, and the station numbers of the accentuated vertical lines, should be in heavy black figures on the left-hand and lower margins respectively. The title should give the information specified in Art. 546 (*a*). The datum should always be given near the title. Practically all the directions for constructing a title for a map (pp. 523 to 525) hold good for profiles.

PART III

SURVEYING INSTRUMENTS*

INTRODUCTORY

564. The Study of Instruments. When the student is first learning to use an instrument it is not necessary that he should study it in detail; on the contrary, it is better for him to confine his attention to the few parts which he has to manipulate. These parts are visible and need no description. When, however, he has become somewhat accustomed to the use of an instrument, he should begin a careful study of the different parts, some of which are not visible after the instrument has been put together. Parts thus concealed are shown in the following pages by drawings or photographs. Thus in the photograph on page 551 a telescope is shown taken apart, and on page 565 a transit is shown in a similar manner. The student should not be content, however, with merely observing the construction of the different parts. He should investigate the properties of lenses, of magnetic needles, of level-bubbles, the theory of the telescope, the defects of construction, and similar subjects. In this way he will become familiar with important facts and fundamental principles which are not evident from inspection. A careful study of this portion of the subject (Chapters XLIV and XLV), should precede the study of the adjustments of instruments (Chapter XLVI).

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* For a more comprehensive treatise on surveying instruments the student is referred to Professor I. O. Baker's excellent book entitled "*Engineers' Surveying Instruments*" (John Wiley & Sons).

CHAPTER XLIV

PRIMARY PARTS OF SURVEYING INSTRUMENTS

THERE are certain primary parts, such as the vernier, the magnetic needle, the level-bubble, and the telescope, which are more or less common to all surveying instruments. These parts are discussed in this chapter preparatory to a study of the instruments themselves in the next chapter. In some cases the discussion is little more than a review of certain fundamental principles of physics.

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565. The Vernier, invented in 1631 by *Pierre Vernier,* is a device for measuring the fractional part of one of the smallest divisions of a graduated scale. It usually consists of a small sliding scale running parallel to and in contact with a fixed scale or a graduated arc. The divisions of the vernier instead of being equal to the divisions on the graduated scale or arc, are either just a little smaller or a little larger. The amount by which the smallest division on the vernier differs from the smallest division on the graduated scale or arc determines the smallest reading or *least count* of the vernier.

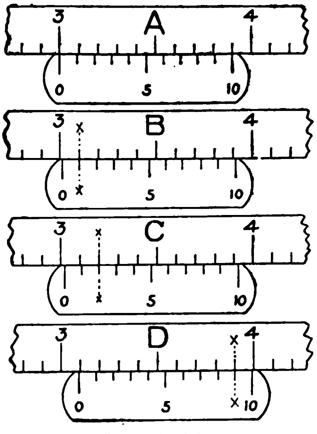

FIG. 565 (*a*).

(*a*) *Illustration.* Fig. 565 (*a*) shows at *A* a portion of a scale divided into inches and tenths of an inch, with a vernier attached. The same scale is shown at *B*, *C*, and *D*, but the vernier has been moved in each case. At *A* the vernier zero is at the 3-inch mark of the scale, and it is seen that 10 divisions of the vernier cover a space equal to 9 divisions on the scale, hence one division on the vernier is equal to 0.9 ÷ 10 = 0.09 inch long, and is 0.01 inch shorter than the smallest divisions on the scale. If, therefore, the vernier is moved along the scale until the first mark on the vernier (not counting the zero) is exactly opposite to the corresponding mark on the scale (as shown at *B*), the vernier zero must be 0.01 inch from the 3-inch mark on the scale. If moved until the second mark on the vernier is opposite to the corresponding mark on the scale (as shown at *C*), the zero will be 0.02 inch from the 3-inch mark. Thus by moving the vernier 0.01 inch at a time, each mark in succession will coincide with a mark on the scale until, when the ninth mark is opposite to the corresponding mark on the scale (as shown at *D*), the vernier zero is 0.09 inch from the 3-inch mark; a movement of another 0.01 inch will then bring the zero of the vernier opposite the 3.1-inch mark, and the process will begin again.

(*b*) Reading the vernier. From the preceding illustration it is evident that when the zero-point of the vernier is between any two adjacent marks on the scale, its position may be determined by observing which mark on the vernier is exactly opposite some mark on the scale. For this reason the zero-mark of a vernier frequently serves as an indicator for the movable part of an instrument to which the vernier is fastened. For example, when a vernier is attached to the target of a leveling-rod (as shown on p. 231), the zero point of the vernier is placed opposite the horizontal center line of the target, so that when this center line (and the vernier zero) stops between two adjacent graduation marks on the rod, the *fractional* reading may be obtained from the vernier. There are always two steps involved in the *whole* reading, *viz.*:

(1) Note the position of the vernier zero with respect to the main (stationary) scale, paying no attention to the other marks of the vernier.

(2) Observe which mark on the vernier is opposite or most nearly opposite a mark on the scale, and add this reading to that obtained in (1).

(*c*) *Remark.* Notice that the only use made of the vernier is in getting the small fractional reading in step (2) and that no attention is paid to the value of the mark on the scale which happens to be opposite a mark on the vernier. In reading any vernier for the first time it will be necessary to determine its least count as explained farther on.

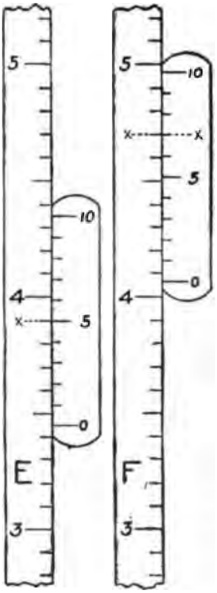

Fig. 565 (*d*).

(*d*) *Illustration.* In Fig. 565 (*d*) the smallest division on the scale is 0.01 ft., and the divisions on the vernier are each equal to 0.009 ft., or 0.001 ft. less than the smallest division on the scale.

On the scale marked *E* the zero of the vernier is just beyond the 0.34-ft. mark and the fifth mark of the vernier is opposite a mark on the scale, hence the reading is 0.34 ft. + 0.005 = 0.345 ft. *i.e.*, the vernier zero or indicator is just half way between 0.34 and 0.35 ft. On the scale marked *F* the reading is 0.40+0.007=0.407 ft. A common mistake would be to call it 0.47 ft.

Remark. In reading a vernier there are certain *danger-points* where mistakes similar to that indicated above are likely to occur. These points for verniers on leveling-rods are pointed out on pages 228–231, and for transit verniers on pages 76–83.

(*e*) DIRECT AND RETROGRADE VERNIERS. A *direct* vernier is one in which the smallest division on the vernier is shorter than the smallest division on the main scale. In a *retrograde* vernier a division on the vernier is longer than a division on the scale. Most verniers on surveying instruments are direct, such as those shown in Figs. 565 (*a*) and 565 (*d*).

(*f*) *Remark.* In reading a retrograde vernier proceed exactly as if it were a direct vernier, observing, however, that the divisions are numbered backward, or in an opposite direction from the numbers on the main scale.

Retrograde verniers are used on leveling-rods which read *downward* from the center of the target instead of upward. (See page 230 for target verniers which are direct and hence read upward.) Retrograde verniers are also used in special forms, such as those described on page 83.

(*g*) TO DETERMINE THE SMALLEST READING or least count of a vernier: (1) *Observe how many divisions on the scale correspond to the same number plus one* (*on a direct vernier*) *or minus one* (*on a retrograde vernier*).

(2) *Divide the value of the smallest division on the scale by the number obtained in* (1).

In algebraic terms let d = length of the smallest division on the scale, v = length of smallest division on the vernier, n = the number of d's which added together equal $(n+1)$ v's for direct or $(n-1)$ v's for retrograde vernier. Then $nd=(n+1)v$ (direct) or $nd=(n-1)v$ (retrograde).

Hence the smallest reading of the vernier is

$$d-v=\frac{d}{n+1} \text{ (direct) or } d-v=\frac{d}{n-1} \text{ (retrograde)}. \quad (1)$$

On many surveying instruments verniers are so constructed that the above rule may be simplified to:

Divide the value of the smallest division on the scale by the number of divisions on the vernier.

(*h*) VERNIERS FOR LEVELING-RODS AND TRANSITS. The principal use of the vernier in surveying is in taking target readings in leveling, and in reading angles in transit work. The verniers on targets are similar to that shown in Fig. 565 (*d*) and are fully illustrated on page 230; they should not be confused with the ordinary scale used on many targets, and illustrated on page 228.

The vernier on the transit is divided into spaces corresponding to angular measure instead of linear; the principle however is exactly the same. Illustrations of transit verniers are given on pages 78 to 82.

566. The Magnetic Needle.* (*a*) DIP OF THE NEEDLE. A needle which is perfectly balanced before it is magnetized will, after it is magnetized, dip towards the pole, and, in the northern hemisphere, the farther north the needle, the more its northern end will be attracted downward. In order to counteract this tendency, a small wire coil is placed around the needle to act as a weight on its southern half. As the dip varies in different localities this little weight may be slipped in or out along the needle to preserve a perfect balance.

(*b*) MAGNETIC DECLINATION. The magnetic needle does not point directly north except in a few localities. In some places it points west of north, in other places east of north. The meridian in which a needle properly magnetized and properly balanced comes to rest at any given place and time is called the *magnetic meridian* for that place and time, and the angle between a magnetic meridian and a true north and south meridian is the corresponding *declination of the needle* or the *magnetic declination*. In some places this declination is practically nothing, while in other places it is very large. Thus in central Ohio it is nearly zero, but in Maine it is 18° west, while in the State of Washington it is 23° east. (See Isogonic Chart.) As the magnetic meridian at any given place is continuously changing there are corresponding *variations* of declination.

VARIATIONS OF DECLINATION. (1) Secular variations, (2) diurnal variations, (3) irregular variations, (4) annual variations, and (5) lunar variations. The last two are so small that they may be neglected.

(*c*) The *secular variation* is the slow variation in the magnetic declination which takes place with the lapse of time, requiring several hundred years for its full development. This change appears to be periodic in its character, like the motion of a pendulum; that is, the compass needle, after moving continuously in one direction for a long term of years, gradually comes to a standstill, and then begins a motion in the opposite direction. Thus at Portland, Maine, the declination appears to have been 12° west in 1700, then decreasing to only 8° west in 1780, then increasing again until now it is more than 14° west.

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* For a treatise on this subject see *Magnetic Declination Tables and Isogonic Charts for 1902 and Principal Facts Relating to the Earth's Magnetism*, by L. A. Bauer (U. S. Coast and Geodetic Survey).

In the work of the surveyor secular variation is very important, and unless otherwise specified it is the variation commonly referred to. The rate of variation is different for different places, and the change per year at a given place is not a constant quantity, as frequently assumed by the surveyor. The change per year (which should not be confused with annual variation) is greatest near the middle point of the swing referred to above, and becomes less and less as the extremes are approached. Thus at San Francisco, where the needle has apparently reached its extreme easterly position, the variation per year at present is almost zero.

In the eastern part of the United States declinations (west) are increasing, while in the western part declinations (east) are decreasing.

The difference in the rate of variation may be shown by taking a few cities at random:

	Approximate Annual Change in Minutes.		
	1895	1900	1902
Bangor, Me.	+1.5	+0.9	+0.1
New Haven, Conn.	+3.3	+3.0	+1.8
New Orleans, La.	+4.3	+4.3	+3.5
San Francisco, Cal.	−0.1	+0.1	+0.1

The declinations at a given place for a number of successive years are sometimes given by an empirical formula, but the various formulas for different places are not always reliable. It is better for a surveyor to determine the declination for himself at the place where he is working, by observations on Polaris, or by some similar method. (See Chapter XXVIII.)

(*d*) *The diurnal variation* is the change in the direction of the needle from hour to hour during the day, from its most easterly position of the needle at about 8 A.M. to its most westerly position at about 1.30 P.M. The needle is in its mean position between 10 A.M. and 12 M., and between 5 and 6 P.M., and hence during those periods the daily variation may usually be ignored. The amount of variation differs for different localities, and for different seasons of the year in the same locality. The total movement in the United States is from 3 to 12 minutes, and is greatest in the months of June, July, and August.

(*e*) *The annual variation* is a *cyclical* change having the year as a period, and should not be confounded with the change per year due to secular variation, for the latter is a *progressive* change. The annual variation amounts in most places to less than one minute and can be neglected for all practical purposes.

(*f*) *Irregular variations* are due to magnetic disturbances or storms which may occur at any time and cannot be predicted. Variations of this kind amounting to more than a degree have been observed.

(*g*) ISOGONIC LINES are imaginary lines passing through places where the magnetic declination is the same at a given time. From inspection of the *isogonic chart* given in the back of this book the isogonic lines are seen to be very irregular.

(*h*) THE AGONIC LINE passes through places where the declination at a given time is zero. In the United States this line passes through South Carolina, Ohio, and Michigan, and is slowly moving westward. For places *east* of this line the declination is *west* and for places *west* it is *east*. This fact is a help in remembering whether the declination for a given place is east or west.

(*i*) LOCAL ATTRACTION is a term used to denote any influence which prevents the needle from pointing to the *magnetic* north in a given locality. In its strict sense it does not include avoidable attraction due to things about the person or to other sources not connected with the place where the needle is read. Some of the sources of local attraction are: Magnetite in the ground, trolley-wires, steel structures, railroad-rails, underground iron pipes, etc. Contrary to a common belief, iron ores do not affect the needle unless they contain magnetite or pyrrhotite. Some of the avoidable sources of attraction are: Keys, knives, steel-bowed spectacles, spectacle cases (which though covered with leather may be solid steel inside), wire in a hat brim, metal buttons, and other bits of iron or steel about the person; also axes, chains, steel tapes, etc., which may be lying on the ground nearby.

Note. In retracing an old survey made many years ago, it was found that the bearings of all the courses were incorrectly given on the original map, and the errors were so large as to indicate that something was wrong with the compass originally used. Fortunately the instrument was still in existence, and an examination disclosed the presence of a steel screw in the compass-box.

567. The Level-bubble. The interior surface of the glass tube of a spirit-level should be ground so that a longitudinal section will show a circular arc. The more uniform this curvature is throughout the length of the tube the more regular will be the motion of the bubble, and the greater the radius of curvature the greater the sensitiveness of the bubble. Within reasonable limits the more sensitive the bubble the more perfect the work, though a very sensitive bubble may be too unsteady for many purposes; on the other hand a sluggish bubble, though it may give the appearance of steadiness to an instrument, and an impression that it "keeps" its adjustment, is incapable of accurate work.

The fluid used in spirit-levels is usually alcohol or ether. Pure alcohol is slow-acting, requiring some time to settle or to obtain its equilibrium, while ether is quick-acting, but is objectionable on account of too great contraction and expansion for slight changes of temperature. Some makers combine the two. Whatever fluid is used the length of the bubble changes with variation in temperature, and as the bubble grows shorter it becomes less sensitive. If the whole tube becomes overheated as it sometimes does when left in the sun, the bubble may become so short as to be nearly useless. If one end of the tube becomes warmer than the other, the tendency is for the bubble to move toward the warmer end. (See experiment, p. 227, and Art. 350 (*a*), p. 272.)

The line tangent to the circular arc of the tube at its middle point, or a line parallel to this tangent, is called the *axis of the bubble-tube*. When this axis is horizontal the bubble will be in the center of the tube. Should the axis become slightly inclined the bubble will move toward the higher end of the tube in proportion to the angle made by the axis with the horizon. The glass tube is usually graduated on top by marks 0.01 ft. apart. The value of a level-bubble is usually expressed by the change which takes place in the inclination of the axis when the bubble moves over a single space. Thus in a 1-minute level for a displacement of one division the inclination changes 1 minute, and in a 20-second level it changes 20 seconds. The value as given by the makers for any particular level is frequently found by actual tests to be erroneous. The engineer can test for himself the sensitiveness of a bubble by the method given below.

When a level-bubble is attached to a telescope the relation between the sensitiveness of the bubble and the magnifying power of the telescope should be such that the smallest vertical movement of the cross-hairs, which is visible to the eye, is followed by an appreciable movement of the bubble. For a good transit the bubble attached to the telescope should have a sensitiveness of not less than 0.01 ft. $=25''$ of arc, while that of the plate-levels should be not less than 0.01 ft. $=40''$ of arc. For a good wye-level or dumpy level the sensitiveness should be not less than 0.01 ft. $=15''$ of arc.

Circular bubbles are sometimes used for the plumbing-levels attached to leveling-rods, and are occasionally found on transits and other instruments.

(*a*) TO TEST THE SENSITIVENESS OF A BUBBLE. Assuming that the bubble-tube is attached to the telescope of a level, the angular value of the bubble may be found as follows: (1) Set up the level and hold a leveling-rod on some point a known distance from the instrument. (2) Take two readings, one with the bubble two or three divisions to one side of the center, the other with the bubble about the same number of divisions the other side of the center. (3) The difference in readings divided by the distance to the rod is the natural tangent of the angle between the two lines of sight. Divide this tangent by the tangent of $1''$ of arc (0.00000485), thus obtaining the value of the angle in seconds of arc. Divide the result by the number of divisions between the two positions of the bubble; the result is the average value in seconds of a single division.

Suggestion. (1) Set the level up in the shade with two diagonally opposite leveling-screws in the line to the leveling-rod. Change the position of the bubble by means of these screws. Probably better results will be obtained if an extra man acts as bubble-tender. (2) In measuring off a base line to the rod it saves arithmetical work to make it an even 200 or 300 ft. (3) Repeat the test, holding the rod at different distances and displacing the bubble by different amounts. (4) Any bubble-tube other than a telescope level may be tested in a similar manner by attaching it rigidly to a telescope, with its axis approximately parallel to the line of sight. (5) There are small instruments called level-triers especially designed for testing levels. (6) Draw a figure and show that if D is the distance from the instrument to the rod, R the radius of curvature of the level-tube, d the length of one division of the scale, and h the difference in two rod-readings when the bubble is moved over n divisions of the scale, then $R=\frac{nd}{h}D$.

568. The Theory of Lenses. In order to understand the construction of the telescope it is necessary first to study some of the most important properties of lenses. The different forms of lenses most used in telescopes are: (1) *Double-convex* (convex on both sides); (2) *plano-convex* (plane on one side, convex on the other); (3) *concavo-convex* (concave on one side, convex on the other). No one of these forms is used singly, but two or more are combined to correct certain defects which could not be overcome in a single lens. It is unnecessary to give here a complete statement of the theory of lenses; it will suffice to point out the principles which apply to the passage of light through a double-convex lens. These principles are in general the same for the other forms.

Remark. The figures in this article are not drawn to scale and are necessarily much distorted.

(*a*) DOUBLE-CONVEX LENSES. Each of the two surfaces of the *double-convex* lens is a portion of a sphere, and the line passing through the centers of these two spheres is the *principal axis*. If the radii of these two surfaces are equal, the *optical center* is at the geometrical center of the lens. If the lens is very thin it may be assumed that any ray of light passing through its center undergoes no deviation. The *principal focus* of a convex lens is the point of its principal axis where rays of light parallel to that axis are converged (approximately) after passing through the lens, and in the ordinary *double-convex lens* the two principal foci are on the principal axis equal distances from the optical center. Thus in Fig. 568 (I), *AA* is the *principal axis*, *C* the *optical center*, and *F* and *F′* the *principal foci*. The distance *FC*, equal to *F′C*, is the *focal length* which, for a given lens, is always constant. The focal length depends upon the curvature, not upon the size of the lens, but as large lenses usually have longer radii of curvature than small lenses their focal lengths are usually greater.

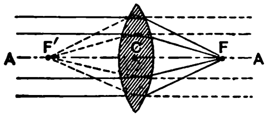

FIG. 568 (I).

Proposition I. *A cone of rays from a point located at either focus becomes a cylinder of rays after passing through the lens, the axis of the cylinder coinciding with the principal axis of the lens. Conversely rays of light parallel to the principal axis converge at a principal focus after passing through the lens.*

Illustration. In Fig. 568 (I), the full lines converge at *F* after passing through the lens and the dotted lines at *F′*.

Illustration. The rays from the sun are practically parallel, and in the familiar experiment of burning a hole in paper with a reading-glass, the rays are converged at the principal focus of the glass.

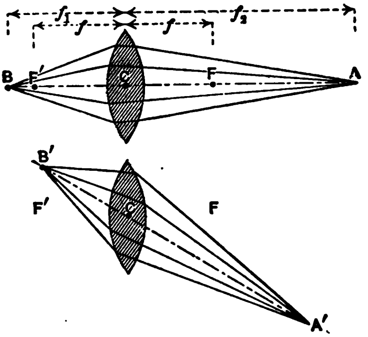

FIG. 568 (II).

Proposition II. *Rays diverging from any point converge approximately at a* REAL *focus after passing through the lens, provided the distance from the optical center to the point is* GREATER *than the focal length of the lens.*

Illustration. In Fig. 568 (II), a cone of rays from any point A beyond the focus F becomes another cone of rays after passing through the lens, converging at a point B *beyond the other focus* F'. Conversely rays from B converge at A.

Conjugate foci. If rays of light from any point converge at a second point after passing through a lens, and if, *vice versa*, rays from the second point converge at the first, the two points are called *conjugate foci*. Thus, for example, A and B in Fig. 568 (II) are conjugate foci as are also A' and B'. The line joining conjugate foci passes through the optical center of the lens. When conjugate foci are not in the principal axis the line joining them is called a *secondary axis, e.g.*, $A'B'$. If f_1 and f_2 are the distances from the optical center to conjugate foci, and f is the focal length of the lens, then from a principle of optics

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From the above equation it is evident that *both* f_1 *and* f_2 *must be greater than the focal length.*

Proposition III. *Rays diverging from any point continue to diverge after passing through the lens as if they came from a so-called* VIRTUAL FOCUS *behind the point, provided the distance from the optical center to the point is* LESS *than the focal length of the lens.*

Illustration. In Fig. 568 (III) rays diverging from any point G at a distance g *less* than f continue to diverge after passing through the lens, though less rapidly than before. They will therefore appear to radiate not from G but from H, a point behind G. H is a conjugate focus, but the rays do not actually pass through it as in the other cases, hence it is called a *virtual conjugate focus.*

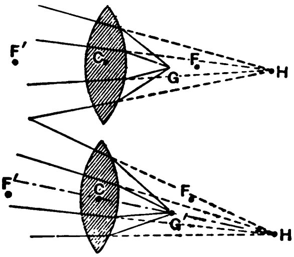

FIG. 568 (III).

(*b*) FORMATION OF IMAGES. If the source of light is an extended surface, then the cone of rays emanating from each point forms its own focus after passing through the lens; and the collection of foci constitutes an image which is *real* and *inverted* if the foci are real, but virtual and erect if the foci are virtual. The relative sizes of the object and image are sensibly proportional, if the lens is thin, to their respective distances from the optical center.

From Proposition III, if an object is between a lens and one of its principal foci, the image is virtual and erect, and is larger than the object. This is the fundamental principle upon which magnifying glasses, microscopes, and eyepieces of telescopes are constructed.

From Proposition I, if an object is at either of the principal foci, no image is formed (unless it is at infinity) for the cone of rays from any point become parallel after passing through the lens.

From Proposition II, if an object is beyond one of the principal foci, its image is real and inverted, and lies on the opposite side of the lens beyond the other principal focus. This is the fundamental principle upon which the object-glass of a telescope is constructed.

From Equation 1, if the distance from the object to the lens is less than *twice* the focal length, the image is larger than the object and at a greater distance from the lens; if the distance is exactly equal to twice the focal length, the object and its image are of equal size and equally distant from the lens; if the distance is more than twice the focal length, the image is smaller than the object and less than twice the focal length from the lens. In all three cases the object and its image are on opposite sides of the lens, and the image is real and inverted. The farther an object recedes from a lens the nearer its image approaches the lens, until with the object at infinity the image is at a principal focus. (Proposition I.)

(c) *Note.* An image formed by a single lens is never perfectly distinct on account of *spherical* and *chromatic aberrations* of the lens. The former is due to the fact that the rays of light passing through the lens near the edge converge at a point slightly nearer the lens than that in which the central rays meet; the latter to the fact that rays of different colors form their foci at different distances. Spherical aberration can be corrected by making the surfaces of forms other than spherical, or by combining two or more lenses properly proportioned; chromatic aberration only by combining two or more convex and concave lenses of different materials. Thus, for example, the objective in the telescope of a surveying instrument is an *achromatic* lens composed of a double-convex *crown-glass* and a concavo-convex *flint glass lens.*

569. The Telescope. (*a*) The telescope shown in the photograph on page 551 is similar to those used on all surveying instruments. The object-glass at *A* gathers the rays of light from an object, and forms an inverted image in the plane of cross-hairs at *DD*. The eyepiece, extending from *E* to *F*, magnifies both the inverted image and the cross-hairs. In some telescopes the image appears to be erect, while in others it remains inverted. The difference lies wholly in the eyepiece, as will be explained farther on.

The eyepiece, shown by itself at *F′G*, is similar to a microscope. When in place, the end *G* moves through a ring at *EE*. A separate view of this ring at *E′E′* shows the four screws that hold it in place. These screws pass through smooth holes (not threaded) in the shell of the telescope, so that by unscrewing and screwing opposite screws the ring may be drawn from one side to the other for the purpose of centering the end *G* of the eyepiece. (See p. 601.) The eyepiece shown is an erecting eyepiece and has four lenses, one at *F′* one at *G*, and two between *F′* and *G*. The eyepiece may be moved in or out for focusing by turning the piece *F′M* so that the projecting head of a screw works in the spiral slot shown at *N*. Some eyepieces are focused by a rack and pinion similar to that used for the object-glass,

The object-glass is set in the end of a tube shown in a separate view at $A'K$. The end K moves through a ring at CC similar to the ring shown at $E'E'$, and it may be centered by adjusting this ring in the manner explained above for the eyepiece ring. The object-glass may be moved in and out for focusing by turning a screw at B. At the inner end of this screw is a pinion (shown at B') which works in a rack shown at H, and thus moves the whole piece that carries the object-glass.

The cross-hair ring at DD is shown in a separate view at $D'D'$. It may be moved up or down and from side to side by turning the capstan screws which pass through smooth holes (unthreaded) in the shell of the telescope. (See p. 587.) The cross-hairs are spider-webs (or very fine platinum wire) almost invisible to the eye, and therefore not visible in the photograph. The whitish lines on the ring at right angles to each other are caused by the shellac which fastens the cross-hairs. Underneath the shellac are very fine scratches to aid in replacing broken hairs. (See p. 616.)

Note. It is very important for purposes of adjustment to understand clearly how the capstan screws control the movement of the cross-hair ring, or, as it is often called, the *reticle*. The cross-hairs must be fastened to this ring at right angles to each other before the ring is placed in the telescope, as they cannot be changed in this respect afterward. Notice, however, that the holes in the telescope through which the screws pass are not threaded; on the contrary, they are a little larger than the screws, so that when the latter are loose the whole ring may be turned slightly by moving the four capstan heads simultaneously around the outside of the telescope until one cross-hair is vertical and the other horizontal. When the capstan screws are tight, each screw presses a curved washer (shown in the photograph) against the outside surface of the telescope. When one screw is loosened and the opposite screw tightened, the whole ring is drawn toward the tightened screw (since the holes in the shell of the telescope are smooth) until the loose screw and its washer are brought into contact again with the outside of the telescope. Notice that before tightening one screw the opposite screw should be loosened, otherwise the ring cannot move and the screw-thread may be stripped. By loosening the lower screw and tightening the upper screw, the whole ring may be drawn upward, or by reversing the process it may be drawn downward. Likewise by working the side screws in a similar manner, it may be drawn to one side or the other. All this may be done without turning the ring, *i.e.*, one hair may be kept vertical, the other horizontal.

(*b*) THE THEORY OF LENSES AS APPLIED TO A TELESCOPE may be studied more in detail by means of Fig. 569 (*b*). Let the upper figure represent a telescope with a non-erecting eyepiece, and the lower figure one with an erecting eyepiece. The former will be considered first. Let GG represent the object-glass and AC any object sighted at. Usually the distance from the object-glass to the object is many times the length of the telescope, and hence the latter is necessarily shown to a greatly exaggerated scale.

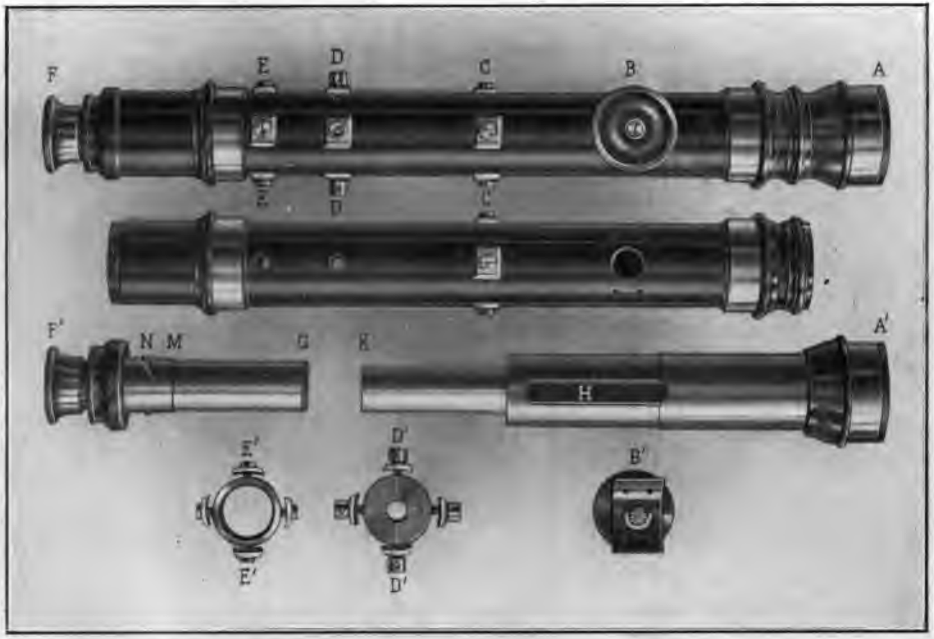

FIG. 569.

1. The cone of rays from *A* becomes another cone of rays after passing through the lens, converging at *a* (Proposition II, p. 547). Likewise rays from *C* converge at *c*, and rays from all points of the object converge at corresponding conjugate foci between *a* and *c*. Thus a very small *inverted* image of the object is formed at *ac* (Art. 568 (*b*)).

2. The distances *d* and *f'* are approximately distances to conjugate foci, both being greater than the focal length of the objective. As *d* is increased *f'* must decrease and *vice versa*, *i.e.*, the farther away the object the nearer must be the inverted image to the object-glass. (From Equation 1, p. 547.) Since, however, it is necessary that the image should lie in the plane of the cross-hairs *WW* (in order to eliminate parallax see p 555) the distance *f'* is changed for objects at different distances by moving the object-glass *in* (nearer to *WW*) for distant objects, and *out* (away from *WW*) for near objects. This explains the rule for focusing on page 86. The focus changes rapidly for objects from 10 to 30 ft. from the telescope, but very slowly after a distance of 100 ft. is reached, the focus for 100 ft. being nearly the same as for greater distances.

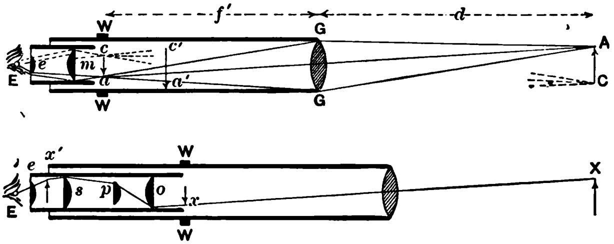

FIG. 569 (*b*).

The inverted image *ac* is between the eyepiece and its principal focus, hence, from Proposition III, p. 548, the image will appear to be larger than it is, as if the rays of light came from virtual foci behind *ac* in some such position as *a'c'*.

In the *erecting eyepiece* extra lenses are inserted which reinvert or erect the image at the cross-hairs before it is magnified. Thus rays from *X* converge at *x* in the plane of the cross-hairs, the lenses of the eyepiece form another image reinverted at *x'*, finally this image as seen by the eye is magnified by the at *e*.

Remark. The chief objection to a *non-erecting* eyepiece is the inconvenience caused to the observer from seeing objects inverted. Thus, for example, a flagman appears to stand on his head, to move to the right when he really moves to the left, and *vice versa*. In the erecting eyepiece, however, the use of two additional lenses causes a loss of light which is a far more serious objection than the slight inconvenience resulting from inversion. Moreover, for a given length of telescope greater magnifying power can be secured without sacrificing illumination by using a non-erecting eyepiece. (Why?) There is but little reason for the use of the erecting eyepiece, although it is more common than the simpler form; it should never be used on instruments intended for work of great precision.

Note. Two special forms of telescope sometimes used for stadia work may be mentioned. The Porro telescope, named after its inventor, has an arrangement of lenses which makes it possible to read stadia distances directly without adding the constant $c+f$. In another form of telescope a flat glass prism at the object-glass makes it possible to measure stadia distances without stadia wires. Neither form is in general use.

(*c*) LINE OF COLLIMATION AND LINE OF SIGHT. The line of collimation of a telescope is usually defined as the line from the optical center of the objective to the intersection of the cross-hairs. Some writers maintain that this definition is correct only when the cross-hairs are in perfect adjustment. Since the engineer is accustomed to the less ambiguous term "*line of sight*" (a line determined by the intersection of the cross-hairs whether they are in adjustment or not), the use of the term "line of collimation" will be avoided in this book.

(*d*) DEFECTS OF TELESCOPES. Spherical and chromatic aberrations, defined in Art. 568 (*c*), p. 549) are inherent defects which the manufacturer should aim to correct by a proper combination of lenses. A third defect is due to the fact that the image of a flat object formed by a lens does not lie in a plane, but is concave toward the lens. This *aberration of sphericity*, as it is termed, is entirely different from spherical aberration. In the case of the object-glass it is inappreciable, but in the eyepiece it is more serious. It is not a source of error in measuring angles and in leveling, since only one point is observed at a time, but in stadia work it may become objectionable. The two plano-convex lenses used in the eyepiece to correct chromatic and spherical aberration correct also aberration of sphericity.

(*e*) QUALITIES OF TELESCOPES that depend upon design and construction are as follows: 1. Definition; 2. Illumination; 3. Size of field; 4. Magnification.

1. *Definition.* If definition is poor the image will not be clear and distinct. Lack of definition is due to inaccuracy in grinding and polishing the lenses or to inaccuracy in centering, i.e., failure to place the component lenses of either the object-glass or the eyepiece so that their axes coincide.

2. *Illumination.* If illumination is poor the image will appear faint, though it may be sharp and definite. Lack of illumination simply means lack of light. The amount of light which reaches the eye through the telescope depends, first of all, upon the size of the *aperture* of the object-glass, which, on account of the diaphragm often placed in the tube, is somewhat smaller than the object-glass itself. The amount of light collected by an objective of a given diameter may be compared with that collected by an objective of different diameter by picturing two squares whose sides are equal respectively to the two diameters, *i.e.*, the amount of light is proportional to the square of the diameter. Of the light which strikes the lens, however, less than 85% enters the telescope, the loss being due to reflection and imperfect transparency. The light which passes through the telescope is distributed over a larger or smaller area by the eyepiece, according to the magnifying power; the greater the magnification the more the illumination is diminished, the brightness of the image varying inversely as the square of the magnifying power. Finally, for the best possible effect, the beam of light which emerges from the eyepiece should be of exactly the same diameter as the pupil of the eye.

3. *Size of field.* The field of view is the whole circular area seen by an observer when looking through the telescope, and the angle at the optical center of the object-glass subtended by the diameter of this circle measures the size of this field. The size of field, however, is not dependent upon the size of the object-glass. It increases as the distance between the object-glass and the eyepiece decreases, it increases as the size of the eyepiece increases, it increases as the magnification decreases. A wide field of view is desirable as it enables the observer to work more rapidly. The use of two plano-convex lenses in the eyepiece, in place of a single lens increases the field of view.

4. *Magnification* depends upon the ratio of the focal length of the object-glass to the focal length of the eyepiece. For example, if the focal length of the objective is 9 inches and that of the eyepiece 0.3 inch, then the magnifying power is 30. The magnifying power should be proportional to the aperture, *i.e.*, to the amount of light which enters the telescope. If the magnifying power is too high for the aperture, ordinary objects will appear too faint, especially under unfavorable atmospheric conditions; and if the magnifying power is too low the objects

will appear too small for accurate sighting. The magnifying power should be such that the least movement of the level bubble or change in the vernier causes an easily perceptible movement of the cross-hairs over the object in the field of view. Except perhaps in stadia work, a higher power than this is a disadvantage since there is a needless loss in the brightness of the image, while a lower power does not develop the full capacity of the instrument. The loss of light in high-power instruments is often mistaken for cloudiness due to poor lenses.

For rapid working the exact focusing required for a high-power telescope is a drawback, a change in focus being required for a small change in distance. For example, in a low-power telescope the focus for distances between 300 and 400 ft. is practically the same, but in high-power telescopes, the change of a few feet requires a change in focus, and if the telescope is not properly focused the image is apt to be less distinct than in low-power telescopes. Low power is better also for focusing upon objects very near the instrument.

Non-erecting eyepieces are not more powerful than the erecting eyepieces, but, being shorter, they allow for a given length of telescope a greater focal length of object-glass, and thus increase the power. They, however, transmit much more light to the eye and have a much larger field.

For transits the average magnifying power is about 24 diameters, while for levels it is from 30 to 36, running up to 50 in some instruments.

Remark. An erecting eyepiece is now made which can be manipulated so as to change the power of the telescope at will. By this device a high power can be used in favorable weather, and when the weather is unfavorable this high power can be converted to a low one with a consequent increase in illumination and field.

(*f*) *Parallax.* If the image formed by the object-glass is not in the same plane with the cross-hairs, any movement of the eye is likely to cause an apparent movement of the image with respect to the cross-hairs. This is called *parallax.* The effect is similar to that produced in looking through a window, where any movement of the eye causes an apparent movement of objects outside. Parallax may render accurate work impossible. To remedy it the image and the cross-hairs must be brought into the same plane. Two steps are necessary.

(1) Point the telescope toward the sky and move the eyepiece in or out until the cross-hairs are as well defined as possible, i.e., in perfect focus. If only one person uses the instrument this focusing may be done once for all (unless the slide has to be moved when the instrument is put in its box), since the proper focus depends simply upon the eye of the observer.

(2) Direct the telescope to the object and focus the object-glass as usual, keeping the eye on the cross-hairs until the image appears in sharp focus. Test by moving the eye from side to side, and if necessary move the object-glass slightly until parallax disappears.

The more accurate the work the more care should be used to eliminate parallax, while the higher the power of the telescope the more difficult it is to do this.

(*g*) TESTS FOR TELESCOPES. A few of the most important tests for telescopes are given below.*

Flatness of field. Draw a heavy-lined square, 6 or 8 inches on a side, on white paper fastened to a board. Hold the board at such a distance that the square nearly fills the field of the telescope. Focus the telescope on it, then the sides of the square should appear perfectly straight. Any curvature except at the extreme limits of the field is especially objectionable in stadia work.

Definition. Focus on small clear print 40 or 50 ft. away. If the definition of the telescope is good the print will be clearly defined and legible.

Size of field. Set two transit poles 200 or 300 ft. from the instrument at such a distance apart that one is in one edge of the field and the other in the opposite edge. Multiply this distance by 57.3 and divide by the distance to the poles 200 or 300 ft., as the case may be. The result is the angular width of field in degrees.

Size of aperture. Focus the telescope on any distant object; then withdraw the eye to a distance of 8 or 10 inches or until the small disc of light which appears in the center of the eyepiece is distinctly seen. Hold a pencil-point at the extreme edge of the object-glass. If it shows itself as a dark spot in the edge of the disc of light, the aperture is equal to the diameter of the glass; if not, move it toward the center until the dark spot appears, measure the distance from the edge of the glass to the pencil-point, and subtract twice this distance from the diameter of the glass. The result is the diameter of the aperture. It is better to use a small piece of moistened paper in place of the pencil-point, sticking it on the glass and moving it into place with the finger-tip.

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* For more complete directions for testing telescopes consult Professor Baker's *Engineers' Surveying Instruments*, p. 80.

Magnifying power. Divide the diameter of the clear aperture by the diameter of its image (the disc of light) formed by the eyepiece. Thus if the image is $\frac{1}{10}$ inch in diameter and the aperture is 1.5 inch, the magnifying power is 15.

Another method: Focus on a coarsely graduated rod held from 125 to 150 ft. from the instrument. Look through the telescope with one eye and along the outside of the telescope with the other eye. One space viewed through the telescope will appear to cover a certain number of spaces seen with the naked eye. This number is approximately the magnifying power.

(*h*) TELESCOPES, SUMMARY.

1. The objective forms an inverted image at the cross-hairs, the eyepiece magnifies this image and the cross-hairs.

2. The objective must be brought nearer to the cross-hairs, as an object sighted at is farther away and *vice versa.*

3. The object-glass is an achromatic lens composed of a double-convex crown-glass and a concavo-convex flint glass.

4. The erecting eyepiece is composed of four lenses; the non-erecting of two. The latter therefore permits more light to reach the eye. It is also shorter and therefore affords space for the use of an objective of longer focus in a telescope of a given length, thus increasing the power.

5. Magnifying power is equal to the focal length of the objective divided by the focal length of the eyepiece.

6. The advantage of high power lies in the apparent increase in the size of the object sighted at. The disadvantages are: loss in illumination, decrease in the size of the field, increased difficulty in focusing. The brightness of an image varies inversely as the square of the magnifying power.

7. The image formed by the objective should lie in the plane of the cross-hairs to avoid parallax.

8. The size of the aperture is not always equal to the diameter of the objective. The amount of light collected by an ob-

jective is proportional to the square of the diameter of the clear aperture.

9. Flatness of field is most important in stadia work.

10. Chromatic aberration, spherical aberration, and aberration of sphericity are defects which are inherent in a single lens, but which can be corrected by compound lenses.

CHAPTER XLV

SURVEYING INSTRUMENTS

As no person is likely to study a surveying instrument unless he has one to use, no attempt is made in this chapter to explain parts of an instrument which are readily understood from inspection. The chief aim is to set forth important requirements which good instruments should fulfil, and to give methods of testing and comparing instruments. Incidentally important features of different instruments are pointed out, and wherever necessary the workings of concealed parts are explained.

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570. Chains and Tapes. (*a*) CHAINS. Each link of a surveyor's chain is made of iron or steel wire, and includes a long piece and a separate ring at each end. There are two kinds of chains: *Gunter's* chains and the *engineer's* chain. The former is 66 ft. long, while the latter is 100 ft. Each has 100 links; hence the length of a link in the Gunter's chain is 0.66 ft., while in the engineer's it is 1.0 ft. Some engineer's chains are only 50 ft. long, but the length of each link is still 1.0 ft. Gunter's chain was made 66 ft. long so that ten square chains should equal one acre. This gives rise to the special units of measurement given at the bottom of page 3. While the chain is still used it is being rapidly replaced by the steel tape. (See *Remark*, p. 33.)

(*b*) TAPES are of three kinds: (1) *Cloth* tapes, (2) *metallic* tapes (linen, woven with fine brass wire), and (3) *steel* tapes. They vary in length from a 3-ft. pocket tape to a 1000-ft. steel tape. Usual lengths are 50 and 100 ft. Cloth tapes are useless except for very rough work. Metallic tapes are somewhat better, but they are affected by moisture and under a strong pull they will sometimes stretch as much as 0.3 ft. in 100 ft. They stretch permanently with continued use and are suitable only for rough work. Steel tapes vary in quality and accuracy of graduation, but even the poorest steel tape is far superior to either a cloth or a metallic tape.

(*c*) STEEL TAPES. A steel tape is a thin ribbon of steel in one piece, from $\frac{1}{8}$ to $\frac{1}{2}$ in. in width, from $\frac{1}{64}$ to $\frac{3}{64}$ in. in thickness.

Band (*or Chain*) *Steel Tape.* Comparatively narrow and thick, divided into feet (sometimes links) with the foot at each end subdivided.

Ribbon Tape. Wider and thinner than the band tape, divided into feet, each foot subdivided; differs from the band tape in being graduated throughout. The ribbon steel tape is one of the most accurate devices known for linear measurements. It is used for city work, for base-line measurements, and for other work where great accuracy is required. Errors due to different sources, such as temperature, pull, sag, etc., can be eliminated or corrected as explained in Chapter V.

Pocket steel tapes in lengths from 3 ft. up, are very useful, and it is a good plan to carry one in the pocket not only in the field but in the office also.

Special tapes may be had fitted with a thermometer and a spring-balance or a compensating handle.

Best size for ordinary work. Band tape $\frac{1}{4}$ to $\frac{3}{8}$ in. wide, a little more than $\frac{1}{64}$ in. thick. *Ribbon* tape $\frac{3}{8}$ to $\frac{1}{2}$ in. wide. Wide thin tapes are easily broken.

Graduation of tapes. Tapes may be graduated to suit the purchaser. Standard graduations are as follows:

(1) Every foot marked, every fifth foot numbered 5–to–100, the foot at the zero end subdivided to tenths of a foot.

(2) Same as (1) except there are two series of numbers, one beginning at either end and numbered throughout in opposite directions, each end foot subdivided.

(3) Same as (1) except numbered symmetrically with respect to middle of tape thus: 0–10–20–30–40–50–40–30–20–10–0, each end foot subdivided.

(4) Graduated from end to end, each foot subdivided, and each foot numbered. This is the best form of graduation. (See *Remark*, p. 34.)

Graduation marks. There are four methods in common use for marking graduations: (1) By small brass rivets, one every foot, with some device for indicating 10-ft. points, as, for example, one rivet at 10 ft., two at 20 ft., three at 30 ft., etc. (2) By short brass sleeves at graduation points, with numbers stamped in the brass. (3) By lumps of solder at graduation points, with numbers stamped in the solder. (4) By etching the division lines on the tape with acid. Rivet holes in (1) and etching in (4) weaken the tape. Numbers etched are apt to become indistinct. Methods (2) and (3) are preferable for rough usage.

Reels and handles. Tapes may be wound up in leather or metal boxes or on some form of open reel. It is better for the tape to use neither, but to do it up in the form of a figure 8 (see p. 31). This allows the tape to dry quickly, avoids sharp bends, and permits the use of non-detachable handles. Most detachable handles come off too easily and are apt to be lost. Fasten such handles on with string or wire. The raw-hide loop handle is not apt to come off, but is more likely to catch in the bushes.

Note. After a steel tape has been done up in a figure 8 it may be twisted into a still more convenient form as follows: Tie the ends of the tape, but leave untied the middle of the 8 where the sides cross, thus allowing the tape to hang in one loop. Grasp opposite sides of the loop where they cross in the 8 and pull them apart, thus stretching the 8 crosswise. If properly done the tape will turn into two circular loops which may be brought together as one circle about the size of one loop of the 8. To undo the tape reverse the process, allowing the tape to spring back into a figure 8. Several trials may be necessary before the "trick" is learned, but the form is such a convenient one for carrying tapes that every chainman should learn to use it.

(*d*) STANDARDIZING TAPES. A steel tape can be sent to the National Bureau of Standards at Washington, where it will be tested for a small fee. The certificate of comparison furnished will state, among other data, the temperature at which comparison was made, the tension at which the tape was compared, and the length corrected for a standard temperature of 62° F. Having thus obtained the data concerning one tape this tape may be kept unused as a standard by which to test other tapes, or a permanent standard may be established as explained below.

Permanent standard. Any device used for testing the graduation of a steel tape should fulfil the following conditions:

(1) The exact distance between permanent marks should be known at the time the tape is tested.

(2) The tape should be supported throughout its length; if supported at intervals the tops of the supports should be level.

(3) Marks on the standard should be very fine, but clear and permanent.

(4) Some provision should be made for keeping the tape steady at a known pull; also some provision for measuring the temperature of the tape.

(5) There should be some device for observing and measuring the discrepancies in length between the standard and the tape tested.

For ordinary surveying a standard for a 100-ft. tape may be established as follows: On a level floor nail pieces of zinc about 2 inches square and 25 ft. apart centers (or closer if desired) for 100 ft. Near each end place a small turn-buckle, one end fastened to the floor, the other end attached in one case to the tape and in the other case to a spring-balance which connects the tape and turn-buckle. Select a day when the temperature *near the floor* is 62° (or whatever may be the standard for the tape), stretch the tape by means of the turn-buckle until the spring-balance reaches 12 lbs. (or whatever may be the standard pull), and make a fine deep scratch on each piece of zinc at the zero-point, the 100-ft. point, and at intermediate points. The scratches may be made with a straight-edge and fine-pointed awl, the work being done under a magnifying glass. When a floor space 100 ft. long cannot be had, the standard may be laid off on a sidewalk, water-table of a building, curbing or other convenient place if proper provisions can be made for marks. Sometimes stone posts with copper plugs or plates in their tops and set so that they cannot be disturbed are used as standards.*

In extensive work where great accuracy is required a more elaborate standard is sometimes necessary. That used for the Boston Water-works consisted of a steel strip a little over 100 ft. long, laid in a wooden trough with a cover to protect it. Graduations were on silver plugs inserted in the strip. A tape laid on the strip for testing would expand or contract for changes of temperature the same amount as the strip itself.†

Testing the tape. Stretch the tape with the turn-buckles and the spring-balance. Bring the zero of the tape exactly opposite the zero of the standard as observed through a magnifying glass, keeping the pull correct. Read the amount the tape is long or short, using a magnifying glass and a steel or boxwood scale divided to 50ths or 100ths of an inch. Note the temperature of the tape. The difference between the length of the tape and the standard should be corrected for the difference between the observed temperature and the standard temperature; the result is the error of the tape in inches and should be reduced to a decimal of a foot. Intermediate points should also be tested, since some of them may be incorrect even though there is no error in the total length of the tape. A tag attached to the handle should give the result of the test in some such manner as this: "Tested by John Smith, Fred Jones, and Ralph Brown, Sept. 14, 1907, Total length of tape 0.005 ft. short. Intermediate marks at 25-ft. points correct."

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* See *Engineering News*, May 6, 1897, p. 284, for a description of an elaborate apparatus for standardizing tapes at Salt Lake City.

† *See a paper on* "The 100-foot Standard of Length of the Boston Water-*works at Chestnut Hill*," by Charles W. Sherman.

571. The Transit. Although there are upward of four hundred parts in an ordinary transit, it may be divided for purposes of explanation into the three parts shown in the photograph on page 565 viz. (1) the *upper plate*, (2) the *lower plate* (or middle part), and (3) the *leveling-head*. The *supporting axis* of the telescope rests at either end upon a *standard*, and these standards are rigidly attached to the upper plate, so that *when the telescope is turned sidewise or horizontally the upper plate must turn with it.* The whole may therefore be considered as *one* part. The *inner spindle A* fits into a socket in the *outer spindle B*, and the latter fits into the socket *C* in the leveling-head. The object of these two spindles is to allow either the upper or the lower plate to be revolved without turning the other. When the transit is set up properly the leveling-head is stationary and the two plates are level. If the upper plate is clamped to the lower plate, both may be revolved together on the spindle *B*. If now the lower plate is fastened to the leveling-head, neither plate can revolve nor can the telescope be turned sidewise. If, however, the upper plate is unclamped from the lower, it can be turned (telescope and all) on the spindle *A*, although the lower plate will remain stationary since it is still fastened to the leveling-head. The upper plate is sometimes called the *vernier plate* because it has attached to it a vernier, the zero of which may be called an indicator. The entire upper part including the telescope is sometimes called the *alidade*. The lower plate or *limb* has on its outer edge the graduated circle. There are three steps in measuring a horizontal angle: (1) Set the indicator of the vernier opposite the zero on the limb and clamp the upper and lower plates together. (2) Turn the telescope toward the backsight, both plates revolving on the spindle *B*, and when the line of sight is on the backsight clamp the lower plate to the leveling-plate. Now neither plate can turn and the indicator is still opposite the zero. (3) Unclamp the upper plate, turn the telescope toward the foresight, the upper plate revolving on the spindle *A*, but the lower plate remaining stationary; the value of the arc which the indicator passes over as the telescope is turned from the backsight to the foresight can be read by observing the point on the limb at which the indicator stops when the line of sight is on the foresight. (See p. 67.) For the methods of reading angles see Chapter VIII, and for directions for setting up and manipulating the transit see Chapter IX.

(*a*) *Note.* Although the transits manufactured by different instrument-makers vary somewhat in design from that shown in the photograph practically all transits work on the general principle of a double spindle, in order to give the lower plate an independent motion. An exception may be noted in the case of the "cyclotomic transit," made by a San Francisco firm, in which the lower plate is *stationary*. Around the periphery of this plate, however, is a 'floating" graduated ring which is not fixed but possesses that independent rotation which the lower plate has in the case of the double spindle. Instead of turning the whole plate around its vertical axis, a narrow metal band is turned around the stationary plate, thus accomplishing the same purpose. See *Engineering News*, March 19, 1896.

(*b*) SET-SCREWS AND TANGENT-SCREWS. The upper plate is clamped to the lower plate by means of the set-screw *D*. This screw is not attached to the lower plate, although it appears to be in the photograph, but it works through an arm *E* which ends in a collar around the outside of the spindle *B*. When the screw *D* is set this collar is clamped to *B*, but when the screw is loose the arm is free to turn about *B*. When the upper and lower plates are put together the arm *E* fits between the *tangent-screw* and the cartridge-spring at the point *G*. Hence when the screw *D* is loose the upper plate can revolve, taking *D* with it, but when *D* is set the upper plate is virtually clamped to the lower. By turning the tangent-screw *F*, however, the upper plate can be made to revolve slowly, *provided D remains fixed;* if *D* is loose, turning *G* will have no effect. For example, the zero of the vernier may be set *approximately* opposite the zero of the limb, the screw *D* may then be tightened, and the upper plate turned slowly by the screw *F* until the vernier zero is *exactly* opposite the other zero. In like manner when a line of sight is *approximately* on a foresight it may be brought *exactly* on, without disturbing the lower plate, by tightening *D* and turning *F*.

The lower plate is clamped to the leveling-head by means of the set-screw *H*. When this is tightened it pushes in a little

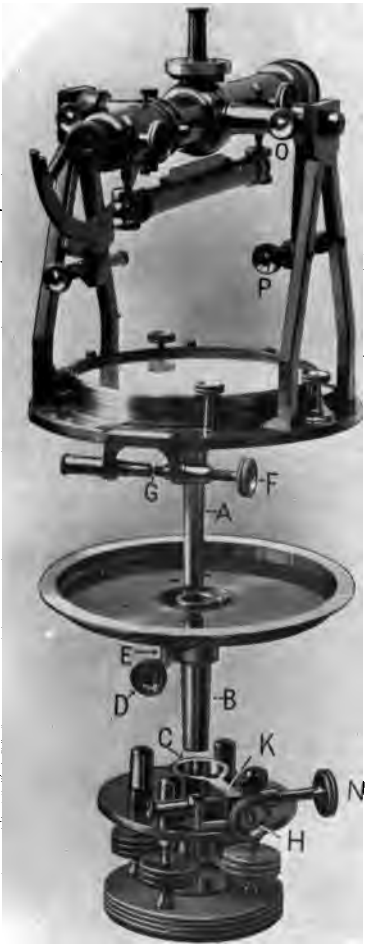

FIG. 571.

key (the little black piece at the inside end of the arm *K*) which binds the spindle *B*, so that the lower plate cannot turn. The collar *C*, however, is not fastened to the leveling-head, but simply rests upon it so that by turning the *lower tangent-screw N* the arm *K* can be moved slowly, the lower plate turning with it, *provided the screw H* is tight; if *H* is loose, turning *N* will move the arm *K* and the collar *C*, but not the spindle *B* (or the lower plate). The screw *H* is used mostly in backsighting, when the upper and lower plates are clamped together at zero; when the line of sight is approximately on the backsight the screw *H* may be set, and the lower plate turned slowly by the screw *N* (taking the upper plate with it) until the line of sight is exactly on the backsight, with both plates still clamped together at zero.

Notice that neither of the tangent-screws *F* nor *H* is of any use until the corresponding set-screw is tightened. For the proper use of these screws in measuring angles see page 91.

Note. In many transits the collar *C* is formed by a bent piece, the two ends of which are prolonged to form the arm *K*. These two ends are held together by a clamp-screw which replaces the set-screw *H*. When the clamp is loosened the two ends spring apart, thus releasing the grip of the collar on the spindle *B*.

To control the movement of the telescope in a *vertical* plane the set-screw *O* works upon the supporting axis of the telescope. When this screw is tight the telescope cannot be moved up or down except by means of the tangent-screw *P*. The set-screw and corresponding tangent-screw shown at the other end of the supporting axis are for the purpose of adjusting the *vertical arc* and are omitted on many transits. (See *Caution*, p. 98.)

Remark. If a transit is in perfect adjustment and properly set up it is impossible to measure with it any except vertical and horizontal angles, for when the telescope is turned up or down it moves in a vertical plane, and when it is turned sidewise it revolves on a *vertical* spindle *A*, the indicator passing over a *horizontal* graduated arc (lower plate).

(*c*) Special attachments for special uses of the transit are as follows:

1. *Solar attachment* for observations on the sun for meridian, longitude, etc.

2. *Gradienter* for running grades, etc. (See p. 245.)

3. *An auxiliary telescope* attached to the end of the supporting axis of the regular telescope for use in mining.

4. *Diagonal* or *prismatic eyepiece* for sighting at very high nts.

5. *A reflector* for illuminating the cross-hairs when working in the dark.

6. *A compass-needle and -box* generally, but not always furnished on transits.

7. *Stadia wires* should be placed in every transit intended for general work. They are often omitted, however, unless specially ordered.

(*d*) Requirements for transits. The most important requirements to be met in an ordinary transit are as follows:

1. The axis of the spindle *B* should be exactly perpendicular to the plane of the graduated circle.

2. The axes of the two spindles *A* and *B* should coincide, the conical surfaces should be parallel and in close contact. The instrument should turn on either spindle without play and without friction.

3. The graduations on the limb should be accurate, the lines should be fine, and for a transit reading to 30″ or less the graduations should be on solid silver. The center of the graduated circle on the limb should be in the axis of rotation. (For different graduations of the limb see Chapter VIII, pp. 67–75.)

4. As it is impossible to fulfil perfectly all of the above requirements, there should be two verniers 180° apart for the purpose of eliminating errors of eccentricity in accurate work. (See pp. 101, 195.)

5. Theoretically, in order to avoid parallax the vernier plate should be in the plane of the graduated arc, and the space between the two should be imperceptible. As the latter condition is impracticable, some makers place the inner plate slightly below the plane of the outer, so that the diagonal line of sight, when it reaches the inner piece, will strike beyond the space between the two plates and give a continuity of lines. If this is done some provision should be made for bringing the eye into the vertical plane passing through the agreeing division of the vernier and the limb. (For different forms of verniers see pp. 78–82.)

6. The relation between the magnifying power of the telescope and the least count of the vernier should be such that the least perceptible movement of the vernier will cause a perceptible movement of the cross-hairs on a distant object and *vice versa.* (See p. 555.) Still greater power is desirable for stadia work. A similar relation should exist between the mag-

nifying power of the telescope and the sensitiveness of the level under the telescope and of the plate-levels. (See p. 545.) For additional requirements for telescopes see pp. 553 to 558.

8. Clamps and set-screws should hold without forcing, and there should be no "back lash" or lost motion in the tangent-screws.

9. The tripod should be as light as is consistent with rigidity, and for convenience in setting up there should be a "shifting-head."

10. There are five additional requirements which are met by adjustments, viz., (1) The plane of the plate-levels should be perpendicular to the vertical axis of the spindles. (2) The line of collimation of the telescope should be perpendicular to the supporting axis. (3) The supporting axis should be parallel to the plane of the plate-levels. (4) The axis of the bubble under the telescope should be parallel to the line of collimation. (5) The vertical arc should read zero when the line of collimation is horizontal. For these adjustments see pp. 584 to 591.

(*e*) Tests for the transit. *Graduation.* Graduating engines have been perfected to such an extent that no errors in graduation are likely to occur which can be detected by any simple test that the surveyor can make. Errors frequently run in periods, so that one or two repetitions, should they fall within these periods, give the same though false results. A second class of errors of graduation, called *accidental* errors, follow no regular law. The surveyor for the most part must depend upon the instrument-maker to eliminate as far as possible both periodic and accidental errors of graduation.

> The vital point of any instrument is the graduation. Defects in other parts may prove an annoyance or inconvenience, and yet the instrument work correctly. With defects of graduation, accuracy, if not impossible, is a mere matter of accident. Defects in ordinary parts of the instrument become evident to the engineer. In the graduations they are unknown, and frequently the only evidence of their existence is the unsatisfactory condition of the finished work, leaving the engineer in doubt whether the unsatisfactory results condemning his work arise from his own carelessness or the imperfections of his instrument. It is not necessary that errors should be found in all parts; the mere fact that an error may exist anywhere is sufficient to throw doubt upon the work.—*Catalog of Young and Sons.*

Eccentricity. The two opposite verniers should read exactly alike in any position. Test by reading both verniers in several positions until the circle has been turned through at least 90°. If readings differ by a constant amount the verniers are not set *tly 180° apart.* If they differ by a variable amount and the graduation on the limb is assumed to be correct, then the center of the upper plate (vernier plate) is not in the center of the graduated circle, i.e., the vertical axes of the inner and outer spindles do not coincide. The total difference in readings between two opposite verniers is generally due to a combination of the two causes. A similar test can be applied to the vertical circle if it has two opposite verniers.

Note. For tests of the levels see page 545, and for the telescope see page 556.

572. The Level in its simplest form is used merely to establish a horizontal line of sight at any point on the earth's surface. All forms except *hand-levels* are mounted on tripods similar to those used for transits.

There are two distinct types of tripod levels, the *wye*-level and the *dumpy* level. Each of these has but one vertical spindle, and is much less complex than the transit. Each has a leveling-head similar to that used on the transit, and the upper part carries a telescope with a level-bubble tube attached. The most essential qualities in the telescope are definition and magnifying power. (See p. 553.) The length of the telescope is not limited as it is in the transit, and varies from 12 to 22 inches in ordinary levels. Hence a higher magnifying power may be obtained without sacrificing illumination or field of view (see p. 554); it should not be less than 30 diameters. Unless the stadia is used flatness of field is not so essential. An inverting eyepiece is better than an erecting eyepiece. The bubble-tube can be made longer than that on the transit telescope, and thus a greater radius of curvature secured, increasing the sensitiveness of the bubble to correspond to the increase in magnifying power. Sensitiveness should not be less than 0.01 ft. = 30″, and may be increased to less than 10″ in the larger instruments. It is, of course, especially important that the level-bubble should have the other good qualities mentioned on page 544. Another requirement in a good level is stability, and this depends mainly upon the leveling-head. Four leveling-screws are furnished on ordinary levels, although three give greater stability and delicacy. The spindle should be long and well fitted to the socket, and the whole upper part of the instrument should be well balanced when the telescope is focused for the average length of sight.

(*a*) THE WYE-LEVEL, so called because the telescope rests in two Y-shaped supports, has two distinguishing characteristics: (1) The telescope may be revolved about its own axis as it rests in the wyes, and (2) it may be taken out of the wyes and replaced, turned end for end. The object of this construction is to facilitate adjustment (see p. 592), but unless the diameters of the rings or collars are exactly the same, perfect adjustment is impossible by the indirect method. Even if the rings are alike to start with they are subject to wear that is seldom the same on the two rings. Hence if the level attached to the telescope is adjusted in the usual manner by reversing it end for end in the wyes, it may be adjusted parallel to the bottom of the collars and not parallel to the line of sight. Most wye-levels are subject to this uncertainty of adjustment unless the "peg method" is used. (See p. 597.) To overcome this objection some makers furnish a "reversion" bubble-tube, the glass of which is ground barrel-shape and graduated on the bottom as well as on the top. The arithmetical mean of two readings, one with the telescope normal, the other with it revolved 180° about its own axis until the bubble-tube is on top, is correct, though the rings may be of unequal diameters. In any form of wye-level there should be some device for stopping the telescope, as it is revolved about its own axis, when the vertical hair becomes truly vertical.

(*b*) THE DUMPY LEVEL is one in which the telescope is attached to the bar in such a way that it cannot be revolved about its own axis or taken out of its supports and turned end for end. Though less convenient to adjust than the wye-level it holds its adjustment longer. As the line of sight is adjusted by the "peg method," the uncertainty of adjustment mentioned in connection with the wye-level is avoided. The dumpy level is more simple and compact than the wye-level, and if equally well made it will do as accurate work. Some of the most perfect levels made and used in work of great precision are of this type.*

(*c*) SPECIAL FORMS OF LEVELS. *Levels of precision* are simply wye-levels or dumpy levels made with the greatest care, and having some modifications and attachments designed to increase the accuracy of the work. (See page 275.) *Hand-levels,* such as *Locke's Hand-level,* are for rough work and are used mostly for locating contours in topographic surveying. (See p. 233.) To this class also belongs the *Clinometer* or *Abney Hand-level,* which is a modification of the Locke level, having a supplementary telescope and graduated arc for reading vertical angles. For work of medium accuracy there are varieties of levels, such as the *Architect's Level,* which are simply cheaper forms of the wye-level or the dumpy level.

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* *See Engineering News*, July 2, 1903, for a description of one of the latest *precise leveling instruments.*

Note. By exercising a little ingenuity home-made levels may be devised, which can be used for rough work. For example, a right angle may be constructed of two wooden pieces and pivoted to a staff; the vertical leg is made vertical by a plumb-line attached to the upper corner, and the line of sight directed along the horizontal leg. Another form is the well-known A-level in which the plumb-line is attached to the apex of the A and the line of sight directed along the horizontal bar. This form may be adjusted by placing the two bottom points of the A on two rigid supports, marking the point at which the plumb-line crosses the horizontal bar, reversing the A on the supports, marking a second point on the bar, and taking a final point half-way between. If the A-level is held so that the plumb-line passes through this final point, the horizontal bar will be truly horizontal. There are also forms of water-levels, such as a U-shaped tube with water in it, or even a bucket of water with a wooden float and two upright sticks to serve as sights. A carpenter's square and plumb-bob are frequently used for establishing a horizontal line.

(*e*) TESTS FOR THE LEVEL. (1) Sensitiveness of the bubble. (See p. 545 for test.) (2) Magnification: The least displacement of the bubble should cause a perceptible movement of the cross-hair, otherwise the magnifying power is too low. (See p. 555.) (3) The optical center should move in the line of collimation as the objective is moved from the shortest to the longest focus. (See *Suggestion* 8, p. 594, for test.) (4) The diameter of the rings on a wye-level should be exactly equal. (See *Remark,* p. 597, for test.) (5) For other tests of telescope see p. 556.).

573. Leveling-rods may be divided into two classes: (1) *target* rods and (2) *self-reading* or *speaking* rods. Although the self-reading rod is usually provided with a target for use in accurate work, the graduation on the rod is such that the levelman can take direct readings through the telescope without the aid of a target. Experiments show that the difference in precision between target readings and direct readings is not as great as is generally supposed, and that it is a waste of time to use a target in a large part of ordinary leveling. (See p. 271.) The two types of leveling-rods most used, the *New York* rod and the *Philadelphia* rod, are illustrated on pages 228 and 230. The former is a target rod and the latter a self-reading rod. Frequently, however, the sights are so short that the New York rod can be used as a self-reading rod, and for this reason the upper half should be graduated on the front face. Many rod-makers fail to do this and it may be necessary for the engineer to do it for himself. There are many patterns of self-reading rods in use. (See Stadia Rods, p. 574.)

(*a*) TARGETS. The *quadrant* target shown on pages 228 and 230 is of the type most used. Another form called the *diamond* target gives better results for long sights. There are several other patterns in use. Targets are provided with a *vernier* as shown on the New York rod, page 230, or with a *scale* as shown on the Philadelphia rod, page 228. The *angle* target, in which the right-hand half is at right angles to the left-hand half, is a special form devised to insure the rod being held perpendicular to the line of sight. (See p. 270.)

(*b*) SPECIAL FORMS OF LEVELING-RODS. A *hinge* rod, in which the upper half is fastened to the lower by a hinge, is sometimes used in place of the ordinary extension rod. Better than either of these for accurate work is the single-piece rod, since the errors that are likely to be caused by any form of joint are avoided. There are various other forms, such as rods on which the target is moved up and down by an endless chain, and flexible rods which are simply canvas strips graduated like a self-reading rod. These strips when rolled up can be carried in the pocket; when tacked to a straight board they answer very well for a self-reading rod.

The *automatic leveling-rod* is a special form of self-reading rod which gives the elevation of points directly without arithmetical work. It consists of an endless steel band passing over rollers at the top and bottom of a wooden rod. The graduation is painted on the band to correspond to that of a self-reading rod, but reads downward instead of upward. In using the rod the band is set for every backsight by moving it until the graduation mark which corresponds to the elevation of the point on which the rod is held comes into the line of sight. For example, if the elevation of the point where a backsight is taken is 94.83, the band is moved until the 4.83-mark comes into the line of sight. If now the rod is held on any other point, the line of sight will strike the tape at a graduation mark corresponding to the elevation of that point, provided the tape has not been disturbed between the backsight and foresight. Thus elevations are read directly, and the backsight and foresight columns are entirely *minated from* the level notes. This form of rod is very useful in taking topography and in cross-section work. Rods made according to the same general principle are described in *Engineering News*, Sept. 13, 1900, and March 12, 1903.

(*c*) THE PLUMBING-LEVEL consists of a round level vial which may be held against the rod or fastened to it. It is used for determining when the rod is perpendicular, and adds greatly to the accuracy of the work. Some such device should always be used in work of great precision.

574. Stadia Rods are of two kinds: target and self-reading. The former has two targets both of which must be set for every sight taken. It may be slightly more accurate, but it is far less convenient than the self-reading rod, and is seldom used in ordinary work. It is sometimes used for exceptionally long sights.

There is a large variety of self-reading stadia rods in common use. Many engineers prefer to graduate their own rods, using the pattern best suited to their work. All of the rods in Fig. 574 are graduated with the foot as a unit. Rods No. 1 and No. 5 are often graduated with the meter as a unit. In Fig. 574 the space between the top and bottom dotted lines corresponds to 10 ft. on the ground, while each of the small spaces between the dotted lines corresponds to 2 ft. on the ground.

The first two rods are suitable for long sights, the last three for short sights. The difference in accuracy, however, is not as great as at first appears, for distances can be estimated on the first two rods almost as closely as they can be read on the last three. Moreover, the large number of subdivisions on each of the last three rods leads to confusion in counting the marks included between the stadia wires.

Whatever pattern is used there should be sufficient white background at any point on the rod to make a stadia wire covering that point very clear and distinct to an observer looking through the telescope. Rods No. 1 and No. 5 (U. S. Coast and Geodetic Survey rods) and rod No. 4 (described by Mr. T. J. McMinn in *Engineering News*, Oct. 22, 1896), are excellent in this respect, while rod No. 2 is defective at those points where the black spaces so nearly cover the width of the rod. Rod No. 2 was used on the Oswego-Mohawk Ship Canal Survey, but the width of the rod was somewhat greater in proportion to the size of the black spaces than that shown in the cut.

(*a*) *Directions for making stadia rods.* The rod should be from 4 to 6 inches wide, about ½ inch thick, and from 10 to 14 ft. long, made of straight-grained white pine free from knots and planed smooth. A thin strip fastened securely to each edge and projecting slightly in front of the face serves to stiffen the rod and also protects the graduation. If the side strips are left off, the corners should be rounded slightly, and a stiffening strip about 1½″ thick by ¾″ screwed to the back. For protection and convenience in transportation, some engineers prefer rods hinged at the middle, so that the upper and lower halves may be folded together, faces inward. A metal shoe may be screwed to the bottom of the rod and thin metal strengthening plates fastened on the back at the top and bottom.

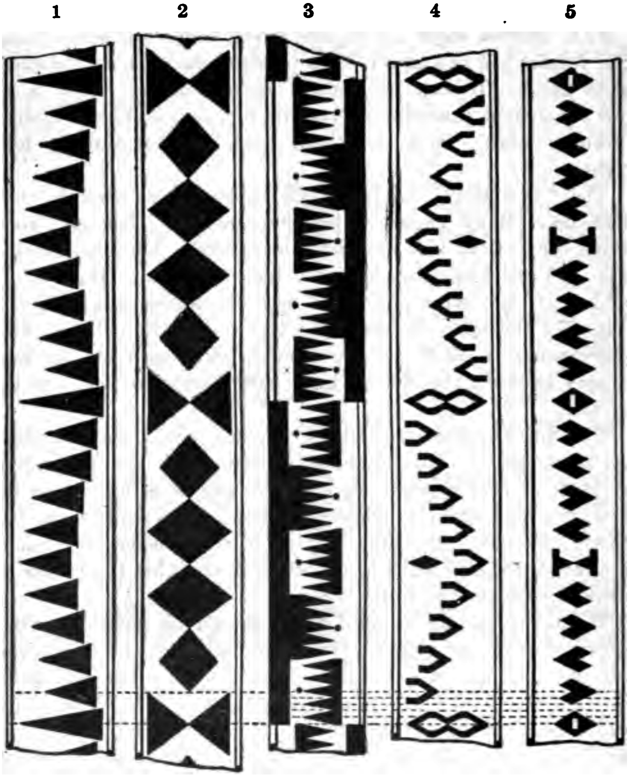

Fig. 574.

At least two good coats of white paint should be given the rod; it is better to give the rod a preliminary coat of shellac, and then several coats of paint, rubbing each coat down after it is dry to a smooth dull finish. If the rod is to be graduated to suit a fixed stadia interval, the unit of length *may be determined* by the method explained on page 304. Ordinarily, *however, the rod is* divided into feet and tenths, or into meters and decimeters, by means of a scale, beginning about 3 inches from the bottom. The pattern should be first penciled, then ruled with a right-line pen filled with paint and finally filled in with a camel's-hair brush. A dull black paint may be used throughout, although some engineers prefer to emphasize certain division-marks, as, for example, the even foot-marks, by painting them red. Sometimes such marks are numbered, but it is doubtful if this is of any advantage. If desired the whole rod may be coated with a durable, elastic varnish rubbed down to a dull finish. The rod should be allowed to dry thoroughly before using.

575. The Surveyor's Compass in its simplest form is mounted on a single staff, but in its usual form it is mounted on a tripod. The upper part consists of a horizontal arm having two uprights or sight vanes, with a compass-box between them. There are various devices for attaching this upper part to the tripod, such as a ball-and-socket joint, a leveling-head similar to that used on transits, and in some instruments a combination of the two. The needle should be so sensitive that when it is attracted to one side by a piece of iron held a foot or so away it will settle in its original position several times in succession. Sensitiveness depends upon the sharpness and perfect form of the pivot, the form and smoothness of the cup, the shape and balance of the needle, and the strength of its magnetism. A good needle properly charged does not often lose its magnetism, and hence in the majority of cases a sluggish needle is due to the dulling of the pivot-point, or to the scratching of the cup. As it is difficult to sharpen a pivot, special precautions should be taken to avoid dulling it, such, for example, as lifting the needle off the pivot when not in use. (See p. 294.) The extreme ends of the needle should come close to the graduated circle and should lie in the same plane with it to avoid parallax in reading. The center of the needle and the two ends should lie in the same vertical plane, so that if the pivot is in the center of the graduated circle the readings of the two ends of the needle will agree. The magnetic axis of the needle should coincide with its geometric axis. The plane passing through the slits of the sight vanes, in which the line of sight is contained, should pass through the zero-points of the graduated circle and be at right angles to the plane of this circle. The above are the most important requirements for a good compass; most of them depend wholly upon the process of manufacture, while a few may be met by proper adjustment of the instrument. (See p. 602.)

(*a*) Special forms of compasses. When the *plain compass* described above has a movable vernier for setting off the magnetic declination it is called a *vernier compass*. The *prismatic compass* used for less accurate work has a triangular glass prism and a mirror by means of which the needle may be read at the same time that a sight is taken; this is an advantage when the instrument is held in the hand instead of being mounted on a Jacob's-staff or a tripod. Compasses are sometimes provided with a graduation for reading angles independently of the needle, but the precision which can be obtained with open sights is hardly sufficient to warrant this. Telescopes sometimes replace the open sights, but if the work is of such a nature as to require a telescope, it is better to use a light transit in place of the compass. The solar compass, formerly used in land surveys for determining the true meridian by an observation on the sun, has now been replaced almost entirely by the solar attachment on the transit which is constructed on the same general principles. (See Appendix I.)

(*b*) TESTS FOR THE COMPASS. (1) Stretch a fine thread through the two sight vanes. The north and south points of the graduated circle and the zero of the declination-arc vernier should be directly underneath this thread. (2) Test the sensitiveness of the needle to see if it will settle several times in succession at the same reading after being attracted to one side. (3) By means of a transit set stakes about 200 ft. from a station at angular intervals of, say, 30° until a circle is completed. Set the compass over the station and sight at each stake in succession, noting the needle readings. If at any point the change in the needle reading is not 30° from that of the preceding reading, and if all sources of attraction outside of the compass have been carefully removed, it is probable that the metal of the compass-box contains some magnetic substance, as, for example, iron mixed with the brass. If a compass has a long declination arc and vernier, the transit may be dispensed with in the above test, the angles being turned off by means of the vernier.

576. The Plane-table. A complete plane-table consists of a drawing-board mounted on a tripod and an *alidade* consisting of a thin flat straight-edge upon which is mounted a telescope and vertical arc.

The *board* varies in size from 15″×15″ for a traverse board up to 24″×30″ for a regulation plane-table. It should be constructed so that it will not warp and its upper surface should be a smooth *plane*, two conditions difficult to fulfil. There are various devices for fastening the paper to the board, of which screw-tacks are perhaps the best. (See 393 (3), p. 327.)

The drawing-board is connected to the tripod by a mechanism which supports the board, yet leaves it free to revolve in a horizontal plane. If the diameter of this support is small the board is likely to be unstable, while if the diameter is large the instrument is likely to be heavy and cumbersome. The maximum of rigidity and strength is probably obtained by the use of radiating arms which support the board. For ordinary work, however, a lighter and more portable instrument with a quick leveling device is desirable. An excellent form is that adopted by the U. S. Geological Survey and known as *Johnson's* plane-table.

(*a*) ALIDADE. The telescope should be equal to that on an ordinary transit. As vertical angles are frequently of great importance in plane-table work, the diameter of the vertical arc should be large and the sensitiveness of the bubble should be in proportion, as pointed out in the specifications for a transit (p. 567). The axis of the telescope should be parallel to the edge of the metal straight-edge.

(*b*) ACCESSORIES. A small compass-box, sometimes called a *declinator*, is required for determining the magnetic meridian. *Levels* for leveling the board are also required. Sometimes they are attached to the ruler of the alidade, sometimes they are combined with the compass-box. A *plumbing-bar* is occasionally required and should be furnished with the instrument. Drawing-instruments, scale, etc., a water-proof cover, a case for the drawing-paper, and a sunshade complete the outfit.

(*c*) TRAVERSE BOARDS are small plane-tables used for rough work, such as sketching in contours in topographic surveys. In place of the telescope two slotted uprights or open sights are used.

(*d*) TESTS FOR PLANE-TABLE. The edge of the ruler which forms the base of the alidade should be straight. (See test for straight-edge, p. 438.) To see if the upper surface of the board is a plane lay a straight-edge on it in all directions. If it is only a little out it may be scraped or planed down, and then smoothed with sandpaper.

577. The Sextant. The distinguishing characteristic of the sextant is the arrangement of mirrors, which enables the observer to sight at two different objects *simultaneously*, and thus to measure an angle at a single observation. While other instruments must be kept stationary during the time between the backsight and the foresight, the sextant may be held in the hand, since the two sights are taken at the same time, and horizontal, vertical, or inclined angles may be measured very accurately even though the instrument is in motion. For this reason the sextant is used for observations at sea. In surveying it is used for measuring angles from a boat to points on shore when locating soundings, buoys, etc., by the method shown in Fig. 197, p. 137; it is also used in reconnaissance work, explorations, geological surveys, and for solar observations.

(*a*) TO MEASURE A HORIZONTAL ANGLE. In Fig. 577 (*a*) let *B* represent the mirror or *index-glass* which is rigidly attached to the *vernier arm BH*. Let *C* represent the *horizon-glass*, the lower portion of which is a mirror, and the upper portion clear glass. The colored glasses may be turned back out of the way except when making observations on the sun, when they will be needed to protect the eye. Usually three eyepieces are furnished: one for ordinary terrestrial work, one for astronomical work, and one without magnifying power.

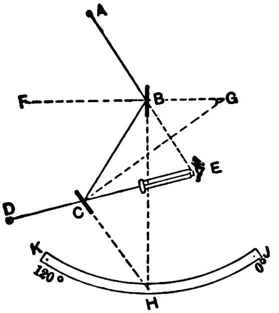

FIG. 577 (*a*).

Let *A* and *D* be the two points sighted at, and *E* the eye of the observer; then the angle required is *AED*.

(1) Holding the sextant in one hand by the handle underneath, bring the plane of the graduated arc *JK* into the plane of the eye and the two points *A* and *D*.

(2) Sight through the upper portion (clear glass) of the horizon-glass *C* directly at *D* and swing the vernier arm *BH* (thus turning the index-glass *B*) until the image of *A* reflected from *B* to the lower half (mirror) of *C* appears superimposed upon *D*.

(3) Clamp the vernier and by means of the tangent-screw bring the images of *A* and *D* into exact coincidence.

(4) Read the vernier. The reading is the angle *AED*, but

unless these three points lie in a horizontal plane the angle is not a true horizontal angle. In most cases, however, where the sextant would be used, the difference between the angle and its horizontal projection would be a negligible quantity.

(*b*) *Practical suggestions.* (1) It is a little difficult at first to bring the two images into coincidence, especially if the observer is in a boat, where on account of the unsteady motion it may be necessary to make quick observations. *In such a case it is often best to dispense with the use of the eyepiece altogether;* simply place the eye at the center of the ring from which the eyepiece has been removed and proceed as if looking through a telescope. This is a good way in which to become accustomed to the use of the sextant, and it will be found that angles can be measured by this method with surprising accuracy. (2) Sight through the telescope directly at the fainter object, so that the brighter object will be the one reflected. To do this it may be necessary to hold the sextant upside down. (3) Do not try to hold the images still in the field of view, but find the final position of the vernier arm by a series of observations, twisting the wrist to produce a regular oscillating motion, and noting the relation of the reflected image to the direct image as they come together. (4) The adjusting screw that moves the telescope to or from the plane of the arc is for the purpose of centering the two images and making them equally bright. (5) Keep the fingers off the graduation, as it tarnishes easily. (6) The degrees on the arc are numbered to correspond to twice their value, that is, each degree is marked two degrees, etc., as explained below. The sextant is so called because the arc is about 60° long, but angles may be measured to 120° or more. (7) The index correction should be determined as accurately as the angle is to be measured. (See p. 606.)

(*c*) VERTICAL ANGLES may be measured by holding the sextant so that its arc lies in a vertical plane. In observations on the sun an artificial horizon, such as a shallow box of mercury, is used; the image of the sun in this reflecting surface and the sun itself are the two objects sighted at, the resulting angle being the *double* altitude of the sun.

(*d*) THE THEORY OF THE SEXTANT. If a ray of light is reflected successively from two plane mirrors the angle between the first and last directions of the ray is twice the angle between the planes of the two mirrors.

Proof. In Fig. 577 (*a*) let *FG* and *CG* be drawn perpendicular to the planes of the mirrors *B* and *C* respectively. Let the ray from *A* be deflected at *B* to *C*, and again at *C* to *E*. Since the angle of incidence equals the angle of reflection,

$ABF = FBC$ and $BCG = GCE$;
also by geometry $BGC = CHB$.
$BEC = 180° - EBC - BCE = ABC - BCE.$
$BGC = 180° - GBC - BCG = \frac{1}{2}ABC - \frac{1}{2}BCE.$
$\therefore BEC = 2BGC = 2CHB.$

When the mirrors are brought into parallel planes *CHB* becomes zero and hence the angle *AED* will be zero, i.e. the rays from *A* and *D* are parallel.[*] This gives the position of the arm for a reading of 0°. If the arm is moved to any point as H, the zero of the vernier will move over the arc an angular distance equal to the angle between the mirrors, which, as shown above, is one half of the required angle BEC or AED. For this reason the arc is numbered to give at once the angle required at E, i.e. each degree is counted as twice its real value.

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[*] A ray which comes from any point to the eye through the horizon-glass is not strictly parallel to a ray from the same point coming to the eye indirectly by reflection, since the index-glass is several inches out of the direct line of sight, but for sights more than 1000 ft. the error would not exceed from 30 to 40 seconds (see table, p. 105), and for sights of a mile or more it is inappreciable.

CHAPTER XLVI

ADJUSTMENTS

The actual work of adjusting a surveying instrument is comparatively simple and easy once the reasons for the various steps are clearly understood. If, on the other hand, the reasons are not clear, and the student is content simply to follow directions such as those printed in catalogs of instruments, he is not likely to work intelligently or to gain confidence in his ability to adjust an instrument. The aim in this chapter, therefore, is to lead the student to a thorough understanding of fundamental principles, so that if he forgets the methods and order of adjustments (as he almost surely will) he will be able to reason them out for himself instead of relying upon his memory or upon printed directions.

The adjustments explained are those of the transit, the level, the compass, the plane-table, and the sextant.

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578. Why Adjustments are Necessary. When an instrument leaves the hands of the maker it should be in complete adjustment. There are certain parts, however, which are likely to be deranged by accident or by continuous use. Accurate work can often be done with an instrument out of adjustment, provided certain methods of eliminating errors are followed, as for example, the method of double reverse, page 92, but such methods involve extra labor, and hence it saves time to keep an instrument in adjustment. Moreover, there are some errors which cannot be eliminated except by adjusting the instrument. A good instrument will sometimes stay in adjustment for months, nevertheless all instruments should be tested frequently. In accurate leveling, for instance, the level should be tested at least once a day. The more accurate the instrument and the more exacting the requirements the more attention should be paid to the adjustments. No man therefore is a competent surveyor who is unable to adjust his instruments.

579. General Suggestions. The following suggestions apply to the adjustment of all surveying instruments: (1) For each instrument there are usually several adjustments. Each adjustment consists of two steps: first a *test* to determine the error, and then the *correction* of this error by turning certain adjusting-screws or nuts. In most cases this is done by inserting an adjusting-pin, hence the terms, capstan screws and capstan nuts. (2) The adjusting-pin should fit snugly into the hole, otherwise the holes will become worn and out of shape. In an emergency a wire nail can be used. (3) If all of the adjustments are badly out, do not perfect the adjustments one at a time, but rather bring the whole instrument to a nice adjustment by repeating the entire series as many times as may be necessary. This is because the adjustment of one part may affect some other part already adjusted. *Hence the final step in the adjustment of any instrument is to test each adjustment to see that none has been disturbed.* For this reason select a place where the adjustments may be made in succession without disturbing the instrument. (4) In making any one adjustment the test and correction should be repeated a number of times. It will save time not to split hairs during the early part of the adjustment, but the more nearly the adjustment is perfected the more carefully it should be made. (5) An instrument will stay in adjustment longer if screws and nuts are left in perfect contact with the surfaces against which they operate. For example, after a screw has come to bearing, a slight turn of the adjusting-pin of, say, 20° will "set" it so that it is not easily disturbed. Undue strain however should be avoided. (6) Before attempting to adjust cross-hairs study carefully the note on page 550 and the photograph on page 551. Tightening the capstan screws which move the cross-hair ring does not affect the tightness of the cross-hairs, but if too tight the whole telescope may be affected. If one pair of screws is fastened tightly while the other pair at right angles is being turned, undue strain may be brought on the first pair if the adjustment is considerable. In such a case loosen all four screws *slightly* before working either pair. (7) In adjusting cross-hairs take the least important first, i.e., the horizontal hair in a transit telescope and the vertical hair in the telescope of a level. (8) In this book the line between the intersection of the cross-hairs and the center of the object-glass is called the *line of sight.* A distinction is made between this line and the *line of collimation* (see p. 553), with which it may or may not coincide. (9) If after adjusting an instrument the intersection of the cross-hairs does not appear to be in the *center of the* field of vision, the eyepiece probably needs "centering" (see page 601). (10) Eliminate parallax (see p. 555). (11) It is well to set up the instrument in the shade during adjustment. This is especially true of the level, as a bright sun affects the bubble.

580. The Method of Reversion. *Every* adjustment depends upon the method of reversion, i.e., the adjustment is tested by reversing the instrument, or some part of it, so that an error will become apparent; this *apparent* error is *double* the *true* error, because reversion simply places the error as much to one side as it was on the opposite side before reversion. This will become evident as the adjustments are explained. The thing to remember is that a *single* reversion *doubles* the error.

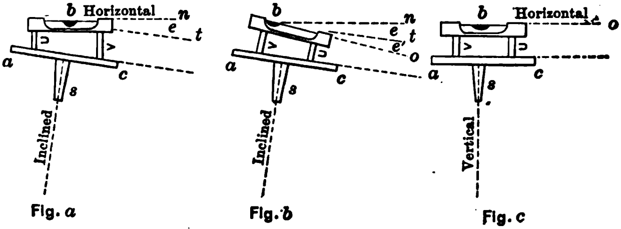

Fig. *a* Fig. *b* Fig. *c*

FIG. 580 (*a*).

(*a*) *Illustration. The reversion of the bubble-tube, end for end*, is an excellent illustration of the method of reversion. Moreover, since it is the basis of three of the six principal adjustments, it is worth while to understand it perfectly.

In Fig. *a* let the bubble be in the center of the tube—its axis *bn* must therefore be horizontal—but let the supports *u* and *v* be of *unequal* length. Then any surface as *ac*, upon which the supports rest, cannot be level, but *a* will be higher than *c*, since *u* is shorter than *v*. Suppose that it is desired to bring the axis of the bubble *bn* parallel to the surface *ac* by making the supports *u* and *v* of equal length; this is equivalent to making *bn* coincide with some line *bt* which has the same inclination *e* with the horizontal that *ac* has. This error *e*, which is to be corrected, is *above* the required line *bt*. Keep the surface *ac* stationary, but take the bubble-tube off, and reverse it end for end (Fig. b). The supports *u* and *v* being attached to the tube, and their lengths as yet being unchanged, the new axis of the bubble *bo* still makes the angle *e* with *ac* and *bt*, but is *below bt* instead of above. In other words, reversion has simply placed the error on the opposite side, and the total error $e+e'=2e$ is *double* the true error. To correct the error therefore change the lengths of the supports *u* and *v* until the bubble moves HALF-way back to the center; *bo* will then coincide with *bt*, and *u* and *v* will be of equal lengths. If now the surface *ac* is made horizontal (Fig. *c*), the bubble will return to the center and its axis *bo* will be horizontal.

(*b*) Let the bubble-tube be *attached* to the surface *ac* by supporting screws *u* and *v*, and let this surface be revolved on a spindle *s*. It is evident that if the axis of this spindle is at right angles to *ac the inclination of ac cannot change during revolution*, and that if the surface is revolved through 180° *the bubble-tube will be turned end for end* just the same as if it had been taken off and reversed. Hence in this case the reasoning and the conclusions of the preceding paragraph still hold good. It is to be noted that before adjustment (Figs. *a* and *b*) the axis of the spindle is inclined; after adjustment (Fig. *c*) it is vertical.

Remark. If the axis of the spindle is not perpendicular to *ac* reversion and correction will still bring the axis of the bubble *perpendicular* to the axis of the spindle.

581. Adjustments of the Transit. THE PRINCIPAL LINES to be considered in the adjustment of the transit are: (1) The line of sight (see p. 553). (2) The axis of the bubble in each plate-level (see p. 544). (3) The supporting axis of the telescope. (4) The vertical axis through the spindles. (5) The axis of the bubble-tube attached to the telescope.

THE PRINCIPAL ADJUSTMENTS are five in number, the first three of which should always be made; the last two are necessary only when the transit is used for measuring vertical angles or for leveling:

I. *The adjustment of the plate-levels,* so that the plates shall lie in a horizontal plane when each bubble is in the center of its tube.

II. *The adjustment of the cross-hairs,* so that the line of sight shall revolve in a plane when the telescope is turned on its supporting axis.

III. *The adjustment of the standards,* so that the line of sight shall revolve in a *vertical* plane when the first two adjustments have been made and the transit is leveled up.

IV. *The adjustment of the telescope level,* so that the axis of the bubble shall be parallel to the line of sight.

V. *The adjustment of the vertical circle,* so that its vernier shall read zero when the line of sight is horizontal.

THE RELATIVE IMPORTANCE OF ADJUSTMENTS. (1) The first adjustment is important in all transit work. (2) The second is important in work which involves plunging the telescope. (3) The third is important in the measurement of *horizontal* angles between points which differ considerably in elevation. Such measurements are also affected to some extent by the first two adjustments. (4) The fourth adjustment is necessary when the transit is used as a level. (5) The fifth adjustment is not essential, but it facilitates the reading of vertical angles.

I. ADJUSTMENT OF THE PLATE-LEVELS. *Object.* To make the axis of each bubble parallel to the plate to which the bubble-tubes are attached, so that when each bubble is in the center of its tube the plate shall be truly horizontal. This is equivalent to bringing the axes of the bubbles perpendicular to the axis of the spindle upon which the plate turns, provided this axis has been made by the instrument-maker perpendicular to the plate.*

Test. The final test is to see that each bubble comes to rest in the center of its tube when the plate is turned on its spindle to any position from 0° to 360°. During adjustment, however, repeated tests for each bubble are made as follows:

(1) Level up and bring the bubble-tube parallel to two diagonally opposite leveling-screws.

(2) Turn the plate half-way around until the bubble-tube is parallel to the same pair of screws, but reversed end for end. If the bubble does not come to rest at the center the tube needs adjusting.

Correction. (1) Bring the bubble half-way back to the center of its tube by turning one and then the other of the capstan screws which support the ends of the tube. (2) Bring it the rest of the way (i.e., to the center) by leveling the plate with the leveling thumb-screws. (3) Repeat the test and the adjustment until the bubble remains in the center when the plate is turned half-way around. (4) Adjust the other bubble-tube in like manner. (5) Apply the final test of turning the plate completely around from 0° to 360°.

Principle involved. The reason why the bubble is brought *half*-way back after reversion is explained in Art. 580, p. 583.

Practical suggestions. (1) Set up the transit so that it is *firm*. If a place is chosen where a 300-ft. sight can be taken each way, and where the telescope can be directed to some high point, all four of the remaining adjustments can be made without disturbing the instrument. (2) It is better to clamp the lower plate and to allow the upper plate to remain unclamped throughout the adjustment. When the final test is satisfactory, clamp both plates together, loosen the lower clamp, and turn the two plates completely around; both bubbles should remain in the centers of their tubes; if they do not it shows that the axes of the inner and outer spindles (see photograph, p. 565) are not parallel, a defect best remedied by the maker. If the axes are not parallel no error will be caused in the measurement of horizontal angles unless they are measured by repetition, provided the plate-levels are adjusted perpendicular to the axis of the inner spindle. (3) If the bubbles are not badly out both may be adjusted simultaneously, taking care to reverse one tube parallel to one pair of leveling-screws and the other tube parallel to the other pair. (4) It will often suffice to turn one capstan screw only, lowering slightly the end of the tube where the bubble lies, or raising the other end. (5) Turning a capstan screw clockwise screws it into the plate and therefore *lowers* the corresponding end of the tube; counter-clockwise unscrews the screw and *raises* the end of the tube.

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* If the axis of the spindle is not perpendicular to the plate the adjustment of the plate-levels does not in any way involve the horizontality of the plate. For this reason it is more correct to consider the relation of the plate-levels to the *vertical axis*, ignoring the plate altogether For the sake of clearness, however, it is better to consider the parallelism of the plate-levels to the plate as the real object of the adjustment, especially as the vertical axis is so nearly perpendicular to the plate in the ordinary transit that the error is inappreciable.

II. ADJUSTMENT OF THE CROSS-HAIRS. This adjustment consists of two steps: (1) adjustment of the horizontal hair and (2) adjustment of the vertical hair. The former affects only leveling operations and measurements of vertical angles, while the latter affects all transit work, especially that in which the telescope is plunged. The adjustment of the horizontal hair is often overlooked or ignored.

Note. Before adjusting the cross-hairs separately it is well to see that the vertical hair is truly vertical; if the two cross-hairs have been put on the ring exactly at right angles to each other the horizontal hair will then be truly horizontal. Test the vertical hair by seeing if it coincides with a plumb-line or the corner of a building, or, another method, with the supporting axis horizontal, sight upon some well-defined point and depress the telescope; the point should appear to move up and down the hair. If the hair is not vertical loosen all four capstan screws slightly take hold of their heads and turn the ring until the hair is vertical. (See note, page 550). Or, better still, tap the loose heads gently with the finger-tip or with a pencil, since the amount which the ring must be removed is slight.

Adjustment of the Horizontal Cross-hair

This adjustment is necessary only when vertical angles are to be measured with more than ordinary accuracy, or when careful leveling is to be done with the transit.

Object. To bring the horizontal hair into the plane of motion of the optical center of the object-glass; if it is not in that plane the line of sight will change as the object-glass is moved in or out for focusing.

Test. (1) Hold a leveling-rod on a stake driven near the transit, clamp the telescope so that the line of sight is *approximately* parallel to the surface of the ground for about 300 ft., bring the center of the target to the line of sight, and note the rod reading. (2) Without moving the telescope read a rod upon a second stake about 300 ft. away. (3) Unclamp, plunge, turn the plates, bring the inverted telescope to the former reading on the first stake, and tighten all clamps. If the horizontal hair does not strike the former reading on the second stake it should be adjusted.

Correction. Set the target at the mean of the two readings taken at the second stake. With the telescope still clamped in the position it was in when the second reading was taken, *move the horizontal* hair by means of the capstan screws until it bisects the target, when the rod is held on the second stake. Loosen the capstan screw on top of the telescope and tighten the opposite screw to move the hair upward *as observed through an* INVERTING *eyepiece*, and *vice versa* to move it downward. (Why? See p. 550.) Repeat the test and correction until the adjustment is perfected. *The principle involved* is that of single reversion; the telescope being inverted *once*, the error is doubled. (See Fig. 582 (I), p. 593.)

Practical suggestions. (1) It is not necessary that the telescope should be level when sighting at the leveling-rod—any line of sight will do. (2) It is better to use two leveling-rods, one for each stake, then the target may be left undisturbed on the rod near the transit. In an emergency two straight sticks or transit poles may be used in place of leveling-rods. A small piece of paper wrapped around each stick and held in place by a rubber band will serve as a target, the band marking the points sighted at. Use three bands for the pole at the farther stake, one for the first sight, one for the second, and one half-way between.

Adjustment of the Vertical Cross-hair

This is often called the *adjustment of the line of collimation.* (See p. 553.)

Object. To make the line of sight *perpendicular* to the supporting axis of the telescope, so that when the telescope is plunged the line of sight will generate a *plane;* if the line of sight is not perpendicular it will generate the surface of a cone. In practice the chief object of the adjustment is to be able to produce a straight line by backsighting and then simply plunging the telescope *once.*

Test. The test corresponds exactly to reversing in azimuth and altitude or double reverse. (See p. 92.) Sight on some permanent backsight, plunge, establish point number *one.* Leave the telescope inverted, turn upper plate, sight on the permanent backsight, plunge, and establish point number *two.* If these two points do not coincide the vertical cross-hair must be adjusted.

Correction. (1) Measuring from point *two* towards point *one,* mark a *third* point *one fourth* of the distance from point *two* to point *one.* (2) By *loosening* one of the capstan screws on the side of the telescope and *tightening* the other, move the cross-hair ring until the vertical hair is on point *three.* (3) Repeat the test and the adjustment until the two points established by the double reverse exactly coincide.

Principle involved. This is a *double* application of the method of reversion. Reversing (plunging) the telescope once doubles the

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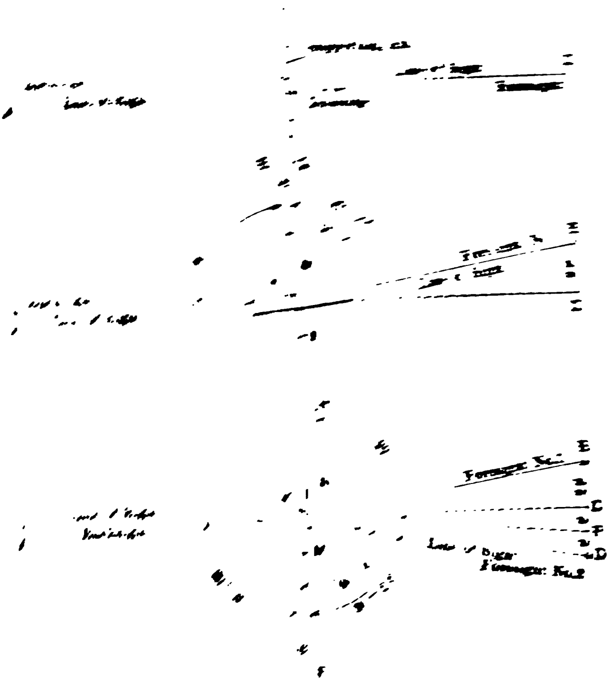

Fig. [illegible] (1).

[illegible] XY [illegible] axis of a telescope (top figure in Fig. [illegible] (1)). If AKX is an exact right angle the line of sight is in adjustment [illegible] on A and plunging the telescope the line of collimation will generate a plane, striking a point C which is in the line AK produced. But suppose the line of sight is out of adjustment [illegible] angle AKB (middle figure) is not 90° but [illegible] the line of sight will generate [illegible]. When the telescope is plunged the [illegible] B instead of C. From the [illegible] strike a point [illegible] and hence BKC = 2e. Leave the [illegible] QKB is 90° − 2e [illegible] on A again. (Bottom figure: [illegible] N, the ends of the supporting [illegible] of M and [illegible] collimation will strike some [illegible] the line of [illegible] DKC = 2e. In practice the point C is not found, but the angle BKD equals $4e$. Hence mark a point P one fourth the distance DB from D and move the cross-hair ring until the vertical hair is on P. Then HKP is a right angle—the object of the adjustment.

Question. Why is the horizontal hair adjusted before the vertical hair?

Question In adjusting the vertical hair is the horizontal hair necessarily thrown out of adjustment? If not, why not? (See note, p. 550.)

Practical suggestions. (1) Set up in some comparatively level place where a sight of about 300 ft. each way can be had; closer adjustment can be made at this than at longer distances. (2) Select some well-defined point for a permanent backsight; if necessary set a very slender stake or establish a backsight in some other way. (3) Each foresight should be marked and numbered. The backsight and the position of the transit can often be chosen so that the foresights will fall on some vertical surface, where pencil-marks can be distinctly seen through the telescope, as, for example, marks on a sheet of paper tacked to the side of a building. (4) When the man who marks the foresights has measured back one fourth the apparent error, he should hold his pencil on the point, and *give some well-understood signal* that the point indicated is the one to which the vertical hair should be adjusted, otherwise points may be confused by the man at the transit. If the transitman is sighting at a vertical surface, confusion may be avoided by drawing a pencil-line which can be seen from the transit. As tests are repeated erase all lines previously drawn, so that for each test there is only one line that the transitman can see, i.e., the line to which the vertical hair should be adjusted. An experienced transitman can often make this adjustment without help, by taking for a foresight some such object as a window-pane, estimating the error each time and repeating the test and correction until the vertical hair bisects the object used as a foresight. (5) On some transits there are three sets of capstan-screws. Those nearest the eye adjust the eyepiece, the next set are for the stadia wires, and the *third* set for the cross-hair ring. In this adjustment, therefore, *use the two side screws farthest from the eyepiece.* (6) If, in looking through the telescope, the vertical hair should *apparently* be moved to the *right* in order to cover the point indicated, *loosen* the capstan screw on the *right-hand* side of the telescope and tighten the opposite screw. This really moves the vertical cross-hair to the *left*, since the cross-hair ring is drawn towards the screw that is tightened (see p. 550), but the eyepiece inverts the movement (p. 552) so that it *appears* to be drawn to the *right*. Hence the rule: *Loosen the screw towards which the cross-hair must* APPARENTLY *be moved, and tighten the opposite screw.** (7) Do not be too particular about getting the cross-hair on the point until after two or three tests by double reverse have been made; the nearer the adjustment approaches completion the more nearly should the hair split the point. (8) The final adjustment can often be made by simply tightening one screw without loosening the other, but screws must not be strained (see *Suggestion* (6), p. 582). (9) When the line of sight is in adjustment it should coincide with the line of collimation, and this in turn should be concentric with the geometrical axis of the telescope. If these conditions are not fulfilled errors may be introduced as the objective is moved in or out for sights of different lengths. Perfect adjustment in this respect depends largely on the maker of the instrument. (10) Since the line of sight is determined solely by the position of the intersection of the cross-hairs with respect to the center of the object-glass, it is not affected by the position of the eyepiece. The latter may be moved, in any direction, or even entirely removed and a new one substituted without affecting the adjustment of the cross-hairs.

III. ADJUSTMENT OF THE STANDARDS. *Object.* By the preceding adjustment of the vertical hair the line of sight has been made to revolve in a *plane;* the object of the adjustment of the standards is to make this plane vertical when the plate to which the standards are attached is level, i.e., to make the supporting axis parallel to the plate by making the standards of equal heights.*

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* This is assuming that the eyepiece is inverting, i.e., it inverts the image so that objects appear right side up. (See p. 552..)

Test. (1) Level up, sight to some high and well-defined point, clamp to prevent motion sidewise, depress the telescope without plunging, and mark point number *one.* (2) Unclamp, plunge, turn plates half-way around, and with inverted telescope backsight again at the high point, depress the telescope without plunging, and establish point number *two;* if it does not coincide with point number *one* the standards should be adjusted.

Correction. (1) Mark a point half-way between the first and second point. (2) Sight on this middle point, clamp, and point the telescope upward; the line of sight will not quite strike the original high point. (3) Make it strike the point by raising or lowering one end of the supporting axis. This is done by turning the capstan screw near the top of the standard, *loosening or tightening at the same time* the screws which fasten the cap of the standard. Repeat the test and correction until the two points exactly coincide after reversion.

Principle involved. Since the telescope is plunged (i.e., reversed) but *once,* this is a case of single reversion, and the apparent error is *double* the true error—not four times the error as in the adjustment of the vertical hair, where the telescope was plunged twice. This is evident from Fig. 581 (III).

Illustration. In the first position of the axis, Fig. 581 (III), the *right-hand* end is the higher; after backsighting on A and depressing the telescope the intersection of the cross-hairs falls on B. Turning the plate half-way around brings the higher end of the axis to the *left,* so that the second point falls at C. The point M is *approximately* where the line of sight should strike when the axis is horizontal, i.e., A and M are in the same *vertical* plane.

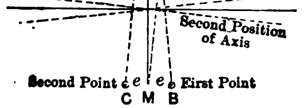

FIG. 581 (III).

Remark. Theoretically the figure does not truly represent the adjustment, and the point M is not half-way between C and B except under certain conditions. Two oblique planes are generated by the line of sight, which are not truly represented by AB and AC. The problem is therefore one in *descriptive geometry.* (See Nugent's Plane Surveying, p. 240.) In practice, however, points B and C are so near together that by taking M half-way between and making the correction $e=CM=NA$ the supporting axis is made *more nearly* horizontal, until after a few trials the adjustment is perfected.

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* More correctly, perpendicular to the vertical axis (see foot-note, p. 585).

Practical suggestions. For a permanent backsight a church spire, a point on some high building or some other well-defined point may be taken. When no such point is visible a comparatively low point 40 or 50 feet above the ground may be chosen, but the transit should be correspondingly near, so that when the telescope is depressed it will describe as large an arc as possible. (2) The usual method of correction is to bring the cross-hairs on *M*. If, however, *M* is nearly on the same level with the telescope, it is better to sight on *M* and then point the telescope upward before making the correction, for the higher the telescope is pointed the more the direction of the line of sight is changed by raising or lowering one end of the supporting axis (*Why?*), and the easier it is to gauge the amount of adjustment. (3) To decide whether the adjustable end of the supporting axis is too high or too low, note in which direction the second plane generated slopes, and remember that the supporting axis is perpendicular to that plane. (See Fig. 581 (III).) (4) If the end of the axis must be raised *loosen first* the screws fastening the cap to the standard before turning the capstan screw.

IV. ADJUSTMENT OF THE TELESCOPE LEVEL. This adjustment is necessary when the transit is used as a level, or when vertical angles with reference to a horizontal plane are to be measured.

Object. To make the axis of the bubble parallel to the line of sight when the latter is horizontal.

Test and correction. Exactly the same as the "peg" adjustment of the level (see p. 597).

V. ADJUSTMENT OF THE VERTICAL CIRCLE AND VERNIER. This adjustment affects the measurement of vertical angles with reference to a horizontal plane. Note that it does not affect the measurement of a vertical angle between *two points*.

Object. To make the vertical circle read zero when the line of sight is horizontal. It is assumed that the transit having been leveled up, the vertical axis is truly vertical because of previous adjustments.

Test. Bring the bubble of the telescope level exactly to the center by clamp- and tangent-screw, and see by inspection if the zero on the vernier and the zero on the vertical circle coincide.

Correction. Shift the *vernier* until the zeroes coincide, by loosening the screws which hold it to the standard. On some transits other provision is made for shifting the vernier or the vertical arc.

Practical suggestion. (1) If the zeroes cannot be made to coincide, note the *amount* and *sign* of the difference. This will be an *index error* to be applied as a correction to each angle of elevation or depression. (2) If there is no level underneath the telescope, make the line of collimation horizontal by the method used in the "peg" adjustment, i.e. set the transit exactly half way between two points, clamp the telescope in as nearly a horizontal position as can be estimated and proceed as directed on page 598.

582. Adjustments of the Wye-level. THE PRINCIPAL LINES to be considered in adjusting a level are:

(1) The *line of sight* from the intersection of the cross-hairs to the optical center of the object-glass.

(2) The *line of collimation,* with which the line of sight should be made to coincide.

(3) The *axis of the collars or rings,* a line joining the centers of the collars.

(4) The *axis of the bubble* (see p. 544).

(5) The *line of the bottom of the wyes,* a line joining the lowest points, where the collars rest on the inner surfaces of the wyes.

(6) The *vertical axis* of the level through the spindle. For sake of clearness another line will be considered, viz.:

(7) The *axis of the level-bar,* a line through the horizontal bar which supports the wyes. This axis is assumed to be *perpendicular to the vertical axis;* it may or may not coincide with the geometrical axis of the level-bar.

THE OBJECT OF THE PRINCIPAL ADJUSTMENTS is to make lines (1), (2), and (3) coincide, and then to bring the first five lines parallel to (7) *and hence perpendicular to* (6). The three principal adjustments may be outlined as follows: Let Fig. 582 represent a level out of adjustment, the errors of course being greatly exaggerated.

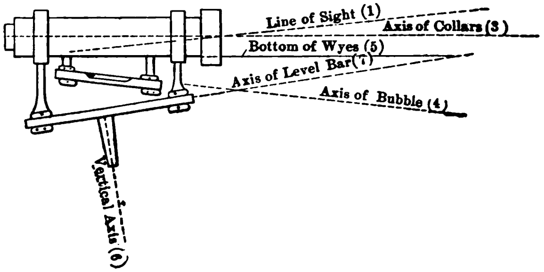

FIG. 582.

I. *The adjustment of the cross-hairs* makes the *line of sight* coincide with the *axis of the collars.* This is done by moving the cross-hair ring, and in a well-made instrument results also in bringing the line of sight into the line of collimation.

II. *The adjustment of the level-tube* brings the *axis of the bubble parallel* to *the* bottom *of the wyes,* whether the collars are of the same size or not. This is done by turning the screws which hold the ends of the tube to the telescope. In a well-made level the collars should be true circles, of the same size and each in perfect contact with the inner surface of the wye; then the *bottom of the wyes*, the *axis of the collars*, the *line of sight*, and *the axis of the bubble* are all parallel as a result of the first two adjustments.

III. The *adjustment of the wyes* brings the *axis of the bubble* parallel to the *axis of the level-bar.* This is done by making the wyes of equal height. All lines should now be *perpendicular* to *the vertical axis*, which is the true object of the three adjustments.

Remark. Before taking up the adjustments in detail the student should thoroughly acquaint himself with the relations which should exist between the principal lines as set forth in this article.

I. Adjustment of the cross-hairs. *Object.* To make the *line of sight* coincide with the *axis of the collars.*

Test. (1) Direct the line of sight to some well-defined point and clamp. (2) Loosen the clips which hold the telescope, rotate the telescope half-way around in the wyes (not end for end) until the bubble-tube is above the telescope. If the line of sight does not strike the point the cross-hairs should be adjusted. In the final test the intersection of the cross-hairs should remain on the point throughout a complete revolution of the telescope.

Correction. If both hairs are off the point, each must be adjusted by bringing it *half-way* back. This is done by moving the cross-hair ring, loosening the upper or lower capstan screw and tightening the opposite screw for the horizontal hair, loosening a side capstan screw, and tightening the opposite screw for the vertical hair (see p. 550). Repeat the test and adjustment until the latter is perfected.

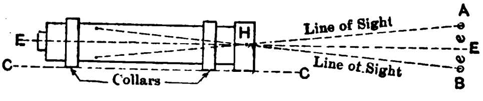

Fig. 582 (I).

The principle involved is that of single reverse. If the line of sight is directed to *A*, Fig. 582 (I), and the telescope is then rotated the line of sight will strike some point at *B*. The angle

AHB is twice the error. In like manner the distance that the vertical hair is to the right or to the left of the point, after reversion, is double the error.

Practical suggestions. (1) It is not necessary that the telescope should be horizontal while making this adjustment so long as the wyes remain in a fixed position. Any two points of support could be used in place of the wyes. (2) A nail in a fence or building about 300 ft. away, or, better yet, a piece of paper tacked up with a fine cross marked on it, will serve for a point to sight at if no other definite point is visible. (3) Loosen the capstan screw *towards* which a cross-hair should be moved (as observed through the ordinary eyepiece), and tighten the opposite screw. (See hint (6), p. 582). (4) The capstan screws farthest from the eyepiece move the cross-hair ring. (*DD* in the photograph p. 551). (5) Adjusting for one hair may disturb the adjustment of the other. (6) Read hints (7), (8), and (10), page 582. (7) It is important in leveling to have the horizontal hair truly horizontal. Test as follows: Level up, close the clips so that the pin will hold the telescope in place (or see that the mark on the collar coincides with the mark on the wye). Mark the point where the line of sight strikes some vertical surface a short distance away. Revolve the telescope on its vertical axis so that the point appears to move from one edge of the field of view to the other; if the point is covered by the cross-hair throughout the movement the hair is horizontal (assuming that the axis of revolution is truly vertical). If the cross-hair is not horizontal loosen slightly all four capstan screws and gently tap their heads to move the cross-hair ring (as directed in the note, p. 586). Repeat the test and correction until the hair is truly horizontal. This will bring the other hair vertical, or nearly enough vertical to plumb the rod by, since the hairs are fastened to the ring at right angles to each other. (8) When the adjustment has been completed for a point 300 ft. away, focus on a point near the instrument, say 20 ft. away, and repeat the test. If the instrument is out of adjustment for this second point, either the object-glass slide does not work parallel to the axis of the collars or the optical center is not in the axis of the collars. On some makes of levels the object-glass slide can be adjusted (see p. 602), but the latter defect can be remedied only by the maker.

II. Adjustment of the bubble-tube. This adjustment consists of *two* steps, neither of which, theoretically, is entirely independent of the other.

First Step.

Object. To bring the *axis of the bubble* into the *same plane* with the *axis of the collars.*

Test. (1) Level up and clamp the telescope over two diagonally opposite leveling-screws. (2) Loosen the clips, rotate the telescope a few degrees in its wyes (not end for end), until the bubble-tube is no longer underneath the telescope. If the bubble does not remain in the center the tube should be adjusted. In the final test the bubble should remain in the center when the tube is swung to either side.

Correction. Bring the bubble *all* the way back to the center *by turning* the capstan screws on each *side* of the level-holder, *ally at the* object-glass end of the tube.

Principle involved. There is no reversion during the test; hence the *apparent* error is equal to the *true* error, not double it. Before adjustment the *axis of the bubble* generates a *hyperperboloid of revolution of one nappe*—the more the adjustment is perfected the more nearly will it generate a cylinder.

Practical suggestions. (1) Since in most of the later instruments the telescope is not free to turn, being held in place by a pin in one of the clips, this adjustment is not important except as it prepares the way for the second step, in which the clips are left open. (2) If, when the telescope is rotated first to one side, then to the other the bubble travels toward the *same* end, it shows that the bubble-tube is conical instead of cylindrical and the adjustment will not correct it. (3) Before beginning the adjustment put on the sunshade. This is not of much importance in this adjustment; but in some of the following adjustments it is, since the maker usually plans to have the telescope in perfect balance when the sunshade is on and the eyepiece and objective each at the average focus. (4) In some instruments it is necessary to loosen the capstan nuts used in the second step before the screw used in this step can be turned.

Remark: Assuming that the axis of the bubble and the axis of the collar are in the same vertical plane, the bubble will still move toward one end of the tube when the telescope is rotated unless the axis of the bubble is parallel to the axis of the collar and to the bottom of the wyes (the object of the second step). Hence it may be necessary to repeat both the first and second step until the two corresponding tests are satisfactory.

Second Step in the Adjustment of the Bubble-tube.

Object. To make the *axis of the bubble* parallel to the *bottom of the wyes.*

Second test. (1) Level up and clamp the telescope over two diagonally opposite leveling-screws. (2) Loosen clips, take the telescope out of its wyes, and replace it *turned end for end.* If the bubble does not return to the center the tube should be adjusted.

Second correction. Bring the bubble half-way back to the center by lowering the end of the tube towards which the bubble has moved, or, what is equivalent, by raising the other end. This is done by turning the capstan *nuts* which hold one end of the tube to the telescope. Level up again, and repeat the test and adjustment until reversion can be made without causing any change in the position of the bubble.

Principle involved. This is a case of single reversion, exactly like that explained on p. 583. Before reversion the *axis of the bubble BB* in Fig. 582 (II) (next page) is horizontal, but *not* parallel to *CC,* the bottom of the collars; or, since the bottom of the collars and the bottom of the wyes are in contact, *BB* is not parallel to the bottom of the wyes, also represented by *CC.* The bottom of the wyes *CC* is kept stationary during the test, and therefore corresponds to *ac*, Fig. 580 (*b*), p. 583. Hence turning the bubble-tube end for end doubles the error.

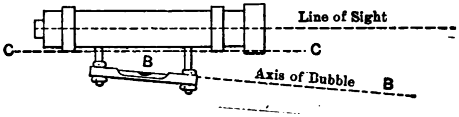

FIG. 582 (II).

Practical suggestions. (1) Great care is necessary in taking the telescope out and replacing it in the wyes to avoid jarring or disturbing the rest of the instrument. (2) Loosen the upper capstan nut first, raise or lower the lower nut as the case requires, and finally bring home the upper nut in completing the adjustment. Usually the nuts are at one end of the bubble-tube only. (3) Notice that this adjustment, like the adjustment of the cross-hairs, could be perfected on any two points of support independent of the wyes. (4) Although by this adjustment the axis of the bubble is made parallel to the bottom of the wyes, it does not follow that it is also parallel to the *line of sight*, even though the latter has been made by the adjustment of the cross-hairs to coincide with the *axis of the collars;* for the collars may not be true circles of equal diameter; if they are not, this method of adjustment fails and the "peg" method must be used. (See p. 599.)

III. ADJUSTMENT OF THE WYES. *Object.* To make the *axis of the bubble* parallel to the *axis of the level-bar,* and thus to make all the *principal lines* of the level *perpendicular* to the *vertical axis.* Practically, the object is to get the two wyes of equal height.

Test. (1) Level up, bring the telescope over two diagonally opposite leveling-screws, and see that the bubble is exactly in the center. (2) Revolve the telescope on its spindle half-way around or until it is over the same pair of leveling-screws, but reversed end for end. If the bubble does not remain in the center the wyes should be adjusted. In the final test the bubble should remain in the center during a complete revolution of the telescope.

Correction. Bring the bubble half-way back by turning the large capstan nuts, i.e., those at the ends of the level-bar, thus changing the heights of the wyes above the axis of the level-bar until they are equal. Level up the instrument again, and repeat the test and the adjustment until the position of the bubble is not changed by revolving the telescope on its spindle.

Caution. On some wye-levels there are one or two small set-screws in the uprights at the end of the level bar; these must be loosened before the capstan nuts can be turned.

Principle involved. This is also a case of single reversion, exactly like that explained on p. 583, also exactly similar to the first adjustment of the transit. The bubble-tube is attached to the level-bar by the supporting wyes. Before reversion the axis of the bubble is horizontal, but since the inclination of the level bar *HH* does not change during revolution it corresponds to *ac*, p. 583, and reversion doubles the error.

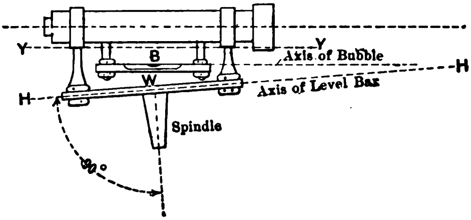

FIG. 582 (III).

Practical suggestions. (1) When the first and second adjustments of the level are "out," it is not apparent unless tests are made; but when the third adjustment is "out" it is known at once, since the bubble will not remain in the center. For this reason undue importance is too often attached to this last adjustment. In reality it does not affect the accuracy of the work, provided the bubble is brought to the center of its tube just before each sight is taken. The adjustment is made simply as a matter of convenience to save this continual leveling up. A good levelman will not depend upon it, but will watch the bubble instinctively and always bring it to the center before taking any important sight. (2) It is well to make the first test with the telescope over one pair of leveling-screws, the next test over the other pair, alternating until the adjustment is nearly perfected, when it may be completed over a single pair of screws. (3) Loosen the upper nut, then raise or lower the lower nut as the case requires. Though nuts are provided at each end of the bar, unless the adjustment is badly out, the adjustment can usually be made wholly by those at one end. (4) When unable to make the adjustment correctly, examine the outside socket to see that it is securely set in the main socket, and also notice that the clamp does not bear on the ring which it encircles. When these are correct and the error is still apparent it is probably due to some imperfection of the interior spindle. (5) Having completed all three adjustments of the level they should be verified by the "peg-method" test. (See *Remark*, below.)

583. The "Peg Adjustment." *Object.* To make the line of sight and the axis of the bubble parallel.

Remark. This adjustment is also called the *direct* adjustment in contrast to the *indirect* adjustment explained in the preceding article. It is used mostly for the adjustment of the dumpy level and for the level on a transit telescope. The chief advantage of the wye-level lies in the ease with which it may be adjusted by the indirect method; yet if certain defects exist—as, for example, inequality of the rings—they are not disclosed by the indirect test. Hence, after the wye-level has been adjusted by the methods of the preceding article, it should be tested by the peg method explained below. (See also suggestion 7 on page 599.)

There are two methods of making the peg adjustment.

FIRST METHOD.

Test. (1) Drive two stakes *A* and *B* (Fig. 583) from 300 to 400 ft. apart, and set up at *C* a third point *exactly* half-way between the other two. (2) Take rod-readings R_1 and R_2 on stakes at *A* and *B*. The difference in the readings will be the *true* difference in elevations d_1 of *A* and *B*, *no matter how much the instrument may be out of adjustment.* Set up again near one of the points, say at *F*, and again take rod-readings on *A* and *B*. If the difference in elevation of *A* and *B* as found from the last two readings agrees with d_1 (i.e., R_2-R_1), the level is in adjustment.

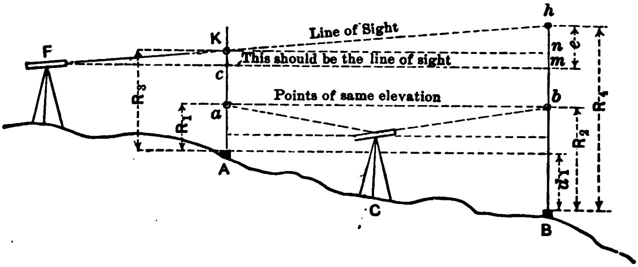

FIG. 583.

Error. If the level is out of adjustment, let R_3 and R_4 be the rod-readings taken when the instrument is at *F*. The line of sight should strike *m*, hence the error is $hm=e$. The error for a distance between stakes *A* and *B* (i.e., distance *Kn*) is *hn*, and from similar triangles $e=\frac{Fm}{cm}(d_2-d_1)$ where $d_2=R_4-R_3$ and $d_1=R_2-R_1$.

Correction. There are two methods of correcting the error—(1) by adjusting the *cross-hairs* and (2) by adjusting the *bubble-tube.*

To correct by adjusting the cross-hairs. (1) Compute *e* from the above formula and set the target at (R_4-e), thus bringing it to *m*. Clamp the instrument (telescope still pointing at the last sight *h*) and make the horizontal hair bisect the target by turning the lower and upper capstan screws that move the *cross-hair ring.* (See *Note*, p. 550.) The *line of sight* should now be in the line Fm. Since the bubble was in the center of its tube when the reading was taken on h, and since the telescope has not been moved, it should still be in the center, and its axis should now be parallel to the line of sight.

To correct by adjusting the bubble-tube. (1) Compute e and set the target at (R_4-e). (2) By means of the leveling-screws incline the telescope until the line of sight strikes m; this throws the bubble out, but makes the line of sight truly *horizontal.* (3) Raise or lower one end of the bubble-tube by turning the capstan nuts that hold it to the telescope (see *Suggestion* (2), p. 596) until the bubble returns to the center.

Practical suggestions:

1. In Fig. 583 the line of sight to h is inclined *upward*; when it is inclined *downward* h falls below m, and e must be *added* to R instead of subtracted.
2. The points ACB and F should be approximately in a straight line.
3. It is well to take AB and FA, so that $\frac{Fm}{cm}$ is some even fraction. For example, if $AB=400$ ft. and $FA=10$ then $\frac{Fm}{cm}=\frac{41}{40}$.
4. When the level is at C, several sets of readings should be taken on A and B to determine accurately the difference in elevation d_1.
5. If the vertical axis is truly vertical; but the line of sight is inclined, the latter generates a cone; the base of which, represented by ab, is horizontal; hence all points in the base, including a and b, have the same elevation.
6. After adjusting for the target at m set it for the correct reading at c. This not only serves as a check, but also detects any error in the movement of the object-glass slide, since the latter must be run far out in focusing on F so close to A, i.e. the line of motion must be *straight* and *parallel* to the axis of the collars.
7. The three adjustments of a wye-level having been made in the usual way, if the level is then tested by the peg method it follows that: (1) There is no error in the movement of the object-glass. (2) The collars are of the same size. (3) That the line of sight is parallel to the axis of the rings (4) That the axis of the bubble is parallel to the bottom of the wyes.
8. If the object-glass slide is not straight the engineer cannot remedy it. If the line of motion is not parallel, on some instruments the object-glass slide can be adjusted (see p. 602). If the other sources of error suggested in (7) exist, the level may be adjusted by the peg method and used as a dumpy level.
9. In the wye-level and in the transit, since the cross-hairs have been adjusted to meet certain conditions, it is better to leave them as they are and to adjust the bubble-tube; with the dumpy level it may be otherwise.

PEG ADJUSTMENT. SECOND METHOD.

In this method the level is not set up half-way between the pegs, as in the preceding method, but it is set up first very near one stake and then very near the other. Two sets of readings are thus obtained, and the true difference in elevation is obtained by taking the average of the two *differences*, as in reciprocal leveling. (See p. 243.)

Test. (1) Set up near one stake so that when the rod is held on the stake the eye-end of the telescope will swing within an inch or two of the face of the rod. (2) Read the rod on this stake with the eye at the object-glass, thus looking through the telescope "*wrong end to.*" (3) Take a reading on the second stake in the usual manner. The difference between the first two readings may be called d_1. (4) Set up in a similar position near the second stake, repeat the process, and obtain a second difference d_2. If d_1 and d_2 are not equal the *line of sight* and the *axis of the bubble* are not parallel.

Correction. The error is $\frac{1}{2}(d_2-d_1)$. Apply this to the last rod-reading, set the target, and make the adjustment by one of the two methods explained on page 598, according to whether it is desired to adjust the *cross-hairs* or the *bubble-tube.*

Principle involved. If R_1 and R_2 are the first two rod-readings, R_3 and R_4 the last two, and e is the error, then the true difference in elevation is $R_1-(R_2-e)$, and also $(R_3-e)-R_4$. But $R_1-(R_2-e)=(R_3-e)-R_4$ hence $e=\dfrac{(R_3-R_4)-(R_2-R_1)}{2}$. Note that R_1 and R_4 are on the first stake and R_3 and R_2 are on the second stake.

Remark. The student should draw a figure illustrating the above principle. The first method of "peg" adjustment is generally preferred to that given here.

PEG ADJUSTMENT. MODIFIED METHOD.

When a strip of nearly level ground can be found the following modification of the peg adjustment can be used to advantage. (1) Set up exactly half-way between two pegs, 300 or 400 ft. apart as in the first method, but drive the higher peg down until the two tops are on the same level, i.e., R_1 equals R_2. (2) Set up the instrument nearly over one peg hold a leveling-rod on the peg and set the target at the height of the cross-hairs as in the second method. (3) Without disturbing the rod-reading, point the telescope at the target as the rod is held on the other peg, and adjust the level tube until the bubble stands in the center. This method avoids all computation since the line of sight has been made parallel to the top of the pegs and is therefore horizontal.

584. Adjustments of the Dumpy-level. PRINCIPAL LINES. Unlike the wye-level, the telescope of the dumpy level cannot be taken from its wyes, but instead it is rigidly attached to two uprights or standards. Hence two lines of the wye-level are eliminated, i.e., the axis of the collars and the bottom of the wyes. The other lines remain the same as in the wye-level. After making the horizontal hair truly horizontal (see *Note* on page 586), and testing the object-glass slide (see p. 602) there are only two adjustments to be made.

I. ADJUSTMENT OF THE CROSS-HAIRS AND THE BUBBLE-TUBE. *Object.* To make the axis of the bubble and the line of sight parallel to each other. *Test and adjustment.* Exactly the same as the "peg adjustment" explained in the preceding article.

Practical suggestions. (1) In some dumpy-levels the bubble-tube cannot be adjusted, hence the engineer has no choice, but must adjust the cross-hairs. In levels where both the bubble-tube and the cross-hairs are adjustable it is better to adjust the bubble-tube (second method of correction, p. 599), since the cross-hairs once placed in the optical axis of the telescope are not disturbed by this adjustment. Many engineers prefer a dumpy level in which the bubble-tube is fixed (non-adjustable), since it can then be placed by the maker once for all parallel to the optical axis, and hence when the line of sight is adjusted it will coincide with or be parallel to that axis.

II. ADJUSTMENT OF THE STANDARDS. *Object.* To make the line of sight and the axis of the bubble parallel to the axis of the level-bar, and hence perpendicular to the vertical axis. *Test and adjustment.* Exactly the same as for the third adjustment of the wye-level (see p. 596), except that in most instruments the screws for raising or lowering the standards or uprights are on the underside of the level-bar, and are to be turned with a screw-driver.

Remark. On some dumpy-levels no provision is made for raising or lowering the standards, in which case the order of adjustments is changed to that given below.

First adjustment. To make the axis of the bubble perpendicular to the vertical axis. Test and correction: Revolve the instrument 180° about its vertical axis, as in the third adjustment of the wye-level, but correct half the error by turning the capstan nuts at the end of the level-tube, and the rest by the thumb-screws of the leveling-head.

Second adjustment. To make the line of sight parallel to the axis of the bubble use the peg method as described for the first adjustment above, but correct the error by moving the cross-hairs instead of the level-tube. (See first method of correction, page 598.)

585. Centering the Eyepiece: An instrument may be in good adjustment and still have the intersection of the cross-hairs appear far out of the center of the field of view. This does not affect the accuracy of the work, although it does affect the seeing power of the telescope. The inverting eyepiece usually has its inner end held in place by a ring which can be moved like the cross-hair ring by means of four screws, until the intersection of the cross-hairs appears in the center of the field.

Practical suggestions. (1) In some instruments the screws have capstan-heads like those that move the cross-hair ring, but they are nearer the eye-end of the telescope. (See *E′E′* in photograph, p. 551.) In other instruments the screws are concealed under a collar, which may be easily unscrewed and slipped along the telescope. The screws can then be turned with a screw-driver. (2) Centering the eyepiece does not affect any of the other adjustments, but it is best to wait until they are completed before the centering is done. It is not often that the adjustment of the eyepiece is necessary; there is no provision for it on non-erecting eyepieces.

586. Adjustment of the Object-glass Slide: .When an instrument leaves the maker's hands the object-glass slide should be perfectly straight, and its line of motion should be parallel to the axis of the telescope. No further adjustment of the object-glass slide should be necessary, at least for a long time. In many instruments no provision is made for this adjustment. In other instruments the rear end of the slide works through a ring which can be adjusted by four screws in the same way that the ring which holds the eye piece is moved (see *CC* in photograph, p. 551). It is necessary first to remove the bubble-tube from the telescope in order to get at the screw directly above the tube. The ring is moved until the test of focusing on a far point and a near point is satisfactory. (See pp. 586, 594.)

587. Adjustments of the Compass. The adjustments of the compass are four in number; they affect the following parts: I. Levels. II. Sights. III. Needle. IV. Pivot-point.

I. ADJUSTMENT OF THE LEVELS. *Object.* To make the axes of the level-bubbles perpendicular to the vertical axis.

Test and correction. Exactly the same as for the similar adjustment of the transit (p. 584), except that after bringing a bubble half-way back it is brought the rest of the way by turning the instrument on the ball-and-socket joint. On some compasses a leveling-head like that on transits is furnished; then the adjustment is exactly the same.

II. ADJUSTMENT OF THE SIGHTS. *Object. To bring the sights into a vertical plane.*

Test. Level up and sight at a suspended plumb-line, first through one of the sight vanes and then through the other, turning the instrument so that each sight vane in turn is nearer the plumb-line than the other. If the slit in either is not vertical the bottom of the vane must be refitted to the plate.

Correction. Either file off one side of the bottom of the sight vane or insert a piece of paper under the other side.

Remark. Ordinarily this adjustment should not be necessary. It is not essential that the plane of the sights should be exactly in the plane of the vertical axis.

III. Adjustment of the needle. *Object.* To straighten the needle so that its two ends shall lie in the same vertical plane with its center.

Test. Let the needle down on its pivot and remove the glass top; with a small splinter of wood bring the south end of the needle exactly opposite some prominent graduation of the circle, as the zero- or ninety-degree mark, and read the north end. Turn the needle on its pivot end for end, bring the north end to the graduation-mark on the circle, and read the south end. If the two readings do not agree the needle is not straight.

Correction. Remove the needle from its pivot and straighten it by bending it in the direction that will move the south end toward the original reading of the north end. The correction should be one half the difference of the two readings; but as this must be estimated, the needle should be replaced on the pivot and the test and correction repeated until the needle is straight.

Suggestions. (1) Instead of turning the needle on its pivot, the whole instrument may be turned if preferred to bring each end of the needle in turn to the same graduation-mark. (2) In addition to the above adjustment it may be necessary to bend the ends of the needle slightly up or down, so that they shall lie in the same plane as the horizontal circle—or, better still, a trifle above. If, however, this brings the ends above or below the plane of the pivot-point the ends of the needle are likely to be unsteady. Continued resharpening of a pivot may bring its point much too low, when the only remedy is a new center-pin.

IV. Adjustment of the Pivot-point. *Object.* To bring the pivot-point exactly in the center of the graduated circle.

Test. Read both ends of the needle in various positions. If for every position of the needle the readings for the two ends agree, the pivot is in the center; otherwise it must be centered.

Correction. Find roughly the position of the needle where the difference in the two readings is the greatest. Remove the needle and bend the pivot at right angles to this direction. Replace the needle and see if the readings of the two ends agree. Repeat the process until the readings agree for any position of the needle.

Suggestions. (1) To bend the pivot apply the little brass wrench, or the flat pliers usually furnished with the instrument, about one eighth of an inch below the top of the pivot. (2) This adjustment should be made after the needle has been straightened in III.

588. **The Adjustments of the Plane-table** are practically the same as some of those already given for the transit and the level; hence it is unnecessary to treat each one in detail. They may be summarized as follows:

I. THE LEVELS. This adjustment is similar to the second step in the second adjustment of the level (p. 595). The levels are placed near the center of the board and reversed, for the test, by taking them off and turning them end for end without moving the board. The bubble is brought half-way back by adjustment, and the rest of the way by leveling the board. The method is the same whether the levels are attached to the ruler of the alidade or are separate.

II. THE UPPER SURFACE OF THE BOARD should be perpendicular to the axis. Place the alidade on the board, bring the bubble under the telescope to the center. turn the table through 180° without disturbing the alidade, and correct half the error by inserting washers between the board and the arms that hold it to the tripod-head. Turn the alidade on the board at right angles to its first position and repeat the test.

III. THE TELESCOPE. The line of sight should be perpendicular to the supporting axis, and the supporting axis should be parallel to the top of the table. The two adjustments corresponding to these two requirements are similar to the second and third adjustments of the transit. (See pp. 586-591.)

Remark. It is desirable though not essential to have the line of sight either in the plane of the edge of the ruler or parallel to it. This may be tested by establishing two points 20 ft. or so away, one in the line of sight of the telescope, the other in the line found by sighting along the edge of the ruler. If these lines of sight do not lie in the same vertical plane or in parallel planes, the telescope may be turned slightly by loosening the screws which attach the standard to the ruler. No error will be introduced if the above requirements are not fulfilled, provided the vertical planes through the lines of sight remain at a fixed angle with each other.

IV. THE TELESCOPE LEVEL. Same as Adjustment IV of the transit. (See p. 591.)

V. THE VERTICAL CIRCLE. Same as Adjustment V of the transit. (See p. 591.)

589. Adjustments of the Sextant. I. ADJUSTMENT OF THE INDEX-GLASS. *Object.* To make the index-glass perpendicular to the plane of the graduated arc.

Test. Hold the sextant with the arc away from the body and move the vernier to a point between the 20° and 40° marks. Hold the eye just above the plane of the arc and slightly to the right of the index-glass. Look obliquely into the right-hand side of the glass in such a way as to see a portion of the arc both directly and by reflection at the same time. If the direct and reflected portions do not appear to form a continuous curve *the index-glass* is not perpendicular to the plane of the arc.

Correction. Change the inclination of the glass by means of the screws at its base, or by whatever means have been provided on that particular instrument, until the above test is fulfilled. It may be necessary to insert thin paper under the base. If the reflected image is below the arc, the glass is leaning backward; if above, forward.

II. ADJUSTMENT OF THE HORIZON-GLASS. *Object.* To make the horizon-glass perpendicular to the plane of the graduated arc. This must be done after the index-glass has been adjusted.

Test. Point the telescope to any well-defined object and move the index-arm slowly back and forth over the zero of the graduated arc. The reflected image will be seen moving back and forth; if it passes exactly over the direct image the two mirrors are perpendicular to the plane of the arc, otherwise the horizon-glass should be adjusted.

Correction. Turn the screws on the back of the horizon-glass until the above test is fulfilled.

Suggestions. A star is the best object to sight at, although the sun will also answer the purpose. In case the latter is used it is well to make the two images of different colors by use of the glass-shades. A clear-cut line of some terrestrial object may be used—as, for example, a vertical edge of a tall chimney, half a mile or more away. In that case the vertical line should appear straight and unbroken.

III. ADJUSTMENT OF TELESCOPE. *Object.* To make the line of sight parallel to the plane of the graduated arc.

Test. Place the sextant face upward on a table or some other firm and level support, point the telescope at some vertical surface about 20 ft. away. Place two objects of equal height on the extreme ends of the limb to serve as temporary sights. Sight across the tops of these objects and mark a point where the line of sight strikes the vertical surface. Near this point mark another point where the line of sight through the telescope strikes the same surface. The difference in elevation between these two points should equal the difference in elevation between the axis of the telescope and the line across the top of the two temporary sights, measured at the sextant; otherwise the telescope must be adjusted.

Correction. Tip the ring or collar that carries the telescope by means of the proper screws until the above test is fulfilled.

Suggestions. If the temporary sights are made of the same height as the telescope axis the two points on the vertical surface should coincide. With some instruments two peep-sights are furnished for this purpose. A discrepancy of ½ inch in 20 feet would cause a maximum error of only 1 second of arc.

IV. Index error. *Object.* To determine the reading of the arc when the direct and reflected image of an object coincide.

Test. Bring the direct and reflected image of a distant point into coincidence. If the vernier does not read zero the error is called the *index error.*

Correction. The error may be corrected by turning the horizon-glass around an axis perpendicular to the plane of the graduated arc. As this adjustment will not remain perfect it is better to determine the index error from time to time without attempting to adjust the glass, noting whether it should be added or subtracted from all angle readings, and then apply it as a correction for each reading when measuring angles.

Suggestions. If the vernier zero is to the left of the zero of the graduated arc the index error is *minus*—i.e., it must be subtracted from all readings; if to the right it is *plus*. Since this index error must be applied as a correction to every angle reading it should be determined with great care. For terrestrial measurements it will suffice to sight at some distant line. as the edge of a chimney. (See suggestion, second adjustment.) For astronomical observations the error should be determined with greater accuracy. For example, the direct and reflected images of the sun may be brought externally tangent to each other in each of the two possible positions. Call the reading to the right *minus*, the one to the left *plus*, and algebraically *divide by two;* the result is the index error with the proper sign. For greater accuracy, repeat the process at least two or three times and take the mean of all the results. In taking readings to the right of the zero, be careful to reverse mentally the numbering of the graduations on the vernier.

CHAPTER XLVII

THE CARE OF INSTRUMENTS.*

In the care of instruments it is more important to know how to avoid unnecessary wear and tear than it is to know how to make repairs, though the surveyor should be able to do both. The most important precautions to be taken in the care of instruments are suggested in this chapter. In addition, directions are given for cleaning instruments, for repairing broken tapes, for replacing broken cross-hairs, for stopping the fretting of slides and centers, and for making such other minor repairs as do not require the services of a skilled instrument-maker.

590. Precautions. By taking proper precautions the wear and tear on instruments may be greatly reduced, and frequently permanent injury to some part of an instrument may be avoided. *"It must be remembered that there are many parts of an instrument which, if once impaired, cannot be restored to their original efficiency."*

A FEW DON'TS.

1. In taking an instrument out of its box do not lift it by the standards or supporting axis; place the hands under the plates or the leveling-base. There is no excuse for certain makers' persisting in furnishing a box so constructed that it is impossible to lift the instrument out properly.
2. Do not leave an instrument unguarded in a street, road, or pasture, or in the close vicinity of blasting.
3. Do not expose an instrument needlessly to dust or dampness. Carry a waterproof hood for transits and levels.
4. Do not let the compass-needle swing needlessly; always lift it from its pivot before starting to carry the instrument.

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* In this chapter the author has quoted frequently from catalogs of instrument-makers, as it is to be presumed that such men are well qualified to give directions for the care of instruments. Extracts reproduced word for word are placed in quotation-marks; but as it is impracticable to credit each quotation, this method is taken of acknowledging the author's indebtedness, especially to the firms of *C. L. Berger & Sons* and *Young & Sons*.

Every precaution should be taken to guard the pivot and keep it sharp.

5. Do not *force* screws or other movable parts of an instrument.

6. Do not continue to use slides or centers which are fretting. If an instrument has met with a fall, bending the centers, it should not be revolved until repaired, lest still further injury be done.

7. Do not allow oil or grease to remain on exposed parts, for it will gather dust.

8. Do not rub lenses with silk or muslin. Avoid rubbing them altogether; use a brush for removing dust.

9. Do not set an instrument on a smooth floor without proper precautions; the tripod-legs are apt to open out and to let the instrument fall.

10. Do not carry an instrument on the shoulder through a doorway or in any place where it is likely to get bumped.

11. Do not take instruments apart unnecessarily. "Even when fittings are perfect it often requires care and experience to place them together properly. A screw left loose or one tightened too hard may interfere with the correct working of the instrument."

12. Do not let a "high" level rod down on the run, or lean the rod against a tree, or leave it in any other position where it is likely to fall.

13. Do not pull on a steel tape when there is a kink in it, or jerk it needlessly, or step on it when it is on soft ground, or allow vehicles to run over it, or bend it into sharp corners. If a tape gets wet do not put it away without wiping it dry.

14. In using a plane-table do not start to carry it until the little compass-box and other accessories have been removed.

A FEW PRECAUTIONS.

1. In carrying an instrument set all clamps lightly to prevent needless wear, yet loose enough so that if the parts are bumped they will yield.

2. In carrying the transit or the level a long distance, it is better to put it in its box; but be careful that there is no looseness in the resting-blocks or in any of the parts of the instrument itself.

3. In packing an instrument for shipment it is best to fill in the hollow spaces with paper or excelsior, and to put the instrument-box in another box about an inch larger all around.

4. When a telescope is not in use keep the cap over the lens.

5. Keep the hands off the vertical circle and other exposed graduations to avoid tarnishing.

6. In case of rain when there is no waterproof bag at hand, turn the telescope object-end up with the cap over the lens to keep the water out.

7. After an instrument has been exposed to moisture, wipe it dry before putting it away.

8. "To protect an instrument from the effects of salt water, when used near the sea-coast, a fine film of watch-oil rubbed over the exposed parts will often prevent the appearance of oxide. To remove such oxide-spots as well as possible, apply some watch-oil and allow it to remain for a few hours, then rub dry with a soft piece of linen. To preserve the outer appearance of an instrument, never use anything for dusting except a fine camel's-hair brush. To remove water and dust spots, first use the camel's-hair brush, and then rub off with fine watch-oil and wipe dry; to let the oil remain would tend to accumulate dust on the instrument."

591. Steel Tapes. *Care.* Wipe tapes clean and dry after using. To remove rust, rub with oil and fine sand or emery. On the better class of tapes use a little mineral oil instead, as sand or emery is likely to efface the etchings. Some simple device for holding the tape is useful when one person does the scouring. (See *Engineering News*, March 9, 1899 and February 22, 1900.)

(*a*) RIVETING A BROKEN TAPE. One of the best devices on the market for mending broken steel tapes is an eyelet tool. This consists of a punch for cutting holes in the tape and a set which clinches or sets the eyelets used as rivets. A piece of scrap-tape an inch or two in length is needed to serve as a splice; if possible this should be of the same width as the broken tape, and with the same graduations. Lay one end of the tape on the splice, taking care to match the graduations if the tape is fully graduated. Clamping the two pieces together, punch a hole about half an inch back from the broken end and *set the eyelet or rivet as soon as the hole is punched*. Usually one additional rivet will be needed. If the tape is wide the two rivets may be set crosswise; if narrow they may be set tandem. Make the other broken end of the tape butt against the end already fastened to the splice and rivet it in the same manner. After riveting, cut away a portion of the original tape at the splice by filing a notch crosswise close to the rivets on each side of the joint; slip a knife under and break off the ends at the file-marks. Unless this is done the tape is apt to break again at the splice. In a similar manner the ends of the splice-piece may be cut off close to the last rivets. The splice should not be less than three fourths of an inch long. In mending a heavy tape the above method must be modified by punching one thickness at a time. Punch one piece, and then clamping it to the other punch the second piece through the holes already made in the first. For thick tapes solid copper rivets fastened with a hammer may be used in place of eyelets.

(*b*) SOLDERING A BROKEN TAPE. Another good method of mending a tape is to wrap a small strip of copper around the broken joint and to fasten it with soft solder. The outfit required consists of a small soldering-iron or a blowpipe, some flux and solder, small flat-nosed pliers, a small flat file and a piece of emery cloth.

Clean the broken ends with the emery cloth until the surfaces are bright, moisten with flux, melt a small piece of solder, spread it on both sides of the tape, and wipe the solder with a piece of cloth while the tape is still hot. Both of the broken ends having been tinned in this way, clean one side of the copper strip with the emery cloth and give it a similar coating. Bring the broken ends in perfect contact, put on the copper sleeve, slowly heat, and when the solder is hot enough to run clamp the joint with the pliers for a moment. When cool trim the copper sleeve, filing off any solder that may remain on the outside, and polish the joint with the emery cloth.

Suggestions. Flux may be made by putting zinc into muriatic acid, and when the bubbles cease to rise, adding all the sal-ammoniac that the acid will dissolve. Stick-flux, or prepared flux, such as electricians use, is more convenient to carry in the field. Resin is also used. (2) If the work has to be done in the field, as is often the case, the method may be modified as follows: Cut a strip of tin about ½ inch to ⅝ inch wide (a piece from an old tin can will answer the purpose), and of such a length that it will reach around the tape to be mended, and just about meet on the flat side—not the edge. Using one end of the tape as a "form," bend the strip into a sleeve, insert the broken ends of the tape until by measuring between graduation-marks they are known to be in perfect contact, clamp the sleeve tightly about the tape with the pliers, heat the joint, and put sufficient flux on to run under the sleeve. Hold the solder on until it has melted and run under and about the sleeve, then allow it to cool without movement of the ends of the tape. A number of sleeves may be made in advance; and these, together with solder, stick-flux, a candle for heating the tape, a small flat file, and a pair of pliers, form an outfit which can be carried in the pocket. Tapes can be mended in this manner in a few minutes and will stand all a man can pull. (3) Sleeves should not be more than, say, [illegible] inch wide. If the break is a long one use a piece of old tape and two sleeves. Too long a sleeve interferes with the pliability of the tape.

(*c*) EMERGENCY REPAIRS. There are several methods of mending a broken tape in the field when no outfit for riveting or soldering is at hand.

The stick-splice. A chain tape may be spliced temporarily as follows: Secure a round stick about 4 in. long and ½ in. in diameter, and split it lengthwise, making two half-round pieces. Across the flat side of one of these pieces and equidistant from the two ends cut two notches about ¼ in. apart, ⅛ in. deep, and at right angles to the axis of the stick. Bend ⅛ in. of each of the broken ends of the tape at right angles to fit the notches in the stick, taking care that when these ends are inserted in the notches the graduations shall be the proper distance apart. Complete the splice by placing the other half-round stick with its flat side against the tape to hold the ends in place, and binding the two halves together with stout string so that they are almost as they were before the stick was split. Trim each end of the stick to slope like a sharpened pencil, so that it will not catch in the brush. Such a splice, made in a few minutes, will frequently last several days, or until more permanent repairs can be made.

Rough riveting. Tapes may be roughly riveted as follows: Drive a darning-needle through a thick piece of wood, allowing each end to project. The needle, supported by the wood throughout most of its length, may then be used as a punch. The broken ends of the tape may be riveted to the splice-piece with large pins cut off and hammered down, or they may be laced together with fine wire; the lacing should be continued until the holes are filled with the wire, and then it should be smoothed and pressed down with pliers or with two blocks of wood. Tapping the wire with a hammer is likely to break the tape.

592. Lubricants. Watch-oil, vaseline, and rendered marrow are all used for lubricating different parts of surveying instruments. For most purposes watch-oil is the best. (See *Care of Centers*, p. 618, for the advantages and disadvantages of different lubricants.) The tendency is to use lubricants too freely. As a rule any part which has been oiled should be wiped off clean; enough oil will remain after an effort has been made to remove it. No part exposed to the dust should be greased. Before applying oil make sure that all dust or foreign matter has been removed from the part to be oiled.

593. To Clean Screws and Screw-holes. Screws may be washed in soap and water applied with a stiff tooth-brush, then oiled and wiped clean. The inside of nuts and other threaded holes may be cleaned by screwing through the hole a piece of soft wood with two sides thinned down, applying a little oil and removing it in the same manner. Unless the oil or grease is almost wholly removed the screw-threads are likely to be injured by dust adhering to the screw.

594. Verniers and Graduated Arcs. The first rule in cleaning graduated arcs is to avoid rubbing the edge of the graduations. Every precaution should be taken to protect the edges, for if they become worn or injured the value of the entire instrument is greatly lessened. "To remove dirt and oxide that may have accumulated on the surface of a solid silver graduation, apply some fine watch-oil and allow it to remain for a few hours; take a soft piece of linen and rub lightly until dry, but without touching the edge of the graduation. If after cleaning the solid silver surface should show alternately brighter spots, which would interfere somewhat with the accurate reading of the graduation, barely moisten the finger with vaseline and apply the same to the surface; then wipe the finger dry and lightly rub it once or twice around the graduation. Avoid touching the edges as much as possible. Such cleaning, however, must be resorted to only when absolutely necessary, and then only with the greatest care, as it is apt to reduce the minuteness of the graduation and to spoil its fine appearance. If after such cleaning dirt and grease have accumulated on the inner edge of the graduation and verniers, gently wipe clean before restoring the vernier-plate to place." Before applying oil or vaseline to graduations brush all dust from the surface with a camel's-hair brush. Scouring the surface with any form of silver polish may seriously injure the graduations.

595. Compass-needles. *To balance the needle.* If the needle is only a trifle out do not meddle with it, as there is danger of doing more harm than good; but if the compass-needle requires to be balanced, proceed as follows: Remove the glass cover by unscrewing the ring which holds it, or in some instruments, by gently prying off the ring with a knife where the two ends come together. Raise the needle off its pivot by the lifter and remove it with a pair of tweezers. Shift the weight-coil slightly and replace the needle on the lifter, then use the latter to lower it to place. If the needle is removed directly from the pivot or replaced without using the lifter, the delicate pivot-point is apt to be injured. It is not always necessary to remove the compass-glass entirely. Frequently, by means of beeswax slightly softened, the glass may be lifted (after the ring has been removed) just enough to take out the needle.

To straighten the needle (See page 603.)

To remagnetize the needle. This should not often be necessary. It may be done as explained on page 295.

596. Level Bubbles and Tubes. It frequently happens that the glass tube of a spirit-level becomes loose in its brass tube, or is broken and has to be replaced. Extra tubes are often provided with the instrument. "Upon finding that the adjustment of a spirit-level in an even temperature is not as stable as desirable, the level fastenings, tube, screws, etc., should be examined, to see if any of them are loose. If the trouble is in the screws, tighten them, but if the spirit-level can be shifted in its tube by a touch of the finger, take it apart; soften the plaster of Paris in water, and remove it with a sharp-pointed stick of wood. Cautiously move the spirit-level with your finger, at first only a trifle to and fro, increasing the length of stroke little by little, until it can be safely taken out without breaking;—clean thoroughly. Cut pieces of white paper, of the width of the radius of the tube, and somewhat shorter than the length of the spirit-level, but longer than the opening in the brass tube, and insert these in sufficient quantity at the bottom of the brass tube, to fill up the space intervening between the glass and the brass tube. The uppermost layer of paper should, however, be so wide, as to envelop the spirit-level up to the opening in the brass tube. Now insert the spirit-level, taking care not to touch the glass ends that are sealed, and place the division or other marks, indicating where the level has been ground to a true curvature, uppermost in the brass tube. The level must be pushed in with sufficient friction to prevent slipping in the tube, yet not so tight as to cause a crack at a subsequent low temperature, as brass will contract more than glass. No part of the spirit-level should touch any part of the metal tube. Now prepare some plaster of Paris with water, of the consistency of paste, and pour in at each end enough to fill up the space between the end-pieces and the glass, stirring it sufficiently to make a perfect contact by it and the glass and the brass, but leaving the spirit-level ends exposed. Now put the level together and adjust as described elsewhere."

597. Telescope. (*a*) CARE OF LENSES. "The lenses of a telescope should not be cleaned too often. Too frequent wipings will scratch the glass and injure the polish, which is more injurious than a little speck of dirt. When it becomes necessary to clean the glass, take a soft, dry piece of chamois-skin or an old piece of linen which by repeated washing has become soft. If the glass is very dirty use a little alcohol, but be careful not to touch the mounting of the lens, as the alcohol dissolves the lacquer. *Dirt on the eyepiece especially on the field-lens,* is far more objectionable than on the objective; hence the former requires more frequent cleaning. Small scratches and bubbles in the objective have no seriously injurious effect, as they only take up a very small portion of light."* Sometimes a film settles between the crown and flint glasses of the objective. If this film is simply moisture it can be evaporated usually by placing the telescope in the sun or in a warm room some distance from a stove. Sometimes it is advisable to remove the eyepiece and to let the telescope stand with a cloth over the open end, in order that the moisture on the inside may evaporate more rapidly. If, however, the haziness is caused by a network of small cracks in the cement between the two glasses, or if it cannot be removed by evaporation, the telescope should be sent to the maker or to an expert. Do not unscrew the lens of the objective if it can be avoided. Small specks of dust or lint should be brushed from the glass with a camel's-hair brush. The glass next to the eye, especially in high powers, frequently becomes dimmed from moisture of the eyelash. It should seldom be necessary to wipe the inside of glasses that are protected; but if the objects seem dim and the outside surfaces of the lenses seem clean, remove the eye piece and look through it. If it is not clear, examine the inner end; very likely the surface of the lens most remote from the eye needs cleaning. While the eyepiece is out look through the telescope to see if the object-glass is clear.

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*From the catalogue of the Bausch, Lomb, Saegmuller Co.

(*b*) CARE OF THE FOCUSING SLIDES. Both the eyepiece and the objective should move freely for focusing. If dust or grit causes either to fret, deep scratches are likely to be cut in the soft metal; hence the slide should be attended to as soon as possible after it begins to fret. "*Once commencing to fret it grows worse rapidly, and oftentimes is then beyond repairing.* Never use emery or emery-paper on slides as this will cause serious trouble afterward. After a thorough cleaning of the slide and tube (taking care not to break the cross-wires), endeavor to smooth carefully the injured parts with the back of a pen-knife, and apply barely enough tallow to grease the surface of the injured part. If this does not remove the trouble, a little scraping of the roughened parts on the slide, and, if accessible, on the inside of the tube, may become necessary; then apply a mere trifle of finely powdered pumice-stone moistened with oil. Replace the slide and grind a little by moving it in and out; *clean thoroughly*, and with a piece of charcoal moistened with oil smooth the parts thus ground on the slide. This process of grinding is a most precarious operation, and generally requires the hand of a skillful workman; it should be resorted to only in case of the utmost necessity. Whenever permissible, recourse should be had to a maker. These remarks apply equally to the centers."

"To prevent the focusing slide from fretting, which is usually due to the rush of air carrying in dust and grit when the slide is being run out and is causing momentarily a rarefied space, wrap a piece of chamois-skin over the barrel in the shape of a tubular form and fasten by means of rubber bands or by sewing. In an emergency fine watch-oil may be used to grease the slide should it continue to fret, until the instrument can be sent to the maker. In case of rain during non-use, place the telescope vertical, object end up, and no water can enter the telescope."

"If the focusing slide seems to work too hard, everything else being right, it is generally caused by the lubricant on the pinion hardening in cold weather, and the same cause may also make the focusing slide work too freely in hot weather by softening, i.e., the slide will not stay in place when in a vertical position. If the slide moves too freely it should be tightened by running out the slide to its full length, then applying a screw-driver to the screw on top of the focusing screw and turning a very small part of a turn until the required friction is obtained. If the slide works too tightly run in the slide, unscrew the top screw one turn, gently tap it by the screw-driver handle to release it, and then tighten to the required stiffness."

"A great deal of annoyance is caused to the engineer if the eyepiece or the object-slide of the telescope moves too freely in its tube, requiring a refocusing of the cross-wires and object at every revolution of the telescope in altitude. If the eyepiece can be retained in its socket, with sufficient friction to keep it focused to the cross-wires, no matter how much it may wabble otherwise, this imperfection (in old instruments) will not lead to any inaccuracy; but if there is not sufficient friction to keep it focused to the wires, a little rendered tallow or marrow applied to its bearing-surfaces in most cases will remedy this evil. Wabbling in the object-slide, however, leading to inaccuracy of collimation, or back-lash in its rack and pinion motion, can be remedied only by a maker; but if the object-slide moves too freely in and out of its tube only, this may be remedied by applying a little tallow to the bearing-parts of the rack and pinion, or by tightening the screw in the pinion-head. If not entirely successful, a thin disk made of parchment, or a thin leather-washer, both greased with tallow and inserted between the flanges of the pinion-head and its socket, will insure the desired result."

Whenever it becomes necessary to remove the object-glass slide, a test should be made for the line of collimation after it is replaced. (See p. 602.)

(*c*) Replacing broken cross-hairs. The whole process of replacing broken cross-hairs is very easy, and can be carried out in less time almost than is required to explain it. (1) Take the cross-hair ring out of the telescope and clean it. The fine scratches which indicate where the cross-hairs are to go should be plainly visible after the old shellac and other foreign matter have been removed from the surface. Lay the ring on a flat surface, marked side uppermost. (See p. 551.) (2) Procure a clean, fine, opaque spider-web and stretch it across the ring so that it coincides with two of the opposite scratches. Fasten one end with a drop of shellac, wait for this to harden, then, making sure that the hair is stretched tight, fasten the other hair in a similar manner. Allow the shellac to harden for several minutes, cut off the two ends of the web, and then proceed in the same manner to replace the other web if that too is broken.

Suggestions. (1) In many telescopes it is necessary to remove the eyepiece ring in order to take out the cross-hair ring. (2) In removing the cross-hair ring proceed as follows: Take out first the two *side* capstan screws that hold the ring in place, and *loosen slightly* the other two screws. Turn the ring at right angles to its usual position so that one of the holes from which a screw has been removed is toward the eye-end of the telescope. Sharpen a piece of soft wood to fit the hole, and screw it in lightly until it passes through both holes. Remove the top and bottom capstan screws and take the ring out on the end of the stick. Leave the stick in the ring to serve as a handle while replacing the cross-hairs, and then use it afterward in putting the ring back in the telescope. (3) There are two ways of procuring spider-webs—from the spider's nest or cocoon and from the spider itself. "The best spider-threads are those of which the spider makes its nest. These nests are yellowish brown balls, which may be found hanging on shrubs, etc., in the late fall or early winter. The nest should be torn open and the eggs removed; if this is not done the young spiders when hatched will eat the threads. The fibers next to the eggs are to be preferred on account of their fineness and darker color. One or two cocoons may be kept on hand, and if desired they may be wrapped in a bit of paper and carried in the pocket, where they are always ready." From the spider's cocoon select two or three webs, each about two inches long and of the same appearance. Attach each end of these webs to a bit of wood or wax to act as weights, and immerse them in water five or ten minutes. Remove one web from the water, and very gently pass it between the fore-finger and thumb nails, holding it vertically to remove any particles of moisture or dirt. Then stretch it across the ring as already explained. The little weights should be about as heavy as the hair will support in order to stretch it tight, and before dropping the shellac on either end examine the hair with a magnifying-glass to see if it exactly coincides with the scratch; if it does not, move it gently with a knife-point. If the ring is placed nearly on a level with the eye and between it and the source of light (as a window), the spider-web will glisten and become plainly visible; otherwise it is sometimes difficult to see it. (4) When a cocoon cannot be procured, a spider can usually be found. Cut a forked stick five or six inches long, get the spider on the end of one of the prongs and gently shake him off. He will usually begin to spin a web, and, as he spins, wind the web around the fork in such a way that no two strands will be close together. If the fork is pressed together *a very little* during the winding it will stretch the webs when released. When one strand is laid across the ring the forked stick will act as a weight to stretch it tight.

(*d*) TO REMOVE DUST FROM CROSS-HAIRS. Take out the eyepiece or object-glass (or both if necessary) and blow gently through the tube. Cover up both ends and wait a few minutes before putting back the eyepiece or object-glass. If the object-glass is removed, a test should be made for the line of collimation (see p. 602.) Unless the small particles of dust are on such a part of the web as to interfere with its use it is preferable to allow them to remain.

598. Care of Tripods. "Legs of tripods, if fitting too loose or too tight, and dull shoes are frequent sources of falls. The test of the proper degree of tightness of the legs is this, that if the leg is raised to a horizontal position and left free, it should gradually sink to the ground. If it drops abruptly it is too loose; if it does not sink it is too tight." The nuts will need adjustment after a period of dry or damp weather or after the tripod has been kept in the dry atmosphere of a room. The steel shoes will loosen from wear and tear unless the screws are tightened from time to time. Keep them somewhat sharp, for the greater the hold on the ground and the more carefully the screws are tightened, the steadier the instrument. When the tripod becomes wet wipe dry and renew the varnish whenever necessary.

599. Care of Centers, Spindles, and Sockets. These should be cleaned and oiled occasionally, the oil being wiped off with a soft cloth or chamois-skin. The outer spindle of a transit will need more attention than the inner spindle, and the two plates should not be taken apart unless it is necessary. So long as centers work freely it is best not to disturb them.

"The centers of a transit should always be lubricated with *fine watch-oil only,* and after a careful cleaning; never apply fresh oil before thoroughly wiping off old grit and oil. Rendered marrow is a most excellent lubricant for instruments made of brass and the many kindred alloys of copper and tin. In the varying climes of our northern latitudes this lubricant becomes rigid in cold weather, and an instrument so treated will often become unmanageable in the field. Its application, particularly to the centers of a transit, is therefore restricted to the warmer zones. The use of watch-oil for the finer parts of an instrument, involving freedom of motion, is *imperative in our latitudes.*"

"Many parts of an instrument, especially those whose metal compositions are closely related to each other, may sometimes cause trouble if simply oiled. If they begin to fret and grind, but are otherwise free from grit, etc., the judicious application of a little marrow may prove very beneficial, but it should be cleaned off again as much as possible. The rack and pinion motion and the telescope clamp should always be greased with marrow, but the clamp-, tangent-, and leveling-screws should receive as little of it as possible in the Northern States."

"Vaseline, not having as great a tendency to rigidity under similar circumstances, may prove an excellent substitute for marrow, and may often be applied to level-centers, where watch-oil would not give the necessary rigidity in the use of the more ordinary instruments, but it must be renewed quite often. In the finer class of leveling instruments, the centers should be lubricated with oil only, as in transits."

Sockets may be cleaned with a stick of about the same taper, around which wash-leather, soaked in oil, has been wrapped, the final cleaning being done with fresh pieces of the leather *without* the oil.

(*a*) "*Fretting of centers* should be treated in the same way as fretting of the slide to the telescope. If unable to stop it, temporary relief may be found on placing between the shoulders of the centers a washer of thin paper or thin card. There will then be a shake in the center, but this is a lesser evil than the entire loss of the use of the instrument."

APPENDIX I

DETERMINATION OF A MERIDIAN BY SOLAR OBSERVATION

IN this appendix three methods of determining a meridian are explained, viz.: (1) Method of *single altitude* by direct observation on the sun. (2) Method involving the use of the *solar attachment*. (3) Method of *equal altitudes* of the sun or of a star. In each case the method of procedure is given in detail, so that it may be easily followed by any one who is accustomed to the use of surveying instruments and who has a knowledge of spherical trigonometry. All three methods, however, will be understood better if studied in connection with a course in field astronomy. On page 632 two common and very simple methods of determining latitudes are outlined.

600. The Astronomical Triangle. When the sun is the only heavenly body observed, the celestial sphere may be considered to have a radius equal in length to the distance

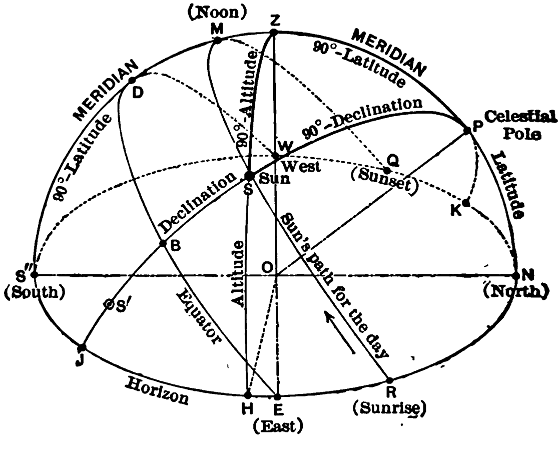


FIG. 600.

from the earth to the sun. If the axis of the earth is produced indefinitely it will intersect the celestial sphere in a point called the *celestial pole* represented by *P* in Fig. 600. If a line from the center of the earth through any point on its surface is produced indefinitely it will pierce the celestial sphere in the *zenith* for that point, and hence observers at different points will have different zeniths, but the celestial pole is the same for all. If in Fig. 600, Z is the zenith for an observer at O then the circle $S''ENW$ is the *horizon.* The arc $EBDW$ represents the *celestial equator,* cut from the celestial sphere by a plane passed through the equator of the earth. In the northern hemisphere the sun is always north of the equator (between it and the pole) from March 21 to September 21, and always south of it between September 21 and March 21. The angular distance from the equator to the sun at any given moment is the sun's *declination for that moment, positive if north, negative if south.* Thus if the sun is at S in Fig. 600, the declination north (or positive) is the arc BS; if the sun were at S' the declination south (or negative) is BS'. The great circle $S''ZPN$, etc., is the meridian for the point on the earth's surface for which Z is the zenith. For any given day the sun appears to move in a circle which is approximately parallel to the equator. Thus for the particular day represented in the figure the sun rose at R is now at S, at noon it will be at M, and will set at Q. (Suppose that the day is in May. Will the apparent path of the sun on the following day be nearer or farther from the celestial equator? Does the apparent path of the sun ever coincide approximately with the celestial equator?)

Remark. It is important to fix in mind the algebraic signs corresponding to north and south declinations, viz., north +, south −.

The spherical triangle ZPS is called the *astronomical triangle.* For any given point on the earth's surface the side ZP is always the same, but the other two sides are always changing, ZS depending upon the time of day (or altitude of sun) and PS on the time of year and day (or declination of sun).

$$ZP = 90^\circ - \text{Latitude};$$
$$ZS = 90^\circ - \text{Altitude};$$
$$PS = 90^\circ - \text{Declination}.$$

The altitude HS can be measured, the declination BS can be ascertained from the Nautical Almanac, and if the latitude PN of the place of observation is known the astronomical triangle can be solved for the angle SZP. This angle is the angular distance of the sun from the meridian at the time of the observation; hence if a transit is at O and is pointed at the sun, the angular distance of the line of sight from the north can be found, for it is equal to NOH, equal to the computed angle SZP. When this angular distance has been determined it is a simple matter to convert it into bearing or azimuth. (See p. 378) The line of sight to the sun may now be used as a reference line for calculating the bearing of other lines, or, if desired, a meridian may be established by turning off the bearing (or azimuth) right or left as the case may be.

It is evident that the astronomical triangle can be used to determine other quantities. For example, the three sides being known as above, the angle ZPS (called the *hour angle*) can be computed; this is the angular distance of the sun from the meridian and is easily converted into time of day. From this the longitude can be found by comparing with Greenwich time. Likewise if the time of observation (hour angle ZPS) and the sun's declination are known and the altitude is measured, the side ZP may be computed, and hence the latitude PN of the place of observation ascertained.

Question. What arc in Fig. 600 corresponds to the time of day?

601. The Sun's Declination is continually changing at rates varying from about zero seconds per hour on June 22 and December 22, to almost one minute of arc per hour on March 22 and September 22. In two of the methods discussed in this appendix the declination is required at the time of the observation. In the *American Ephemeris* and *Nautical Almanac*, published by the Bureau of Equipment, U. S. Navy Department, the apparent declination of the sun is given for every day in the year at the instant of Greenwich Mean Noon. The rate of change or difference for one hour is also given, so that if the number of hours that have elapsed from Greenwich Mean Noon to the time of observation is known, the change in declination is easily computed and applied as a correction to the declination given in the almanac.

(*a*) *Standard time.* In the United States watches are usually set to keep "standard time." There are four time belts. The *eastern* belt extends from the Atlantic Ocean to an irregular line drawn from Detroit, Mich., to Charleston, S. C.; the *central* belt from the last-named line to an irregular line drawn from Bismarck, N. D., to the mouth of the Rio Grande; the *mountain* belt from the last-named line to the western boundary of Montana, Idaho, Utah, and Arizona; the *western* belt from the last-named line to the Pacific coast. The difference in time between adjoining sections is one hour. The time in the *eastern* belt is that of the 75th meridian, i.e., it is noon (standard time) in this belt 5 hours after it is noon at Greenwich; in the *central* belt the time is that of the 90th meridian, or 6 hours slow; in the *mountain* belt the time is that of the 105th, or 7 hours slow; in the *western* belt the time is that of the 120th meridian, or 8 hours slow. The *true local* time at any place will be faster or slower than standard (or watch) time as the place is east or west of the time meridian. Thus the true local time at Boston, Mass., is about 16 minutes faster than standard time, while at Buffalo, N. Y., it is about 16 minutes slower, the 75th meridian being approximately half-way between those two cities. Local and standard time nearly agree at Denver, Colo., as Denver is almost on the 105th meridian.

(*b*) *To look up the declination.* (1) Add to the standard (or watch-time) of the observation 5, 6, 7, or 8 hours, according to the time belt in which the place of observation is located. The result is the Greenwich time of observation. (Since observations in the U. S. should not be taken before 8 A.M., the Greenwich time of observation will be at least an hour past noon.) (2) Look up the sun's apparent declination at Greenwich Mean Noon for the month and day of the observation. (3) To this declination add *algebraically* a correction for the change in declination between Greenwich Mean Noon and the Greenwich time of observation. This correction is found by multiplying the difference for one hour given in the table by the Greenwich time (in hours) of the observation.

Illustration. The time of an observation at New Haven, Conn., was, according to the watch, 9.30 A.M. on May 20, 1907, hence the Greenwich time was 2.30 P.M. on the same day. The sun's apparent declination as given in the almanac for Greenwich Mean Noon on May 20 is $+19^\circ\ 47'\ 38.''1$ and the change per hour is $+32.''05$, hence the declination at the time of the observation or 2.30 P.M. Greenwich time was $(2\tfrac{1}{2}\times 32.05)+19^\circ\ 47'\ 38.''1 = +19^\circ\ 48'\ 58.''2$.

Illustration. The time of an observation at Chicago, Ill., was 2.30 P.M., Oct. 15, 1907, hence the Greenwich time was 8.30 P.M.

The sun's apparent declination for Greenwich Mean Noon Oct. 15 was $-8^\circ\ 11'\ 29.5''$, and the change per hour $-55.83''$, hence the declination at the time of the observation (8.30 P.M. Greenwich time) was $(-55''.83\times 8.5)+(-8^\circ\ 11'\ 29.5'') = -8^\circ\ 19'\ 24.1''$.

602. Refraction. On account of refraction the sun appears to be higher than it really is. Hence all vertical angles found by sighting at the sun should be reduced by *subtracting* the correction for refraction. It will be seen from the following table that the nearer the sun is to the horizon the greater the correction for refraction. Below say 20 or 30° the correction for refraction is very uncertain, and hence any observations depending principally upon a single altitude of the sun should be taken only after the sun is from 20 to 30° above the horizon. The correction for refraction varies with the temperature, height of barometer, etc. The following mean values for a temperature of 50° F. and barometer reading of 29.9 inches are taken from Hayford's Geodetic Astronomy.

MEAN REFRACTION CORRECTIONS.

Altitude.	Correction.	Altitude.	Correction.	Altitude.	Correction.
0°	34′ 08″	16°	3′ 20″	40°	1′ 09″
5°	9′ 52″	17°	3′ 08″	45°	0′ 58″
10°	5′ 19″	18°	2′ 57″	50°	0′ 49″
11°	4′ 51″	19°	2′ 48″	55°	0′ 40″
12°	4′ 27″	20°	2′ 39″	60°	0′ 34″
13°	4′ 07″	25°	2′ 04″	70°	0′ 21″
14°	3′ 49″	30°	1′ 41″	80°	0′ 10″
15°	3′ 34″	35°	1′ 23″	90°	0′ 0″

603. Determination of the Meridian by the Sun's Altitude. This method may be divided into two parts: (1) The *observation* or the measurement of the sun's distance above the horizon. (2) The *computation* or the solution of the spherical triangle to find the angle *SZP* (see p. 620).

(*a*) Method of procedure for the observation. (1) Set up the transit over some point *M*, and with verniers set at zero backsight on some point *N*. The line *MN* may be any convenient line, the object of the observation being to find the true azimuth of some line of reference, thus establishing indirectly the direction of a true meridian.

(2) Loosen the upper clamp and foresight on the sun, bringing its image tangent to the cross-hairs in the lower left-hand quadrant as shown in Fig. 603. Read and record the horizontal and vertical angles and note the time of the observation.

(3) Loosen the upper clamp, plunge the telescope and again foresight on the sun, but this time bring the image tangent to the cross-hairs in the upper right-hand quadrant as shown in Fig. 603(*a*). Read and record the horizontal and vertical angles, and note the time of the observation.

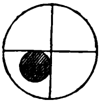


Fig. 603.

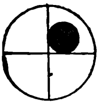


Fig. 603 *a*.

The mean horizontal angle, the mean vertical angle, and the mean time of observation as found from the results obtained in steps (1) and (2) will be used in computation as explained farther on.

Practical suggestions. (1) If observations are taken too early in the morning (before the sun reaches an altitude of at least 20 to 30°) the correction for refraction will be large and uncertain, while if they are taken too near noon the altitude of the sun is changing too slowly and the astronomical triangle becomes poorly proportioned for definite results. Hence the best time of day for observations is from 8 to 10 A.M. or from 2 to 4 P.M.; they should never be taken within an hour of noon. (2) In determining the meridian for any survey it is well to set up over a regular station and backsight on some other distant station, thus making the reference line *MN* coincide with one of the main lines of the survey. (3) In setting up, level the instrument with great care, using the level under the telescope for the final test in at least two positions at right angles to each other. Of course, the instrument should be in good adjustment. If the vernier of the vertical circle is not adjustable do not forget to record the amount and *sign* of the index error (see p. 98). (4) The object in sighting at the sun twice, once with the telescope normal and once with it reversed, is to eliminate instrumental errors. The object in bringing the cross-hairs tangent to the sun's image first in one quadrant and then in the opposite quadrant is to avoid the error which might occur in attempting to sight directly at the sun's center as judged by the eye. Of course the time between the two observations should be as short as possible, yet there is no need of undue haste. No appreciable error will occur if the pointings are not more than, say, two or three minutes apart. (5) As a further precaution both the *A* and *B* verniers on the limb may be read to eliminate errors of eccentricity, as is done in triangulation. (See p. 195). Likewise if there are two verniers on the vertical circle the two readings may be noted. (6) Prismatic eyepieces with colored shades are furnished with transits for use in solar observations. If no such eyepiece is at hand and the telescope has an erecting eyepiece, proceed as follows: Point the telescope approximately at the sun, hold a white card three or four inches behind the eyepiece, and draw the latter out until the image of the sun and the cross-hairs are both sharply focused on the card. The image may then be brought tangent to the wires almost as accurately as if the observer were looking through the telescope. If, however, the sun is high, the limb of the transit may interfere with holding the card in this manner. If the eyepiece is non-erecting the card method cannot be used, and hence a prismatic eyepiece is essential. (7) The maximum error in determining an azimuth by the altitude of the sun should not exceed three minutes, and under favorable conditions it should not exceed one minute. By taking several sets of observations both morning and afternoon the error can be reduced still more.

(*b*) MODIFIED METHOD. If the transit does not have a vertical circle, or if for any other reason it is not feasible to take two sights, direct and indirect, proceed as before, omitting the second sight. The altitude as observed in (1) must be corrected for the sun's semi-diameter, i.e., subtract 16′ of arc if the horizontal hair was made tangent to the upper edge of the sun. The computed azimuth must then be corrected by the quantity 16′ × secant of altitude, i.e., sun's semi-diameter divided by cosine of altitude.

Remark. The angle subtended by the sun's semi-diameter varies from 15′ 45″ about July 1, to 16′ 18″ about January 1.

(*c*) METHOD OF PROCEDURE FOR COMPUTATIONS. (1) Find the mean value of the observed altitudes. Look up in the table on page 624 the correction for refraction which corresponds to the mean altitude and subtract it from the mean altitude. Correct the result for the index error if there is any such error. (See suggestion (3), p. 625). Call the final corrected mean altitude .

Remark. If the highest accuracy is desired the altitude should be corrected also for parallax, i.e., the angle between a line from the sun to the center of the earth and a line from the sun to the point on the earth's surface where the observation is made. As this correction never exceeds 9 seconds of arc it may be ignored in observations made with transits reading to minutes.

(2) Find the sun's declination for the day and hour of the observation as explained on page 623.

(3) Subtract the final corrected declination of the sun from 90°, thus getting the *polar distance* p of the sun at the time of the observation.

(4) The latitude, the polar distance, and the corrected mean altitude being known, the angle Z of the spherical triangle can be found by spherical trigonometry, using any one of several formulas. Either of the following formulas is recommended:

h = corrected mean altitude; p = polar distance of the sun;
L = latitude of the observer and $S=\frac{1}{2}(h+L+p)$.

$$\tan \tfrac{1}{2}Z=\sqrt{\frac{\sin (S-L) \sin (S-h)}{\cos S \cos (S-p)}}. \quad . \quad . \quad . \quad (1)$$

Another formula:

$$\cos \tfrac{1}{2}Z=\sqrt{\frac{\cos S \cos (S-p)}{\cos L \cos h}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

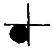
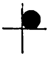
(5) The angle Z found from either of the above formulas is the angular distance of the sun east or west of the meridian, or it is equal to the *angular distance* of the line of sight to the center of the sun measured from the NORTH point. For example, in Fig. 600, p. 620, the angle $Z=SZP$ is measured by the arc NH equal to the angle NOH, i.e., the *horizontal* angle that a line of sight from the place of observation O to the sun S makes with a north and south line ON. This angular distance $Z=NOH$ may easily be converted into azimuth or bearing as explained on page 378.

Remark. In astronomical work azimuths are measured from the south, and hence the left-hand members of equations (1) and (2) are frequently given as *cot* $\frac{1}{2}Z$ and *sin* $\frac{1}{2}Z$ respectively. In that case Z is equal to the angular distance of the sun measured from the *south*.

Questions. Assume azimuths to be measured clockwise from the north and the value of Z to be found from formulas (1) and (2). (1) If the altitude of the sun is found from a morning observation is the corresponding value of Z equal to the *azimuth* of the line of sight to the sun? (2) If the altitude is found from an afternoon observation is Z equal to the azimuth? Assuming azimuths to be measured clockwise from the south and Z to be found from the cotangent or sine instead of from the tangent or cosine, answer the same questions (1) and (2).

(*d*) EXAMPLE ILLUSTRATING SINGLE ALTITUDE METHOD.

Record of Observations, Feb. 21, 1907, New Haven, Conn.

Sun.	Telescope.	Horizontal Angles.		Vertical Angles.	Time.
		Vernier A.	Vernier B.		
	Nor.	38° 07′ 40″	38° 08′ 20″	22° 34′ 30″	8 h. 57 m. 30 s.
	Inv.	38° 33′ 50″	38° 34′ 10″	22° 36′ 0″	9 h. 02 m. 30 s.
	Mean	38° 20′ 45″	38° 21′ 15″	22° 35′ 15″	9 h. 0 m. 0 s.

COMPUTATION

$2\text{ h. }0\text{ m. }0\text{ s.} = (9\text{-}0\text{-}0) + (5\text{-}0\text{-}0) =$ Greenwich mean time of observation.
$-\ 10^\circ\ 52'\ 0'' =$ Apparent declination at Greenwich mean noon.
$+1'\ 48'' = +54'' \times 2 =$ correction for 2 hours.

$-\ 10^\circ\ 50'\ 12'' =$ Apparent declination at time of observation.
$22^\circ\ 33'\ 0'' = h = 22^\circ\ 35'\ 15'' - 2'\ 15''$ (ref.) = corrected altitude.
$41^\circ\ 18'\ 36'' = L =$ Latitude of place of observation.
$100^\circ\ 50'\ 12'' = p = [90^\circ - (-10^\circ\ 50'\ 12'') =$ polar distance.

$164^\circ\ 41'\ 48'' = 2S.$ I.
$82^\circ\ 20'\ 54'' = S$ co-log $\cos S = 0.875658$
$41^\circ\ 02'\ 18'' = S - L$ log $\sin S - L = 9.817277$
$59^\circ\ 47'\ 54'' = S - h$ log $\sin S - h = 9.936645$
$-\ 18^\circ\ 29'\ 18'' = S - p$ co-log $\cos S - p = 0.023014$
$2|0.652594$
$64^\circ\ 44'\ 40'' = \frac{1}{2}Z$ log $\tan \frac{1}{2} Z = 0.326297$
$129^\circ\ 29'\ 20'' = Z =$ azimuth of sun from the north.
$-\ 38^\circ\ 21'\ 0'' =$ Angle between sun and MN.

$91^\circ\ 08'\ 20'' =$ Azimuth of MN from the north.

604. The Solar Attachment, frequently used in connection with an engineer's transit, is a device for determining a true meridian by an observation on the sun. Its use is based on the same general principles involved in the method of the preceding article, except that instead of solving the astronomical triangle by spherical trigonometry the instrument is made to solve it graphically, so that at the end of the observation the line of sight through the transit telescope lies in the true meridian.

Two of the forms of solar attachments in common use are explained below; all forms are alike in principle.

(*a*) THE BURT SOLAR ATTACHMENT. The solar attachment shown in Fig. 604 (*a*) is essentially the solar apparatus of Burt (see p. 576) placed upon the cross-bar of an ordinary transit. The principal parts are: (1) The *arm* AB, which has a small lens at A and a small silver disk at B; (2) the *polar axis* and (3) the *declination arc* marked in the cut; (4) the *hour circle* marked with Roman numerals. The little framework carrying the arm AB and the *declination* arc is free to revolve about the polar axis. The observation on the sun is made by bringing the image of the sun formed by the lens into a square ruled on the silver disk. A shutter at A is moved over the lens when the sun is very bright, otherwise it is difficult to tell when the image is exactly centered in the square; when the sun is dull the shutter is moved back, leaving the clear lens. When the declination is south (sun below the equator) the arm AB is reversed or turned end for end from the position shown in the cut; for this position there is a lens at B and a silver disk at A.

Attention is called to the following facts:*

1. If the telescope is pointed north and depressed until the reading on the vertical circle is equal to the co-latitude of the place where the transit stands, the polar axis will be parallel to the axis of the earth, and may be said to point to the celestial pole.

2. If when the telescope is in the above position the line AB were made parallel to the telescope it could be directed to the sun by merely turning the arm about the polar axis, *provided the sun were in the celestial equator.* The sun, however, is in this equator only twice during the year. Suppose that for a given day its declination is 17° north, i.e., it may be considered to move in a circle approximately parallel to the equator but 17° north of it. (See p. 620.) Then if the end A is raised until the declination arc reaches 17°, as shown in the engraving, the line AB could be directed approximately to the sun at any time during the day by merely revolving the arm about the polar axis, *provided the polar axis is kept stationary and pointing to the pole.* If the sun's declination were 17° south the only difference would be that the line from B to A should be directed

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* Written for observations in the northern hemisphere.

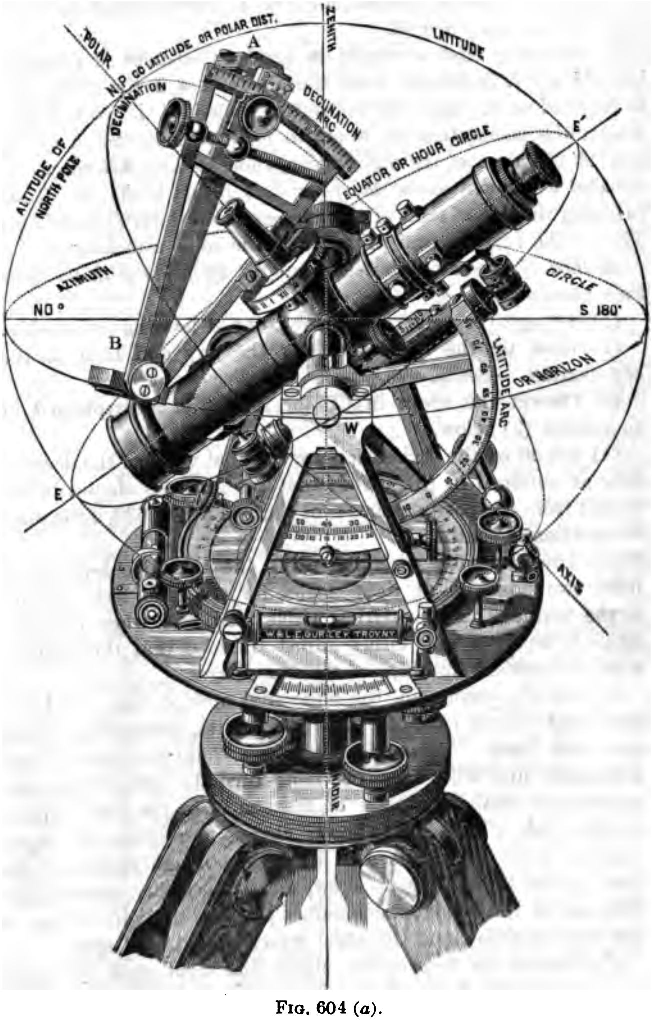

FIG. 604 (*a*).

This cut is kindly furnished by W. and L. E. Gurley of Troy, N. Y.

downward instead of upward, and this is accomplished by turning the arm end for end.

3. Assuming that a transit in good adjustment is properly leveled up, it is evident from the above that if the polar axis is inclined at an angle to the vertical equal to the co-latitude, and if the declination arc is set to read the declination of the sun at the time of the observation, then *the line AB cannot be directed to the sun unless the polar axis points to the pole* (thus bringing the line of sight of the telescope into the true meridian). This is the fundamental principle of the solar attachment.

Method of procedure in determining a meridian with the Burt solar attachment.

(1) Set up over some convenient point *M* leveling up roughly.

(2) Look up the declination for the day and hour of the observation. (See p. 623.)

(3) Correct the result in (2) for refraction as explained in suggestion (2) below.

(4) Set off on the declination arc the final value of the declination as obtained in (3). Note that the arm *AB* should be reversed from the position shown in Fig. 604 (*a*), p. 629, if the sun's declination is south.

(5) Level up very carefully, using the telescope level in at least two positions at right angles for the final test. Notice if the verniers on the vertical circle read zero when the telescope is level; if not, either adjust them so that they will or note the amount and *sign* of the index error.

(6) Set the verniers of the horizontal limb at zero, but leave the lower clamp loose so that both plates, clamped together, can turn freely. With one hand turn these two plates (and telescope), and with the other hand turn the arm of the solar attachment until the image of the sun is in the ruled square on the disk. Tighten the lower clamp, and bring the image exactly in the center (as seen through a magnifying-glass) by turning the lower tangent-screw, and if necessary turning also the arm of the solar attachment. The line of sight is now in the true meridian and the plate zeroes should read zero.

(7) Loosen the upper-plate clamp and sight at *N* the other end of a reference line *MN*. The azimuth of the line *MN* is the angle turned off, i.e., the reading on the limb. If desired a meridian *MO* may be established before the upper-plate clamp is loosened by lining in a point *O* on the ground.

Practical suggestions. (1) Many of the suggestions on page 625 apply equally well to the use of the solar attachment, especially that pertaining to the best time for observations. (2) In correcting the declination for refraction in step (3) that table on page 649 should be used which most nearly corresponds to the latitude the observer. Note the time of observation in hours before or after noon, and opposite this number in the table find the correction in the column corresponding most nearly to the declination. Notice that if the declination is north (+) the correction is to be added while if it is south (−) it is to be subtracted, or, if given the proper sign, the correction is always added algebraically (Northern hemisphere). (3) When the image has been centered in the ruled square it will at once move out, but if no mistake has been made it may be kept approximately in the square by simply revolving the solar attachment about the polar axis. If it is necessary to disturb the main part of the transit it shows that the polar axis is not pointing exactly at the pole or that some mistake was made in setting off the declination. (4) When the observation is completed the pointer which moves around the hour circle should indicate approximately the time of the observation. (5) The image should be examined through a magnifying-glass during the final steps of the observation. It is said that an error of one-quarter of a minute in azimuth or latitude can be detected by thus observing the sun's image; but, nevertheless, the instrument under the best of conditions cannot be relied upon to give as accurate results as may be obtained by the direct method of observation, page 624. (6) If at the end of step (6) the plate verniers do not read zero it is likely that the wrong tangent-screw has been turned; but no harm has been done, as the reading can be noted and subtracted from the final reading when sighting at *N*.

(*b*) THE SAEGMULLER SOLAR ATTACHMENT is in many respects superior to the Burt attachment. In place of the arm carrying the lens and silver disk, there is a small auxiliary telescope with a prismatic eyepiece through which the sun may be observed directly. The declination arc is usually omitted, and in its place a spirit-level is attached to the auxiliary telescope to aid in setting off the declination as explained below. The method of procedure is so similar to that just explained for the Burt attachment that only a brief outline is given.

Method of procedure for determining a true meridian with the Saegmuller solar attachment. (1) Find the final corrected value for the sun's declination at the time of observation. (See step (2) and (3) for the Burt attachment, p. 630.)

(2) Having leveled up carefully over some point *M* and noted whether there is any index error as in step (5), p. 630, back sight on some point *N*. Point the smaller telescope on the same backsight, thus bringing both telescopes into the same vertical plane.

(3) Incline the larger telescope until the reading on the vertical circle is equal to the corrected value of the sun's declination found in (1). If this declination is north depress the object-glass end of the telescope; if south elevate it. Without disturbing the larger telescope, bring the smaller telescope to a

horizontal position by means of its level. The angle between the two telescopes is now equal to the sun's declination.

(4) Without disturbing the relative positions of the two telescopes unclamp the vertical circle and set it to the co-latitude of the place of observation, making the object-glass end of the telescope the higher.

(5) Unclamp the upper horizontal plate, turn it (and the larger telescope) around the vertical axis, and at the same time turn the smaller telescope around the polar axis (see step 6, p. 630) until the image of the sun is brought into the field of the smaller telescope; when the image is accurately bisected the large telescope must be in the meridian *with the object-glass end toward the south.* The bearing of MN is then known from the reading on the horizontal limb, and if desired a true meridian may be staked out on the ground. (See step 7, p. 630)

Remark. If desired the plate verniers may be left clamped at zero until the sight has been taken at the sun, and then the angle turned off from the point N as directed in step (7), page 630.

Practical suggestions. Read the practical suggestions on pages 625 and 631.

605. Other Methods of Determining a Meridian. There are two other methods of determining a meridian, viz.: (1) Sight at the sun or a star when it is some distance from the meridian, carefully measuring its altitude, and then sight at it again when it has crossed the meridian and is at exactly the same altitude. In the case of a star the meridian is halfway between the two sights, and in the case of the sun it is nearly so, being found by applying a small correction. This method is called the method of equal altitudes, and is explained in detail in Mr. Barber's article at the end of this appendix.

(2) A second method is to sight at the sun at exactly noon. This method involves the determination of the true local time of the place of observation, i.e., the difference between the watch-time and sun-time. The method is described in *Engineering News*, March 31, 1904.

606. To Determine Latitude. (*a*) BY OBSERVATION ON POLARIS. (1) Observe polaris at culmination and correct the vertical angle for refraction (p. 623) and index error if there is any (see p. 98). The result is the true altitude. (2) If the star is at upper culmination subtract the polar distance of Polaris for the given date as given in the table below; if at lower culmination add the polar distance. The result is the required latitude. (Why? See p. 620.)

MEAN POLAR DISTANCE OF POLARIS FOR THE BEGINNING OF FICTITIOUS YEAR

Year.	Mean Polar Distance.	Year.	Mean Polar Distance.	Year.	Mean Polar Distance.
1906	1° 11′ 41.05″	1911	1° 10′ 07.82″	1916	1° 08′ 34.97″
1907	1° 11′ 22.37″	1912	1° 09′ 49.22″	1917	1° 08′ 16.45″
1908	1° 11′ 03.71″	1913	1° 09′ 30.64″	1918	1° 07′ 57.94″
1909	1° 10′ 45.07″	1914	1° 09′ 12.07″	1919	1° 07′ 39.45″
1910	1° 10′ 26.44″	1915	1° 08′ 53.51″	1920	1° 07′ 20.98″

Note. The above table is based upon data furnished for this book by the Superintendent of the U. S. Geodetic and Coast Survey. Each value is found by subtracting from 90° the *mean* declination of Polaris for the *beginning* of the corresponding fictitious year. If desired, the polar distance for any day in the year may be found from the apparent declination of Polaris as given for that day in the Nautical Almanac. For 1907 the apparent declination decreases from 88° 48′ 51.6″ on January 15 to 88° 48″ 19.8″ on July 1, and then increases to 88° 49′ 9.1″ on December 31.

The beginning of the *fictitious* (Besselian) year is the instant at which the mean sun is 280° from the vernal equinox. It differs from the beginning of the ordinary year by a fraction of a day which varies for different years. The *mean* place of a star at a given instant differs from the true place by the effect of nutation at that instant ,and from the apparent place by the effects of both nutation and aberration.

(*b*) BY OBSERVATION ON THE SUN AT NOON. (1) Sight at the lower or upper limb of the sun a little before noon, allowing for the difference between standard time and local mean time, and follow the sun until it reaches its highest point, i.e., when it crosses the meridian. (2) Correct the vertical angle for refraction (p. 346) and index error if there is any. The result is the true altitude of the sun's limb. If it is the lower limb add the sun's semi-diameter (about 16′, but given more exactly in the Nautical Almanac); if it is the upper limb subtract the semi-diameter. This corrected result is the true altitude of the sun's center. (3) From the true altitude of the sun's center found in (2) subtract *algebraically* the sun's declination. The result is the required latitude. (Why? See Fig. 600, p. 620, where $S''M$ represents the altitude at noon.)

(*c*) LATITUDE WITH SOLAR ATTACHMENT. The latitude of a place may be determined very easily with a solar attachment by observing the sun at noon.

(1) Set off the co-latitude (as nearly as can be estimated) and the corrected declination of the sun in the usual manner.

(2) Point the smaller telescope at the sun. (The two telescopes are in the same vertical plane but not parallel, the angle between them being equal to the corrected declination of the sun.) By means of the tangent screw which changes the inclination of the *larger* telescope, keep the *smaller* telescope pointing exactly at the sun until it ceases to move upward and begins to move downward. The reading of the vertical circle at this highest altitude is equal to the co-latitude.

607. Determination of the Meridian in High Latitudes —Observations by Equal Altitudes.*

There is no operation in the field-work of a land surveyor that is fundamentally more important than the establishment of a correct meridian line at the place of survey, from which to deflect the courses of boundaries.

The rules for surveying public lands prohibit a surveyor from assuming a probable "variation" of the magnetic compass to govern his work. They also prohibit him from depending on a variation of his needle throughout his work, though he may have obtained it by proper means at the outset, because all the successive courses may be influenced by unknown attraction.

A large number of surveys throughout the country are known to have been recorded with untrue courses, regardless of these requirements, causing many legal complications.

From a desire to aid surveyors to overcome the difficulties known to impede precision in this element of their work, especially in high latitudes like those of Alaska, and to determine exact meridians by astronomic means without very elaborate instruments, some suggestions are here offered as auxiliary to the means which are adapted to all places.

The use of equal altitudes has received slight notice in a few works on surveying, but has not been set forth in the detailed manner applied to the other processes. But since preparing this article and laying it aside for consideration, it has been learned that this principle was recommended to the deputies in one of the States by instructions issued some years ago by a surveyor-general. It was given them "as being available in emergencies, calling their attention to the advantages in such cases of being able to take the forenoon observation, go on with the survey till the afternoon time approached, and then complete the set. It is available also for P.M. followed by A.M. observation."

Thus the writer now finds, from correspondence, that one of the best authorities on these subjects then gave full indorsement to the equal-altitude method for using either sun or star, as being correct, practical, and of important utility in many contingencies.

The instructions now generally provided require that the meridian be ascertained by observation of either Polaris or the sun; and that the correctness of all solar work must be checked and confirmed by Polaris.

VARIOUS OBSTACLES

to compliance with these rules have been found in Alaska, and are evident:—

Polaris being from 55° to 70° above the horizon, it is too high to be observed without a reflecting eyepiece.

Its position so near the zenith greatly increases any instrumental error, in bringing a vertical line to the ground.

It is obscured by daylight during several months in the summer season.

Fogs and mists are said to be often prevalent, even after clear days.

The sun in summer reaches only a small elevation, moving in a low arc, with high and uncertain refraction, both causes unfavorable to accurate results.

The ordinary solar compass or transit is not constructed for use in such latitudes, and special constructions are inconvenient.

Yielding to these difficulties, it appears to have been a common but unauthorized practice to disregard accuracy, assume a magnetic declination, and work by the needle only

A SIMPLE AND PRACTICAL METHOD

is therefore very desirable, to enable surveys to be made with courses that will bear the test of a correct examiner. Such a method is now suggested, which may be used by any careful observer having a good common transit with vertical arc.

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* This article is a reprint of a paper by Mr. A. W. Barber, published in *Michigan Engineer the Proceedings of the Michigan Engineering Society* for 1905.

This process requires no reflecting eyepiece; no full vertical circle; no mathematical tables; no calculations of local mean time, standard time, sidereal time, or astronomical and civil day. It does not depend on the date nor the hour of the day, nor any reckoning of culmination, hour-angle, elongation, or polar azimuth. Neither does the sun's declination or the atmospheric refraction enter at all into the calculation.

Moreover, should there be unsuspected error in the graduation or setting of the vertical arc, or some defect of collimation in the telescope, it would equally affect both parts of the observer's work, and produce no effect in the resulting meridian.

THE METHOD BY EQUAL ALTITUDES

depends on the fact that the circumpolar stars describe invariable circular arcs below the true pole; and that when a peg is set on the ground to mark the course to a certain star when west of north at an altitude of say 35°, and another peg is afterward set to mark the position of that star when it has passed to the northeast and risen to the same precise elevation, the meridian of the transit will lie exactly midway between the pegs. Hence the surveyor may be advised to proceed as follows in any latitude or time of the year when the stars are sufficiently discernible:—

1. Choose a suitable station, set transit firmly and level precisely, by the telescope level turned in all directions.
2. Set the index arm for vertical angles at zero, and keep it tightly clamped for reading elevation angles.
3. Provide suitable illumination for cross-wires, and also for reading angles, horizontal and vertical.
4. Select a conspicuous star, perhaps 30° or 40° from the pole, which appears two hours more or less before its lower culmination—that is, which stands west of north and is rapidly descending. Identify this star beyond all chance of error, noting it on a diagram for certainty some hours later.
5. Direct the telescope to this point, fixing the star at the intersection of the cross-wires, and clamp the axis so it will retain the altitude shown on the arc. Read and note down the angle of elevation, and read more than once.
6. Unclamp the axis, bring the telescope to the earth, and have an assistant drive a peg in line with the cross-wire, from 3 to 5 chains distant. A candle held there before a white surface will exhibit the wire and give the exact point for a tack.
7. Repeat the observation once or twice, at intervals of 10 or 15 minutes, for confirmation of results, marking successive pegs, A, B, etc., with degrees and minutes of elevation found.
8. Be ready to observe the upward path of the same star after it has passed east below the pole. Correct the leveling, set the vertical index successively at each altitude previously noted (beginning with the lowest); and when the star (diagonally ascending in the field of the glass) approaches the horizontal wire, bring the vertical wire also upon the star at the intersection, using the slow-motion screw of the horizontal plate.
9. Keep the plate at that point, bring the telescope down, and set peg in line as before. Repeat the process for each observation A, B, etc., taken before midnight, marking each peg B, A, etc., with the elevation in figures.
10. Measure the arc between pegs A A denoting equal altitude, and take one half. Lay off this half from either peg, and set a peg and tack for the true meridian. As a test of correctness, the middle point between pegs B B and between C C should be found to coincide with the one first found. A single pair is sufficient, except for confirmation.

BY EQUAL ALTITUDES OF THE SUN.

In this operation a reflecting eyepiece with dark glass will be necessary. The sun's large image in the field can be centered as truly as a star. It is therefore found best to place the intersection of the wires at the lower limb (apparently the sun's upper edge, as reversed by the mirror) and at the precise point of tangency, when the sun is just leaving the horizontal wire, apparently descending, with the vertical wire bisecting its disk.

This is convenient for the forenoon observations; hence at the corresponding times after noon, with the proper altitude of telescope, one must be ready at the moment the sun (now apparently ascending) first reaches the level wire, having the vertical one bisect the sun by the point of tangency, as before

The pair of pegs in this case will be set southeast and southwest of the station. The center or bisecting line of the included arc would be the meridian, were it not for the sun's change of declination in the intervening time. This slight change requires a calculation for correction, which the star process avoids. The formula calls for a short arithmetical process, with a table of natural sines.

WORKING BY A REFERENCE MARK

Instead of using pegs and tacks for day-work in this line, the writer has found it easier to use a reference point or mark.

On April 17, after careful leveling, I set the horizontal plates at zero with the telescope directed at a distant spire for my mark. Clamp the lower plate fast, and direct the telescope to the sun, observing it as above shown. Find by the horizontal angle that the sun's azimuth to the left or east of the spire is 30° 27'; and by the vertical arc I find his altitude 47° 09'. (The semi-diameter may be disregarded in each pair of observations if the same limb of the sun is used each time.)

For the corresponding afternoon observation, I have the index of altitude fixed at 47° 09', and watch the sun rise (apparently) to the proper position. At the right moment clamp the plate, use the slow motion, and when the disk is in position find from the horizontal plate that the sun is 84° 49' west of the spire or mark.

The whole arc is 30° 27' + 84° 49' = 115° 16'; and the bisecting meridian is 57° 38' from either position of the sun. From this one half I subtract the first azimuth of the sun from the mark; 57° 38' − 30° 27' = 27° 11' as the true bearing of the mark from the uncorrected meridian, and it apparently bears S. 27° 11' E. from the transit.

THE CORRECTION FOR DECLINATION

at or near the times of the solstices, will be merely theoretical, as an hourly difference of declination less than 10 seconds will be quite negligible. But during the rest of the year it should be ascertained, for it would amount to as much as a change of 10' were observations taken six hours apart on September 25, in latitude 65°.

To calculate this correction: Take one half the change in declination between observations at equal altitude; divide these minutes of change by the product of the cosine of the latitude by the sine of half the difference in time expressed in degrees (15° per hour); the quotient will be the minutes of arc for the correction. This is to be applied from south to west from June 21 to December 21 (declination decreasing) and from south to east the rest of the year.

EXAMPLE OF ABOVE OPERATION

on April 17, in latitude 39°, using standard watch-time; second and third pairs of observations, the first pair being already noted.

B. 10 : 01 A.M. altitude 49° 36' azimuth E. from mark 26° 07'
C. 10 : 16 A.M. altitude 51° 48' azimuth E. from mark 21° 38'
C. 1 : 59 P.M. altitude 51° 48' azimuth W. from mark 76° 00'
B. 2 : 14 P.M. altitude 49° 36' azimuth W. from mark 80° 29'

The sum of the measured arcs of the B B positions, 26° 07' + 80° 29' = 106° 36'. The middle point for meridian is at 53° 18'. As the reference mark is 26° 07' from the forenoon sun, its arc from the south meridian point must be 53° 18' − 26° 07', and its course from the transit is again found S. 27° 11' E. (Uncorrected.)

The third pair, C C, gives 21° 38' + 76° 00' = 97° 38'. One half of this, or 48° 49', less 21° 38', gives the same resulting arc, 27° 11'.

CORRECTION FOR CHANGE OF DECLINATION

One half of the change on that day in 4¼ hours was 1.9'. The cosine of the latitude 39° is .78; the sine of half the difference in time (2⅛ hours = about 32°) is .53; their product is .78 × .53 = .41. Dividing 1.9' by .41, the quotient is 4½ minutes of arc for correction of the south meridian point eastward. This gives the bearing of the spire or mark from the true meridian, S. 27° 06′ 30″ E.

THE DIRECT TRANSIT OBSERVATION OF THE SUN

for meridian is an operation highly commended by many; and with a perfect instrument used by an unerring workman it should be wholly reliable. Yet the writer meets numerous capable engineers who object to its use and will not depend on it.

There are so many uncertainties attending its use by unknown persons with the possibly disordered instruments, and so many lines nominally established by it have been found out of limits, that the method has not been approved in the Manual of Instructions unless the results are verified by the independent check of Polaris. The latter being direct and simple and having no possible connection with the solar methods, cannot be dispensed with as the court of last resort to prove the accuracy of the tedious and intricate "direct solar" problems, where the mean of repeated observations is apt to involve repetition of error.

To use the sun in that manner, one must sacrifice much time, in the best working hours, in observing and in solving his spherical triangles, usually under conditions of physical discomfort, while his party waits; or else he must proceed to work and secure his solution at another time and place.

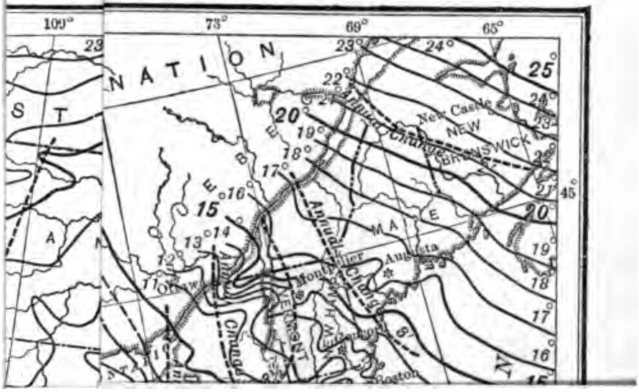
Of all the methods of taking an observation for meridian, this appears most thickly beset with chances for error. The resuit is certainly affected by any index error in the vertical arc, or in the collimation of the telescope, unless one has a full circle annd eliminates the error by reversing, repeating, and averaging. Any arithmetical error in logarithmic readings or written work will vitiate the results.

The surveyor is obliged to pay due regard to latitude and co-latitude, altitude and zenith distance, declination and co-declination. He must take care of his standard time, longitude, and azimuth from reference mark. The exact refraction should be used, with due regard to temperature, barometer, and humidity. Having safely determined all these, he is then ready to perform his calculation; yet at last mav well suspect whether some error has crept in, unless he has some independent test.

Such a check, suited to the latitude of Alaska or elsewhere, is believed to be found in the method of equal altitudes. A surveyor may set a tack beneath his star at ten in the evening, noting its altitude; and should fog and storm set in for a day or a week, it will only delay but not defeat his completion of the work. Whenever it clears off he can secure his counter observation about two in the morning, with no doubt of finding the correct meridian.

The foregoing pages will not appeal strongly to surveyors that are indifferent to certain precision, and averse to losing sleep; but they have been approved by several of a different class—men who have kept many a long vigil of the small hours to secure the most critical use of Polaris, and lay courses from which no competent inspector could materially differ.

GNETIC FOR 1905



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TRIGONOMETRIC FUNCTIONS.

Let A (Fig. 107) = angle BAC = arc BF, and let the radius $AF = AB = AH = 1$.

We then have

$\sin A = BC$
$\cos A = AC$
$\tan A = DF$
$\cot A = HG$
$\sec A = AD$
$\operatorname{cosec} A = AG$
$\operatorname{versin} A = CF = BE$
$\operatorname{covers} A = BK = HL$
$\operatorname{exsec} A = BD$
$\operatorname{coexsec} A = BG$
$\operatorname{chord} A = BF$
$\operatorname{chord} 2A = BI = 2BC$

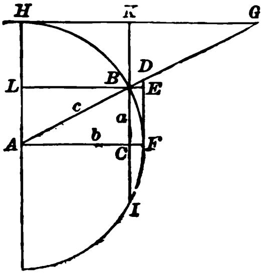

FIG. 107.

In the right-angled triangle ABC (Fig. 107)
Let $AB = c$, $AC = b$, and $BC = a$.
We then have:

1. $\sin A = \dfrac{a}{c} = \cos B$
2. $\cos A = \dfrac{b}{c} = \sin B$
3. $\tan A = \dfrac{a}{b} = \cot B$
4. $\cot A = \dfrac{b}{a} = \tan B$
5. $\sec A = \dfrac{c}{b} = \operatorname{cosec} B$
6. $\operatorname{cosec} A = \dfrac{c}{a} = \sec B$
7. $\operatorname{vers} A = \dfrac{c-b}{c} = \operatorname{covers} B$
8. $\operatorname{exsec} A = \dfrac{c-b}{b} = \operatorname{coexsec} B$
9. $\operatorname{covers} A = \dfrac{c-a}{c} = \operatorname{versin} B$
10. $\operatorname{coexsec} A = \dfrac{c-a}{a} = \operatorname{exsec} B$
11. $a = c \sin A = b \tan A$
12. $b = c \cos A = a \cot A$
13. $c = \dfrac{a}{\sin A} = \dfrac{b}{\cos A}$
14. $a = c \cos B = b \cot B$
15. $b = c \sin B = a \tan B$
16. $c = \dfrac{a}{\cos B} = \dfrac{b}{\sin B}$
17. $a = \sqrt{(c+b)(c-b)}$
18. $b = \sqrt{(c+a)(c-a)}$
19. $c = \sqrt{a^2 + b^2}$
20. $C = 90^\circ = A + B$

21. area $= \dfrac{ab}{2}$

SOLUTION OF OBLIQUE TRIANGLES.

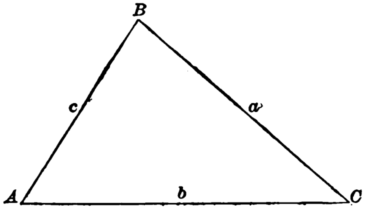

FIG. 108.

	GIVEN.	SOUGHT.	FORMULÆ.
22	A, B, a	C, b, c	$C = 180° - (A + B)$, $b = \frac{a}{\sin A} \cdot \sin B$, $c = \frac{a}{\sin A} \sin (A + B)$
23	A, a, b	B, C, c	$\sin B = \frac{\sin A}{a} \cdot b$, $C = 180° - (A + B)$, $c = \frac{a}{\sin A} \cdot \sin C$.
24	C, a, b	$\frac{1}{2}(A + B)$	$\frac{1}{2}(A + B) = 90° - \frac{1}{2} C$
25		$\frac{1}{2}(A - B)$	$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B)$
26		A, B	$A = \frac{1}{2}(A + B) + \frac{1}{2}(A - B)$, $B = \frac{1}{2}(A + B) - \frac{1}{2}(A - B)$
27		c	$c = (a + b) \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = (a - b) \frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}$
28		area	$K = \frac{1}{2} a b \sin C$.
29	a, b, c	A	Let $s = \frac{1}{2}(a + b + c)$; $\sin \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$
30			$\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$; $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$
31			$\sin A = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{bc}$; $\text{vers } A = \frac{2(s-b)(s-c)}{bc}$
32		area	$K = \sqrt{s(s-a)(s-b)(s-c)}$
33	A, B, C, a	area	$K = \frac{a^2 \sin B \cdot \sin C}{2 \sin A}$

TABLE I.

ELEVATIONS CORRESPONDING TO BAROMETRIC READINGS.*

B.	A.	Diff. for .01	B.	A.	Diff. for .01.	B.	A.	Diff. for .01.
Inches	Feet	Feet	Inches	Feet	Feet	Inches	Feet	Feet
18.0	13,918	−15.1	22.4	7,960	−12.2	26.7	3,175	−10.2
18.1	13,767	15.0	22.5	7,838	12.1	26.8	3,073	10.1
18.2	13,617	14.9	22.6	7,717	12.0	26.9	2,972	10.1
18.3	13,468	14.9	22.7	7,597	12.0	27.0	2,871	10.1
18.4	13,319	14.7	22.8	7,477	11.9	27.1	2,770	10.0
18.5	13,172	14.7	22.9	7,358	11.9	27.2	2,670	10.0
18.6	13,025	14.6	23.0	7,239	11.8	27.3	2,570	10.0
18.7	12,879	14.6	23.1	7,121	11.7	27.4	2,470	9.9
18.8	12,733	14.4	23.2	7,004	11.7	27.5	2,371	9.9
18.9	12,589	14.4	23.3	6,887	11.7	27.6	2,272	9.9
19.0	12,445	14.3	23.4	6,770	11.6	27.7	2,173	9.8
19.1	12,302	14.2	23.5	6,654	11.6	27.8	2,075	9.8
19.2	12,160	14.2	23.6	6,538	11.5	27.9	1,977	9.7
19.3	12,018	14.1	23.7	6,423	11.5	28.0	1,880	9.7
19.4	11,877	14.0	23.8	6,308	11.4	28.1	1,783	9.7
19.5	11,737	13.9	23.9	6,194	11.4	28.2	1,686	9.7
19.6	11,598	13.9	24.0	6,080	11.3	28.3	1,589	9.6
19.7	11,459	13.8	24.1	5,967	11.3	28.4	1,493	9.6
19.8	11,321	13.7	24.2	5,854	11.3	28.5	1,397	9.5
19.9	11,184	13.7	24.3	5,741	11.2	28.6	1,302	9.5
20.0	11,047	13.6	24.4	5,629	11.1	28.7	1,207	9.5
20.1	10,911	13.5	24.5	5,518	11.1	28.8	1,112	9.4
20.2	10,776	13.4	24.6	5,407	11.1	28.9	1,018	9.4
20.3	10,642	13.4	24.7	5,296	11.0	29.0	924	9.4
20.4	10,508	13.3	24.8	5,186	10.9	29.1	830	9.4
20.5	10,375	13.3	24.9	5,077	10.9	29.2	736	9.3
20.6	10,242	13.2	25.0	4,968	10.9	29.3	643	9.3
20.7	10,110	13.1	25.1	4,859	10.8	29.4	550	9.2
20.8	9,979	13.1	25.2	4,751	10.8	29.5	458	9.2
20.9	9,848	13.0	25.3	4,643	10.8	29.6	366	9.2
21.0	9,718	12.9	25.4	4,535	10.7	29.7	274	9.2
21.1	9,589	12.9	25.5	4,428	10.7	29.8	182	9.1
21.2	9,460	12.8	25.6	4,321	10.6	29.9	91	9.1
21.3	9,332	12.8	25.7	4,215	10.6	30.0	00	9.1
21.4	9,204	12.7	25.8	4,109	10.5	30.1	−91	9.0
21.5	9,077	12.6	25.9	4,004	10.5	30.2	181	9.0
21.6	8,951	12.6	26.0	3,899	10.5	30.3	271	9.0
21.7	8,825	12.5	26.1	3,794	10.4	30.4	361	9.0
21.8	8,700	12.5	26.2	3,690	10.4	30.5	451	8.9
21.9	8,575	12.4	26.3	3,586	10.3	30.6	540	8.9
22.0	8,451	12.4	26.4	3,483	10.3	30.7	629	8.8
22.1	8,327	12.3	26.5	3,380	10.3	30.8	717	8.8
22.2	8,204	12.2	26.6	3,277	−10.2	30.9	805	−8.8
22.3	8,082	−12.2	26.7	3,175		31.0	−893	
22.4	7,960							

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* Taken from Appendix 10, Report of U. S. Coast and Geodetic Survey for 1881.

TABLE II.

CORRECTION COEFFICIENTS TO BAROMETRIC ELEVATIONS FOR TEMPERATURE AND HUMIDITY.*

t+*t′*.	*C*.	Diff. for 1°	*t*+*t′*.	*C*.	Diff. for 1°.	*t*+*t′*.	*C*.	Diff. for 1°.
0°	−0 1250	10.9	60°	−0.0380	10.7	120°	+0.0262	10.6
10°	− .0915	10.9	70°	− .0273	10.7	130°	+ .0368	10.4
20°	− .0806	10.8	80°	− .0166	10.8	140°	+ .0472	10.3
30°	− .0698	10.6	90°	− .0058	10.7	150	+ .0575	10.2
40	− .0592	10.6	100°	+ .0049	10.7	160°	+ .0677	10.2
50°	− .0486	10.6	110°	+ .0156	10.6	170°	+ .0779	10.0
60°	− .0380		120°	+ .0262		180°	+ .0879	

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* Compiled from Tables I and IV, Report of U. S. Coast and Geodetic Survey for 1881.

TABLE III.

AMOUNT AND VARIATION OF THE MAGNETIC NEEDLE FROM ITS MEAN DAILY POSITION.

The letters E and W indicate which side of the mean position the needle points.

Season and Position in Latitude.	Local Mean Time: Morning Hours.						
	6 h.	7 h.	8 h.	9 h.	10 h.	11 h.	12 h.
December, January, February:	Min.	Min.	Min.	Min.	Min.	Min.	Min.
Latitude 37° to 49°........	0.7 E	1.1 E	1.9 E	2.2 E	1.5 E	0.1 W	1.8 W
Latitude 25° to 37°........	0.1 W	0.1 E	1.0 E	2.0 E	2.2 E	1.1 E	0.5 W
March, April, May:							
Latitude 37° to 49°........	2.6 E	3.8 E	4.4 E	3.5 E	1.2 E	1.6 E	3.8 W
Latitude 25° to 37°........	1.6 E	2.8 E	3.3 E	2.6 E	1.1 E	0.6 W	1.9 W
June, July, August:							
Latitude 37° to 49°........	4.0 E	5.6 E	5.7 E	4.5 E	1.7 E	1.6 E	4.1 W
Latitude 25° to 37°........	2.4 E	4.0 E	4.2 E	2.9 E	0.5 E	1.6 W	2.8 W
September, October, November:							
Latitude 37° to 49°........	1.8 E	2.6 E	3.1 E	2.5 E	1.0 E	1.5 E	3.3 W
Latitude 25° to 37°........	0.9 E	2.1 E	2.6 E	2.1 E	0.6 E	0.9 E	2.1 W

Season and Position in Latitude.	Local Mean Time: Afternoon Hours.						
	0 h.	1 h.	2 h.	3 h.	4 h.	5 h.	6 h.
December, January February:	Min.	Min.	Min.	Min.	Min.	Min.	Min.
Latitude 37° to 49°........	1.8 W	2.9 W	2.8 W	2.1 W	1.3 W	0.7 W	0.2 W
Latitude 25° to 37°........	0.5 W	1.5 W	1.8 W	1.6 W	1.0 W	0.4 W	0.1 W
March, April, May:							
Latitude 37° to 49°........	3.8 W	4.8 W	4.6 W	3.8 W	2.5 W	1.4 W	0.7 W
Latitude 25° to 37°........	1.9 W	2.6 W	2.8 W	2.4 W	1.6 W	0.9 W	0.5 W
June, July, August:							
Latitude 37° to 49°........	4.1 W	5.6 W	5.6 W	4.6 W	3.0 W	1.4 W	0.6 W
Latitude 25° to 37°........	2.8 W	3.2 W	3.1 W	2.4 W	1.5 W	0.8 W	0.4 W
September, October, November:							
Latitude 37° to 49°........	3.3 W	4.0 W	3.4 W	2.3 W	1.2 W	0.6 W	0.1 W
Latitude 25° to 37°........	2.1 W	2.3 W	1.9 W	1.2 W	0.7 W	0.4 W	0.2 W

TABLE IV.

LOCAL CIVIL TIME OF THE ELONGATIONS OF POLARIS IN THE YEAR 1905.

(Computed for Latitude 40° north and Longitude 90° or 6 h. west of Greenwich.)

Date.	Eastern Elongation.	Western Elongation.
1905.	h. m.	h. m.
January 1	12 46.4 P.M.	12 39.3 A.M.
January 15	11 51.1 A.M.	11 41.1 P.M.
February 1	10 44.0 A.M.	10 34.0 P.M.
February 15	9 48.7 A.M.	9 38.7 P.M.
March 1	8 53.5 A.M.	8 43.5 P.M.
March 15	7 58.3 A.M.	7 48.3 P.M.
April 1	6 51.3 A.M.	6 41.3 P.M.
April 15	5 56.3 A.M.	5 46.3 P.M.
May 1	4 53.5 A.M.	4 43.5 P.M.
May 15	3 58.5 A.M.	3 48.5 P.M.
June 1	2 51.9 A.M.	2 41.9 P.M.
June 15	1 57.1 A.M.	1 47.1 P.M.
July 1	12 54.4 A.M.	12 44.4 P.M.
July 15	11 55.7 P.M.	11 49.6 A.M.
August 1	10 49.1 P.M.	10 43.1 A.M.
August 15	9 54.3 P.M.	9 48.2 A.M.
September 1	8 47.7 P.M.	8 41.6 A.M.
September 15	7 52.8 P.M.	7 46.7 A.M.
October 1	6 50.0 P.M.	6 43.9 A.M.
October 15	5 55.0 P.M.	5 48.9 A.M.
November 1	4 48.2 P.M.	4 42.1 A.M.
November 15	3 53.0 P.M.	3 47.0 A.M.
December 1	2 50.0 P.M.	2 43.9 A.M.
December 15	1 54.8 P.M.	1 48.7 A.M.

To find the time of culmination. The time between either elongation and upper culmination is about 5 hrs. 55 m., while between either elongation and lower culmination it is about 6 hrs. 3 m. (See p. 358.)

A. To refer the above tabular quantities to years subsequent to 1905:

For year 1907	add	2.5	minutes	
1908	"	3.8	"	up to March 1
1908	subtract	0.1	"	on and after March 1
1909	add	1.1	"	
1910	"	2.3	"	
1911	"	3.6	"	
1912	"	5.0	"	up to March 1
1912	"	1.1	"	on and after March 1
1913	"	2.5	"	
1914	"	3.9	"	
1915	"	5.4	"	
1916	"	6.9	"	up to March 1
1916	"	3.0	"	on and after March 1
1917	"	4.6	"	
	"	6.2	"	
	"	7.8	"	
	"	9.4	"	up to March 1
	"	5.5	"	on and after March 1

B. *To refer to any calendar day other than the first and fifteenth of each month,* SUBTRACT *the quantities below from the tabular quantity for the* PRECEDING DATE *or* ADD *to the tabular quantity for the* FOLLOWING DATE:

Day of Month.	Minutes.	No. of Days Elapsed.
2 or 16	3.9	1
3 17	7.9	2
4 18	11.8	3
5 19	15.7	4
6 20	19.6	5
7 21	23.6	6
8 22	27.5	7
9 23	31.4	8
10 24	35.4	9
11 25	39.3	10
12 26	43.2	11
13 27	47.2	12
14 28	51.1	13
29	55.0	14
30	58.9	15
31	62.9	16

For the tabular year, two eastern elongations occur on July 14, and two western elongations on January 11.

C. *To refer the table to standard time.* ADD to the tabular quantities four minutes for every degree of longitude the place is west of the standard meridian and SUBTRACT when the place is east of the standard meridian.

D. *To refer to any other than the tabular latitude between the limits of* 25° *and* 50° *North:* ADD to the time of WEST elongation $0^{\text{M}}.12$ for every degree south of 40° and SUBTRACT from the time of WEST elongation $0^{\text{M}}.17$ for every degree north of 40°. For EASTERN elongations SUBTRACT $0^{\text{M}}.12$ for every degree south of 40° and ADD $0^{\text{M}}.17$ for every degree north of 40°.

E. *To refer to any other than the tabular longitude;* ADD $0^{\text{M}}.16$ for each 15° east of the ninetieth meridian and SUBTRACT $0^{\text{M}}.16$ for each 15° west of the ninetieth meridian.

The deduced time of elongation will seldom be in error more than $0'.3$, especially before 1911.

Note. In the *Practical Suggestions* on page 357 are given complete directions for applying the above rules, together with an example illustrating the method of finding the time of elongation.

Table IV and the accompanying rules have been kindly furnished for this book by the Superintendent of the U. S. Coast and Geodetic Survey.

TABLE

AZIMUTH OF POLARIS WHEN AT ELONGATION

Latitude.	1905	1906	1907	1908	1909	1910	1911	1912
25°	1° 19′.4	1° 19′.1	1° 18′.7	1° 18′.4	1° 18′.1	1° 17′.7	1° 17′.4	1° 17′.0
26	20 .1	19 .8	19 .4	19 .1	18 .7	18 4	18 .0	17 .7
27	20 .8	20 .5	20 .1	19 .8	19 .4	19 .1	19 .1	18 .4
28	21 .6	21 .3	20. 9	20 .5	20 .1	19 .8	19 .4	19 .1
29	22 .4	22 .1	21 .7	21 .3	20 .9	20 .5	20 .2	19 .8
30	23 .1	22 .8	22 .4	22 .1	21 .7	21 .3	21 .0	20 .6
31	24 .0	23 .6	23 .2	22 .5	22 .2	21 .2	21 .8	21 .5
32	24 .9	24 .5	24 .1	23 .8	23 .4	23 .1	22 .7	22 .3
33	25 .9	25 .5	25 .1	24 .7	24 .3	24 .0	23 .6	23 .3
34	26 .9	26 .5	26 .1	25 .7	25 .3	25 .0	24 .6	24 .2
35	27 .9	27 .5	27 .1	26 .8	26 .4	26 .0	25 .6	25 .2
36	29 .0	28 .6	28 .2	27 .9	27 .5	27 .1	26 .7	26 .1
37	30 .1	29 .7	29 .3	29 .0	28 .6	28 .2	27 .8	27 .4
38	31 .4	31 .0	30 .6	30 .2	29 .8	29 .4	29 .0	28 .6
39	32 .7	32 .3	31 .8	31 .4	31 .0	30 .6	30 .2	29 .8
40	34 .0	33 .6	33 .2	32 .8	32 .4	32 .0	31 .6	31 .1
41	35 .4	35 .0	34 .6	34 2	33 .8	33 .4	32 .9	32 .5
42	36 .9	36 5	36 .0	35 .6	35 2	34 .8	34 .2	34 .0
43	38 .5	38 .1	37 .6	37 .2	36 .8	36 .3	35 .9	35 .5
44	40 .1	39 .7	39 .2	38 .8	38 .4	37 .9	37 .5	37 .1
45	41 .8	41 .4	40 .9	40 .5	40 .1	39 .6	39 .2	38 .7
46	43 .7	43 .2	42 .7	42 .3	41 .9	41 .4	41 .0	40 .5
47	45 .6	45 .1	44 .6	44 .2	43 .7	43 .3	42 .8	42 .4
48	47 .7	47 .2	46 .7	46 .3	45 .8	45 .3	44 .8	44 .4
49	49 .8	49 .3	48 .8	48 .4	47 .9	47 .4	46 .9	46 .4
50	1° 52′.0	1° 51′.5	1° 51′.0	1° 50′.6	1° 50′.1	1° 49′.6	1° 49′.1	1° 48′.1

The above Table was computed with mean declination of Polaris for each year. A more accurate result will be had by applying to the tabular values the following correction, which depends on the difference of the mean and the apparent place of the star. (See Note in Art. 605.) The deduced azimuth will, in general, be correct within 0′.3.

The azimuths in Table V are reckoned clockwise from the true north for the eastern elongation, and counter-clockwise from the true north for the western elongation. For intermediate latitudes values may be obtained by interpolation without in general exceeding an error of 0′.3. Table V and the supplementary table have been kindly furnished by the Superintendent of the U. S. Coast and Geodetic Survey for publication in this book.

V

FOR ANY YEAR BETWEEN 1905 AND 1920

Latitude.	1913	1914	1915	1916	1917	1918	1919	1920
25°	1° 16′ .7	1° 16′ .4	1° 16′ .0	1° 15′ .7	1° 15′ .3	1° 15′ .0	1° 14′ .7	1° 14′ .3
26	17 .3	16 .6	16 .3	16 .3	16 .0	15 .5	15 .3	14 .9
27	18 .0	17 .7	17 .3	17 .0	16 .6	16 .3	15 .9	15 .6
28	18 .7	18 .4	18 .0	17 .7	17 .3	17 .0	16 .6	16 .3
29	19 .5	19 .1	18 .8	18 .4	18 .1	17 .7	17 .4	17 .0
30	20 .3	19 .9	19 .6	19 .2	18 .8	18 .5	18 .1	17 .8
31	21 .1	20 .7	20 .4	20 .0	19 .7	19 .3	18 .9	18 .6
32	22 .0	21 .6	21 .2	20 .9	20 .5	20 .1	19 .8	19 .4
33	22 .9	22 .5	22 .1	21 .8	21 .4	21 .0	20 .7	20 .3
34	23 .8	23 .5	23 .1	22 .7	22 .4	22 .0	21 .6	21 .2
35	24 .9	24 .5	24 .1	23 .7	23 .3	23 .0	22 .6	22 .2
36	25 .9	25 .5	25 .2	24 .8	24 .4	24 .0	23 .6	23 .3
37	27 .0	26 .7	26 .3	25 .9	25 .5	25 .7	24 .7	24 .3
38	28 .2	27 .8	27 .4	27 .0	26 .0	26 .2	25 .9	25 .5
39	29 .4	29 .0	28 .6	28 .2	27 .8	27 .5	27 .1	26 .7
40	30 .7	30 .3	29 .9	29 .5	29 .1	28 .7	28 .3	27 .9
41	32 .1	31 .7	31 .3	30 .9	30 .4	30 .0	29 .6	29 .1
42	33 .5	33 .1	32 .7	32 .3	31 .9	31 .5	31 .0	30 .5
43	35 .0	34 .6	34 .2	33 .8	33 .4	32 .9	32 .5	32 .1
44	36 .6	36 .2	35 .8	35 .3	34 .9	34 .5	34 .1	33 .6
45	38 .3	37 .8	37 .4	37 .0	36 .1	36 .1	35 .7	35 .3
46	40 .1	39 .6	39 .2	38 .7	38 .3	37 .8	37 .4	37 .0
47	41 .9	41 .5	41 .0	40 .6	40 .1	39 .7	39 .2	38 .8
48	43 .9	43 .4	43 .0	42 .5	42 .0	41 .6	41 .1	40 .7
49	46 .0	45 .5	45 .0	44 .5	44 .1	43 .6	43 .1	42 .7
50	1° 48 .2	1° 47′ .7	1° 47′ .2	1° 46′ .2	1° 46′ .2	1° 45′ .7	1° 45′ .3	1° 44′ .8

For Middle of	Correction.	For Middle of	Correction.
January	−0′.5	July	+0′.2
February	−0 .4	August	+0 .1
March	−0 .3	September	−0 .1
April	−0 .1	October	−0 .4
May	+0 .1	November	−0 .6
June	+0 .2	December	−0 .8

TO REDUCE DEGREES TO TIME.									TO REDUCE TIME TO DEGREES.							
°	H.	M.	°	H.	M.	Degrees.	Hours.	Minutes.	Hours.	Degrees.	M.	°	′	M.	°	′
′	M.	S.	′	M.	S.						S.	′	″	S.	′	″
″	S.	T.	″	S.	T.						T.	″	‴	T.	″	‴
1	0	4	51	3	24	101	6	44	1	15	1	0	15	51	12	45
2	0	8	52	3	28	102	6	48	1½	22½	2	0	30	52	13	0
3	0	12	53	3	32	103	6	52	2	30	3	0	45	53	13	15
4	0	16	54	3	36	104	6	56	2½	37½	4	1	0	54	13	30
5	0	20	55	3	40	105	7	0	3	45	5	1	15	55	13	45
6	0	24	56	3	44	106	7	4	3½	52½	6	1	30	56	14	0
7	0	28	57	3	48	107	7	8	4	60	7	1	45	57	14	15
8	0	32	58	3	52	108	7	12	4½	67½	8	2	0	58	14	30
9	0	36	59	3	56	109	7	16	5	75	9	2	15	59	14	45
10	0	40	60	4	0	110	7	20	5½	82½	10	2	30	60	15	0
11	0	44	61	4	4	115	7	40	6	90	11	2	45	61	15	15
12	0	48	62	4	8	120	8	0	6½	97½	12	3	0	62	15	30
13	0	52	63	4	12	125	8	20	7	105	13	3	15	63	15	45
14	0	56	64	4	16	130	8	40	7½	112½	14	3	30	64	16	0
15	1	0	65	4	20	135	9	0	8	120	15	3	45	65	16	15
16	1	4	66	4	24	140	9	20	8½	127½	16	4	0	66	16	30
17	1	8	67	4	28	145	9	40	9	135	17	4	15	67	16	45
18	1	12	68	4	32	150	10	0	9½	142½	18	4	30	68	17	0
19	1	16	69	4	36	155	10	20	10	150	19	4	45	69	17	15
20	1	20	70	4	40	160	10	40	10½	157½	20	5	0	70	17	30
21	1	24	71	4	44	165	11	0	11	165	21	5	15	71	17	45
22	1	28	72	4	48	170	11	20	11½	172½	22	5	30	72	18	0
23	1	32	73	4	52	175	11	40	12	180	23	5	45	73	18	15
24	1	36	74	4	56	180	12	0	12½	187½	24	6	0	74	18	30
25	1	40	75	5	0	185	12	20	13	195	25	6	15	75	18	45
26	1	44	76	5	4	190	12	40	13½	202½	26	6	30	76	19	0
27	1	48	77	5	8	195	13	0	14	210	27	6	45	77	19	15
28	1	52	78	5	12	200	13	20	14½	217½	28	7	0	78	19	30
29	1	56	79	5	16	205	13	40	15	225	29	7	15	79	19	45
30	2	0	80	5	20	210	14	0	15½	232½	30	7	30	80	20	0
31	2	4	81	5	24	215	14	20	16	240	31	7	45	81	20	15
32	2	8	82	5	28	220	14	40	16½	247½	32	8	0	82	20	30
33	2	12	83	5	32	225	15	0	17	255	33	8	15	83	20	45
34	2	16	84	5	36	230	15	20	17½	262½	34	8	30	84	21	0
35	2	20	85	5	40	235	15	40	18	270	35	8	45	85	21	15
36	2	24	86	5	44	240	16	0	18½	277½	36	9	0	86	21	30
37	2	28	87	5	48	245	16	20	19	285	37	9	15	87	21	45
38	2	32	88	5	52	250	16	40	19½	292½	38	9	30	88	22	0
39	2	36	89	5	56	255	17	0	20	300	39	9	45	89	22	15
40	2	40	90	6	0	260	17	20	20½	307½	40	10	0	90	22	30
41	2	44	91	6	4	270	18	0	21	315	41	10	15	91	22	45
42	2	48	92	6	8	280	18	40	21½	322½	42	10	30	92	23	0
43	2	52	93	6	12	290	19	20	22	330	43	10	45	93	23	15
44	2	56	94	6	16	300	20	0	22½	337½	44	11	0	94	23	30
45	3	0	95	6	20	310	20	40	23	345	45	11	15	95	23	45
46	3	4	96	6	24	320	21	20	23½	352½	46	11	30	96	24	0
47	3	8	97	6	28	330	22	0	24	360	47	11	45	97	24	15
48	3	12	98	6	32	340	22	40			48	12	0	98	24	30
49	3	16	99	6	36	350	23	20			49	12	15	99	24	45
50	3	20	100	6	40	360	24	0			50	12	30	100	25	0

TABLE VII.

REFRACTIONS IN TERMS OF LATITUDE, HOUR-ANGLE, AND SUN'S DECLINATION.

A TABLE OF MEAN REFRACTIONS IN DECLINATION TO BE USED WITH THE SOLAR ATTACHMENT.

Apply to the Declination as Found in the Ephemeris.

Hour-angle.	Declinations.								
	For Latitude 20°.								
	+20°	+15°	+10°	+5°	0°	−5°	−10°	−15°	−20°
0 h	0″	05″	10″	15″	21″	27″	33″	40″	48″
2	03	07	13	18	24	30	36	44	52
3	06	13	18	24	30	36	44	52	1′ 02
4	17	22	28	35	42	50	1′ 00	1′ 11	1 26
5	39	47	57	1′ 07	1′ 20	1′ 37	2 00	2 32	3 25
	For Latitude 30°.								
0 h	10″	15″	21″	27″	33″	40″	48″	57″	1′ 08″
2	14	19	25	31	38	46	54	1′ 05	1 18
3	20	26	32	39	47	55	1′ 06	1 19	1 36
4	32	39	46	52	1′ 06	1′ 19	1 35	1 57	2 29
5	1′ 00	1′ 10	1′ 24	1′ 52	2 07	2 44	3 46	5 43	13 06
	For Latitude 40°.								
0 h	21″	27″	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″
2	25	32	39	46	52	1′ 06	1 19	1 35	1 57
3	33	40	48	57	1′ 08	1 21	1 38	2 02	2 36
4	47	55	1′ 06	1′ 19	1 36	1 58	2 30	3 21	4 59
5	1′ 15	1′ 31	1 51	2 20	3 05	4 25	7 34	25 18	
	For Latitude 50°								
0 h	33″	40″	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″
2	38	46	55	1′ 06	1 18	1 35	1 57	2 28	3 19
3	47	56	1′ 06	1 19	1 36	2 29	2 31	3 23	5 02
4	1′ 02	1′ 14	1 29	1 48	2 16	2 58	4 18	6 59	19 47
5	1 30	1 51	2 19	3 04	4 22	7 28	24 10		
	For Latitude 60°.								
0 h	48″	57″	1′ 08″	1′ 21″	1′ 39″	2′ 02″	2′ 36″	3′ 33″	5′ 23″
2	54	1′ 04	1 17	1 33	1 54	2 24	3 12	4 38	8 15
3	1′ 03	1 15	1 30	1 51	2 20	3 04	4 24	7 31	24 44
4	1 18	1 34	1 56	2 28	3 18	4 50	8 53		
5	1 45	2 11	2 50	3 57	6 21	15 32			

DECIMALS OF A FOOT FOR EACH 32D OF AN INCH.

In.	0	1	2	3	4	5	6	7	8	9	10	11	In.
0	Foot	.0833	.1667	.2500	.3333	.4167	.5000	.5833	.6667	.7500	.8333	.9167	0
1-32	.0026	.0859	.1693	.2526	.3359	.4193	.5026	.5859	.6693	.7526	.8359	.9193	1-32
1-16	.0052	.0885	.1719	.2552	.3385	.4219	.5052	.5885	.6719	.7552	.8385	.9219	1-16
3-32	.0078	.0911	.1745	.2578	.3411	.4245	.5078	.5911	.6745	.7578	.8411	.9245	3-32
1-8	.0104	.0938	.1771	.2604	.3438	.4271	.5104	.5938	.6771	.7604	.8438	.9271	1-8
5-32	.0130	.0964	.1797	.2630	.3464	.4297	.5130	.5964	.6797	.7630	.8464	.9297	5-32
3-16	.0156	.0990	.1823	.2656	.3490	.4323	.5156	.5990	.6823	.7656	.8490	.9323	3-16
7-32	.0182	.1016	.1849	.2682	.3516	.4349	.5182	.6016	.6849	.7682	.8516	.9349	7-32
1-4	.0208	.1042	.1875	.2708	.3542	.4375	.5208	.6042	.6875	.7708	.8542	.9375	1-4
9-32	.0234	.1068	.1901	.2734	.3568	.4401	.5234	.6068	.6901	.7734	.8568	.9401	9-32
5-16	.0260	.1094	.1927	.2760	.3594	.4427	.5260	.6094	.6927	.7760	.8594	.9427	5-16
11-32	.0286	.1120	.1953	.2786	.3620	.4453	.5286	.6120	.6953	.7786	.8620	.9453	11-32
3-8	.0313	.1146	.1979	.2813	.3646	.4479	.5313	.6146	.6979	.7813	.8646	.9479	3-8
13-32	.0339	.1172	.2005	.2839	.3672	.4505	.5339	.6172	.7005	.7839	.8672	.9505	13-32
7-16	.0365	.1198	.2031	.2865	.3698	.4531	.5365	.6198	.7031	.7865	.8698	.9531	7-16
15-32	.0391	.1224	.2057	.2891	.3724	.4557	.5391	.6224	.7057	.7891	.8724	.9557	15-32
1-2	.0417	.1250	.2083	.2917	.3750	.4583	.5417	.6250	.7083	.7917	.8750	.9583	1-2
17-32	.0443	.1276	.2109	.2943	.3776	.4609	.5443	.6276	.7109	.7943	.8776	.9609	17-32
9-16	.0469	.1302	.2135	.2969	.3802	.4635	.5469	.6302	.7135	.7969	.8802	.9635	9-16
19-32	.0495	.1328	.2161	.2995	.3828	.4661	.5495	.6328	.7161	.7995	.8828	.9661	19-32
5-8	.0521	.1354	.2188	.3021	.3854	.4688	.5521	.6354	.7188	.8021	.8854	.9688	5-8
21-32	.0547	.1380	.2214	.3047	.3880	.4714	.5547	.6380	.7214	.8047	.8880	.9714	21-32
11-16	.0573	.1406	.2240	.3073	.3906	.4740	.5573	.6406	.7240	.8073	.8906	.9740	11-16
23-32	.0599	.1432	.2266	.3099	.3932	.4766	.5599	.6432	.7266	.8099	.8932	.9766	23-32
3-4	.0625	.1458	.2292	.3125	.2958	.4792	.5625	.6458	.7292	.8125	.8958	.9792	3-4
25-32	.0651	.1484	.2318	.3151	.3984	.4818	.5651	.6484	.7318	.8151	.8984	.9818	25-32
13-16	.0677	.1510	.2344	.3177	.4010	.4844	.5677	.6510	.7344	.8177	.9010	.9844	13-16
27-32	.0703	.1536	.2370	.3203	.4036	.4870	.5703	.6536	.7370	.8203	.9036	.9870	27-32
7-8	.0729	.1563	.2396	.3229	.4063	.4896	.5729	.6563	.7396	.8229	.9063	.9896	7-8
29-32	.0755	.1589	.2422	.3255	.4089	.4922	.5755	.6589	.7422	.8255	.9089	.9922	29-32
15-16	.0781	.1615	.2448	.3281	.4115	.4948	.5781	.6615	.7448	.8281	.9115	.9948	15-16
31-32	.0807	.1641	.2474	.3307	.4141	.4974	.5807	.6641	.7474	.8307	.9141	.9974	31-32
	C	1	2	3	4	5	6	7	8	9	10	11	

TABLE IX.

REDUCTION OF INCLINED DISTANCES TO THE HORIZONTAL.

Inclined Distance = 100 feet.

Slope.	Correction.	Horizontal Distance.	Slope.	Correction.	Horizontal Distance.
0° 00′	....	100.000	8° 00′	0.973	99.027
30	0.004	99.996	30	1.096	98.902
1 00	0.015	99.985	9 00	1.231	98.769
30	0.034	99.966	30	1.371	98.629
2 00	0.061	99.939	10 00	1.519	98.481
30	0.095	99.905	30	1.675	98.325
3 00	0.137	99.863	11 00	1.837	98.163
30	0.187	99.813	30	2.008	97.992
4 00	0.244	99.756	12 00	2.185	97.814
30	0.308	99.692	30	2.370	97.630
5 00	0.381	99.619	13 00	2.563	97.437
30	0.460	99.540	30	2.763	97.237
6 00	0.548	99.452	14 00	2.970	97.030
30	0.643	99.357	30	3.185	96.815
7 00	0.745	99.255	15 00	3.407	96.593
30	0.856	99.144	30	3.637	96.363

TABLE X.

REDUCTION OF STADIA READINGS

TO

HORIZONTAL DISTANCES

AND TO

DIFFERENCES OF ELEVATION.

This table was computed by Professor Arthur Winslow, State Geologist of Missouri.

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Dif Ele
0′	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.2
2	"	.06	"	1.80	99.87	3.55	99.72	5.2
4	"	.12	"	1.86	"	3.60	99.71	5.3
6	"	.17	99.96	1.92	"	3.66	"	5.4
8	"	.23	"	1.98	99.86	3.72	99.70	5.4
10	"	.29	"	2.04	"	3.78	99.69	5.5
12	"	.35	"	2.09	99.85	3.84	"	5.5
14	"	.41	99.95	2.15	"	3.90	99.68	5.6
16	"	.47	"	2.21	99.84	3.95	"	5.6
18	"	.52	"	2.27	"	4.01	99.67	5.7
20	"	.58	"	2.33	99.83	4.07	99.66	5.8
22	"	.64	99.94	2.38	"	4.13	"	5.8
24	"	.70	"	2.44	99.82	4.18	99.65	5.9
26	99.99	.76	"	2.50	"	4.24	99.64	5.9
28	"	.81	99.93	2.56	99.81	4.30	99.63	6.0
30	"	.87	"	2.62	"	4.36	"	6.0
32	"	.93	"	2.67	99.80	4.42	99.62	6.1
34	"	.99	"	2.73	"	4.48	"	6.2
36	"	1.05	99.92	2.79	99.79	4.53	99.61	6.2
38	"	1.11	"	2.85	"	4.59	99.60	6.3
40	"	1.16	"	2.91	99.78	4.65	99.59	6.3
42	"	1.22	99.91	2.97	"	4.71	"	6.4
44	99.98	1.28	"	3.02	99.77	4.76	99.58	6.5
46	"	1.34	99.90	3.08	"	4.82	99.57	6.5
48	"	1.40	"	3.14	99.76	4.88	99.56	6.6
50	"	1.45	"	3.20	"	4.94	"	6.6
52	"	1.51	99.89	3.26	99.75	4.99	99.55	6.7
54	"	1.57	"	3.31	99.74	5.05	99.54	6.7
56	99.97	1.63	"	3.37	"	5.11	99.53	6.8
58	"	1.69	99.88	3.43	99.73	5.17	99.52	6.9
60	"	1.74	"	3.49	"	5.23	99.51	6.9
$c+f=$ .75	.75	.01	.75	.02	.75	.03	.75	.0
$c+f=$ 1.00	1.00	.01	1.00	.03	1.00	.04	1.00	.0
$c+f=$ 1.25	1.25	.02	1.25	.03	1.25	.05	1.25	.0

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	"	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.48	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.26
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.42	12.43
14	"	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.29	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.23	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
$+f = .75$	.75	.06	.75	.07	.75	.08	.74	.10
$+f = 1.00$	1.00	.08	.99	.09	.99	.11	.99	.13
$+f = 1.25$	1.25	.10	1.24	.11	1.24	.14	1.24	.16

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Dif Ele
0′	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.
$c + f =$ .75	.74	.11	.74	.12	.74	.14	.73	.
$c + f =$ 1.00	.99	.15	.99	.16	.98	.18	.98	.
$c + f =$ 1.25	1.23	.18	1.23	.21	1.23	.22	1.22	.

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.81	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
$c+f=$ .75	.73	.16	.73	.17	.73	.19	.72	.20
$c+f=$ 1.00	.98	.22	.97	.23	.97	.25	.96	.27
$c+f=$ 1.25	1.22	.27	1.21	.29	1.21	.31	1.20	.34

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0'	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.97	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.19	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.75	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.66	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.48	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
c + f = .75	.72	.21	.72	.23	.71	.24	.71	.25
c + f = 1.00	.96	.28	.95	.30	.95	.32	.94	.33
c + f = 1.25	1.20	.36	1.19	.38	1.19	.40	1.18	.42

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	20°		21°		22°		23°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0′	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
$c + f = .75$	.70	.26	.70	.27	.69	.29	.69	.30
$c + f = 1.00$	.94	.35	.93	.37	.92	.38	.92	.40
$c + f = 1.25$	1.17	.44	1.16	.46	1.15	.48	1.15	.50

TABLE X.

STADIA REDUCTIONS FOR READING 100.

Minutes.	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Dif Ele
0′	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.4
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.4
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.5
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.5
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.
16	83.11	37.47	81.78	38 60	80.41	39.69	79.01	40.
18	83 07	37.51	81.74	38.64	80.37	39.72	78.96	40.
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.4
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.4
$c+f=$ .75	.68	.31	.68	.32	.67	.33	.66	.
$c+f=$ 1.00	.91	.41	.90	.43	.89	.45	.89	.4
$c+f=$ 1.25	1.14	.52	1.13	.54	1.12	.56	1.11	.

′	0″	10″	15″	20″	30″	40″	45″	50″	′
0	.00000	00278	.00417	.00556	.00833	.01111	.01250	.01389	0
1	.01667	.01944	.02083	.02222	.02500	.02778	.02917	.03055	1
2	.03333	.03611	.03750	.03889	.04167	.04444	.04583	.04722	2
3	.05000	.05278	.05417	.05556	.05833	.06111	.06250	.06389	3
4	.06667	.06944	.07083	.07222	.07500	.07778	.07917	.08056	4
5	.08333	.08611	.08750	.08889	.09167	.09444	.09583	.09722	5
6	.10000	.10278	.10417	.10556	.10833	.11111	.11250	.11389	6
7	.11667	.11944	.12083	.12222	.12500	.12778	.12917	.13056	7
8	.13333	.13611	.13750	.13889	.14167	.14444	.14583	.14722	8
9	.15000	.15278	.15417	.15556	.15833	.16111	.16250	.16389	9
10	.16667	.16944	.17083	.17222	.17500	.17778	.17917	.18056	10
11	.18333	.18611	.18750	.18889	.19167	.19444	.19583	.19722	11
12	.20000	.20278	.20417	.20556	.20833	.21111	.21250	.21389	12
13	.21667	.21944	.22083	.22222	.22500	.22778	.22917	.23056	13
14	.23333	.23611	.23750	.23889	.24167	.24444	.24583	.24722	14
15	.25000	.25278	.25417	.25556	.25833	.26111	.26250	.26389	15
16	.26667	.26944	.27083	.27222	.27500	.27778	.27917	.28056	16
17	.28333	.28611	.28750	.28889	.29167	.29444	.29583	.29722	17
18	.30000	.30278	.30417	.30556	.30833	.31111	.31250	.31389	18
19	.31667	.31944	.32083	.32222	.32500	.32778	.32917	.33056	19
20	.33333	.33611	.33750	.33889	.34167	.34444	.34583	.34722	20
21	.35000	.35278	.35417	.35556	.35833	.36111	.36250	.36389	21
22	.36667	.36944	.37083	.37222	.37500	.37778	.37917	.38056	22
23	.38333	.38611	.38750	.38889	.39167	.39444	.39583	.39722	23
24	.40000	.40278	.40417	.40556	.40833	.41111	.41250	.41389	24
25	.41667	.41944	.42083	.42222	.42500	.42778	.42917	.43056	25
26	.43333	.43611	.43750	.43889	.44167	.44444	.44583	.44722	26
27	.45000	.45278	.45417	.45556	.45833	.46111	.46250	.46389	27
28	.46667	.46944	.47083	.47222	.47500	.47778	.47917	.48056	28
29	.48333	.48611	.48750	.48889	.49167	.49444	.49583	.49722	29
30	.50000	.50278	.50417	.50556	.50833	.51111	.51250	.51389	30
31	.51667	.51944	.52083	.52222	.52500	.52778	.52917	.53056	31
32	.53333	.53611	.53750	.53889	.54167	.54444	.54583	.54722	32
33	.55000	.55278	.55417	.55556	.55833	.56111	.56250	.56389	33
34	.56667	.56944	.57083	.57222	.57500	.57778	.57917	.58056	34
35	.58333	.58611	.58750	.58889	.59167	.59444	.59583	.59722	35
36	.60000	.60278	.60417	.60556	.60833	.61111	.61250	.61389	36
37	.61667	.61944	.62083	.62222	.62500	.62778	.62917	.63056	37
38	.63333	.63611	.63750	.63889	.64167	.64444	.64583	.64722	38
39	.65000	.65278	.65417	.65556	.65833	.66111	.66250	.66389	39
40	.66667	.66944	.67083	.67222	.67500	.67778	.67917	.68056	40
41	.68333	.68611	.68750	.68889	.69167	.69444	.69583	.69722	41
42	.70000	.70278	.70417	.70556	.70833	.71111	.71250	.71389	42
43	.71667	.71944	.72083	.72222	.72500	.72778	.72917	.73056	43
44	.73333	.73611	.73750	.73889	.74167	.74444	.74583	.74722	44
45	.75000	.75278	.75417	.75556	.75833	.76111	.76250	.76389	45
46	.76667	.76944	.77083	.77222	.77500	.77778	.77917	.78056	46
47	.78333	.78611	.78750	.78889	.79167	.79444	.79583	.79722	47
48	.80000	.80278	.80417	.80556	.80833	.81111	.81250	.81389	48
49	.81667	.81944	.82083	.82222	.82500	.82778	.82917	.83056	49
50	.83333	.83611	.83750	.83889	.84167	.84444	.84583	.84722	50
51	.85000	.85278	.85417	.85556	.85833	86111	.86250	.86389	51
52	.86667	.86944	.87083	.87222	.87500	.87778	.87917	.88056	52
53	.88333	.88611	.88750	.88889	.89167	.89444	.89583	.89722	53
54	.90000	.90278	.90417	.90556	.90833	.91111	.91250	.91389	54
55	.91667	.91944	.92083	.92222	.92500	.92778	.92917	.93056	55
56	.93333	.93611	.93750	.93889	.94167	.94444	.94583	.94722	56
57	.95000	.95278	.95417	.95556	.95833	.96111	.96250	.96389	57
58	.96667	.96944	.97083	.97222	.97500	.97778	.97917	.98056	58
59	.98333	.98611	.98750	.98889	.99167	.99444	.99583	.99722	59
′	0″	10″	15″	20″	30″	40″	45″	50″	′

TABLE XII.

CONVERSION OF ENGLISH INCHES INTO CENTIMETRES.

Ins.	0	1	2	3	4	5	6	7	8	9
	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.	Cm.
0	0.000	2.540	5.080	7.620	10.16	12.70	15.24	17.78	20.32	22.86
10	25.40	27.94	30.48	33.02	35.56	38.10	40.64	43.18	45.72	48.26
20	50.80	53.34	55.88	58.42	60.96	63.50	66.04	68.58	71.12	73.66
30	76.20	78.74	81.28	83.82	86.36	88.90	91.44	93.98	96.52	99.06
40	101.60	104.14	106.68	109.22	111.76	114.30	116.84	119.38	121.92	124.46
50	127.00	129.54	132.08	134.62	137.16	139.70	142.24	144.78	147.32	149.86
60	152.40	154.94	157.48	160.02	162.56	165.10	167.64	170.18	172.72	175.26
70	177.80	180.34	182.88	185.42	187.96	190.50	193.04	195.58	198.12	200.66
80	203.20	205.74	208.28	210.82	213.36	215.90	218.44	220.98	223.52	226.06
90	228.60	231.14	233.68	236.22	238.76	241.30	243.84	246.38	248.92	251.46
100	254.00	256.54	259.08	261.62	264.16	266.70	269.24	271.78	274.32	276.86

CONVERSION OF CENTIMETRES INTO ENGLISH INCHES.

Cm.	0	1	2	3	4	5	6	7	8	9
	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.	Ins.
0	0.000	0.394	0.787	1.181	1.575	1.969	2.362	2.756	3.150	3.543
10	3.937	4.331	4.742	5.118	5.512	5.906	6.299	6.693	7.087	7.480
20	7.874	8.268	8.662	9.055	9.449	9.843	10.236	10.630	11.024	11.418
30	11.811	12.205	12.599	12.992	13.386	13.780	14.173	14.567	14.961	15.355
40	15.748	16.142	16.536	16.929	17.323	17.717	18.111	18.504	18.898	19.292
50	19.685	20.079	20.473	20.867	21.260	21.654	22.048	22.441	22.835	23.229
60	23.622	24.016	24.410	24.804	25.197	25.591	25.985	26.378	26.772	27.166
70	27.560	27.953	28.347	28.741	29.134	29.528	29.922	30.316	30.709	31.103
80	31.497	31.890	32.284	32.678	33.071	33.465	33.859	34.253	34.646	35.040
90	35.434	35.827	36.221	36.615	37.009	37.402	37.796	38.190	38.583	38.977
100	39.370	39.764	40.158	40.552	40.945	41.339	41.733	42.126	42.520	42.914

CONVERSION OF ENGLISH FEET INTO METRES.

Feet.	0	1	2	3	4	5	6	7	8	9
	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.	Met.
0	0.000	0.3048	0.6096	0.9144	1.2192	1.5239	1.8287	2.1335	2.4383	2.7431
10	3.0479	3.3527	3.6575	3.9623	4.2671	4.5719	4.8767	5.1815	5.4863	5.7911
20	6.0959	6.4006	6.7055	7.0102	7.3150	7.6198	7.9246	8.2294	8.5342	8.8390
30	9.1438	9.4486	9.7534	10.058	10.363	10.668	10.972	11.277	11.582	11.887
40	12.192	12.496	12.801	13.106	13.411	13.716	14.020	14.325	14.630	14.935
50	15.239	15.544	15.849	16.154	16.459	16.763	17.068	17.373	17.678	17.983
60	18.287	18.592	18.897	19.202	19.507	19.811	20.116	20.421	20.726	21.031
70	21.335	21.640	21.945	22.250	22.555	22.859	23.164	23.469	23.774	24.079
80	24.383	24.688	24.993	25.298	25.602	25.907	26.212	26.517	26.822	27.126
90	27.431	27.736	28.041	28.346	28.651	28.955	29.260	29.565	29.870	30.174
100	30.479	30.784	31.089	31.394	31.698	32.003	32.308	32.613	32.918	33.222

CONVERSION OF METRES INTO ENGLISH FEET.

Met.	0	1	2	3	4	5	6	7	8	9
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0	0.000	3.2809	6.5618	9.8427	13.123	16.404	19.685	22.966	26.247	29.528
10	32.809	36.090	39.371	42.651	45.932	49.213	52.494	55.775	59.056	62.337
20	65.618	68.899	72.179	75.461	78.741	82.022	85.303	88.584	91.865	95.146
30	98.427	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.96
40	131.24	134.52	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50	164.04	167.33	170.61	173.89	177.17	180.45	183.73	187.01	190.29	193.57
60	196.85	200.13	203.42	206.70	209.98	213.26	216.54	219.82	223.10	226.38
70	229.66	232.94	236.22	239.51	242.79	246.07	249.35	252.63	255.91	259.19
80	262.47	265.75	269.03	272.31	275.60	278.88	282.16	285.44	288.72	292.00
90	295.28	298.56	301.84	305.12	308.40	311.69	314.97	318.25	321.53	324.81
100	328.09	331.37	334.65	337.93	341.21	344.49	347.78	351.06	354.34	357.62

CONVERSION OF ENGLISH STATUTE-MILES INTO KILOMETRES.

Miles.	0	1	2	3	4	5	6	7	8	9
	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.	Kilo.
0	0.0000	1.6093	3.2186	4.8279	6.4372	8.0465	9.6558	11.2652	12.8745	14.4848
10	16.093	17.702	19.312	20.921	22.530	24.139	25.749	27.358	28.967	30.577
20	32.186	33.795	35.405	37.014	38.623	40.232	41.842	43.451	45.060	46.670
30	48.279	49.888	51.498	53.107	54.716	56.325	57.935	59.544	61.153	62.763
40	64.372	65.981	67.591	69.200	70.809	72.418	74.028	75.637	77.246	78.856
50	80.465	82.074	83.684	85.293	86.902	88.511	90.121	91 730	93.339	94.949
60	96 558	98.167	99.777	101.39	102.99	104.60	106.21	107.82	109.43	111.04
70	112.65	114.26	115.87	117.48	119.08	120.69	122.30	123.91	125.52	127.13
80	128.74	130.35	131.96	133.57	135.17	136.78	138.39	140.00	141.61	143.22
90	144.85	146.44	148.05	149.66	151.26	152.87	154.48	156.09	157.70	159.31
100	160.93	162.53	164 14	165 75	167.35	168.96	170.57	172.18	173.79	175.40

CONVERSION OF KILOMETRES INTO ENGLISH STATUTE-MILES.

Kilom.	0	1	2	3	4	5	6	7	8	9
	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.	Miles.
0	0.0000	0.6214	1.2427	1.8641	2.4855	3.1069	3.7282	4.3497	4.9711	5.5924
10	6.2138	6.8352	7.4565	8.0780	8.6994	9.3208	9.9421	10.562	11.185	11.805
20	12.427	13.049	13.670	14.292	14.913	15.534	16.156	16.776	17.399	18.019
30	18.641	19.263	19.884	20.506	21.127	21.748	23.370	22.990	23.613	24.233
40	24.855	25.477	26.098	26.720	27.341	27.962	28.584	29.204	29.827	30.447
50	31.069	31.690	32.311	32.933	33.554	34.175	34.797	35.417	36.040	36.660
60	37.282	37.904	38.525	39.147	39.768	40.389	41.011	41.631	42.254	42.874
70	43.497	44.118	44.739	45.361	45.982	46.603	47.225	47.845	48.468	49.088
80	49.711	50.332	50.953	51.575	52.196	52.817	53.439	54.059	54.682	55.302
90	55.924	56.545	57.166	57.788	58.409	59.080	59.652	60.272	60.895	61.515
100	62.138	62.759	63.380	64.002	64.623	65.244	65.866	66.486	67.109	67.729

TABLE XIII.

LENGTH IN FEET OF 1′ ARCS OF LATITUDE AND LONGITUDE.

Lat.	1′ Lat.	1′ Long.	Lat.	1′ Lat.	1′ Long.
1°	6045	6085	31°	6061	5222
2°	6045	6083	32°	6062	5166
3°	6045	6078	33°	6063	5109
4°	6045	6071	34°	6064	5051
5°	6045	6063	35°	6065	4991
6°	6045	6053	36°	6066	4930
7°	6046	6041	37°	6067	4867
8°	6046	6027	38°	6068	4802
9°	6046	6012	39°	6070	4736
10°	6047	5994	40°	6071	4669
11°	6047	5975	41°	6072	4600
12°	6048	5954	42°	6073	4530
13°	6048	5931	43°	6074	4458
14°	6049	5907	44°	6075	4385
15°	6049	5880	45°	6076	4311
16°	6050	5852	46°	6077	4235
17°	6050	5822	47°	6078	4158
18°	6051	5790	48°	6079	4080
19°	6052	5757	49°	6080	4001
20°	6052	5721	50°	6081	3920
21°	6053	5684	51°	6082	3838
22°	6054	5646	52°	6084	3755
23°	6054	5605	53°	6085	3671
24°	6055	5563	54°	6086	3586
25°	6056	5519	55°	6087	3499
26°	6057	5474	56°	6088	3413
27°	6058	5427	57°	6089	3323
28°	6059	5378	58°	6090	3233
29°	6060	5327	59°	6091	3142
30°	6061	5275	60°	6092	3051

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19008	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25001	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.95094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.23737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93421	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	*.35837*	*.93358*	.37461	.92718	.39072	.92050	.40674	.91355	.42262	.90631	0
′	*Cosin*	*Sine*	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	43051	.90259	.44620	.89493	.46175	88701	.47716	.87882	.49242	87026	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	40
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62220	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.82310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59905	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	23
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

′	4°		5°		6°		7°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.23190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

′	8°		9°		10°		11°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.21375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	81°		80°		79°		78°		

′	12°		13°		14°		15°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.62048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	2.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	1
59	.30541	3.27426	.22460	3.08073	.34400	2.90696	.36364	2.74997	0
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	2
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53132	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.18639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95127	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03276	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.48459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91552	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91414	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91288	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91142	23
28	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91017	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

′	28°		29°		30°		31°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58083	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	*.55431*	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	*Cotang*	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70170	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.63314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	57°		56°		55°		54°		

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79724	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

′	40°		41°		42°		43°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87238	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	2
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	1
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	1
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	1
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	1
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	1
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	1
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	1
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	1
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	1
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	1
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	
	Cotang	Tang			Cotang	Tang			Cotang	Tang	
′	45°		′	′	45°		′	′	45°		′

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS.

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
1	1	1	1.0000000	1.0000000	1.000000000
2	4	8	1.4142136	1.2599210	.500000000
3	9	27	1.7320508	1.4422496	.333333333
4	16	64	2.0000000	1.5874011	.250000000
5	25	125	2.2360680	1.7099759	.200000000
6	36	216	2 4494897	1.8171206	.166666667
7	49	343	2.6457513	1.9129312	.142857143
8	64	512	2.8284271	2.0000000	.125000000
9	81	729	3.0000000	2.0800837	.111111111
10	100	1000	3.1622777	2.1544347	.100000000
11	121	1331	3.3166248	2.2239801	.090909091
12	144	1728	3.4641016	2.2894286	.083333333
13	169	2197	3.6055513	2.3513347	.076923077
14	196	2744	3.7416574	2.4101422	.071428571
15	225	3375	3.8729833	2.4662121	.066666667
16	256	4096	4.0000000	2.5198421	.062500000
17	289	4913	4.1231056	2.5712816	.058823529
18	324	5832	4.2426407	2.6207414	.055555556
19	361	6859	4.3588989	2.6684016	.052631579
20	400	8000	4.4721360	2.7144177	.050000000
21	441	9261	4.5825757	2.7589243	.047619048
22	484	10648	4.6904158	2.8020393	.045454545
23	529	12167	4.7958315	2.8438670	.043478261
24	576	13824	4.8989795	2.8844991	.041666667
25	625	15625	5.0000000	2.9240177	.040000000
26	676	17576	5.0990195	2.9624960	.038461538
27	729	19683	5.1961524	3.0000000	.037037037
28	784	21952	5.2915026	3.0365889	.035714286
29	841	24389	5.3851648	3.0723168	.034482759
30	900	27000	5.4772256	3.1072325	.033333333
31	961	29791	5.5677644	3.1413806	.032258065
32	1024	32768	5.6568542	3.1748021	.031250000
33	1089	35937	5.7445626	3.2075343	.030303030
34	1156	39304	5.8309519	3.2396118	.029411765
35	1225	42875	5.9160798	3.2710663	.028571429
36	1296	46656	6.0000000	3.3019272	.027777778
37	1369	50653	6.0827625	3.3322218	.027027027
38	1444	54872	6.1644140	3.3619754	.026315789
39	1521	59319	6.2449980	3.3912114	.025641026
40	1600	64000	6.3245553	3.4199519	.025000000
41	1681	68921	6.4031242	3.4482172	.024390244
42	1764	74088	6.4807407	3.4760266	.023809524
43	1849	79507	6.5574385	3.5033981	.023255814
44	1936	85184	6.6332496	3.5303483	.022727273
45	2025	91125	6.7082039	3.5568933	.022222222
46	2116	97336	6.7823300	3.5830479	.021739130
47	2209	103823	6.8556546	3.6088261	.021276600
48	2304	110592	6.9282032	3.6342411	.020833333
49	2401	117649	7.0000000	3.6593057	.020408163
50	2500	125000	7.0710678	3.6840314	.020000000
51	2601	132651	7.1414284	3.7084298	.019607843
52	2704	140608	7.2111026	3.7325111	.019230769
53	2809	148877	7.2801099	3.7562858	.018867925
54	2916	157464	7.3484692	3.7797631	.018518519
55	3025	166375	7.4161985	3.8029525	.018181818
56	3136	175616	7.4833148	3.8258624	.017857143
57	3249	185193	7.5498344	3.8485011	.017543860
58	3364	195112	7.6157731	3.8708766	.017241379
59	3481	205379	7.6811457	3.8929965	.016949153
60	3600	216000	7.7459667	3.9148676	.016666667
61	3721	226981	7.8102497	3.9364972	.016393443
62	3844	238328	7.8740079	3.9578915	.016129032

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
63	3969	250047	7.9372539	3.9790571	.015873016
64	4096	262144	8.0000000	4.0000000	.015625000
65	4225	274625	8.0622577	4.0207256	.015384615
66	4356	287496	8.1240384	4.0412401	.015151515
67	4489	300763	8.1853528	4.0615480	.014925373
68	4624	314432	8.2462113	4.0816551	.014705882
69	4761	328509	8.3066239	4.1015661	.014492754
70	4900	343000	8.3666003	4.1212853	.014285714
71	5041	357911	8.4261498	4.1408178	.014084507
72	5184	373248	8.4852814	4.1601676	.013888889
73	5329	389017	8.5440037	4.1793390	.013698630
74	5476	405224	8.6023253	4.1983364	.013513514
75	5625	421875	8.6602540	4.2171633	.013333333
76	5776	438976	8.7177979	4.2358236	.013157895
77	5929	456533	8.7749644	4.2543210	.012987013
78	6084	474552	8.8317609	4.2726586	.012820513
79	6241	493039	8.8881944	4.2908404	.012658228
80	6400	512000	8.9442719	4.3088695	.012500000
81	6561	531441	9.0000000	4.3267487	.012345679
82	6724	551368	9.0553851	4.3444815	.012195122
83	6889	571787	9.1104336	4.3620707	.012048193
84	7056	592704	9.1651514	4.3795191	.011904762
85	7225	614125	9.2195445	4.3968296	.011764706
86	7396	636056	9.2736185	4.4140049	.011627907
87	7569	658503	9.3273791	4.4310476	.011494253
88	7744	681472	9.3808315	4.4479602	.011363636
89	7921	704969	9.4339811	4.4647451	.011235955
90	8100	729000	9.4868330	4.4814047	.011111111
91	8281	753571	9.5393920	4.4979414	.010989011
92	8464	778688	9.5916630	4.5143574	.010869565
93	8649	804357	9.6436508	4.5306549	.010752688
94	8836	830584	9.6953597	4.5468359	.010638298
95	9025	857375	9.7467943	4.5629026	.010526316
96	9216	884736	9.7979590	4.5788570	.010416667
97	9409	912673	9.8488578	4.5947009	.010309278
98	9604	941192	9.8994949	4.6104363	.010204082
99	9801	970299	9.9498744	4.6260650	.010101010
100	10000	1000000	10.0000000	4.6415888	.010000000
101	10201	1030301	10.0498756	4.6570095	.009900990
102	10404	1061208	10.0995049	4.6723287	.009803922
103	10609	1092727	10.1488916	4.6875482	.009708738
104	10816	1124864	10.1980390	4.7026694	.009615385
105	11025	1157625	10.2469508	4.7176940	.009523810
106	11236	1191016	10.2956301	4.7326235	.009433962
107	11449	1225043	10.3440804	4.7474594	.009345794
108	11664	1259712	10.3923048	4.7622032	.009259259
109	11881	1295029	10.4403065	4.7768562	.009174312
110	12100	1331000	10.4880885	4.7914199	.009090909
111	12321	1367631	10.5356538	4.8058955	.009009009
112	12544	1404928	10.5830052	4.8202845	.008928571
113	12769	1442897	10.6301458	4.8345881	.008849558
114	12996	1481544	10.6770783	4.8488076	.008771930
115	13225	1520875	10.7238053	4.8629442	.008695652
116	13456	1560896	10.7703296	4.8769990	.008620690
117	13689	1601613	10.8166538	4.8909732	.008547009
118	13924	1643032	10.8627805	4.9048681	.008474576
119	14161	1685159	10.9087121	4.9186847	.008403361
120	14400	1728000	10.9544512	4.9324242	.008333333
121	14641	1771561	11.0000000	4.9460874	.008264463
122	14884	1815848	11.0453610	4.9596757	.008196721
123	*15129*	1860867	11.0905365	4.9731898	.008130081
124	*15376*	1906624	11.1355287	4.9866310	.008064516

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS.

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
125	15625	1953125	11.1803399	5.0000000	.008000000
126	15876	2000376	11.2249722	5.0132979	.007936508
127	16129	2048383	11.2694277	5.0265257	.007874016
128	16384	2097152	11.3137085	5.0396842	.007812500
129	16641	2146689	11.3578167	5.0527743	.007751938
130	16900	2197000	11.4017543	5.0657970	.007692308
131	17161	2248091	11.4455231	5.0787531	.007633588
132	17424	2299968	11.4891253	5.0916434	.007575758
133	17689	2352637	11.5325626	5.1044687	.007518797
134	17956	2406104	11.5758369	5.1172299	.007462687
135	18225	2460375	11.6189500	5.1299278	.007407407
136	18496	2515456	11.6619038	5.1425632	.007352941
137	18769	2571353	11.7046999	5.1551367	.007299270
138	19044	2628072	11.7473401	5.1676493	.007246377
139	19321	2685619	11.7898261	5.1801015	.007194245
140	19600	2744000	11.8321596	5.1924941	.007142857
141	19881	2803221	11.8743421	5.2048279	.007092199
142	20164	2863288	11.9163753	5.2171034	.007042254
143	20449	2924207	11.9582607	5.2293215	.006993007
144	20736	2985984	12.0000000	5.2414828	.006944444
145	21025	3048625	12.0415946	5.2535879	.006896552
146	21316	3112136	12.0830460	5.2656374	.006849315
147	21609	3176523	12.1243557	5.2776321	.006802721
148	21904	3241792	12.1655251	5.2895725	.006756757
149	22201	3307949	12.2065556	5.3014592	.006711409
150	22500	3375000	12.2474487	5.3132928	.006666667
151	22801	3442951	12.2882057	5.3250740	.006622517
152	23104	3511808	12.3288280	5.3368033	.006578047
153	23409	3581577	12.3693169	5.3484812	.006535948
154	23716	3652264	12.4096736	5.3601084	.006493506
155	24025	3723875	12.4498996	5.3716854	.006451613
156	24336	3796416	12.4899960	5.3832126	.006410256
157	24649	3869893	12.5299641	5 3946907	.006369427
158	24964	3944312	12.5698051	5.4061202	.006329114
159	25281	4019679	12.6095202	5.4175015	.006289308
160	25600	4096000	12.6491106	5.4288352	.006250000
161	25921	4173281	12.6885775	5.4401218	.006211180
162	26244	4251528	12.7279221	5.4513618	.006172840
163	26569	4330747	12.7671453	5.4625556	.006134969
164	26896	4410944	12.8062485	5.4737037	.006097561
165	27225	4492125	12.8452326	5.4848066	.006060606
166	27556	4574296	12.8840987	5.4958647	.006024096
167	27889	4657463	12.9228480	5.5068784	.005988024
168	28224	4741632	12.9614814	5.5178484	.005952381
169	28561	4826809	13.0000000	5.5287748	.005917160
170	28900	4913000	13.0384048	5.5396583	.005882353
171	29241	5000211	13.0766968	5.5504991	.005847953
172	29584	5088448	13.1148770	5.5612978	.005813953
173	29929	5177717	13.1529464	5.5720546	.005780347
174	30276	5268024	13.1909060	5.5827702	.005747126
175	30625	5359375	13.2287566	5.5934447	.005714286
176	30976	5451776	13.2664992	5.6040787	.005681818
177	31329	5545233	13.3041347	5.6146724	.005649718
178	31684	5639752	13.3416641	5.6252263	.005617078
179	32041	5735339	13.3790882	5.6357408	.005586592
180	32400	5832000	13.4164079	5.6462162	.005555556
181	32761	5929741	13.4536240	5.6566528	.005524862
182	33124	6028568	13.4907376	5.6670511	.005494505
183	33489	6128487	13.5277493	5.6774114	.005464481
184	33856	6229504	13.5646600	5.6877340	.005434783
185	34225	6331625	13.6014705	5.6980192	.005405405
186	34596	6434856	13.6381817	5.7082675	.005376344

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
187	34969	6539203	13.6747943	5.7184791	.005347594
188	35344	6644672	13.7113092	5.7286543	.005319149
189	35721	6751269	13.7477271	5.7387936	.005291005
190	36100	6859000	13.7840488	5.7488971	.005263158
191	36481	6967871	13.8202750	5.7589652	.005235602
192	36864	7077888	13.8564065	5.7689982	.005208333
193	37249	7189057	13.8924440	5.7789966	.005181347
194	37636	7301384	13.9283883	5.7889604	.005154639
195	38025	7414875	13.9642400	5.7988900	.005128205
196	38416	7529536	14.0000000	5.8087857	.005102041
197	38809	7645373	14.0356688	5.8186479	.005076142
198	39204	7762392	14.0712473	5.8284767	.005050505
199	39601	7880599	14.1067360	5.8382725	.005025126
200	40000	8000000	14.1421356	5.8480355	.005000000
201	40401	8120601	14.1774469	5.8577660	.004975124
202	40804	8242408	14.2126704	5.8674643	.004950495
203	41209	8365427	14.2478068	5.8771307	.004926108
204	41616	8489664	14.2828569	5.8867653	.004901961
205	42025	8615125	14.3178211	5.8963685	.004878049
206	42436	8741816	14.3527001	5.9059406	.004854369
207	42849	8869743	14.3874946	5.9154817	.004830918
208	43264	8998912	14.4222051	5.9249921	.004807692
209	43681	9129329	14.4568323	5.9344721	.004784689
210	44100	9261000	14.4913767	5.9439220	.004761905
211	44521	9393931	14.5258390	5.9533418	.004739336
212	44944	9528128	14.5602198	5.9627320	.004716981
213	45369	9663597	14.5945195	5.9720926	.004694836
214	45796	9800344	14.6287388	5.9814240	.004672897
215	46225	9938375	14.6628783	5.9907264	.004651163
216	46656	10077696	14.6969385	6.0000000	.004629630
217	47089	10218313	14.7309199	6.0092450	.004608295
218	47524	10360232	14.7648231	6.0184617	.004587156
219	47961	10503459	14.7986486	6.0276502	.004566210
220	48400	10648000	14.8323970	6.0368107	.004545455
221	48841	10793861	14.8660687	6.0459435	.004524887
222	49284	10941048	14.8996644	6.0550489	.004504505
223	49729	11089567	14.9331845	6.0641270	.004484305
224	50176	11239424	14.9666295	6.0731779	.004464286
225	50625	11390625	15.0000000	6.0822020	.004444444
226	51076	11543176	15.0332964	6.0911994	.004424779
227	51529	11697083	15.0665192	6.1001702	.004405286
228	51984	11852352	15.0996689	6.1091147	.004385965
229	52441	12008989	15.1327460	6.1180332	.004366812
230	52900	12167000	15.1657509	6.1269257	.004347826
231	53361	12326391	15.1986842	6.1357924	.004329004
232	53824	12487168	15.2315462	6.1446337	.004310345
233	54289	12649337	15.2643375	6.1534495	.004291845
234	54756	12812904	15.2970585	6.1622401	.004273504
235	55225	12977875	15.3297097	6.1710058	.004255319
236	55696	13144256	15.3622915	6.1797466	.004237288
237	56169	13312053	15.3948043	6.1884628	.004219409
238	56644	13481272	15.4272486	6.1971544	.004201681
239	57121	13651919	15.4596248	6.2058218	.004184100
240	57600	13824000	15.4919334	6.2144650	.004166667
241	58081	13997521	15.5241747	6.2230843	.004149378
242	58564	14172488	15.5563492	6.2316797	.004132231
243	59049	14348907	15.5884573	6.2402515	.004115226
244	59536	14526784	15.6204994	6.2487998	.004098361
245	60025	14706125	15.6524758	6.2573248	.004081633
246	*60516*	14886936	15.6843871	6.2658266	.004065041
247	*61009*	15069223	15.7162336	6.2743054	.004048583
248	*61504*	15252992	15.7480157	6.2827613	.004032258

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
249	62001	15438249	15.7797338	6.2911946	.004016064
250	62500	15625000	15.8113883	6.2996053	.004000000
251	63001	15813251	15.8429795	6.3079935	.003984064
252	63504	16003008	15.8745079	6.3163596	.003968254
253	64009	16194277	15.9059737	6.3247035	.003952569
254	64516	16387064	15.9373775	6.3330256	.003937008
255	65025	16581375	15.9687194	6.3413257	.003921569
256	65536	16777216	16.0000000	6.3496042	.003906250
257	66049	16974593	16.0312195	6.3578611	.003891051
258	66564	17173512	16.0623784	6.3660968	.003875969
259	67081	17373979	16.0934769	6.3743111	.003861004
260	67600	17576000	16.1245155	6.3825043	.003846154
261	68121	17779581	16.1554944	6.3906765	.003831418
262	68644	17984728	16.1864141	6.3988279	.003816794
263	69169	18191447	16.2172747	6.4069585	.003802281
264	69696	18399744	16.2480768	6.4150687	.003787879
265	70225	18609625	16.2788206	6.4231583	.003773585
266	70756	18821096	16.3095064	6.4312276	.003759398
267	71289	19034163	16.3401346	6.4392767	.003745318
268	71824	19248832	16.3707055	6.4473057	.003731343
269	72361	19465109	16.4012195	6.4553148	.003717472
270	72900	19683000	16.4316767	6.4633041	.003703704
271	73441	19902511	16.4620776	6.4712736	.003690037
272	73984	20123648	16.4924225	6.4792236	.003676471
273	74529	20346417	16.5227116	6.4871541	.003663004
274	75076	20570824	16.5529454	6.4950653	.003649635
275	75625	20796875	16.5831240	6.5029572	.003636364
276	76176	21024576	16.6132477	6.5108300	.003623188
277	76729	21253933	16.6433170	6.5186839	.003610108
278	77284	21484952	16.6733320	6.5265189	.003597122
279	77841	21717639	16.7032931	6.5343351	.003584229
280	78400	21952000	16.7332005	6.5421326	.003571429
281	78961	22188041	16.7630546	6.5499116	.003558719
282	79524	22425768	16.7928556	6.5576722	.003546099
283	80089	22665187	16.8226038	6.5654144	.003533569
284	80656	22906304	16.8522995	6.5731385	.003521127
285	81225	23149125	16.8819430	6.5808443	.003508772
286	81796	23393656	16.9115345	6.5885323	.003496503
287	82369	23639903	16.9410743	6.5962023	.003484321
288	82944	23887872	16.9705627	6.6038545	.003472222
289	83521	24137569	17.0000000	6.6114890	.003460208
290	84100	24389000	17.0293864	6.6191060	.003448276
291	84681	24642171	17.0587221	6.6267054	.003436426
292	85264	24897088	17.0880075	6.6342874	.003424658
293	85849	25153757	17.1172428	6.6418522	.003412969
294	86436	25412184	17.1464282	6.6493998	.003401361
295	87025	25672375	17.1755640	6.6569302	.003389831
296	87616	25934336	17.2046505	6.6644437	.003378378
297	88209	26198073	17.2336879	6.6719403	.003367003
298	88804	26463592	17.2626765	6.6794200	.003355705
299	89401	26730899	17.2916165	6.6868831	.003344482
300	90000	27000000	17.3205081	6.6943295	.003333333
301	90601	27270901	17.3493516	6.7017593	.003322259
302	91204	27543608	17.3781472	6.7091729	.003311258
303	91809	27818127	17.4068952	6.7165700	.003300330
304	92416	28094464	17.4355958	6.7239508	.003289474
305	93025	28372625	17.4642492	6.7313155	.003278689
306	93636	28652616	17.4928557	6.7386641	.003267974
307	94249	28934443	17.5214155	6.7459967	.003257329
308	94864	29218112	17.5499288	6.7533134	.003246753
309	95481	29503629	17.5783958	6.7606143	.003236246
310	*96100*	*29791000*	17.6068169	6.7678995	.003225806

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
311	96721	30080231	17.6351921	6.7751690	.003215434
312	97344	30371328	17.6635217	6.7824229	.003205128
313	97969	30664297	17.6918060	6.7896613	.003194888
314	98596	30959144	17.7200451	6.7968844	.003184713
315	99225	31255875	17.7482393	6.8040921	.003174603
316	99856	31554496	17.7763888	6.8112847	.003164557
317	100489	31855013	17.8044938	6.8184620	.003154574
318	101124	32157432	17.8325545	6.8256242	.003144654
319	101761	32461759	17.8605711	6.8327714	.003134796
320	102400	32768000	17.8885438	6.8399037	.003125000
321	103041	33076161	17.9164729	6.8470213	.003115265
322	103684	33386248	17.9443584	6.8541240	.003105590
323	104329	33698267	17.9722008	6.8612120	.003095975
324	104976	34012224	18.0000000	6.8682855	.003086420
325	105625	34328125	18.0277564	6.8753443	.003076923
326	106276	34645976	18.0554701	6.8823888	.003067485
327	106929	34965783	18.0831413	6.8894188	.003058104
328	107584	35287552	18.1107703	6.8964345	.003048780
329	108241	35611289	18.1383571	6.9034359	.003039514
330	108900	35937000	18.1659021	6.9104232	.003030303
331	109561	36264691	18.1934054	6.9173964	.003021148
332	110224	36594368	18.2208672	6.9243556	.003012048
333	110889	36926037	18.2482876	6.9313008	.003003003
334	111556	37259704	18.2756669	6.9382321	.002994012
335	112225	37595375	18.3030052	6.9451496	.002985075
336	112896	37933056	18.3303028	6.9520533	.002976190
337	113569	38272753	18.3575598	6.9589434	.002967359
338	114244	38614472	18.3847763	6.9658198	.002958580
339	114921	38958219	18.4119526	6.9726826	.002949853
340	115600	39304000	18.4390889	6.9795321	.002941176
341	116281	39651821	18.4661853	6.9863681	.002932551
342	116964	40001688	18.4932420	6.9931906	.002923977
343	117649	40353607	18.5202592	7.0000000	.002915452
344	118336	40707584	18.5472370	7.0067962	.002906977
345	119025	41063625	18.5741756	7.0135791	.002898551
346	119716	41421736	18.6010752	7.0203490	.002890173
347	120409	41781923	18.6279360	7.0271058	.002881844
348	121104	42144192	18.6547581	7.0338497	.002873563
349	121801	42508549	18.6815417	7.0405806	.002865330
350	122500	42875000	18.7082869	7.0472987	.002857143
351	123201	43243551	18.7349940	7.0540041	.002849003
352	123904	43614208	18.7616630	7.0606967	.002840909
353	124609	43986977	18.7882942	7.0673767	.002832861
354	125316	44361864	18.8148877	7.0740440	.002824859
355	126025	44738875	18.8414437	7.0806988	.002816901
356	126736	45118016	18.8679623	7.0873411	.002808989
357	127449	45499293	18.8944436	7.0939709	.002801120
358	128164	45882712	18.9208879	7.1005885	.002793296
359	128881	46268279	18.9472953	7.1071937	.002785515
360	129600	46656000	18.9736660	7.1137866	.002777778
361	130321	47045881	19.0000000	7.1203674	.002770083
362	131044	47437928	19.0262976	7.1269360	.002762431
363	131769	47832147	19.0525589	7.1334925	.002754821
364	132496	48228544	19.0787840	7.1400370	.002747253
365	133225	48627125	19.1049732	7.1465695	.002739726
366	133956	49027896	19.1311265	7.1530901	.002732240
367	134689	49430863	19.1572441	7.1595988	.002724796
368	135424	49836032	19.1833261	7.1660957	.002717391
369	136161	50243409	19.2093727	7.1725809	.002710027
370	136900	50653000	19.2353841	7.1790544	.002702703
371	137641	51064811	19.2613603	7.1855162	.002695418
372	138384	51478848	19.2873015	7.1919663	.002688172

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
373	139129	51895117	19.3132079	7.1984050	.002680965
374	139876	52313624	19.3390796	7.2048322	.002673797
375	140625	52734375	19.3649167	7.2112479	.002666667
376	141376	53157376	19.3907194	7.2176522	.002659574
377	142129	53582633	19.4164878	7.2240450	.002652520
378	142884	54010152	19.4422221	7.2304268	.002645503
379	143641	54439939	19.4679223	7.2367972	.002638522
380	144400	54872000	19.4935887	7.2431565	.002631579
381	145161	55306341	19.5192213	7.2495045	.002624672
382	145924	55742968	19.5448203	7.2558415	.002617801
383	146689	56181887	19.5703858	7.2621675	.002610966
384	147456	56623104	19.5959179	7.2684824	.002604167
385	148225	57066625	19.6214169	7.2747864	.002597403
386	148996	57512456	19.6468827	7.2810794	.002590674
387	149769	57960603	19.6723156	7.2873617	.002583979
388	150544	58411072	19.6977156	7.2936330	.002577320
389	151321	58863869	19.7230829	7.2998936	.002570694
390	152100	59319000	19.7484177	7.3061436	.002564103
391	152881	59776471	19.7737199	7.3123828	.002557545
392	153664	60236288	19.7989899	7.3186114	.002551020
393	154449	60698457	19.8242276	7.3248295	.002544529
394	155236	61162984	19.8494332	7.3310369	.002538071
395	156025	61629875	19.8746069	7.3372339	.002531646
396	156816	62099136	19.8997487	7.3434205	.002525253
397	157609	62570773	19.9248588	7.3495966	.002518892
398	158404	63044792	19.9499373	7.3557624	.002512563
399	159201	63521199	19.9749844	7.3619178	.002506266
400	160000	64000000	20.0000000	7.3680630	.002500000
401	160801	64481201	20.0249844	7.3741979	.002493766
402	161604	64964808	20.0499377	7.3803227	.002487562
403	162409	65450827	20.0748599	7.3864373	.002481390
404	163216	65939264	20.0997512	7.3925418	.002475248
405	164025	66430125	20.1246118	7.3986363	.002469136
406	164836	66923416	20.1494417	7.4047206	.002463054
407	165649	67419143	20.1742410	7.4107950	.002457002
408	166464	67917312	20.1990099	7.4168595	.002450980
409	167281	68417929	20.2237484	7.4229142	.002444988
410	168100	68921000	20.2484567	7.4289589	.002439024
411	168921	69426531	20.2731349	7.4349938	.002433090
412	169744	69934528	20.2977831	7.4410189	.002427184
413	170569	70444997	20.3224014	7.4470342	.002421308
414	171396	70957944	20.3469899	7.4530399	.002415459
415	172225	71473375	20.3715488	7.4590359	.002409639
416	173056	71991296	20.3960781	7.4650223	.002403846
417	173889	72511713	20.4205779	7.4709991	.002398082
418	174724	73034632	20.4450483	7.4769664	.002392344
419	175561	73560059	20.4694895	7.4829242	.002386635
420	176400	74088000	20.4939015	7.4888724	.002380952
421	177241	74618461	20.5182845	7.4948113	.002375297
422	178084	75151448	20.5426386	7.5007406	.002369668
423	178929	75686967	20.5669638	7.5066607	.002364066
424	179776	76225024	20 5912603	7.5125715	.002358491
425	180625	76765625	20.6155281	7.5184730	.002352941
426	181476	77308776	20.6397674	7.5243652	.002347418
427	182329	77854483	20.6639783	7.5302482	.002341920
428	183184	78402752	20.6881609	7.5361221	.002336449
429	184041	78953589	20.7123152	7.5419867	.002331002
430	184900	79507000	20.7364414	7.5478423	.002325581
431	185761	80062991	20.7605395	7.5536888	.002320186
432	186624	80621568	20.7846097	7.5595263	.002314815
433	187489	81182737	20.8086520	7.5653548	.002309469
434	188356	81746504	20.8326667	7.5711743	.002304147

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
435	189225	82312875	20.8566536	7.5769849	.002298851
436	190096	82881856	20.8806130	7.5827865	.002293578
437	190969	83453453	20.9045450	7.5885793	.002288330
438	191844	84027672	20.9284495	7.5943633	.002283105
439	192721	84604519	20.9523268	7.6001385	.002277904
440	193600	85184000	20.9761770	7.6059049	.002272727
441	194481	85766121	21.0000000	7.6116626	.002267574
442	195364	86350888	21.0237960	7.6174116	.002262443
443	196249	86938307	21.0475652	7.6231519	.002257336
444	197136	87528384	21.0713075	7.6288837	.002252252
445	198025	88121125	21.0950231	7.6346067	.002247191
446	198916	88716536	21.1187121	7.6403213	.002242152
447	199809	89314623	21.1423745	7.6460272	.002237136
448	200704	89915392	21.1660105	7.6517247	.002232143
449	201601	90518849	21.1896201	7.6574138	.002227171
450	202500	91125000	21.2132034	7.6630943	.002222222
451	203401	91733851	21.2367606	7.6687665	.002217295
452	204304	92345408	21.2602916	7.6744303	.002212389
453	205209	92959677	21.2837967	7.6800857	.002207506
454	206116	93576664	21.3072758	7.6857328	.002202643
455	207025	94196375	21.3307290	7.6913717	.002197802
456	207936	94818816	21.3541565	7.6970023	.002192982
457	208849	95443993	21.3775583	7.7026246	.002188184
458	209764	96071912	21.4009346	7.7082388	.002183406
459	210681	96702579	21.4242853	7.7138448	.002178649
460	211600	97336000	21.4476106	7.7194426	.002173913
461	212521	97972181	21.4709106	7.7250325	.002169197
462	213444	98611128	21.4941853	7.7306141	.002164502
463	214369	99252847	21.5174348	7.7361877	.002159827
464	215296	99897344	21.5406592	7.7417532	.002155172
465	216225	100544625	21.5638587	7.7473109	.002150538
466	217156	101194696	21.5870331	7.7528606	.002145923
467	218089	101847563	21.6101828	7.7584023	.002141328
468	219024	102503232	21.6333077	7.7639361	.002136752
469	219961	103161709	21.6564078	7.7694620	.002132196
470	220900	103823000	21.6794834	7.7749801	.002127660
471	221841	104487111	21.7025344	7.7804904	.002123142
472	222784	105154048	21.7255610	7.7859928	.002118644
473	223729	105823817	21.7485632	7.7914875	.002114165
474	224676	106496424	21.7715411	7.7969745	.002109705
475	225625	107171875	21.7944947	7.8024538	.002105263
476	226576	107850176	21.8174242	7.8079254	.002100840
477	227529	108531333	21.8403297	7.8133892	.002096436
478	228484	109215352	21 8632111	7.8188456	.002092050
479	229441	109902239	21.8860686	7.8242942	.002087683
480	230400	110592000	21.9089023	7.8297353	.002083333
481	231361	111284641	21.9317122	7.8351688	.002079002
482	232324	111980168	21.9544984	7.8405949	.002074689
483	233289	112678587	21.9772610	7.8460134	.002070393
484	234256	113379904	22.0000000	7.8514244	.002066116
485	235225	114084125	22.0227155	7.8568281	.002061856
486	236196	114791256	22.0454077	7.8622242	.002057613
487	237169	115501303	22.0680765	7.8676130	.002053388
488	238144	116214272	22.0907220	7.8729944	.002049180
489	239121	116930169	22.1133444	7 8783684	.002044990
490	240100	117649000	22.1359436	7.8837352	.002040816
491	241081	118370771	22.1585198	7.8890946	.002036660
492	242064	119095488	22.1810730	7.8944468	.002032520
493	243049	119823157	22.2036033	7.8997917	.002028398
494	244036	120553784	22.2261108	7.9051294	.002024291
495	245025	121287375	22.2485955	7.9104599	.002020202
496	246016	122023936	22.2710575	7.9157832	.002016129

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS,

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
497	247009	122763473	22.2934968	7.9210994	.002012072
498	248004	123505992	22.3159136	7.9264085	.002008032
499	249001	124251499	22.3383079	7.9317104	.002004008
500	250000	125000000	22.3606798	7.9370053	.002000000
501	251001	125751501	22.3830293	7.9422931	.001996008
502	252004	126506008	22.4053565	7.9475739	.001992032
503	253009	127263527	22.4276615	7.9528477	.001988072
504	254016	128024064	22.4499443	7.9581144	.001984127
505	255025	128787625	22.4722051	7.9633743	.001980198
506	256036	129554216	22.4944438	7.9686271	.001976285
507	257049	130323843	22.5166605	7.9738731	.001972387
508	258064	131096512	22.5388553	7.9791122	.001968504
509	259081	131872229	22.5610283	7.9843444	.001964637
510	260100	132651000	22.5831796	7.9895697	.001960784
511	261121	133432831	22.6053091	7.9947883	.001956947
512	262144	134217728	22.6274170	8.0000000	.001953125
513	263169	135005697	22.6495033	8.0052049	.001949318
514	264196	135796744	22.6715681	8.0104032	.001945525
515	265225	136590875	22.6936114	8.0155946	.001941748
516	266256	137388096	22.7156334	8.0207794	.001937984
517	267289	138188413	22.7376340	8.0259574	.001934236
518	268324	138991832	22.7596134	8.0311287	.001930502
519	269361	139798359	22.7815715	8.0362935	.001926782
520	270400	140608000	22.8035085	8.0414515	.001923077
521	271441	141420761	22.8254244	8.0466030	.001919386
522	272484	142236648	22.8473193	8.0517479	.001915709
523	273529	143055667	22.8691933	8.0568862	.001912046
524	274576	143877824	22.8910463	8.0620180	.001908397
525	275625	144703125	22.9128785	8.0671432	.001904762
526	276676	145531576	22.9346899	8.0722620	.001901141
527	277729	146363183	22.9564806	8.0773743	.001897533
528	278784	147197952	22.9782506	8.0824800	.001893939
529	279841	148035889	23.0000000	8.0875794	.001890359
530	280900	148877000	23.0217289	8.0926723	.001886792
531	281961	149721291	23.0434372	8.0977589	.001883239
532	283024	150568768	23.0651252	8.1028390	.001879699
533	284089	151419437	23.0867928	8.1079128	.001876173
534	285156	152273304	23.1084400	8.1129803	.001872659
535	286225	153130375	23.1300670	8.1180414	.001869159
536	287296	153990656	23.1516738	8.1230962	.001865672
537	288369	154854153	23.1732605	8.1281447	.001862197
538	289444	155720872	23.1948270	8.1331870	.001858736
539	290521	156590819	23.2163735	8.1382230	.001855288
540	291600	157464000	23.2379001	8.1432529	.001851852
541	292681	158340421	23.2594067	8.1482765	.001848429
542	293764	159220088	23.2808935	8.1532939	.001845018
543	294849	160103007	23.3023604	8.1583051	.001841621
544	295936	160989184	23.3238076	8.1633102	.001838235
545	297025	161878625	23.3452351	8.1683092	.001834862
546	298116	162771336	23.3666429	8.1733020	.001831502
547	299209	163667323	23.3880311	8.1782888	.001828154
548	300304	164566592	23.4093998	8.1832695	.001824818
549	301401	165469149	23.4307490	8.1882441	.001821494
550	302500	166375000	23.4520788	8.1932127	.001818182
551	303601	167284151	23.4733892	8.1981752	.001814882
552	304704	168196608	23.4946802	8.2031319	.001811594
553	305809	169112377	23.5159520	8.2080825	.001808318
554	306916	170031464	23.5372046	8.2130271	.001805054
555	308025	170953875	23.5584380	8.2179657	.001801802
556	309136	171879616	23.5796522	8.2229985	.001798561
557	310249	172808693	23.6008474	8.2278254	.001795332
558	311364	173741112	23.6220236	8.2327463	.001792115

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
559	312481	174676879	23.6431808	8.2376614	.001788909
560	313600	175616000	23.6643191	8.2425706	.001785714
561	314721	176558481	23.6854386	8.2474740	.001782531
562	315844	177504328	23.7065392	8.2523715	.001779359
563	316969	178453547	23.7276210	8.2572633	.001776199
564	318096	179406144	23.7486842	8.2621492	.001773050
565	319225	180362125	23.7697286	8.2670294	.001769912
566	320356	181321496	23.7907545	8.2719039	.001766784
567	321489	182284263	23.8117618	8.2767726	.001763668
568	322624	183250432	23.8327506	8.2816355	.001760563
569	323761	184220000	23.8537209	8.2864928	.001757469
570	324900	185193000	23.8746728	8.2913444	.001754386
571	326041	186169411	23.8956063	8.2961903	.001751313
572	327184	187149248	23.9165215	8.3010304	.001748252
573	328329	188132517	23.9374184	8.3058651	.001745201
574	329476	189119224	23.9582971	8.3106941	.001742160
575	330625	190109375	23.9791576	8.3155175	.001739130
576	331776	191102976	24.0000000	8.3203353	.001736111
577	332929	192100033	24.0208243	8.3251475	.001733102
578	334084	193100552	24.0416306	8.3299542	.001730104
579	335241	194104539	24.0624188	8.3347553	.001727116
580	336400	195112000	24.0831891	8.3395509	.001724138
581	337561	196122941	24.1039416	8.3443410	.001721170
582	338724	197137368	24.1246762	8.3491256	.001718213
583	339889	198155287	24.1453929	8.3539047	.001715266
584	341056	199176704	24.1660919	8.3586784	.001712329
585	342225	200201625	24.1867732	8.3634466	.001709402
586	343396	201230056	24.2074369	8.3682095	.001706485
587	344569	202262003	24.2280829	8.3729668	.001703578
588	345744	203297472	24.2487113	8.3777188	.001700680
589	346921	204336469	24.2693222	8.3824653	.001697793
590	348100	205379000	24.2899156	8.3872065	.001694915
591	349281	206425071	24.3104916	8.3919423	.001692047
592	350464	207474688	24.3310501	8.3966729	.001689189
593	351649	208527857	24.3515913	8.4013981	.001686341
594	352836	209584584	24.3721152	8.4061180	.001683502
595	354025	210644875	24.3926218	8.4108326	.001680672
596	355216	211708736	24.4131112	8.4155419	.001677852
597	356409	212776173	24.4335834	8.4202460	.001675042
598	357604	213847192	24.4540385	8.4249448	.001672241
599	358801	214921799	24.4744765	8.4296383	.001669449
600	360000	216000000	24.4948974	8.4343267	.001666667
601	361201	217081801	24.5153013	8.4390098	.001663894
602	362404	218167208	24.5356883	8.4436877	.001661130
603	363609	219256227	24.5560583	8.4483605	.001658375
604	364816	220348864	24.5764115	8.4530281	.001655629
605	366025	221445125	24.5967478	8.4576906	.001652893
606	367236	222545016	24.6170673	8.4623479	.001650165
607	368449	223648543	24.6373700	8.4670001	.001647446
608	369664	224755712	24.6576560	8.4716471	.001644737
609	370881	225866529	24.6779254	8.4762892	.001642036
610	372100	226981000	24.6981781	8.4809261	.001639344
611	373321	228099131	24.7184142	8.4855579	.001636661
612	374544	229220928	24.7386338	8.4901848	.001633987
613	375769	230346397	24.7588368	8.4948065	.001631321
614	376996	231475544	24.7790234	8.4994233	.001628664
615	378225	232608375	24.7991935	8.5040350	.001626016
616	379456	233744896	24.8193473	8.5086417	.001623377
617	380689	234885113	24.8394847	8.5132435	.001620746
618	381924	236029032	24.8596058	8.5178403	.001618123
619	383161	237176659	24.8797106	8.5224321	.001615509
620	384400	238328000	24.8997992	8.5270189	.001612903

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
621	385641	239483061	24.9198716	8.5316009	.001610306
622	386884	240641848	24.9399278	8.5361780	.001607717
623	388129	241804367	24.9599679	8.5407501	.001605136
624	389376	242970624	24.9799920	8.5453173	.001602564
625	390625	244140625	25.0000000	8.5498797	.001600000
626	391876	245314376	25.0199920	8.5544372	.001597444
627	393129	246491883	25.0399681	8.5589899	.001594896
628	394384	247673152	25.0599282	8.5635377	.001592357
629	395641	248858189	25.0798724	8.5680807	.001589825
630	396900	250047000	25.0998008	8.5726189	.001587302
631	398161	251239591	25.1197134	8.5771523	.001584786
632	399424	252435968	25.1396102	8.5816809	.001582278
633	400689	253636137	25.1594913	8.5862047	.001579779
634	401956	254840104	25.1793566	8.5907238	.001577287
635	403225	256047875	25.1992063	8.5952380	.001574803
636	404496	257259456	25.2190404	8.5997476	.001572327
637	405769	258474853	25.2388589	8.6042525	.001569859
638	407044	259694072	25.2586619	8.6087526	.001567398
639	408321	260917119	25.2784493	8.6132480	.001564945
640	409600	262144000	25.2982213	8.6177388	.001562500
641	410881	263374721	25.3179778	8.6222248	.001560062
642	412164	264609288	25.3377189	8.6267063	.001557632
643	413449	265847707	25.3574447	8.6311830	.001555210
644	414736	267089984	25.3771551	8.6356551	.001552795
645	416025	268336125	25.3968502	8.6401226	.001550388
646	417316	269586136	25.4165301	8.6445855	.001547988
647	418609	270840023	25.4361947	8.6490437	.001545595
648	419904	272097792	25.4558441	8.6534974	.001543210
649	421201	273359449	25.4754784	8.6579465	.001540832
650	422500	274625000	25.4950976	8.6623911	.001538462
651	423801	275894451	25.5147016	8.6668310	.001536098
652	425104	277167808	25.5342907	8.6712665	.001533742
653	426409	278445077	25.5538647	8.6756974	.001531394
654	427716	279726264	25.5734237	8.6801237	.001529052
655	429025	281011375	25.5929678	8.6845456	.001526718
656	430336	282300416	25.6124969	8.6889630	.001524390
657	431649	283593393	25.6320112	8.6933759	.001522070
658	432964	284890312	25.6515107	8.6977843	.001519757
659	434281	286191179	25.6709953	8.7021882	.001517451
660	435600	287496000	25.6904652	8.7065877	.001515152
661	436921	288804781	25.7099203	8.7109827	.001512859
662	438244	290117528	25.7293607	8.7153734	.001510574
663	439569	291434247	25.7487864	8.7197596	.001508296
664	440896	292754944	25.7681975	8.7241414	.001506024
665	442225	294079625	25.7875939	8.7285187	.001503759
666	443556	295408296	25.8069758	8.7328918	.001501502
667	444889	296740963	25.8263431	8.7372604	.001499250
668	446224	298077632	25.8456960	8.7416246	.001497006
669	447561	299418309	25.8650343	8.7459846	.001494768
670	448900	300763000	25.8843582	8.7503401	.001492537
671	450241	302111711	25.9036677	8.7546913	.001490313
672	451584	303464448	25.9229628	8.7590383	.001488095
673	452929	304821217	25.9422435	8.7633809	.001485884
674	454276	306182024	25.9615100	8.7677192	.001483680
675	455625	307546875	25.9807621	8.7720532	.001481481
676	456976	308915776	26.0000000	8.7763830	.001479290
677	458329	310288733	26.0192237	8.7807084	.001477105
678	459684	311665752	26.0384331	8.7850296	.001474926
679	461041	313046839	26.0576284	8.7893466	.001472754
680	462400	314432000	26.0768096	8.7936593	.001470588
681	463761	315821241	26.0959767	8.7979679	.001468429
682	465124	317214568	26.1151297	8.8022721	.001466276

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
683	466489	318611987	26.1342687	8.8065722	.001464129
684	467856	320013504	26.1533937	8.8108681	.001461988
685	469225	321419125	26.1725047	8.8151598	.001459854
686	470596	322828856	26.1916017	8.8194474	.001457726
687	471969	324242703	26.2106848	8.8237307	.001455604
688	473344	325660672	26.2297541	8.8280099	.001453488
689	474721	327082769	26.2488095	8.8322850	.001451379
690	476100	328509000	26.2678511	8.8365559	.001449275
691	477481	329939371	26.2868789	8.8408227	.001447178
692	478864	331373888	26.3058929	8.8450854	.001445087
693	480249	332812557	26.3248932	8.8493440	.001443001
694	481636	334255384	26.3438797	8.8535985	.001440922
695	483025	335702375	26.3628527	8.8578489	.001438849
696	484416	337153536	26.3818119	8.8620952	.001436782
697	485809	338608873	26.4007576	8.8663375	.001434720
698	487204	340068392	26.4196896	8.8705757	.001432665
699	488601	341532099	26.4386081	8.8748099	.001430615
700	490000	343000000	26.4575131	8.8790400	.001428571
701	491401	344472101	26.4764046	8.8832661	.001426534
702	492804	345948408	26.4952826	8.8874882	.001424501
703	494209	347428927	26.5141472	8.8917063	.001422475
704	495616	348913664	26.5329983	8.8959204	.001420455
705	497025	350402625	26.5518361	8.9001304	.001418440
706	498436	351895816	26.5706605	8.9043366	.001416431
707	499849	353393243	26.5894716	8.9085387	.001414427
708	501264	354894912	26.6082694	8.9127369	.001412429
709	502681	356400829	26 6270539	8.9169311	.001410437
710	504100	357911000	26.6458252	8.9211214	.001408451
711	505521	359425431	26.6645833	8.9253078	.001406470
712	506944	360944128	26.6833281	8.9294902	.001404494
713	508369	362467097	26.7020598	8.9336687	.001402525
714	509796	363994344	26.7207784	8.9378433	.001400560
715	511225	365525875	26.7394839	8.9420140	.001398601
716	512656	367061696	26.7581763	8.9461809	.001396648
717	514089	368601813	26.7768557	8.9503438	.001394700
718	515524	370146232	26.7955220	8.9545029	.001392758
719	516961	371694959	26.8141754	8.9586581	.001390821
720	518400	373248000	26.8328157	8.9628095	.001388889
721	519841	374805361	26.8514432	8.9669570	.001386963
722	521284	376367048	26.8700577	8.9711007	.001385042
723	522729	377933067	26.8886593	8.9752406	.001383126
724	524176	379503424	26.9072481	8.9793766	.001381215
725	525625	381078125	26.9258240	8.9835089	.001379310
726	527076	382657176	26.9443872	8.9876373	.001377410
727	528529	384240583	26.9629375	8.9917620	.001375516
728	529984	385828352	26.9814751	8.9958829	.001373626
729	531441	387420489	27.0000000	9.0000000	.001371742
730	532900	389017000	27.0185122	9.0041134	.001369863
731	534361	390617891	27.0370117	9.0082229	.001367989
732	535824	392223168	27.0554985	9.0123288	.001366120
733	537289	393832837	27.0739727	9.0164309	.001364256
734	538756	395446904	27.0924344	9.0205293	.001362398
735	540225	397065375	27.1108834	9.0246239	.001360544
736	541696	398688256	27.1293199	9.0287149	.001358696
737	543169	400315553	27.1477439	9.0328021	.001356852
738	544644	401947272	27.1661554	9.0368857	.001355014
739	546121	403583419	27.1845544	9.0409655	.001353180
740	547600	405224000	27.2029410	9.0450419	.001351351
741	549081	406869021	27.2213152	9.0491142	.001349528
742	550564	408518488	27.2396769	9.0531831	.001347709
743	552049	410172407	27.2580263	9.0572482	.001345895
744	553536	411830784	27.2763634	9.0613098	.001344086

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
745	555025	413493625	27.2946881	9.0653677	.001342282
746	556516	415160936	27.3130006	9.0694220	.001340483
747	558009	416832723	27.3313007	9.0734726	.001338688
748	559504	418508992	27.3495887	9.0775197	.001336898
749	561001	420189749	27.3678644	9.0815631	.001335113
750	562500	421875000	27.3861279	9.0856030	.001333333
751	564001	423564751	27.4043792	9.0896392	.001331558
752	565504	425259008	27.4226184	9.0936719	.001329787
753	567009	426957777	27.4408455	9.0977010	.001328021
754	568516	428661064	27.4590604	9.1017265	.001326260
755	570025	430368875	27.4772632	9.1057485	.001324503
756	571536	432081216	27.4954542	9.1097669	.001322751
757	573049	433798093	27.5136330	9.1137818	.001321004
758	574564	435519512	27.5317998	9.1177931	.001319261
759	576081	437245479	27.5499546	9.1218010	.001317523
760	577600	438976000	27.5680975	9.1258053	.001315789
761	579121	440711081	27.5862284	9.1298061	.001314060
762	580644	442450728	27.6043475	9.1338034	.001312336
763	582169	444194947	27.6224546	9.1377971	.001310616
764	583696	445943744	27.6405499	9.1417874	.001308901
765	585225	447697125	27.6586334	9.1457742	.001307190
766	586756	449455096	27.6767050	9.1497576	.001305483
767	588289	451217663	27.6947648	9.1537375	.001303781
768	589824	452984832	27.7128129	9.1577139	.001302083
769	591361	454756609	27.7308492	9.1616869	.001300390
770	592900	456533000	27.7488739	9.1656565	.001298701
771	594441	458314011	27.7668868	9.1696225	.001297017
772	595984	460099648	27.7848880	9.1735852	.001295337
773	597529	461889917	27.8028775	9.1775445	.001293661
774	599076	463684824	27.8208555	9.1815003	.001291990
775	600625	465484375	27.8388218	9.1854527	.001290323
776	602176	467288576	27.8567766	9.1894018	.001288660
777	603729	469097433	27.8747197	9.1933474	.001287001
778	605284	470910952	27.8926514	9.1972897	.001285347
779	606841	472729139	27.9105715	9.2012286	.001283697
780	608400	474552000	27.9284801	9.2051641	.001282051
781	609961	476379541	27.9463772	9.2090962	.001280410
782	611524	478211768	27.9642629	9.2130250	.001278772
783	613089	480048687	27.9821372	9.2169505	.001277139
784	614656	481890304	28.0000000	9.2208726	.001275510
785	616225	483736625	28.0178515	9.2247914	.001273885
786	617796	485587656	28.0356915	9.2287068	.001272265
787	619369	487443403	28.0535203	9.2326189	.001270648
788	620944	489303872	28.0713377	9.2365277	.001269036
789	622521	491169069	28.0891438	9.2404333	.001267427
790	624100	493039000	28.1069386	9.2443355	.001265823
791	625681	494913671	28.1247222	9.2482344	.001264223
792	627264	496793088	28.1424946	9.2521300	.001262626
793	628849	498677257	28.1602557	9.2560224	.001261034
794	630436	500566184	28.1780056	9.2599114	.001259446
795	632025	502459875	28.1957444	9.2637973	.001257862
796	633616	504358336	28.2134720	9.2676798	.001256281
797	635209	506261573	28.2311884	9.2715592	.001254705
798	636804	508169592	28.2488938	9.2754352	.001253133
799	638401	510082399	28.2665881	9.2793081	.001251564
800	640000	512000000	28.2842712	9.2831777	.001250000
801	641601	513922401	28.3019434	9.2870440	.001248439
802	643204	515849608	28.3196045	9.2909072	.001246883
803	644809	517781627	28.3372546	9.2947671	.001245330
804	646416	519718464	28.3548938	9.2986239	.001243781
805	648025	521660125	28.3725219	9.3024775	.001242236
806	649636	523606616	28.3901391	9.3063278	.001240695

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
807	651249	525557943	28.4077454	9.3101750	.001239157
808	652864	527514112	28.4253408	9.3140190	.001237624
809	654481	529475129	28.4429253	9.3178599	.001236094
810	656100	531441000	28.4604989	9.3216975	.001234568
811	657721	533411731	28.4780617	9.3255320	.001233046
812	659344	535387328	28.4956137	9.3293634	.001231527
813	660969	537367797	28.5131549	9.3331916	.001230012
814	662596	539353144	28.5306852	9.3370167	.001228501
815	664225	541343375	28.5482048	9.3408386	.001226994
816	665856	543338496	28.5657137	9.3446575	.001225490
817	667489	545338513	28.5832119	9.3484731	.001223990
818	669124	547343432	28.6006993	9.3522857	.001222494
819	670761	549353259	28.6181760	9.3560952	.001221001
820	672400	551368000	28.6356421	9.3599016	.001219512
821	674041	553387661	28.6530976	9.3637049	.001218027
822	675684	555412248	28.6705424	9.3675051	.001216545
823	677329	557441767	28.6879766	9.3713022	.001215067
824	678976	559476224	28.7054002	9.3750963	.001213592
825	680625	561515625	28.7228132	9.3788873	.001212121
826	682276	563559976	28.7402157	9.3826752	.001210654
827	683929	565609283	28.7576077	9.3864600	.001209190
828	685584	567663552	28.7749891	9.3902419	.001207729
829	687241	569722789	28.7923601	9.3940206	.001206273
830	688900	571787000	28.8097206	9.3977964	.001204819
831	690561	573856191	28.8270706	9.4015691	.001203369
832	692224	575930368	28.8444102	9.4053387	.001201923
833	693889	578009537	28.8617394	9.4091054	.001200480
834	695556	580093704	28.8790582	9.4128690	.001199041
835	697225	582182875	28.8963666	9.4166297	.001197605
836	698896	584277056	28.9136646	9.4203873	.001196172
837	700569	586376253	28.9309523	9.4241420	.001194743
838	702244	588480472	28.9482297	9.4278936	.001193317
839	703921	590589719	28.9654967	9.4316423	.001191895
840	705600	592704000	28.9827535	9.4353880	.001190476
841	707281	594823321	29.0000000	9.4391307	.001189061
842	708964	596947688	29.0172363	9.4428704	.001187648
843	710649	599077107	29.0344623	9.4466072	.001186240
844	712336	601211584	29.0516781	9.4503410	.001184834
845	714025	603351125	29.0688837	9.4540719	.001183432
846	715716	605495736	29.0860791	9.4577999	.001182033
847	717409	607645423	29.1032644	9.4615249	.001180638
848	719104	609800192	29.1204396	9.4652470	.001179245
849	720801	611960049	29.1376046	9.4689661	.001177856
850	722500	614125000	29.1547595	9.4726824	.001176471
851	724201	616295051	29.1719043	9.4763957	.001175088
852	725904	618470208	29.1890390	9.4801061	.001173709
853	727609	620650477	29.2061637	9.4838136	.001172333
854	729316	622835864	29.2232784	9.4875182	.001170960
855	731025	625026375	29.2403830	9.4912200	.001169591
856	732736	627222016	29.2574777	9.4949188	.001168224
857	734449	629422793	29.2745623	9.4986147	.001166861
858	736164	631628712	29.2916370	9.5023078	.001165501
859	737881	633839779	29.3087018	9.5059980	.001164144
860	739600	636056000	29.3257566	9.5096854	.001162791
861	741321	638277381	29.3428015	9.5133699	.001161440
862	743044	640503928	29.3598365	9.5170515	.001160093
863	744769	642735647	29.3768616	9.5207303	.001158749
864	746496	644972544	29.3938769	9.5244063	.001157407
865	748225	647214625	29.4108823	9.5280794	.001156069
866	749956	649461896	29.4278779	9.5317497	.001154734
867	751689	651714363	29.4448637	9.5354172	.001153403
868	753424	653972032	29.4618397	9.5390818	.001152074

TABLE XVI.—SQUARES, CUBES, SQUARE ROOTS.

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
869	755161	656234909	29.4788059	9.5427437	.001150748
870	756900	658503000	29.4957624	9.5464027	.001149425
871	758641	660776311	29.5127091	9.5500589	.001148106
872	760384	663054848	29.5296461	9.5537123	.001146789
873	762129	665338617	29.5465734	9.5573630	.001145475
874	763876	667627624	29.5634910	9.5610108	.001144165
875	765625	669921875	29.5803989	9.5646559	.001142857
876	767376	672221376	29.5972972	9.5682982	.001141553
877	769129	674526133	29.6141858	9.5719377	.001140251
878	770884	676836152	29.6310648	9.5755745	.001138952
879	772641	679151439	29.6479342	9.5792085	.001137656
880	774400	681472000	29.6647939	9.5828397	.001136364
881	776161	683797841	29.6816442	9.5864682	.001135074
882	777924	686128968	29.6984848	9.5900939	.001133787
883	779689	688465387	29.7153159	9.5937169	.001132503
884	781456	690807104	29.7321375	9.5973373	.001131222
885	783225	693154125	29.7489496	9.6009548	.001129944
886	784996	695506456	29.7657521	9.6045696	.001128668
887	786769	697864103	29.7825452	9.6081817	.001127396
888	788544	700227072	29.7993289	9.6117911	.001126126
889	790321	702595369	29.8161030	9.6153977	.001124859
890	792100	704969000	29.8328678	9.6190017	.001123596
891	793881	707347971	29.8496231	9.6226030	.001122334
892	795664	709732288	29.8663690	9.6262016	.001121076
893	797449	712121957	29.8831056	9.6297975	.001119821
894	799236	714516984	29.8998328	9.6333907	.001118568
895	801025	716917375	29.9165506	9.6369812	.001117318
896	802816	719323136	29.9332591	9.6405690	.001116071
897	804609	721734273	29.9499583	9.6441542	.001114827
898	806404	724150792	29.9666481	9.6477367	.001113586
899	808201	726572699	29.9833287	9.6513166	.001112347
900	810000	729000000	30.0000000	9.6548938	.001111111
901	811801	731432701	30.0166620	9.6584684	.001109878
902	813604	733870808	30.0333148	9.6620403	.001108647
903	815409	736314327	30.0499584	9.6656096	.001107420
904	817216	738763264	30.0665928	9.6691762	.001106195
905	819025	741217625	30.0832179	9.6727403	.001104972
906	820836	743677416	30.0998339	9.6763017	.001103753
907	822649	746142643	30.1164407	9.6798604	.001102536
908	824464	748613312	30.1330383	9.6834166	.001101322
909	826281	751089429	30.1496269	9.6869701	.001100110
910	828100	753571000	30.1662063	9.6905211	.001098901
911	829921	756058031	30.1827765	9.6940694	.001097695
912	831744	758550528	30.1993377	9.6976151	.001096491
913	833569	761048497	30.2158899	9.7011583	.001095290
914	835396	763551944	30.2324329	9.7046989	.001094092
915	837225	766060875	30.2489669	9.7082369	.001092896
916	839056	768575296	30.2654919	9.7117723	.001091703
917	840889	771095213	30.2820079	9.7153051	.001090513
918	842724	773620632	30.2985148	9.7188354	.001089325
919	844561	776151559	30.3150128	9.7223631	.001088139
920	846400	778688000	30.3315018	9.7258883	.001086957
921	848241	781229961	30.3479818	9.7294109	.001085776
922	850084	783777448	30.3644529	9.7329309	.001084599
923	851929	786330467	30.3809151	9.7364484	.001083423
924	853776	788889024	30.3973683	9.7399634	.001082251
925	855625	791453125	30.4138127	9.7434758	.001081081
926	857476	794022776	30.4302481	9.7469857	.001079914
927	859329	796597983	30.4466747	9.7504930	.001078749
928	861184	799178752	30.4630924	9.7539979	.001077586
929	863041	801765089	30.4795013	9.7575002	.001076426
930	864900	804357000	30.4959014	9.7610001	.001075269

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
931	866761	806954491	30.5122926	9.7644974	.001074114
932	868624	809557568	30.5286750	9.7679922	.001072961
933	870489	812166237	30.5450487	9.7714845	.001071811
934	872356	814780504	30.5614136	9.7749743	.001070664
935	874225	817400375	30.5777697	9.7784616	.001069519
936	876096	820025856	30.5941171	9.7819466	.001068376
937	877969	822656953	30.6104557	9.7854288	.001067236
938	879844	825293672	30.6267857	9.7889087	.001066098
939	881721	827936019	30.6431069	9.7923861	.001064963
940	883600	830584000	30.6594194	9.7958611	.001063830
941	885481	833237621	30.6757233	9.7993336	.001062699
942	887364	835896888	30.6920185	9.8028036	.001061571
943	889249	838561807	30.7083051	9.8062711	.001060445
944	891136	841232384	30.7245830	9.8097362	.001059322
945	893025	843908625	30.7408523	9.8131989	.001058201
946	894916	846590536	30.7571130	9.8166591	.001057082
947	896809	849278123	30.7733651	9.8201169	.001055966
948	898704	851971392	30.7896086	9.8235723	.001054852
949	900601	854670349	30.8058436	9.8270252	.001053741
950	902500	857375000	30.8220700	9.8304757	.001052632
951	904401	860085351	30.8382879	9.8339238	.001051525
952	906304	862801408	30.8544972	9.8373695	.001050420
953	908209	865523177	30.8706981	9.8408127	.001049318
954	910116	868250664	30.8868904	9.8442536	.001048218
955	912025	870983875	30.9030743	9.8476920	.001047120
956	913936	873722816	30.9192497	9.8511280	.001046025
957	915849	876467493	30.9354166	9.8545617	.001044932
958	917764	879217912	30.9515751	9.8579929	.001043841
959	919681	881974079	30.9677251	9.8614218	.001042753
960	921600	884736000	30.9838668	9.8648483	.001041667
961	923521	887503681	31.0000000	9.8682724	.001040583
962	925444	890277128	31.0161248	9.8716941	.001039501
963	927369	893056347	31.0322413	9.8751135	.001038422
964	929296	895841344	31.0483494	9.8785305	.001037344
965	931225	898632125	31.0644491	9.8819451	.001036269
966	933156	901428696	31.0805405	9.8853574	.001035197
967	935089	904231063	31.0966236	9.8887673	.001034126
968	937024	907039232	31.1126984	9.8921749	.001033058
969	938961	909853209	31.1287648	9.8955801	.001031992
970	940900	912673000	31.1448230	9.8989830	.001030928
971	942841	915498611	31.1608729	9.9023835	.001029866
972	944784	918330048	31.1769145	9.9057817	.001028807
973	946729	921167317	31.1929479	9.9091776	.001027749
974	948676	924010424	31.2089731	9.9125712	.001026694
975	950625	926859375	31.2249900	9.9159624	.001025641
976	952576	929714176	31.2409987	9.9193513	.001024590
977	954529	932574833	31.2569992	9.9227379	.001023541
978	956484	935441352	31.2729915	9.9261222	.001022495
979	958441	938313739	31.2889757	9.9295042	.001021450
980	960400	941192000	31.3049517	9.9328839	.001020408
981	962361	944076141	31.3209195	9.9362613	.001019368
982	964324	946966168	31.3368792	9.9396363	.001018330
983	966289	949862087	31.3528308	9.9430092	.001017294
984	968256	952763904	31.3687743	9.9463797	.001016260
985	970225	955671625	31.3847097	9.9497479	.001015228
986	972196	958585256	31.4006369	9.9531138	.001014199
987	974169	961504803	31.4165561	9.9564775	.001013171
988	976144	964430272	31.4324673	9.9598389	.001012146
989	978121	967361669	31.4483704	9.9631981	.001011122
990	980100	970299000	31.4642654	9.9665549	.001010101
991	982081	973242271	31.4801525	9.9699095	.001009082
	984064	976191488	31.4960315	9.9732619	.001008065

No.	Squares.	Cubes.	Square Roots.	Cube Roots.	Reciprocals.
993	986049	979146657	31.5119025	9.9766120	.001007049
994	988036	982107784	31.5277655	9.9799599	.001006036
995	990025	985074875	31.5436206	9.9833055	.001005025
996	992016	988047936	31.5594677	9.9866488	.001004016
997	994009	991026973	31.5753068	9.9899900	.001003009
998	996004	994011992	31.5911380	9.9933289	.001002004
999	998001	997002999	31.6069613	9.9966656	.001001001
1000	1000000	1000000000	31.6227766	10.0000000	.001000000
1001	1002001	1003003001	31.6385840	10.0033322	.0009990010
1002	1004004	1006012008	31.6543836	10.0066622	.0009980040
1003	1006009	1009027027	31.6701752	10.0099899	.0009970090
1004	1008016	1012048064	31.6859590	10.0133155	.0009960159
1005	1010025	1015075125	31.7017349	10.0166389	.0009950249
1006	1012036	1018108216	31.7175030	10.0199601	.0009940358
1007	1014049	1021147343	31.7332633	10.0232791	.0009930487
1008	1016064	1024192512	31.7490157	10.0265958	.0009920635
1009	1018081	1027243729	31.7647603	10.0299104	.0009910803
1010	1020100	1030301000	31.7804972	10.0332228	.0009900990
1011	1022121	1033364331	31.7962262	10.0365330	.0009891197
1012	1024144	1036433728	31.8119474	10.0398410	.0009881423
1013	1026169	1039509197	31.8276609	10.0431469	.0009871668
1014	1028196	1042590744	31.8433666	10.0464506	.0009861933
1015	1030225	1045678375	31.8590646	10.0497521	.0009852217
1016	1032256	1048772096	31.8747549	10.0530514	.0009842520
1017	1034289	1051871913	31.8904374	10.0563485	.0009832842
1018	1036324	1054977832	31.9061123	10.0596435	.0009823183
1019	1038361	1058089859	31.9217794	10.0629364	.0009813543
1020	1040400	1061208000	31.9374388	10.0662271	.0009803922
1021	1042441	1064332261	31.9530906	10.0695156	.0009794319
1022	1044484	1067462648	31.9687347	10.0728020	.0009784736
1023	1046529	1070599167	31.9843712	10.0760863	.0009775171
1024	1048576	1073741824	32.0000000	10.0793684	.0009765625
1025	1050625	1076890625	32.0156212	10.0826484	.0009756098
1026	1052676	1080045576	32.0312348	10.0859262	.0009746589
1027	1054729	1083206683	32.0468407	10.0892019	.0009737098
1028	1056784	1086373952	32.0624391	10.0924755	.0009727626
1029	1058841	1089547389	32.0780298	10.0957469	.0009718173
1030	1060900	1092727000	32.0936131	10.0990163	.0009708738
1031	1062961	1095912791	32.1091887	10.1022835	.0009699321
1032	1065024	1099104768	32.1247568	10.1055487	.0009689922
1033	1067089	1102302937	32.1403173	10.1088117	.0009680542
1034	1069156	1105507304	32.1558704	10.1120726	.0009671180
1035	1071225	1108717875	32.1714159	10.1153314	.0009661836
1036	1073296	1111934656	32.1869539	10.1185882	.0009652510
1037	1075369	1115157653	32.2024844	10.1218428	.0009643202
1038	1077444	1118386872	32.2180074	10.1250953	.0009633911
1039	1079521	1121622319	32.2335229	10.1283457	.0009624639
1040	1081600	1124864000	32.2490310	10.1315941	.0009615385
1041	1083681	1128111921	32.2645316	10.1348403	.0009606148
1042	1085764	1131366088	32.2800248	10.1380845	.0009596929
1043	1087849	1134626507	32.2955105	10.1413266	.0009587738
1044	1089936	1137893184	32.3109888	10.1445667	.0009578544
1045	1092025	1141166125	32.3264598	10.1478047	.0009569378
1046	1094116	1144445336	32.3419233	10.1510406	.0009560229
1047	1096209	1147730823	32.3573794	10.1542744	.0009551098
1048	1098304	1151022592	32.3728281	10.1575062	.0009541985
1049	1100401	1154320649	32.3882695	10.1607359	.0009532888
1050	1102500	1157625000	32.4037035	10.1639636	.0009523810
1051	1104601	1160935651	32.4191301	10.1671893	.0009514748
1052	1106704	1164252608	32.4345495	10.1704129	.0009505703
1053	1108809	1167575877	32.4499615	10.1736344	.0009496676
1054	1110916	1170905464	32.4653662	10.1768539	.0009487666

N	0	1	2	3	4	5	6	7	8	9
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389
1	0432	0475	0518	0561	0604	0647	0689	0732	0775	0817
2	0860	0903	0945	0988	1030	1072	1115	1157	1199	1242
3	1284	1326	1368	1410	1452	1494	1536	1578	1620	1662
4	1703	1745	1787	1828	1870	1912	1953	1995	2036	2078
5	2119	2160	2202	2243	2284	2325	2366	2407	2449	2490
6	2531	2572	2612	2653	2694	2735	2776	2816	2857	2898
7	2938	2979	3019	3060	3100	3141	3181	3222	3262	3302
8	3342	3383	3423	3463	3503	3543	3583	3623	3663	3703
9	3743	3782	3822	3862	3902	3941	3981	4021	4060	4100
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493
1	4532	4571	4610	4650	4689	4727	4766	4805	4844	4883
2	4922	4961	4999	5038	5077	5115	5154	5192	5231	5269
3	5308	5346	5385	5423	5461	5500	5538	5576	5614	5652
4	5690	5729	5767	5805	5843	5881	5918	5956	5994	6032
5	6070	6108	6145	6183	6221	6258	6296	6333	6371	6408
6	6446	6483	6521	6558	6595	6633	6670	6707	6744	6781
7	6819	6856	6893	6930	6967	7004	7041	7078	7115	7151
8	7188	7225	7262	7298	7335	7372	7408	7445	7482	7518
9	7555	7591	7628	7664	7700	7737	7773	7809	7846	7882
120	07918	07954	07990	08027	08063	08099	08135	08171	08207	08243
1	8279	8314	8350	8386	8422	8458	8493	8529	8565	8600
2	8636	8672	8707	8743	8778	8814	8849	8884	8920	8955
3	8991	9026	9061	9096	9132	9167	9202	9237	9272	9307
4	9342	9377	9412	9447	9482	9517	9552	9587	9621	9656
5	9691	9726	9760	9795	9830	9864	9899	9934	9968	10003
6	10037	10072	10106	10140	10175	10209	10243	10278	10312	0346
7	0380	0415	0449	0483	0517	0551	0585	0619	0653	0687
8	0721	0755	0789	0823	0857	0890	0924	0958	0992	1025
9	1059	1093	1126	1160	1193	1227	1261	1294	1327	1361
130	11394	11428	11461	11494	11528	11561	11594	11628	11661	11694
1	1727	1760	1793	1826	1860	1893	1926	1959	1992	2024
2	2057	2090	2123	2156	2189	2222	2254	2287	2320	2352
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7	6732	6761	6791	6820	6850	6879	6909	6938	6967	6997
8	7026	7056	7085	7114	7143	7173	7202	7231	7260	7289
9	7319	7348	7377	7406	7435	7464	7493	7522	7551	7580
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5	9831	9845̲	9859	9872	9886	9900	9914	9927	9941	9955̲
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320	50515̲	50529	50542	50556	50569	50583	50596	50610	50623	50637
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5	2504	2517	2530	2543	2556	2569	2582	2595	2608	2621
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7	2763	2776	2789	2802	2815̲	2827	2840	2853	2866	2879
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8	6585	6597	6608	6620	6632	6644	6656	6667	6679	6691
9	6703	6714	6726	6738	6750	6761	6773	6785	6797	6808
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5	8546	8557	8569	8580	8591	8602	8614	8625	8636	8647
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7	8771	8782	8794	8805	8816	8827	8838	8850	8861	8872
8	8883	8894	8906	8917	8928	8939	8950	8961	8973	8984
9	8995	9006	9017	9028	9040	9051	9062	9073	9084	9095
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4	0243	0252	0260	0269	0278	0286	0295	0303	0312	0321
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8	3078	3086	3094	3102	3111	3119	3127	3135	3143	3151
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2	7960	7967	7974	7981	7988	7996	8003	8010	8017	80
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6	8247	8254	8262	8269	8276	8283	8290	8297	830$\underline{5}$	83
7	8319	8326	8333	8340	8347	835$\underline{5}$	8362	8369	8376	83
8	8390	8398	840$\underline{5}$	8412	8419	8426	8433	8440	8447	84
9	8462	8469	8476	8483	8490	8497	8504	8512	8519	85
610	78533	78540	78547	78554	78561	78569	78576	78583	78590	785
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2	8675	8682	8689	8696	8704	8711	8718	872$\underline{5}$	8732	87
3	8746	8753	8760	8767	8774	8781	8789	8796	8803	88
4	8817	8824	8831	8838	8845	8852	8859	8866	8873	88
5	8888	889$\underline{5}$	8902	8909	8916	8923	8930	8937	8944	89
6	8958	8965	8972	8979	8986	8993	9000	9007	9014	90
7	9029	9036	9043	90$\underline{5}$0	9057	9064	9071	9078	908$\underline{5}$	90
8	9099	9106	9113	9120	9127	9134	9141	9148	9155	91
9	9169	9176	9183	9190	9197	9204	9211	9218	9225	92
620	79239	79246	79253	79260	79267	79274	79281	79288	79295	793
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2	9379	9386	9393	9400	9407	9414	9421	9428	943$\underline{5}$	94
3	9449	9456	9463	9470	9477	9484	9491	9498	950$\underline{5}$	95
4	9518	9525	9532	9539	9546	9553	9560	9567	9574	95
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6	9657	9664	9671	9678	9685	9692	9699	9706	9713	97
7	9727	9734	9741	9748	9754	9761	9768	9775	9782	97
8	9796	9803	9810	9817	9824	9831	9837	9844	9851	98
9	9865	9872	9879	9886	9893	9900	9906	9913	9920	99
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4	0209	0216	0223	0229	0236	0243	0250	0257	0264	02
5	0277	0284	0291	0298	030$\underline{5}$	0312	0318	0325	0332	03
6	0346	0353	0359	0366	0373	0380	0387	0393	0400	04
7	0414	0421	0428	0434	0441	0448	045$\underline{5}$	0462	0468	04
8	0482	0489	0496	0502	0509	0516	0523	0530	0536	05
9	0550	0557	0564	0570	0577	0584	0591	0598	0604	06
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4	0889	0895	0902	0909	0916	0922	0929	0936	0943	09
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8	1158	1164	1171	1178	1184	1191	1198	1204	1211	12
9	1224	1231	1238	124$\underline{5}$	1251	1258	126$\underline{5}$	1271	1278	12
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7	3696	3702	3708	3715	3721	3727	3734	3740	3746	3753
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8	5612	5618	5625	5631	5637	5643	5649	5655	5661	5667
9	5673	5679	5685	5691	5697	5703	5709	5715	5721	5727
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6	6094	6100	6106	6112	6118	6124	6130	6136	6141	6147
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8	6213	6219	6225	6231	6237	6243	6249	6255	6261	6267
9	6273	6279	6285	6291	6297	6303	6308	6314	6320	6326
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8	6806	6812	6817	6823	6829	6835	6841	6847	6853	6859
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5	7216	7221	7227	7233	7239	7245	7251	7256	7262	7268
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7	7332	7338	7344	7349	7355	7361	7367	7373	7379	7384
8	7390	7396	7402	7408	7413	7419	7425	7431	7437	7442
9	7448	7454	7460	7466	7471	7477	7483	7489	7495	7500
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8	7967	7973	7978	7984	7990	7996	8001	8007	8013	8018
9	8024	8030	8036	8041	8047	8053	8058	8064	8070	8076
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2	8195	8201	8207	8213	8218	8224	8230	8235	8241	8247
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5	8366	8372	8377	8383	8389	8395	8400	8406	8412	8417
6	8423	8429	8434	8440	8446	8451	8457	8463	8468	8474
7	8480	8485	8491	8497	8502	8508	8513	8519	8525	8530
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9	8593	8598	8604	8610	8615	8621	8627	8632	8638	8643
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9	9154	9159	9165	9170	9176	9182	9187	9193	9198	9204
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9	9708	9713	9719	9724	9730	9735	9741	9746	9752	9757
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8	0200	0206	0211	0217	0222	0227	0233	0238	0244	0249
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9	1328	1334	1339	1344	1350	1355	1360	1365	1371	1376
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6	2221	2226	2231	2236	2241	2247	2252	2257	2262	2267
7	2273	2278	2283	2288	2293	2298	2304	2309	2314	2319
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2	2531	2536	2542	2547	2552	2557	2562	2567	2572	2578
3	2583	2588	2593	2598	2603	2609	2614	2619	2624	2629
4	2634	2639	2645	2650	2655	2660	2665	2670	2675	2681
5	2686	2691	2696	2701	2706	2711	2716	2722	2727	2732
6	2737	2742	2747	2752	2758	2763	2768	2773	2778	2783
7	2788	2793	2799	2804	2809	2814	2819	2824	2829	2834
8	2840	2845	2850	2855	2860	2865	2870	2875	2881	2886
9	2891	2896	2901	2906	2911	2916	2921	2927	2932	2937
	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988

N	0	1	2	3	4	5	6	7	8	9
850	92942	92947	92952	92957	92962	92967	92973	92978	92983	92988
1	2993	2998	3003	3008	3013	3018	3024	3029	3034	3039
2	3044	3049	3054	3059	3064	3069	3075	3080	3085	3090
3	3095	3100	3105	3110	3115	3120	3125	3131	3136	3141
4	3146	3151	3156	3161	3166	3171	3176	3181	3186	3192
5	3197	3202	3207	3212	3217	3222	3227	3232	3237	3242
6	3247	3252	3258	3263	3268	3273	3278	3283	3288	3293
7	3298	3303	3308	3313	3318	3323	3328	3334	3339	3344
8	3349	3354	3359	3364	3369	3374	3379	3384	3389	3394
9	3399	3404	3409	3414	3420	3425	3430	3435	3440	3445
860	93450	93455	93460	93465	93470	93475	93480	93485	93490	93495
1	3500	3505	3510	3515	3520	3526	3531	3536	3541	3546
2	3551	3556	3561	3566	3571	3576	3581	3586	3591	3596
3	3601	3606	3611	3616	3621	3626	3631	3636	3641	3646
4	3651	3656	3661	3666	3671	3676	3682	3687	3692	3697
5	3702	3707	3712	3717	3722	3727	3732	3737	3742	3747
6	3752	3757	3762	3767	3772	3777	3782	3787	3792	3797
7	3802	3807	3812	3817	3822	3827	3832	3837	3842	3847
8	3852	3857	3862	3867	3872	3877	3882	3887	3892	3897
9	3902	3907	3912	3917	3922	3927	3932	3937	3942	3947
870	93952	93957	93962	93967	93972	93977	93982	93987	93992	93997
1	4002	4007	4012	4017	4022	4027	4032	4037	4042	4047
2	4052	4057	4062	4067	4072	4077	4082	4086	4091	4096
3	4101	4106	4111	4116	4121	4126	4131	4136	4141	4146
4	4151	4156	4161	4166	4171	4176	4181	4186	4191	4196
5	4201	4206	4211	4216	4221	4226	4231	4236	4240	4245
6	4250	4255	4260	4265	4270	4275	4280	4285	4290	4295
7	4300	4305	4310	4315	4320	4325	4330	4335	4340	4345
8	4349	4354	4359	4364	4369	4374	4379	4384	4389	4394
9	4399	4404	4409	4414	4419	4424	4429	4433	4438	4443
880	94448	94453	94458	94463	94468	94473	94478	94483	94488	94493
1	4498	4503	4507	4512	4517	4522	4527	4532	4537	4542
2	4547	4552	4557	4562	4567	4571	4576	4581	4586	4591
3	4596	4601	4606	4611	4616	4621	4626	4630	4635	4640
4	4645	4650	4655	4660	4665	4670	4675	4680	4685	4689
5	4694	4699	4704	4709	4714	4719	4724	4729	4734	4738
6	4743	4748	4753	4758	4763	4768	4773	4778	4783	4787
7	4792	4797	4802	4807	4812	4817	4822	4827	4832	4836
8	4841	4846	4851	4856	4861	4866	4871	4876	4880	4885
9	4890	4895	4900	4905	4910	4915	4919	4924	4929	4934
890	94939	94944	94949	94954	94959	94963	94968	94973	94978	94983
1	4988	4993	4998	5002	5007	5012	5017	5022	5027	5032
2	5036	5041	5046	5051	5056	5061	5066	5071	5075	5080
3	5085	5090	5095	5100	5105	5109	5114	5119	5124	5129
4	5134	5139	5143	5148	5153	5158	5163	5168	5173	5177
5	5182	5187	5192	5197	5202	5207	5211	5216	5221	5226
6	5231	5236	5240	5245	5250	5255	5260	5265	5270	5274
7	5279	5284	5289	5294	5299	5303	5308	5313	5318	5323
8	5328	5332	5337	5342	5347	5352	5357	5361	5366	5371
9	5376	5381	5386	5390	5395	5400	5405	5410	5415	5419
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95

N	0	1	2	3	4	5	6	7	8	9
900	95424	95429	95434	95439	95444	95448	95453	95458	95463	95468
1	5472	5477	5482	5487	5492	5497	5501	5506	5511	5516
2	5521	5525	5530	5535	5540	5545	5550	5554	5559	5564
3	5569	5574	5578	5583	5588	5593	5598	5602	5607	5612
4	5617	5622	5626	5631	5636	5641	5646	5650	5655	5660
5	5665	5670	5674	5679	5684	5689	5694	5698	5703	5708
6	5713	5718	5722	5727	5732	5737	5742	5746	5751	5756
7	5761	5766	5770	5775	5780	5785	5789	5794	5799	5804
8	5809	5813	5818	5823	5828	5832	5837	5842	5847	5852
9	5856	5861	5866	5871	5875	5880	5885	5890	5895	5899
910	95904	95909	95914	95918	95923	95928	95933	95938	95942	95947
1	5952	5957	5961	5966	5971	5976	5980	5985	5990	5995
2	5999	6004	6009	6014	6019	6023	6028	6033	6038	6042
3	6047	6052	6057	6061	6066	6071	6076	6080	6085	6090
4	6095	6099	6104	6109	6114	6118	6123	6128	6133	6137
5	6142	6147	6152	6156	6161	6166	6171	6175	6180	6185
6	6190	6194	6199	6204	6209	6213	6218	6223	6227	6232
7	6237	6242	6246	6251	6256	6261	6265	6270	6275	6280
8	6284	6289	6294	6208	6303	6308	6313	6317	6322	6327
9	6332	6336	6341	6346	6350	6355	6360	6365	6369	6374
920	96379	96384	96388	96393	96398	96402	96407	96412	96417	96421
1	6426	6431	6435	6440	6445	6450	6454	6459	6464	6468
2	6473	6478	6483	6487	6492	6497	6501	6506	6511	6515
3	6520	6525	6530	6534	6539	6544	6548	6553	6558	6562
4	6567	6572	6577	6581	6586	6591	6595	6600	6605	6609
5	6614	6619	6624	6628	6633	6638	6642	6647	6652	6656
6	6661	6666	6670	6675	6680	6685	6689	6694	6699	6703
7	6708	6713	6717	6722	6727	6731	6736	6741	6745	6750
8	6755	6759	6764	6769	6774	6778	6783	6788	6792	6797
9	6802	6806	6811	6816	6820	6825	6830	6834	6839	6844
930	96848	96853	96858	96862	96867	96872	96876	96881	96886	96890
1	6895	6900	6904	6909	6914	6918	6923	6928	6932	6937
2	6942	6946	6951	6956	6960	6965	6970	6974	6979	6984
3	6988	6993	6997	7002	7007	7011	7016	7021	7025	7030
4	7035	7039	7044	7049	7053	7058	7063	7067	7072	7077
5	7081	7086	7090	7095	7100	7104	7109	7114	7118	7123
6	7128	7132	7137	7142	7146	7151	7155	7160	7165	7169
7	7174	7179	7183	7188	7192	7197	7202	7206	7211	7216
8	7220	7225	7230	7234	7239	7243	7248	7253	7257	7262
9	7267	7271	7276	7280	7285	7290	7294	7299	7304	7308
940	97313	97317	97322	97327	97331	97336	97340	97345	97350	97354
1	7359	7364	7368	7373	7377	7382	7387	7391	7396	7400
2	7405	7410	7414	7419	7424	7428	7433	7437	7442	7447
3	7451	7456	7460	7465	7470	7474	7479	7483	7488	7493
4	7497	7502	7506	7511	7516	7520	7525	7529	7534	7539
5	7543	7548	7552	7557	7562	7566	7571	7575	7580	7585
6	7589	7594	7598	7603	7607	7612	7617	7621	7626	7630
7	7635	7640	7644	7649	7653	7658	7663	7667	7672	7676
8	7681	7685	7690	7695	7699	7704	7708	7713	7717	7722
9	7727	7731	7736	7740	7745	7749	7754	7759	7763	7768
	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813

N	0	1	2	3	4	5	6	7	8	9
950	97772	97777	97782	97786	97791	97795	97800	97804	97809	97813
1	7818	7823	7827	7832	7836	7841	7845	7850	7855	7859
2	7864	7868	7873	7877	7882	7886	7891	7896	7900	7905
3	7909	7914	7918	7923	7928	7932	7937	7941	7946	7950
4	7955	7959	7964	7968	7973	7978	7982	7987	7991	7996
5	8000	8005	8009	8014	8019	8023	8028	8032	8037	8041
6	8046	8050	8055	8059	8064	8068	8073	8078	8082	8087
7	8091	8096	8100	8105	8109	8114	8118	8123	8127	8132
8	8137	8141	8146	8150	8155	8159	8164	8168	8173	8177
9	8182	8186	8191	8195	8200	8204	8209	8214	8218	8223
960	98227	98232	98236	98241	98245	98250	98254	98259	98263	98268
1	8272	8277	8281	8286	8290	8295	8299	8304	8308	8313
2	8318	8322	8327	8331	8336	8340	8345	8349	8354	8358
3	8363	8367	8372	8376	8381	8385	8390	8394	8399	8403
4	8408	8412	8417	8421	8426	8430	8435	8439	8444	8448
5	8453	8457	8462	8466	8471	8475	8480	8484	8489	8493
6	8498	8502	8507	8511	8516	8520	8525	8529	8534	8538
7	8543	8547	8552	8556	8561	8565	8570	8574	8579	8583
8	8588	8592	8597	8601	8605	8610	8614	8619	8623	8628
9	8632	8637	8641	8646	8650	8655	8659	8664	8668	8673
970	98677	98682	98686	98691	98695	98700	98704	98709	98713	98717
1	8722	8726	8731	8735	8740	8744	8749	8753	8758	8762
2	8767	8771	8776	8780	8784	8789	8793	8798	8802	8807
3	8811	8816	8820	8825	8829	8834	8838	8843	8847	8851
4	8856	8860	8865	8869	8874	8878	8883	8887	8892	8896
5	8900	8905	8909	8914	8918	8923	8927	8932	8936	8941
6	8945	8949	8954	8058	8963	8967	8972	8976	8981	8985
7	8989	8994	8998	9003	9007	9012	9016	9021	9025	9029
8	9034	9038	9043	9047	9052	9056	9061	9065	9069	9074
9	9078	9083	9087	9092	9096	9100	9105	9109	9114	9118
980	99123	99127	99131	99136	99140	99145	99149	99154	99158	99162
1	9167	9171	9176	9180	9185	9189	9193	9198	9202	9207
2	9211	9216	9220	9224	9229	9233	9238	9242	9247	9251
3	9255	9260	9264	9269	9273	9277	9282	9286	9291	9295
4	9300	9304	9308	9313	9317	9322	9326	9330	9335	9339
5	9344	9348	9352	9357	9361	9366	9370	9374	9379	9383
6	9388	9392	9396	9401	9405	9410	9414	9419	9423	9427
7	9432	9436	9441	9445	9449	9454	9458	9463	9467	9471
8	9476	9480	9484	9489	9493	9498	9502	9506	9511	9515
9	9520	9524	9528	9533	9537	9542	9546	9550	9555	9559
990	99564	99568	99572	99577	99581	99585	99590	99594	99599	99603
1	9607	9612	9616	9621	9625	9629	9634	9638	9642	9647
2	9651	9656	9660	9664	9669	9673	9677	9682	9686	9691
3	9695	9699	9704	9708	9712	9717	9721	9726	9730	9734
4	9739	9743	9747	9752	9756	9760	9765	9769	9774	9778
5	9782	9787	9791	9795	9800	9804	9808	9813	9817	9822
6	9826	9830	9835	9839	9843	9848	9852	9856	9861	9865
7	9870	9874	9878	9883	9887	9891	9896	9900	9904	9909
8	9913	9917	9922	9926	9930	9935	9939	9944	9948	9952
9	9957	9961	9965	9970	9974	9978	9983	9987	9991	9996
1000	00000	00004	00009	00013	00017	00022	00026	00030	00035	00039

USE OF THE COLUMNS HEADED $(q-l)$ AND $(q+l)$, PAGES 719, 720.

The ordinary methods of interpolation for seconds of arc are not accurate for *sines*, *tangents*, and *cotangents* when angles are between 0° and 2°, or for *cosines* when angles are between 88° and 90°. When, therefore, great accuracy is desired the methods given below may be used

I. *Given: Angle A. Required: log sin A.*
$\log \sin A = \log A$ (in seconds) $+ (q-l)$ for sin.*

II. *Given: log sin A. Required: Angle A.*
$\log A$ (in seconds) $= \log \sin A - (q-l)$ for sin.*

Illustration I:
To find log sin 0° 36′ 25″.
$\log A$ in sec. (2185″) = 3.339451
$(q-l)$ for 0° 36′ (sin)* = 4.685567
log sin 0° 36′ 25″ = 8.025018

Illustration II:
log sin A = 8.096340. **Find A.**
log sin A = 8.096340
$(q-l)$ for sin* = 4.685564
log A (in sec.) = 3.410776
A (in sec.) = 2575″ or A = 0° 42′ 55″

III. *Given: Angle A. Required: log tan A.*
$\log \tan A = \log A$ (in seconds) $+ (q-l)$ for tan.*

IV. *Given: log tan A. Required: Angle A.*
$\log A$ (in seconds) $= \log \tan A - (q-l)$ for tan.*

Illustration III:
To find log tan 0° 36′ 25″.
$\log A$ in sec. (2185″) = 3.339451
$(q-l)$ for 0° 36′ (tan)* = 4.685591
log tan 0° 36′ 25″ = 8.025042

Illustration IV:
log tan A = 8.220000. **Find A.**
log tan A = 8.220000
$(q-l)$ for tan* = 4.685615
log A (in sec.) = 3.534385
A (in sec.) = 3423″ or A = 0° 57′ 03″

V. *Given: Angle A. Required: log cot A.*
$\log \cot A = (q+l) - \log A$ (in seconds).

VI. *Given: log cot A. Required: Angle A.*
$\log A$ (in seconds) $= (q+l) - \log \cot A$.

Illustration V:
To find log cot 0° 56′ 12″.
$(q+l)$ = 15.314387
$\log A$ in sec. (3372″) = 3.527888
log cot A = 11.786499

Illustration VI:
log cot A = 12.000 000. **Find A.**
$(q+l)$ = 15.314 411
log cot A = 12.000 000
log A (in sec.) = 3.314411
A (in sec.) = 2063″ or A = 0° 34′ 23″

Cosines.—When the cosine of an angle from 88° to 90° is given or required, use the sine of 90° minus the angle, proceeding as above.

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* For *all* angles up to 2° the first four figures of $(q-l)$ are 4.685 and the first five figures of $(q+l)$ are 15.314. The last three figures are to be found in the column headed $(q-l)$ or $(q+l)$ opposite the angle most nearly corresponding to A. Note that the $(q-l)$ column is double, the left-hand half being used for *sines* and the right-hand half for *tangents*.

0° TABLE XVIII.—LOGARITHMIC SINES. 179°

″	′	Sine.	$q-l$		Tang.	Cotang.	$q+l$	D 1″	Cosine.	′
			4.685				15.314			
0	0	Inf. neg.	575	575	Inf. neg.	Inf. pos.	425		ten	60
60	1	6.463726	575	575	6.463726	13.536274	425		ten	59
120	2	.764756	575	575	.764756	.235244	425		ten	58
180	3	6.940847	575	575	6.940847	13.059153	425		ten	57
240	4	7.065786	575	575	7.065786	12.934214	425		ten	56
300	5	.162696	575	575	.162696	.837304	425		ten	55
360	6	.241877	575	575	.241878	.758122	425	.02	9.999999	54
420	7	.308824	575	575	.308825	.691175	425	.00	.999999	53
480	8	.366816	574	576	.366817	.633183	424	.00	.999999	52
540	9	.417968	574	576	.417970	.582030	424	.00	.999999	51
600	10	.463726	574	576	.463727	.536273	424	.02	.999998	50
660	11	7.505118	574	576	7.505120	12.494880	424	.00	9.999998	49
720	12	.542906	574	577	.542909	.457091	423	.02	.999997	48
780	13	.577668	574	577	.577672	.422328	423	.00	.999997	47
840	14	.609853	574	577	.609857	.390143	423	.02	.999996	46
900	15	.639816	573	578	.639820	.360180	422	.00	.999996	45
960	16	.667845	573	578	.667849	.332151	422	.02	.999995	44
1020	17	.694173	573	578	.694179	.305821	422	.00	.999995	43
1080	18	.718997	573	579	.719003	.280997	421	.02	.999994	42
1140	19	.742478	573	579	.742484	.257516	421	.02	.999993	41
1200	20	.764754	572	580	.764761	.235239	420	.00	.999993	40
1260	21	7.785943	572	580	7.785951	12.214049	420	.02	9.999992	39
1320	22	.806146	572	581	.806155	.193845	419	.02	.999991	38
1380	23	.825451	572	581	.825460	.174540	419	.02	.999990	37
1440	24	.843934	571	582	.843944	.156056	418	.02	.999989	36
1500	25	.861662	571	583	.861674	.138326	417	.00	.999989	35
1560	26	.878695	571	583	.878708	.121292	417	.02	.999988	34
1620	27	.895085	570	584	.895099	.104901	416	.02	.999987	33
1680	28	.910879	570	584	.910894	.089106	416	.02	.999986	32
1740	29	.926119	570	585	.926134	.073866	415	.02	.999985	31
1800	30	.940842	569	586	.940858	.059142	414	.03	.999983	30
1860	31	7.955082	569	587	7.955100	12.044900	413	.02	9.999982	29
1920	32	.968870	569	587	.968889	.031111	413	.02	.999981	28
1980	33	.982233	568	588	.982253	.017747	412	.02	.999980	27
2040	34	7.995198	568	589	7.995219	12.004781	411	.02	.999979	26
2100	35	8.007787	567	590	8.007809	11.992191	410	.03	.999977	25
2160	36	.020021	567	591	.020044	.979956	409	.02	.999976	24
2220	37	.031919	566	592	.031945	.968055	408	.02	.999975	23
2280	38	.043501	566	593	.043527	.956473	407	.03	.999973	22
2340	39	.054781	566	593	.054809	.945191	407	.02	.999972	21
2400	40	.065776	565	594	.065806	.934194	406	.02	.999971	20
2460	41	8.076500	565	595	8.076531	11.923469	405	.03	9.999969	19
2520	42	.086965	564	596	.086997	.913003	404	.02	.999968	18
2580	43	.097183	564	598	.097217	.902783	402	.03	.999966	17
2640	44	.107167	563	599	.107203	.892797	401	.03	.999964	16
2700	45	.116926	562	600	.116963	.883037	400	.02	.999963	15
2760	46	.126471	562	601	.126510	.873490	399	.03	.999961	14
2820	47	.135810	561	602	.135851	.864149	398	.03	.999959	13
2880	48	.144953	561	603	.144996	.855004	397	.02	.999958	12
2940	49	.153907	560	604	.153952	.846048	396	.03	.999956	11
3000	50	.162681	560	605	.162727	.837273	395	.03	.999954	10
3060	51	8.171280	559	607	8.171328	11.828672	393	.03	9.999952	9
3120	52	.179713	558	608	.179763	.820237	392	.03	.999950	8
3180	53	.187985	558	609	.188036	.811964	391	.03	.999948	7
3240	54	.196102	557	611	.196156	.803844	389	.03	.999946	6
3300	55	.204070	556	612	.204126	.795874	388	.03	.999944	5
3360	56	.211895	556	613	.211953	.788047	387	.03	.999942	4
3420	57	.219581	555	615	.219641	.780359	385	.03	.999940	3
3480	58	.227134	554	616	.227195	.772805	384	.03	.999938	2
3540	59	.234557	554	618	.234621	.765379	382	.03	.999936	1
3600	60	8.241855	553	619	8.241921	11.758079	381	.03	9.999934	0
			4.685				15.314			
″	′	Cosine.	$q-l$		Cotang.	Tang.	$q+l$	D 1″	Sine.	′

90° 719

″	′	Sine.	q − l		Tang.	Cotang.	q + l	D 1″	Cosine.	′
			4.685				15.314			
3600	0	8.241855	553	619	8.241921	11.758079	381		9.999934	60
3660	1	.249033	552	620	.249102	.750898	380	.05	.999932	59
3720	2	.256094	551	622	.256165	.743835	378	.05	.999929	58
3780	3	263042	551	623	.263115	.736885	377	.03	.999927	57
3840	4	.269881	550	625	.269956	.730044	375	.03	.999925	56
3900	5	.276614	549	627	.276691	.723309	373	.05	.999922	55
3960	6	.283243	548	628	.283323	.716677	372	.03	.999920	54
4020	7	.289773	547	630	.289856	.710144	370	.03	.999918	53
4080	8	.296207	546	632	.296292	.703708	368	.05	.999915	52
4140	9	.302546	546	633	.302634	.697366	367	.03	.999913	51
4200	10	.308794	545	635	.308884	691116	365	.05	.999910	50
4260	11	8.314954	544	637	8.315046	11.684954	363	.05	9.999907	49
4320	12	.321027	543	638	.321122	.678878	362	.03	.999905	48
4380	13	.327016	542	640	.327114	.672886	360	.05	.999902	47
4440	14	.332924	541	642	.333025	.666975	358	.05	.999899	46
4500	15	.338753	540	644	.338856	.661144	356	.03	.999897	45
4560	16	.344504	539	646	.344610	.655390	354	.05	.999894	44
4620	17	.350181	539	648	.350289	.649711	352	.05	.999891	43
4680	18	.355783	538	649	.355895	.644105	351	.05	.999888	42
4740	19	.361315	537	651	.361430	.638570	349	.05	.999885	41
4800	20	.366777	536	653	.366895	.633105	347	.05	.999882	40
4860	21	8.372171	535	655	8.372292	11.627708	345	.05	9.999879	39
4920	22	.377499	534	657	.377622	.622378	343	.05	.999876	38
4980	23	.382762	533	659	.382889	.617111	341	.05	.999873	37
5040	24	.387962	532	661	.388092	.611908	339	.05	.999870	36
5100	25	.393101	531	663	.393234	.606766	337	.05	.999867	35
5160	26	.398179	530	666	.398315	.601685	334	.05	.999864	34
5220	27	.403199	529	668	.403338	.596662	332	.05	.999861	33
5280	28	.408161	527	670	.408304	.591696	330	.05	.999858	32
5340	29	.413068	526	672	.413213	.586787	328	.07	.999854	31
5400	30	.417919	525	674	.418068	.581932	326	.05	.999851	30
5460	31	8.422717	524	676	8.422869	11.577131	324	.05	9.999848	29
5520	32	.427462	523	679	.427618	.572382	321	.07	.999844	28
5580	33	.432156	522	681	.432315	.567685	319	.05	.999841	27
5640	34	.436800	521	683	.436962	.563038	317	.05	.999838	26
5700	35	.441394	520	685	.441560	.558440	315	.07	.999834	25
5760	36	.445941	518	688	.446110	.553890	312	.05	.999831	24
5820	37	.450440	517	690	.450613	.549387	310	.07	.999827	23
5880	38	.454893	516	693	.455070	.544930	307	.05	.999824	22
5940	39	.459301	515	695	.459481	.540519	305	.07	.999820	21
6000	40	.463665	514	697	.463849	.536151	303	.07	.999816	20
6060	41	8.467985	512	700	8.468172	11.531828	300	.05	9.999813	19
6120	42	.472263	511	702	.472454	.527546	298	.07	.999809	18
6180	43	.476498	510	705	.476693	.523307	295	.07	.999805	17
6240	44	.480693	509	707	.480892	.519108	293	.07	.999801	16
6300	45	.484848	507	710	.485050	.514950	290	.07	.999797	15
6360	46	.488963	506	713	.489170	.510830	287	.05	.999794	14
6420	47	.493040	505	715	.493250	.506750	285	.07	.999790	13
6480	48	.497078	503	718	.497293	.502707	282	.07	.999786	12
6540	49	.501080	502	720	.501298	.498702	280	.07	.999782	11
6600	50	.505045	501	723	.505267	.494733	277	.07	.999778	10
6660	51	8.508974	499	726	8.509200	11.490800	274	.07	9.999774	9
6720	52	.512867	498	729	.513098	.486902	271	.08	.999769	8
6780	53	.516726	497	731	.516961	.483039	269	.07	.999765	7
6840	54	.520551	495	734	.520790	.479210	266	.07	.999761	6
6900	55	.524343	494	737	524586	.475414	263	.07	.999757	5
6960	56	.528102	492	740	.528349	.471651	260	.07	.999753	4
70	57	.531828	491	743	.532080	.467920	257	.08	.999748	3
7	58	.535523	490	745	.535779	.464221	255	.07	.999744	2
	59	.539186	488	748	.539447	.460553	252	.07	.999740	1
	60	8.542819	487	751	8.543084	11.456916	249	.08	9.999735	0
			4.685				15.314			
	′	Cosine.	q − l		Cotang.	Tang.	q + l	D 1″	Sine.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	8.542819	60.05	9.999735	.07	8.543084	60.12	11.456916	60
1	.546422	59.55	.999731	.08	.546691	59.62	.453309	59
2	.549995	59.07	.999726	.07	.550268	59.15	.449732	58
3	.553539	58.58	.999722	.08	.553817	58.65	.446183	57
4	.557054	58.10	.999717	.07	.557336	58.20	.442664	56
5	.560540	57.65	.999713	.08	.560828	57.72	.439172	55
6	.563999	57.20	.999708	.07	.564291	57.27	.435709	54
7	.567431	56.75	.999704	.08	.567727	56.83	.432273	53
8	.570836	56.30	.999699	.08	.571137	56.38	.428863	52
9	.574214	55.87	.999694	.08	.574520	55.95	.425480	51
10	.577566	55.43	.999689	.07	.577877	55.52	.422123	50
11	8.580892	55.02	9.999685	.08	8.581208	55.10	11.418792	49
12	.584193	54.60	.999680	.08	.584514	54.68	.415486	48
13	.587469	54.20	.999675	.08	.587795	54.27	.412205	47
14	.590721	53.78	.999670	.08	.591051	53.87	.408949	46
15	.593948	53.40	.999665	.08	.594283	53.48	.405717	45
16	.597152	53.00	.999660	.08	.597492	53.08	.402508	44
17	.600332	52.62	.999655	.08	.600675	52.70	.399325	43
18	.603489	52.23	.999650	.08	.603839	52.32	.396161	42
19	.606623	51.85	.999645	.08	.606978	51.93	.393022	41
20	.609734	51.48	.999640	.08	.610094	51.58	.389906	40
21	8.612823	51.13	9.999635	.10	8.613189	51.22	11.386811	39
22	.615891	50.77	.999629	.08	.616262	50.85	.383738	38
23	.618937	50.42	.999624	.08	.619313	50.50	.380687	37
24	.621962	50.05	.999619	.08	.622343	50.15	.377657	36
25	.624965	49.72	.999614	.10	.625352	49.80	.374648	35
26	.627948	49.38	.999608	.08	.628340	49.47	.371660	34
27	.630911	49.05	.999603	.10	.631308	49.13	.368692	33
28	.633854	48.70	.999597	.08	.634256	48.80	.365744	32
29	.636776	48.40	.999592	.10	.637184	48.48	.362816	31
30	.639680	48.05	.999586	.08	.640093	48.15	.359907	30
31	8.642563	47.75	9.999581	.10	8.642982	47.85	11.357018	29
32	.645428	47.43	.999575	.08	.645853	47.52	.354147	28
33	.648274	47.13	.999570	.10	.648704	47.22	.351296	27
34	.651102	46.82	.999564	.10	.651537	46.92	.348463	26
35	.653911	46.52	.999558	.08	.654352	46.62	.345648	25
36	.656702	46.22	.999553	.10	.657149	46.32	.342851	24
37	.659475	45.92	.999547	.10	.659928	46.02	.340072	23
38	.662230	45.63	.999541	.10	.662689	45.73	.337311	22
39	.664968	45.35	.999535	.10	.665433	45.45	.334567	21
40	.667689	45.07	.999529	.08	.668160	45.17	.331840	20
41	8.670393	44.78	9.999524	.10	8.670870	44.88	11.329130	19
42	.673080	44.52	.999518	.10	.673563	44.60	.326437	18
43	.675751	44.23	.999512	.10	.676239	44.35	.323761	17
44	.678405	43.97	.999506	.10	.678900	44.07	.321100	16
45	.681043	43.70	.999500	.12	.681544	43.80	.318456	15
46	.683665	43.45	.999493	.10	.684172	43.53	.315828	14
47	.686272	43.18	.999487	.10	.686784	43.28	.313216	13
48	.688863	42.92	.999481	.10	.689381	43.03	.310619	12
49	.691438	42.67	.999475	.10	.691963	42.77	.308037	11
50	.693998	42.42	.999469	.10	.694529	42.53	.305471	10
51	8.696543	42.17	9.999463	.12	8.697081	42.27	11.302919	9
52	.699073	41.93	.999456	.10	.699617	42.03	.300383	8
53	.701589	41.68	.999450	.12	.702139	41.78	.297861	7
54	.704090	41.45	.999443	.10	.704646	41.57	.295354	6
55	.706577	41.20	.999437	.10	.707140	41.30	.292860	5
56	.709049	40.97	.999431	.12	.709618	41.08	.290382	4
57	.711507	40.75	.999424	.10	.712083	40.85	.287917	3
58	.713952	40.52	.999418	.12	.714534	40.63	.285466	2
59	.716383	40.28	.999411	.12	.716972	40.40	.283028	1
60	8.718800		9.999404		8.719396		11.280604	0
′	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.718800	40.07	9.999404	.10	8.719396	40.17	11.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37.77	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36.18	.234754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	.780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9 999181	.12	8.788554	34.32	11.211446	29
32	.789787	34.02	.999174	.13	790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.55	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	.812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821343	31 62	.999044	.13	822298	31.78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.13	.826103	31.48	.173897	10
51	8.827011	31.22	9.999019	.15	8.827992	31.37	11.172008	9
52	.828884	31.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.13	.835471	30.83	.164529	5
56	[illegible]	30.55	.998976	.15	.837321	30.70	.162679	4
57	[illegible]	30.43	.998967	.15	.839163	30.58	.160837	3
58		30.30	.998958	.13	.840998	30.45	.159002	2
		30.18	.998950	.15	.842825	30.32	.157175	1
			9.998941		8.844644		11.155356	0
		1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29.00	.137567	50
11	8.863014	28.73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	.998708	.15	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26.50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.65	9.998548	.18	8.913401	25.83	11.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.18	.916495	25.63	.083505	17
44	.916550	25.38	.998516	.17	.918034	25.57	.081966	16
45	.918073	25.30	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	9.998442	.18	8.928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		11.058048	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.718800	40.07	9.999404	.10	8.719396	40.17	11.280604	60
1	.721204	39.85	.999398	.12	.721806	39.97	.278194	59
2	.723595	39.62	.999391	.12	.724204	39.73	.275796	58
3	.725972	39.42	.999384	.10	.726588	39.52	.273412	57
4	.728337	39.18	.999378	.12	.728959	39.30	.271041	56
5	.730688	38.98	.999371	.12	.731317	39.10	.268683	55
6	.733027	38.78	.999364	.12	.733663	38.88	.266337	54
7	.735354	38.55	.999357	.12	.735996	38.68	.264004	53
8	.737667	38.37	.999350	.12	.738317	38.48	.261683	52
9	.739969	38.17	.999343	.12	.740626	38.27	.259374	51
10	.742259	37.95	.999336	.12	.742922	38.08	.257078	50
11	8.744536	37.77	9.999329	.12	8.745207	37.87	11.254793	49
12	.746802	37.55	.999322	.12	.747479	37.68	.252521	48
13	.749055	37.37	.999315	.12	.749740	37.48	.250260	47
14	.751297	37.18	.999308	.12	.751989	37.30	.248011	46
15	.753528	36.98	.999301	.12	.754227	37.10	.245773	45
16	.755747	36.80	.999294	.12	.756453	36.92	.243547	44
17	.757955	36.60	.999287	.13	.758668	36.73	.241332	43
18	.760151	36.43	999279	.12	.760872	36.55	.239128	42
19	.762337	36.23	.999272	.12	.763065	36.35	.236935	41
20	.764511	36.07	.999265	.13	.765246	36.18	.234754	40
21	8.766675	35.88	9.999257	.12	8.767417	36.02	11.232583	39
22	.768828	35.70	.999250	.13	.769578	35.82	.230422	38
23	.770970	35.52	.999242	.12	.771727	35.65	.228273	37
24	.773101	35.37	.999235	.13	.773866	35.48	.226134	36
25	.775223	35.17	.999227	.12	.775995	35.32	.224005	35
26	.777333	35.02	.999220	.13	.778114	35.13	.221886	34
27	.779434	34.83	.999212	.12	780222	34.97	.219778	33
28	.781524	34.68	.999205	.13	.782320	34.80	.217680	32
29	.783605	34.50	.999197	.13	.784408	34.63	.215592	31
30	.785675	34.35	.999189	.13	.786486	34.47	.213514	30
31	8.787736	34.18	9.999181	.12	8.788554	34.32	11.211446	29
32	.789787	34.02	.999174	.13	790613	34.15	.209387	28
33	.791828	33.85	.999166	.13	792662	33.98	.207338	27
34	.793859	33.70	.999158	.13	.794701	33.83	.205299	26
35	.795881	33.55	.999150	.13	.796731	33.68	.203269	25
36	.797894	33.38	.999142	.13	.798752	33.52	.201248	24
37	.799897	33.25	.999134	.13	.800763	33.37	.199237	23
38	.801892	33.07	.999126	.13	.802765	33.22	.197235	22
39	.803876	32.93	.999118	.13	.804758	33.07	.195242	21
40	.805852	32.78	.999110	.13	.806742	32.92	.193258	20
41	8.807819	32.63	9.999102	.13	8.808717	32.77	11.191283	19
42	.809777	32.48	.999094	.13	.810683	32.63	.189317	18
43	.811726	32.35	.999086	.15	812641	32.47	.187359	17
44	.813667	32.20	.999077	.13	.814589	32.33	.185411	16
45	.815599	32.05	.999069	.13	.816529	32.20	.183471	15
46	.817522	31.90	.999061	.13	.818461	32.05	.181539	14
47	.819436	31.78	.999053	.15	.820384	31.90	.179616	13
48	.821343	31 62	.999044	.13	822298	31.78	.177702	12
49	.823240	31.50	.999036	.15	.824205	31.63	.175795	11
50	.825130	31.35	.999027	.13	.826103	31.48	.173897	10
51	8.827011	31.22	9.999019	.15	8.827992	31.37	11.172008	9
52	.828884	31.08	.999010	.13	.829874	31.23	.170126	8
53	.830749	30.97	.999002	.15	.831748	31.08	.168252	7
54	.832607	30.82	.998993	.15	.833613	30.97	.166387	6
55	.834456	30.68	.998984	.13	.835471	30.83	.164529	5
56	.836297	30.55	.998976	.15	.837321	30.70	.162679	4
57	.838130	30.43	.998967	.15	.839163	30.58	.160837	3
58	.839956	30.30	.998958	.13	.840998	30.45	.159002	2
59	.841774	30.18	.998950	.15	.842825	30.32	.157175	1
60	8.843585		9.998941		8.844644		11.155356	0
′	Cosine.	D 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.843585	30.03	9.998941	.15	8.844644	30.18	11.155356	60
1	.845387	29.93	.998932	.15	.846455	30.08	.153545	59
2	.847183	29.80	.998923	.15	.848260	29.95	.151740	58
3	.848971	29.67	.998914	.15	.850057	29.82	.149943	57
4	.850751	29.57	.998905	.15	.851846	29.70	.148154	56
5	.852525	29.43	.998896	.15	.853628	29.58	.146372	55
6	.854291	29.30	.998887	.15	.855403	29.47	.144597	54
7	.856049	29.20	.998878	.15	.857171	29.35	.142829	53
8	.857801	29.08	.998869	.15	.858932	29.23	.141068	52
9	.859546	28.95	.998860	.15	.860686	29.12	.139314	51
10	.861283	28.85	.998851	.17	.862433	29.00	.137567	50
11	8.863014	28.73	9.998841	.15	8.864173	28.88	11.135827	49
12	.864738	28.62	.998832	.15	.865906	28.77	.134094	48
13	.866455	28.50	.998823	.17	.867632	28.65	.132368	47
14	.868165	28.38	.998813	.15	.869351	28.55	.130649	46
15	.869868	28.28	.998804	.15	.871064	28.43	.128936	45
16	.871565	28.17	.998795	.17	.872770	28.32	.127230	44
17	.873255	28.05	.998785	.15	.874469	28.22	.125531	43
18	.874938	27.95	.998776	.17	.876162	28.12	.123838	42
19	.876615	27.83	.998766	.15	.877849	28.00	.122151	41
20	.878285	27.73	.998757	.17	.879529	27.88	.120471	40
21	8.879949	27.63	9.998747	.15	8.881202	27.78	11.118798	39
22	.881607	27.52	.998738	.17	.882869	27.68	.117131	38
23	.883258	27.42	.998728	.17	.884530	27.58	.115470	37
24	.884903	27.32	.998718	.17	.886185	27.47	.113815	36
25	.886542	27.20	.998708	.15	.887833	27.38	.112167	35
26	.888174	27.12	.998699	.17	.889476	27.27	.110524	34
27	.889801	27.00	.998689	.17	.891112	27.17	.108888	33
28	.891421	26.90	.998679	.17	.892742	27.07	.107258	32
29	.893035	26.80	.998669	.17	.894366	26.97	.105634	31
30	.894643	26.72	.998659	.17	.895984	26.87	.104016	30
31	8.896246	26.60	9.998649	.17	8.897596	26.78	11.102404	29
32	.897842	26.50	.998639	.17	.899203	26.67	.100797	28
33	.899432	26.42	.998629	.17	.900803	26.58	.099197	27
34	.901017	26.32	.998619	.17	.902398	26.48	.097602	26
35	.902596	26.22	.998609	.17	.903987	26.38	.096013	25
36	.904169	26.12	.998599	.17	.905570	26.28	.094430	24
37	.905736	26.02	.998589	.18	.907147	26.20	.092853	23
38	.907297	25.93	.998578	.17	.908719	26.10	.091281	22
39	.908853	25.85	.998568	.17	.910285	26.02	.089715	21
40	.910404	25.75	.998558	.17	.911846	25.92	.088154	20
41	8.911949	25.65	9.998548	.18	8.913401	25.83	11.086599	19
42	.913488	25.57	.998537	.17	.914951	25.73	.085049	18
43	.915022	25.47	.998527	.18	.916495	25.63	.083505	17
44	.916550	25.38	.998516	.17	.918034	25.57	.081966	16
45	.918073	25.30	.998506	.18	.919568	25.47	.080432	15
46	.919591	25.20	.998495	.17	.921096	25.38	.078904	14
47	.921103	25.12	.998485	.18	.922619	25.28	.077381	13
48	.922610	25.03	.998474	.17	.924136	25.22	.075864	12
49	.924112	24.95	.998464	.18	.925649	25.12	.074351	11
50	.925609	24.85	.998453	.18	.927156	25.03	.072844	10
51	8.927100	24.78	9.998442	.18	8.928658	24.95	11.071342	9
52	.928587	24.68	.998431	.17	.930155	24.87	.069845	8
53	.930068	24.60	.998421	.18	.931647	24.78	.068353	7
54	.931544	24.52	.998410	.18	.933134	24.70	.066866	6
55	.933015	24.43	.998399	.18	.934616	24.62	.065384	5
56	.934481	24.35	.998388	.18	.936093	24.53	.063907	4
57	.935942	24.27	.998377	.18	.937565	24.45	.062435	3
58	.937398	24.20	.998366	.18	.939032	24.37	.060968	2
59	.938850	24.10	.998355	.18	.940494	24.30	.059506	1
60	8.940296		9.998344		8.941952		11.058048	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	8.940296	24.03	9.998344	.18	8.941952	24.20	11.058048	60
1	.941738	23.93	.998333	.18	.943404	24.13	.056596	59
2	.943174	23.87	.998322	.18	.944852	24.05	.055148	58
3	.944606	23.80	.998311	.18	.946295	23.98	.053705	57
4	.946034	23.70	.998300	.18	.947734	23.90	.052266	56
5	.947456	23.63	.998289	.20	.949168	23.82	.050832	55
6	.948874	23.55	.998277	.18	.950597	23.73	.049403	54
7	.950287	23.48	.998266	.18	.952021	23.67	.047979	53
8	.951696	23.40	.998255	.20	.953441	23.58	.046559	52
9	.953100	23.32	.998243	.18	.954856	23.52	.045144	51
10	.954499	23.25	.998232	.20	.956267	23.45	.043733	50
11	8.955894	23.17	9.998220	.18	8.957674	23.35	11.042326	49
12	.957284	23.10	.998209	.20	.959075	23.30	.040925	48
13	.958670	23.03	.998197	.18	.960473	23.22	.039527	47
14	.960052	22.95	.998186	.20	.961866	23.15	.038134	46
15	.961429	22.87	.998174	.18	.963255	23.07	.036745	45
16	.962801	22.82	.998163	.20	.964639	23.00	.035361	44
17	.964170	22.73	.998151	.20	.966019	22.92	.033981	43
18	.965534	22.65	.998139	.18	.967394	22.87	.032606	42
19	.966893	22.60	.998128	.20	.968766	22.78	.031234	41
20	.968249	22.52	.998116	.20	.970133	22.72	.029867	40
21	8.969600	22.45	9.998104	.20	8.971496	22.65	11.028504	39
22	.970947	22.37	.998092	.20	.972855	22.57	.027145	38
23	.972289	22.32	.998080	.20	.974209	22.52	.025791	37
24	.973628	22.23	.998068	.20	.975560	22.43	.024440	36
25	.974962	22.18	.998056	.20	.976906	22.37	.023094	35
26	.976293	22.10	.998044	.20	.978248	22.30	.021752	34
27	.977619	22.03	.998032	.20	.979586	22.25	.020414	33
28	.978941	21.97	.998020	.20	.980921	22.17	.019079	32
29	.980259	21.90	.998008	.20	.982251	22.10	.017749	31
30	.981573	21.83	.997996	.20	.983577	22.03	.016423	30
31	8.982883	21.77	9.997984	.20	8.984899	21.97	11.015101	29
32	.984189	21.72	.997972	.22	.986217	21.92	.013783	28
33	.985491	21.63	.997959	.20	.987532	21.83	.012468	27
34	.986789	21.57	.997947	.20	.988842	21.78	.011158	26
35	.988083	21.52	.997935	.22	.990149	21.70	.009851	25
36	.989374	21.43	.997922	.20	.991451	21.65	.008549	24
37	.990660	21.38	.997910	.22	.992750	21.58	.007250	23
38	.991943	21.32	.997897	.20	.994045	21.53	.005955	22
39	.993222	21.25	.997885	.22	.995337	21.45	.004663	21
40	.994497	21.18	.997872	.20	.996624	21.40	.003376	20
41	8.995768	21.13	9.997860	.22	8.997908	21.33	11.002092	19
42	.997036	21.05	.997847	.20	8.999188	21.28	11.000812	18
43	.998299	21.02	.997835	.22	9.000465	21.22	10.999535	17
44	8.999560	20.93	.997822	.22	.001738	21.15	.998262	16
45	9.000816	20.88	.997809	.20	.003007	21.08	.996993	15
46	.002069	20.82	.997797	.22	.004272	21.03	.995728	14
47	.003318	20.75	.997784	.22	.005534	20.97	.994466	13
48	.004563	20.70	.997771	.22	.006792	20.92	.993208	12
49	.005805	20.65	.997758	.22	.008047	20.85	.991953	11
50	.007044	20.57	.997745	.22	.009298	20.80	.990702	10
51	9.008278	20.53	9.997732	.22	9.010546	20.73	10.989454	9
52	.009510	20.45	.997719	.22	.011790	20.68	.988210	8
53	.010737	20.42	.997706	.22	.013031	20.62	.986969	7
54	.011962	20.33	.997693	.22	.014268	20.57	.985732	6
55	.013182	20.30	.997680	.22	.015502	20.50	.984498	5
56	.014400	20.22	.997667	.22	.016732	20.45	.983268	4
57	.015613	20.18	.997654	.22	.017959	20.40	.982041	3
58	.016824	20.12	.997641	.22	.019183	20.33	.980817	2
59	.018031	20.07	.997628	.23	.020403	20.28	.979597	1
60	9.019235		9.997614		9.021620		10.978380	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.019235	20.00	9.997614	.22	9.021620	20.23	10.978380	60
1	.020435	19.95	.997601	.22	.022834	20.17	.977166	59
2	.021632	19.88	.997588	.23	.024044	20.12	.975956	58
3	.022825	19.85	.997574	.22	.025251	20.07	.974749	57
4	.024016	19.78	.997561	.23	.026455	20.00	.973545	56
5	.025203	19.72	.997547	.22	.027655	19.95	.972345	55
6	.026386	19.68	.997534	.23	.028852	19.90	.971148	54
7	.027567	19.62	.997520	.22	.030046	19.85	.969954	53
8	.028744	19.57	.997507	.23	.031237	19.80	.968763	52
9	.029918	19.52	.997493	.22	.032425	19.73	.967575	51
10	.031089	19.47	.997480	.23	.033609	19.70	.966391	50
11	9.032257	19.40	9.997466	.23	9.034791	19.63	10.965209	49
12	.033421	19.35	.997452	.22	.035969	19.58	.964031	48
13	.034582	19.32	.997439	.23	.037144	19.53	.962856	47
14	.035741	19.25	.997425	.23	.038316	19.48	.961684	46
15	.036896	19.20	.997411	.23	.039485	19.43	.960515	45
16	.038048	19.15	.997397	.23	.040651	19.37	.959349	44
17	.039197	19.08	.997383	.23	.041813	19.33	.958187	43
18	.040342	19.05	.997369	.23	.042973	19.28	.957027	42
19	.041485	19.00	.997355	.23	.044130	19.23	.955870	41
20	.042625	18.95	.997341	.23	.045284	19.17	.954716	40
21	9.043762	18.88	9.997327	.23	9.046434	19.13	10.953566	39
22	.044895	18.85	.997313	.23	.047582	19.08	.952418	38
23	.046026	18.80	.997299	.23	.048727	19.03	.951273	37
24	.047154	18.75	.997285	.23	.049869	18.98	.950131	36
25	.048279	18.68	.997271	.23	.051008	18.93	.948992	35
26	.049400	18.65	.997257	.25	.052144	18.88	.947856	34
27	.050519	18.60	.997242	.23	.053277	18.83	.946723	33
28	.051635	18.57	.997228	.23	.054407	18.80	.945593	32
29	.052749	18.50	.997214	.25	.055535	18.73	.944465	31
30	.053859	18.45	.997199	.23	.056659	18.70	.943341	30
31	9.054966	18.42	9.997185	.25	9.057781	18.65	10.942219	29
32	.056071	18.35	.997170	.23	.058900	18.60	.941100	28
33	.057172	18.32	.997156	.25	.060016	18.57	.939984	27
34	.058271	18.27	.997141	.23	.061130	18.50	.938870	26
35	.059367	18.22	.997127	.25	.062240	18.47	.937760	25
36	.060460	18.18	.997112	.23	.063348	18.42	.936652	24
37	.061551	18.13	.997098	.25	.064453	18.38	.935547	23
38	.062639	18.08	.997083	.25	.065556	18.32	.934444	22
39	.063724	18.03	.997068	.25	.066655	18.28	.933345	21
40	.064806	17.98	.997053	.23	.067752	18.25	.932248	20
41	9.065885	17.95	9.997039	.25	9.068846	18.20	10.931154	19
42	.066962	17.90	.997024	.25	.069938	18.15	.930062	18
43	.068036	17.85	.997009	.25	.071027	18.10	.928973	17
44	.069107	17.82	.996994	.25	.072113	18.07	.927887	16
45	.070176	17.77	.996979	.25	.073197	18.02	.926803	15
46	.071242	17.73	.996964	.25	.074278	17.97	.925722	14
47	.072306	17.67	.996949	.25	.075356	17.93	.924644	13
48	.073366	17.63	.996934	.25	.076432	17.88	.923568	12
49	.074424	17.60	.996919	.25	.077505	17.85	.922495	11
50	.075480	17.55	.996904	.25	.078576	17.80	.921424	10
51	9.076533	17.50	9.996889	.25	9.079644	17.77	10.920356	9
52	.077583	17.47	.996874	.27	.080710	17.72	.919290	8
53	.078631	17.42	.996858	.25	.081773	17.67	.918227	7
54	.079676	17.38	.996843	.27	.082833	17.63	.917167	6
55	.080719	17.33	.996828	.27	.083891	17.60	.916109	5
56	.081759	17.30	.996812	.25	.084947	17.55	.915053	4
57	.082797	17.25	.996797	.25	.086000	17.50	.914000	3
58	.083832	17.20	.996782	.27	.087050	17.47	.912950	2
59	.084864	17.17	.996766	.25	.088098	17.43	.911902	1
60	9.085894		9.996751		9.089144		10.910856	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.085894	17.13	9.996751	.27	9.089144	17.38	10.910856	60
1	.086922	17.08	.996735	.25	.090187	17.35	.909813	59
2	.087947	17.05	.996720	.27	.091228	17.30	.908772	58
3	.088970	17.00	.996704	.27	.092266	17.27	.907734	57
4	.089990	16.97	.996688	.25	.093302	17.23	.906698	56
5	.091008	16.93	.996673	.27	.094336	17.18	.905664	55
6	.092024	16.88	.996657	.27	.095367	17.13	.904633	54
7	.093037	16.83	.996641	.27	.096395	17.12	.903605	53
8	.094047	16.82	.996625	.25	.097422	17.07	.902578	52
9	.095056	16.77	.996610	.27	.098446	17.03	.901554	51
10	.096062	16.72	.996594	.27	.099468	16.98	.900532	50
11	9.097065	16.68	9.996578	.27	9.100487	16.95	10.899513	49
12	.098066	16.65	.996562	.27	.101504	16.92	.898496	48
13	.099065	16.62	.996546	.27	.102519	16.88	.897481	47
14	.100062	16.57	.996530	.27	.103532	16.83	.896468	46
15	.101056	16.53	.996514	.27	.104542	16.80	.895458	45
16	.102048	16.48	.996498	.27	.105550	16.77	.894450	44
17	.103037	16.47	.996482	.28	.106556	16.72	.893444	43
18	.104025	16.42	.996465	.27	.107559	16.68	.892441	42
19	.105010	16.37	.996449	.27	.108560	16.65	.891440	41
20	.105992	16.35	.996433	.27	.109559	16.62	.890441	40
21	9.106973	16.30	9.996417	.28	9.110556	16.58	10.889444	39
22	.107951	16.27	.996400	.27	.111551	16.53	.888449	38
23	.108927	16.23	.996384	.27	.112543	16.50	.887457	37
24	.109901	16.20	.996368	.28	.113533	16.47	.886467	36
25	.110873	16.15	.996351	.27	.114521	16.43	.885479	35
26	.111842	16.12	.996335	.28	.115507	16.40	.884493	34
27	.112809	16.08	.996318	.27	.116491	16.35	.883509	33
28	.113774	16.05	.996302	.28	.117472	16.33	.882528	32
29	.114737	16.02	.996285	.27	.118452	16.28	.881548	31
30	.115698	15.97	.996269	.28	.119429	16.25	.880571	30
31	9.116656	15.95	9.996252	.28	9.120404	16.22	10.879596	29
32	.117613	15.90	.996235	.27	.121377	16.18	.878623	28
33	.118567	15.87	.996219	.28	.122348	16.15	.877652	27
34	.119519	15.83	.996202	.28	.123317	16.12	.876683	26
35	.120469	15.80	.996185	.28	.124284	16.08	.875716	25
36	.121417	15.75	.996168	.28	.125249	16.03	.874751	24
37	.122362	15.73	.996151	.28	.126211	16.02	.873789	23
38	.123306	15.70	.996134	.28	.127172	15.97	.872828	22
39	.124248	15.65	.996117	.28	.128130	15.95	.871870	21
40	.125187	15.63	.996100	.28	.129087	15.90	.870913	20
41	9.126125	15.58	9.996083	.28	9.130041	15.88	10.869959	19
42	.127060	15.55	.996066	.28	.130994	15.83	.869006	18
43	.127993	15.53	.996049	.28	.131944	15.82	.868056	17
44	.128925	15.48	.996032	.28	.132893	15.77	.867107	16
45	.129854	15.45	.996015	.28	.133839	15.75	.866161	15
46	.130781	15.42	.995998	.30	.134784	15.70	.865216	14
47	.131706	15.40	.995980	.28	.135726	15.68	.864274	13
48	.132630	15.35	.995963	.28	.136667	15.63	.863333	12
49	.133551	15.32	.995946	.30	.137605	15.62	.862395	11
50	.134470	15.28	.995928	.28	.138542	15.57	.861458	10
51	9.135387	15.27	9.995911	.28	9.139476	15.55	10.860524	9
52	.136303	15.22	.995894	.30	.140409	15.52	.859591	8
53	.137216	15.20	.995876	.28	.141340	15.48	.858660	7
54	.138128	15.15	.995859	.30	.142269	15.45	.857731	6
55	.139037	15.12	.995841	.30	.143196	15.42	.856804	5
56	.139944	15.10	.995823	.28	.144121	15.38	.855879	4
57	.140850	15.07	.995806	.30	.145044	15.37	.854956	3
58	.141754	15.02	.995788	.28	.145966	15.32	.854034	2
59	.142655	15.00	.995771	.30	.146885	15.30	.853115	1
60	9.143555		9.995753		9.147803		10.852197	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	′
0	9.143555	14.97	9.995753	.30	9.147803	15.25	10.852197	60
1	.144453	14.93	.995735	.30	.148718	15.23	.851282	59
2	.145349	14.90	.995717	.30	.149632	15.20	.850368	58
3	.146243	14.88	.995699	.30	.150544	15.17	.849456	57
4	.147136	14.83	.995681	.28	.151454	15.15	.848546	56
5	.148026	14.82	.995664	.30	.152363	15.10	.847637	55
6	.148915	14.78	.995646	.30	.153269	15.08	.846731	54
7	.149802	14.73	.995628	.30	.154174	15.05	.845826	53
8	.150686	14.72	.995610	.32	.155077	15.02	.844923	52
9	.151569	14.70	.995591	.30	.155978	14.98	.844022	51
10	.152451	14.65	.995573	.30	.156877	14.97	.843123	50
11	9.153330	14.63	9.995555	.30	9.157775	14.93	10.842225	49
12	.154208	14.58	.995537	.30	.158671	14.90	.841329	48
13	.155083	14.57	.995519	.30	.159565	14.87	.840435	47
14	.155957	14.55	.995501	.32	.160457	14.83	.839543	46
15	.156830	14.50	.995482	.30	.161347	14.82	.838653	45
16	.157700	14.48	.995464	.30	.162236	14.78	.837764	44
17	.158569	14.43	.995446	.32	.163123	14.75	.836877	43
18	.159435	14.43	.995427	.30	.164008	14.73	.835992	42
19	.160301	14.38	.995409	.32	.164892	14.70	.835108	41
20	.161164	14.35	.995390	.30	.165774	14.67	.834226	40
21	9.162025	14.33	9.995372	.32	9.166654	14.63	10.833346	39
22	.162885	14.30	.995353	.32	.167532	14.62	.832468	38
23	.163743	14.28	.995334	.30	.168409	14.58	.831591	37
24	.164600	14.23	.995316	.32	.169284	14.55	.830716	36
25	.165454	14.22	.995297	.32	.170157	14.53	.829843	35
26	.166307	14.20	.995278	.30	.171029	14.50	.828971	34
27	.167159	14.15	.995260	.32	.171899	14.47	.828101	33
28	.168008	14.13	.995241	.32	.172767	14.45	.827233	32
29	.168856	14.10	.995222	.32	.173634	14.42	.826366	31
30	.169702	14.08	.995203	.32	.174499	14.38	.825501	30
31	9.170547	14.03	9.995184	.32	9.175362	14.37	10.824638	29
32	.171389	14.02	.995165	.32	.176224	14.33	.823776	28
33	.172230	14.00	.995146	.32	.177084	14.30	.822916	27
34	.173070	13.97	.995127	.32	.177942	14.28	.822058	26
35	.173908	13.93	.995108	.32	.178799	14 27	.821201	25
36	.174744	13.90	.995089	.32	.179655	14.22	.820345	24
37	.175578	13.88	.995070	.32	.180508	14.20	.819492	23
38	.176411	13.85	.995051	.32	.181360	14.18	.818640	22
39	.177242	13.83	.995032	.32	.182211	14.13	.817789	21
40	.178072	13.80	.995013	.33	.183059	14.13	.816941	20
41	9.178900	13.77	9.994993	.32	9.183907	14.08	10.816093	19
42	.179726	13.75	.994974	.32	.184752	14.08	.815248	18
43	.180551	13.72	.994955	.33	.185597	14.03	.814403	17
44	.181374	13.70	.994935	.32	.186439	14.02	.813561	16
45	.182196	13.67	994916	.33	.187280	14.00	.812720	15
46	.183016	13.63	.994896	.32	.188120	13.97	.811880	14
47	.183834	13.62	.994877	.33	.188958	13.93	.811042	13
48	.184651	13.58	.994857	.32	.189794	13.92	.810206	12
49	.185466	13.57	.994838	.33	.190629	13.88	.809371	11
50	.186280	13.53	.994818	.33	.191462	13.87	.808538	10
51	9.187092	13.52	9.994798	.32	9.192294	13.83	10.807706	9
52	.187903	13.48	.994779	.33	.193124	13.82	.806876	8
53	.188712	13.45	.994759	.33	.193953	13.78	.806047	7
54	.189519	13.43	.994739	.32	.194780	13.77	.805220	6
55	.190325	13.42	.994720	.33	.195606	13.73	.804394	5
56	.191130	13.38	.994700	.33	.196430	13.72	.803570	4
57	.191933	13.35	.994680	.33	.197253	13.68	.802747	3
58	.192734	13.33	.994660	.33	.198074	13.67	.801926	2
59	.193534	13.30	.994640	.22	.198894	13.65	.801106	1
60	9.194332		9.994620		9.199713		10.800287	0
′	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.194332	13.28	9.994620	.33	9.199713	13.60	10.800287	60
1	.195129	13.27	.994600	.33	.200529	13.60	.799471	59
2	.195925	13.23	.994580	.33	.201345	13.57	.798655	58
3	.196719	13.20	.994560	.33	.202159	13.53	.797841	57
4	.197511	13.18	.994540	.35	.202971	13.52	.797029	56
5	.198302	13.15	.994519	.33	.203782	13.50	.796218	55
6	.199091	13.13	.994499	.33	.204592	13.47	.795408	54
7	.199879	13.12	.994479	.33	.205400	13.45	.794600	53
8	.200666	13.08	.994459	.35	.206207	13.43	.793793	52
9	.201451	13.05	.994438	.33	.207013	13.40	.792987	51
10	.202234	13.05	.994418	.33	.207817	13.37	.792183	50
11	9.203017	13.00	9.994398	.35	9.208619	13.35	10.791381	49
12	.203797	13.00	.994377	.33	.209420	13.33	.790580	48
13	.204577	12.95	.994357	.35	.210220	13.30	.789780	47
14	.205354	12.95	.994336	.33	.211018	13.28	.788982	46
15	.206131	12.92	.994316	.35	.211815	13.27	.788185	45
16	.206906	12.88	.994295	.35	.212611	13.23	.787389	44
17	.207679	12.88	.994274	.33	.213405	13.22	.786595	43
18	.208452	12.83	.994254	.35	.214198	13.18	.785802	42
19	.209222	12.83	.994233	.35	.214989	13.18	.785011	41
20	.209992	12.80	.994212	.35	.215780	13.13	.784220	40
21	9.210760	12.77	9.994191	.33	9.216568	13.13	10.783432	39
22	.211526	12.75	.994171	.35	.217356	13.10	.782644	38
23	.212291	12.73	.994150	.35	.218142	13.07	.781858	37
24	.213055	12.72	.994129	.35	.218926	13.07	.781074	36
25	.213818	12.68	.994108	.35	.219710	13.03	.780290	35
26	.214579	12.65	.994087	.35	.220492	13.00	.779508	34
27	.215338	12.65	.994066	.35	.221272	13.00	.778728	33
28	.216097	12.62	.994045	.35	.222052	12.97	.777948	32
29	.216854	12.58	.994024	.35	.222830	12.95	.777170	31
30	.217609	12.57	.994003	.35	.223607	12.92	.776393	30
31	9.218363	12.55	9.993982	.37	9.224382	12.90	10.775618	29
32	.219116	12.53	.993960	.35	.225156	12.88	.774844	28
33	.219868	12.50	.993939	.35	.225929	12.85	.774071	27
34	.220618	12.48	.993918	.35	.226700	12.85	.773300	26
35	.221367	12.47	.993897	.37	.227471	12.80	.772529	25
36	.222115	12.43	.993875	.35	.228239	12.80	.771761	24
37	.222861	12.42	.993854	.37	.229007	12.77	.770993	23
38	.223606	12.38	.993832	.35	.229773	12.77	.770227	22
39	.224349	12.38	.993811	.37	.230539	12.72	.769461	21
40	.225092	12.35	.993789	.35	.231302	12.72	.768698	20
41	9.225833	12.33	9.993768	.37	9.232065	12.68	10.767935	19
42	.226573	12.30	.993746	.35	.232826	12.67	.767174	18
43	.227311	12.28	.993725	.37	.233586	12.65	.766414	17
44	.228048	12.27	.993703	.37	.234345	12.63	.765655	16
45	.228784	12.23	.993681	.35	.235103	12.60	.764897	15
46	.229518	12.23	.993660	.37	.235859	12.58	.764141	14
47	.230252	12.20	.993638	.37	.236614	12.57	.763386	13
48	.230984	12.18	.993616	.37	.237368	12.53	.762632	12
49	.231715	12.15	.993594	.37	.238120	12.53	.761880	11
50	.232444	12.13	.993572	.37	.238872	12.50	.761128	10
51	9.233172	12.12	9.993550	.37	9.239622	12.48	10.760378	9
52	.233899	12.10	.993528	.37	.240371	12.45	.759629	8
53	.234625	12.07	.993506	.37	.241118	12.45	.758882	7
54	.235349	12.07	.993484	.37	.241865	12.42	.758135	6
55	.236073	12.03	.993462	.37	.242610	12.40	.757390	5
56	.236795	12.00	.993440	.37	.243354	12.38	.756646	4
57	.237515	12.00	.993418	.37	.244097	12.37	.755903	3
58	.238235	11.97	.993396	.37	.244839	12.33	.755161	2
59	.238953	11.95	.993374	.38	.245579	12.33	.754421	1
60	9.239670		9.993351		9.246319		10.753681	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.239670	11.93	9.993351	.37	9.246319	12.30	10.753681	60
1	.240386	11.92	.993329	.37	.247057	12.28	.752943	59
2	.241101	11.88	.993307	.38	.247794	12.27	.752206	58
3	.241814	11.87	.993284	.37	.248530	12.23	.751470	57
4	.242526	11.85	.993262	.37	.249264	12.23	.750736	56
5	.243237	11.83	.993240	.38	.249998	12.20	.750002	55
6	.243947	11.82	.993217	.37	.250730	12.18	.749270	54
7	.244656	11.78	.993195	.38	.251461	12.17	.748539	53
8	.245363	11.77	.993172	.38	.252191	12.15	.747809	52
9	.246069	11.77	.993149	.37	.252920	12.13	.747080	51
10	.246775	11.72	.993127	.38	.253648	12.10	.746352	50
11	9.247478	11.72	9.993104	.38	9.254374	12.10	10.745626	49
12	.248181	11.70	.993081	.37	.255100	12.07	.744900	48
13	.248883	11.67	.993059	.38	.255824	12.05	.744176	47
14	.249583	11.65	.993036	.38	.256547	12.03	.743453	46
15	.250282	11.63	.993013	.38	.257269	12.02	.742731	45
16	.250980	11.62	.992990	.38	.257990	12.00	.742010	44
17	.251677	11.60	.992967	.38	.258710	11.98	.741290	43
18	.252373	11.57	.992944	.38	.259429	11.95	.740571	42
19	.253067	11.57	.992921	.38	.260146	11.95	.739854	41
20	.253761	11.53	.992898	.38	.260863	11.92	.739137	40
21	9.254453	11.52	9.992875	.38	9.261578	11.90	10.738422	39
22	.255144	11.50	.992852	.38	.262292	11.88	.737708	38
23	.255834	11.48	.992829	.38	.263005	11.87	.736995	37
24	.256523	11.47	.992806	.38	.263717	11.85	.736283	36
25	.257211	11.45	.992783	.40	.264428	11.83	.735572	35
26	.257898	11.42	.992759	.38	.265138	11.82	.734862	34
27	.258583	11.42	.992736	.38	.265847	11.80	.734153	33
28	.259268	11.38	.992713	.38	.266555	11.77	.733445	32
29	.259951	11.37	.992690	.40	.267261	11.77	.732739	31
30	.260633	11.35	.992666	.38	.267967	11.73	.732033	30
31	9.261314	11.33	9.992643	.40	9.268671	11.73	10.731329	29
32	.261994	11.32	.992619	.38	.269375	11.70	.730625	28
33	.262673	11.30	.992596	.40	.270077	11.70	.729923	27
34	.263351	11.27	.992572	.38	.270779	11.67	.729221	26
35	.264027	11.27	.992549	.40	.271479	11.65	.728521	25
36	.264703	11.23	.992525	.40	.272178	11.63	.727822	24
37	.265377	11.23	.992501	.38	.272876	11.62	.727124	23
38	.266051	11.20	.992478	.40	.273573	11.60	.726427	22
39	.266723	11.20	.992454	.40	.274269	11.58	.725731	21
40	.267395	11.17	.992430	.40	.274964	11.57	.725036	20
41	9.268065	11.15	9.992406	.40	9.275658	11.55	10.724342	19
42	.268734	11.13	.992382	.38	.276351	11.53	.723649	18
43	.269402	11.12	.992359	.40	.277043	11.52	.722957	17
44	.270069	11.10	.992335	.40	.277734	11.50	.722266	16
45	.270735	11.08	.992311	.40	.278424	11.48	.721576	15
46	.271400	11.07	.992287	.40	.279113	11.47	.720887	14
47	.272064	11.03	.992263	.40	.279801	11.45	.720199	13
48	.272726	11.03	.992239	.42	.280488	11.43	.719512	12
49	.273388	11.02	.992214	.40	.281174	11.40	.718826	11
50	.274049	10.98	.992190	.40	.281858	11.40	.718142	10
51	9.274708	10.98	9.992166	.40	9.282542	11.38	10.717458	9
52	.275367	10.97	.992142	.40	.283225	11.37	.716775	8
53	.276025	10.93	.992118	.42	.283907	11.35	.716093	7
54	.276681	10.93	.992093	.40	.284588	11.33	.715412	6
55	.277337	10.90	.992069	.42	.285268	11.32	.714732	5
56	.277991	10.90	.992044	.40	.285947	11.28	.714053	4
57	.278645	10.87	.992020	.40	.286624	11.28	.713376	3
58	.279297	10.85	.991996	.42	.287301	11.27	.712699	2
59	.279948	10.85	.991971	.40	.287977	11.25	.712023	1
60	9.280599		9.991947		9.288652		10.711348	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

TABLE XVIII.—LOGARITHMIC SINES.

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.280599	10.82	9.991947	.42	9.288652	11.23	10.711348	60
1	.281248	10.82	.991922	.42	289326	11.22	.710674	59
2	.281897	10.78	.991897	.40	.289999	11.20	.710001	58
3	.282544	10.77	.991873	.42	.290671	11.18	.709329	57
4	.283190	10.77	.991848	.42	.291342	11.18	.708658	56
5	.283836	10.73	.991823	.40	.292013	11.15	.707987	55
6	.284480	10.73	.991799	.42	.292682	11.13	.707318	54
7	.285124	10.70	.991774	.42	.293350	11.12	.706650	53
8	.285766	10.70	.991749	.42	.294017	11.12	.705983	52
9	.286408	10.67	.991724	.42	.294684	11.08	.705316	51
10	.287048	10.67	.991699	.42	.295349	11.07	.704651	50
11	9.287688	10.63	9.991674	.42	9.296013	11.07	10.703987	49
12	.288326	10.63	.991649	.42	.296677	11.03	.703323	48
13	.288964	10.60	.991624	.42	.297339	11.03	.702661	47
14	.289600	10.60	.991599	.42	.298001	11.02	.701999	46
15	.290236	10.57	.991574	.42	.298662	11.00	.701338	45
16	.290870	10.57	.991549	.42	.299322	10.97	.700678	44
17	.291504	10.55	.991524	.43	.299980	10.97	.790020	43
18	.292137	10.52	.991498	.42	.300638	10.95	.699362	42
19	.292768	10.52	.991473	.42	.301295	10.93	.698705	41
20	.293399	10.50	.991448	.43	.301951	10.93	.698049	40
21	9.294029	10.48	9.991422	.42	9.302607	10.90	10.697393	39
22	.294658	10.47	.991397	.42	.303261	10.88	.696739	38
23	.295286	10.45	.991372	.43	.303914	10.88	.696086	37
24	.295913	10.43	.991346	.42	.304567	10.85	.695433	36
25	.296539	10.42	.991321	.43	.305218	10.85	.694782	35
26	.297164	10.40	.991295	.42	.305869	10.83	.694131	34
27	.297788	10.40	.991270	.43	.306519	10.82	.693481	33
28	.298412	10.37	.991244	.43	.307168	10.80	.692832	32
29	.299034	10.35	.991218	.42	.307816	10.78	.692184	31
30	.299655	10.35	.991193	.43	.308463	10.77	.691537	30
31	9.300276	10.32	9.991167	.43	9.309109	10.75	10.690891	29
32	.300895	10.32	.991141	.43	.309754	10.75	.690246	28
33	.301514	10.30	.991115	.42	.310399	10.72	.689601	27
34	.302132	10.27	.991090	.43	.311042	10.72	.688958	26
35	.302748	10.27	.991064	.43	.311685	10.70	.688315	25
36	.303364	10.25	.991038	.43	.312327	10.68	.687673	24
37	.303979	10.23	.991012	.43	.312968	10.67	.687032	23
38	.304593	10.23	.990986	.43	.313608	10.65	.686392	22
39	.305207	10.20	.990960	.43	.314247	10.63	.685753	21
40	.305819	10.18	.990934	.43	.314885	10.63	.685115	20
41	9.306430	10.18	9.990908	.43	9.315523	10.60	10.684477	19
42	.307041	10.15	.990882	.45	.316159	10.60	.683841	18
43	.307650	10.15	.990855	.43	.316795	10.58	.683205	17
44	.308259	10.13	.990829	.43	.317430	10.57	.682570	16
45	.308867	10.12	.990803	.43	.318064	10.55	.681936	15
46	.309474	10.10	.990777	.45	.318697	10.55	.681303	14
47	.310080	10.08	.990750	.43	319330	10.52	.680670	13
48	.310685	10.07	.990724	.45	.319961	10.52	.680039	12
49	.311289	10.07	.990697	.43	.320592	10.50	.679408	11
50	.311893	10.03	.990671	.43	.321222	10.48	.678778	10
51	9.312495	10.03	9.990645	.45	9.321851	10.47	10.678149	9
52	.313097	10.02	.990618	.45	.322479	10.45	.677521	8
53	.313698	9.98	.990591	.43	.323106	10.45	.676894	7
54	.314297	10.00	.990565	.45	.323733	10.42	.676267	6
55	.314897	9.97	.990538	.45	.324358	10.42	.675642	5
56	.315495	9.95	.990511	.43	.324983	10.40	.675017	4
57	.316092	9.95	.990485	.45	.325607	10.40	.674393	3
58	.316689	9.92	.990458	.45	.326231	10.37	.673769	2
59	.317284	9.92	.990431	.45	.326853	10.37	.673147	1
60	9.317879		9.990404		9.327475		10.672525	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.317879	9.90	9.990404	.43	9.327475	10.33	10.672525	60
1	.318473	9.88	.990378	.45	.328095	10.33	.671905	59
2	.319066	9.87	.990351	.45	.328715	10.32	.671285	58
3	.319658	9.85	.990324	.45	.329334	10.32	.670666	57
4	.320249	9.85	.990297	.45	.329953	10.28	.670047	56
5	.320840	9.83	.990270	.45	.330570	10.28	.669430	55
6	.321430	9.82	.990243	.47	.331187	10.27	.668813	54
7	.322019	9.80	.990215	.45	.331803	10.25	.668197	53
8	.322607	9.78	.990188	.45	.332418	10.25	.667582	52
9	.323194	9.77	.990161	.45	.333033	10.22	.666967	51
10	.323780	9.77	.990134	.45	.333646	10.22	.666354	50
11	9.324366	9.73	9.990107	.47	9.334259	10.20	10.665741	49
12	.324950	9.73	.990079	.45	.334871	10.18	.665129	48
13	.325534	9.72	.990052	.45	.335482	10.18	.664518	47
14	.326117	9.72	.990025	.47	.336093	10.15	.663907	46
15	.326700	9.68	.989997	.45	.336702	10.15	.663298	45
16	.327281	9.68	.989970	.47	.337311	10.13	.662689	44
17	.327862	9.67	.989942	.45	.337919	10.13	.662081	43
18	.328442	9.65	.989915	.47	.338527	10.10	.661473	42
19	.329021	9.63	.989887	.45	.339133	10.10	.660867	41
20	.329599	9.62	.989860	.47	.339739	10.08	.660261	40
21	9.330176	9.62	9.989832	.47	9.340344	10.07	10.659656	39
22	.330753	9.60	.989804	.45	.340948	10.07	.659052	38
23	.331329	9.57	.989777	.47	.341552	10.05	.658448	37
24	.331903	9.58	.989749	.47	.342155	10.03	.657845	36
25	.332478	9.55	.989721	.47	.342757	10.02	.657243	35
26	.333051	9.55	.989693	.47	.343358	10.00	.656642	34
27	.333624	9.52	.989665	.47	.343958	10.00	.656042	33
28	.334195	9.53	.989637	.45	.344558	9.98	.655442	32
29	.334767	9.50	.989610	.47	.345157	9.97	.654843	31
30	.335337	9.48	.989582	.48	.345755	9.97	.654245	30
31	9.335906	9.48	9.989553	.47	9.346353	9.93	10.653647	29
32	.336475	9.47	.989525	.47	.346949	9.93	.653051	28
33	.337043	9.45	.989497	.47	.347545	9.93	.652455	27
34	.337610	9.43	.989469	.47	.348141	9.90	.651859	26
35	.338176	9.43	.989441	.47	.348735	9.90	.651265	25
36	.338742	9.42	.989413	.47	.349329	9.88	.650671	24
37	.339307	9.40	.989385	.48	.349922	9.87	.650078	23
38	.339871	9.38	.989356	.47	.350514	9.87	.649486	22
39	.340434	9.37	.989328	.47	.351106	9.85	.648894	21
40	.340996	9.37	.989300	.48	.351697	9.83	.648303	20
41	9.341558	9.35	9.989271	.47	9.352287	9.82	10.647713	19
42	.342119	9.33	.989243	.48	.352876	9.82	.647124	18
43	.342679	9.33	.989214	.47	.353465	9.80	.646535	17
44	.343239	9.30	.989186	.48	.354053	9.78	.645947	16
45	.343797	9.30	.989157	.48	.354640	9.78	.645360	15
46	.344355	9.28	.989128	.47	.355227	9.77	.644773	14
47	.344912	9.28	.989100	.48	.355813	9.75	.644187	13
48	.345469	9.25	.989071	.48	.356398	9.73	.643602	12
49	.346024	9.25	.989042	.47	.356982	9.73	.643018	11
50	.346579	9.25	.989014	.48	.357566	9.72	.642434	10
51	9.347134	9.22	9.988985	.48	9.358149	9.70	10.641851	9
52	.347687	9.22	.988956	.48	.358731	9.70	.641269	8
53	.348240	9.20	.988927	.48	.359313	9.67	.640687	7
54	.348792	9.18	.988898	.48	.359893	9.68	.640107	6
55	.349343	9.17	.988869	.48	.360474	9.65	.639526	5
56	.349893	9.17	.988840	.48	.361053	9.65	.638947	4
57	.350443	9.15	.988811	.48	.361632	9.63	.638368	3
58	.350992	9.13	.988782	.48	.362210	9.62	.637790	2
59	.351540	9.13	.988753	.48	.362787	9.62	.637213	1
60	9.352088		9.988724		9.363364		10.636636	0
'	Cosine.	D. 1".	Sine,	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.352088	9.12	9.988724	.48	9.363364	9.60	10.636636	60
1	.352635	9.10	.988695	.48	.363940	9.58	.636060	59
2	.353181	9.08	.988666	.50	.364515	9.58	.635485	58
3	.353726	9.08	.988636	.48	.365090	9.57	.634910	57
4	.354271	9.07	.988607	.48	.365664	9.55	.634336	56
5	.354815	9.05	.988578	.50	.366237	9.55	.633763	55
6	.355358	9.05	.988548	.48	.366810	9.53	.633190	54
7	.355901	9.03	.988519	.50	.367382	9.52	.632618	53
8	.356443	9.02	.988489	.48	.367953	9.52	.632047	52
9	.356984	9.00	.988460	.50	.368524	9.50	.631476	51
10	.357524	9.00	.988430	.48	.369094	9.48	.630906	50
11	9.358064	8.98	9.988401	.50	9.369663	9.48	10.630337	49
12	.358603	8.97	.988371	.48	.370232	9.45	.629768	48
13	.359141	8.95	.988342	.50	.370799	9.47	.629201	47
14	.359678	8.95	.988312	.50	.371367	9.43	.628633	46
15	.360215	8.95	.988282	.50	.371933	9.43	.628067	45
16	.360752	8.92	.988252	.48	.372499	9.42	.627501	44
17	.361287	8.92	.988223	.50	.373064	9.42	.626936	43
18	.361822	8.90	.988193	.50	.373629	9.40	.626371	42
19	.362356	8.88	.988163	.50	.374193	9.38	.625807	41
20	.362889	8.88	.988133	.50	.374756	9.38	.625244	40
21	9.363422	8.87	9.988103	.50	9.375319	9.37	10.624681	39
22	.363954	8.85	.988073	.50	.375881	9.35	.624119	38
23	.364485	8.85	.988043	.50	.376442	9.35	.623558	37
24	.365016	8.83	.988013	.50	.377003	9.33	.622997	36
25	.365546	8.82	.987983	.50	.377563	9.32	.622437	35
26	.366075	8.82	.987953	.52	.378122	9.32	.621878	34
27	.366604	8.78	.987922	.50	.378681	9.30	.621319	33
28	.367131	8.80	.987892	.50	.379239	9.30	.620761	32
29	.367659	8.77	.987862	.50	.379797	9.28	.620203	31
30	.368185	8.77	.987832	.52	.380354	9.27	.619646	30
31	9.368711	8.75	9.987801	.50	9.380910	9.27	10.619090	29
32	.369236	8.75	.987771	.52	.381466	9.23	.618534	28
33	.369761	8.72	.987740	.50	.382020	9.25	.617980	27
34	.370285	8.72	.987710	.52	.382575	9.23	.617425	26
35	.370808	8.70	.987679	.50	.383129	9.22	.616871	25
36	.371330	8.70	.987649	.52	.383682	9.20	.616318	24
37	.371852	8.68	.987618	.50	.384234	9.20	.615766	23
38	.372373	8.68	.987588	.52	.384786	9.18	.615214	22
39	.372894	8.67	.987557	.52	.385337	9.18	.614663	21
40	.373414	8.65	.987526	.50	.385888	9.17	.614112	20
41	9.373933	8.65	9.987496	.52	9.386438	9.15	10.613562	19
42	.374452	8.63	.987465	.52	.386987	9.15	.613013	18
43	.374970	8.62	.987434	.52	.387536	9.13	.612464	17
44	.375487	8.60	.987403	.52	.388084	9.12	.611916	16
45	.376003	8.60	.987372	.52	.388631	9.12	.611369	15
46	.376519	8.60	.987341	.52	.389178	9.10	.610822	14
47	.377035	8.57	.987310	.52	.389724	9.10	.610276	13
48	.377549	8.57	.987279	.52	.390270	9.08	.609730	12
49	.378063	8.57	.987248	.52	.390815	9.08	.609185	11
50	.378577	8.53	.987217	.52	.391360	9.05	.608640	10
51	9·379089	8.53	9.987186	.52	9.391903	9.07	10.608097	9
52	.379601	8.53	.987155	.52	.392447	9.03	.607553	8
53	.380113	8.52	.987124	.53	.392989	9.03	.607011	7
54	.380624	8.50	.987092	.52	.393531	9.03	.606469	6
55	.381134	8.48	.987061	.52	.394073	9.02	.605927	5
56	.381643	8.48	.987030	.53	.394614	9.00	.605386	4
57	.382152	8.48	.986998	.52	.395154	9.00	.604846	3
58	.382661	8.45	.986967	.52	.395694	8.98	.604306	2
59	.383168	8.45	.986936	.53	.396233	8.97	.603767	1
60	9.383675		9.986904		9.396771		10.603229	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.383675	8.45	9.986904	.52	9.396771	8.97	10.603229	60
1	.384182	8.42	.986873	.53	.397309	8.95	.602691	59
2	.384687	8.42	.986841	.53	.397846	8.95	.602154	58
3	.385192	8.42	.986809	.52	.398383	8.93	.601617	57
4	.385697	8.40	.986778	.53	.398919	8.93	.601081	56
5	.386201	8.38	.986746	.53	.399455	8.92	.600545	55
6	.386704	8.38	.986714	.52	.399990	8.90	.600010	54
7	.387207	8.37	.986683	.53	.400524	8.90	.599476	53
8	.387709	8.35	.986651	.53	.401058	8.88	.598942	52
9	.388210	8.35	.986619	.53	.401591	8.88	.598409	51
10	.388711	8.33	.986587	.53	.402124	8.87	.597876	50
11	9.389211	8.33	9.986555	.53	9.402656	8.85	10.597344	49
12	.389711	8.32	.986523	.53	.403187	8.85	.596813	48
13	.390210	8.30	.986491	.53	.403718	8.85	.596282	47
14	.390708	8.30	.986459	.53	.404249	8.82	.595751	46
15	.391206	8.28	.986427	.53	.404778	8.83	.595222	45
16	.391703	8.27	.986395	.53	.405308	8.80	.594692	44
17	.392199	8.27	.986363	.53	.405836	8.80	.594164	43
18	.392695	8.27	.986331	.53	.406364	8.80	.593636	42
19	.393191	8.23	.986299	.55	.406892	8.78	.593108	41
20	.393685	8.23	.986266	.53	.407419	8.77	.592581	40
21	9.394179	8.23	9.986234	.53	9.407945	8.77	10.592055	39
22	.394673	8.22	.986202	.55	.408471	8.75	.591529	38
23	.395166	8.20	.986169	.53	.408996	8.75	.591004	37
24	.395658	8.20	.986137	.55	.409521	8.73	.590479	36
25	.396150	8.18	.986104	.53	.410045	8.73	.589955	35
26	.396641	8.18	.986072	.55	.410569	8.72	.589431	34
27	.397132	8.15	.986039	.53	.411092	8.72	.588908	33
28	.397621	8.17	.986007	.55	.411615	8.70	.588385	32
29	.398111	8.15	.985974	.53	.412137	8.68	.587863	31
30	.398600	8.13	.985942	.55	412658	8.68	.587342	30
31	9.399088	8.12	9.985909	.55	9.413179	8.67	10.586821	29
32	.399575	8.12	.985876	.55	.413699	8.67	.586301	28
33	.400062	8.12	.985843	.53	.414219	8.65	.585781	27
34	.400549	8.10	.985811	.55	.414738	8.65	.585262	26
35	.401035	8.08	.985778	.55	.415257	8.63	.584743	25
36	.401520	8.08	.985745	.55	.415775	8.63	.584225	24
37	.402005	8.07	.985712	.55	.416293	8.62	.583707	23
38	.402480	8.05	.985679	.55	.416810	8.60	.583190	22
39	.402972	8.05	.985646	.55	.417326	8.60	.582674	21
40	.403455	8.05	.985613	.55	.417842	8.60	.582158	20
41	9.403938	8.03	9.985580	.55	9.418358	8.58	10.581642	19
42	.404420	8.02	.985547	.55	.418873	8.57	.581127	18
43	.404901	8.02	.985514	.57	.419387	8.57	.580613	17
44	.405382	8.00	.985480	.55	.419901	8.57	.580099	16
45	.405862	7.98	.985447	.55	.420415	8.55	.579585	15
46	.406341	7.98	.985414	.55	.420927	8.55	.579073	14
47	.406820	7.98	.985381	.57	.421440	8.53	.578560	13
48	.407299	7.97	.985347	.55	.421952	8.52	.578048	12
49	.407777	7.95	.985314	.57	.422463	8.52	.577537	11
50	.408254	7.95	.985280	.55	.422974	8.50	.577026	10
51	9.408731	7.93	9.985247	.57	9.423484	8.48	10.576516	9
52	.409207	7.93	.985213	.55	.423993	8.50	.576007	8
53	.409682	7.92	.985180	.57	.424503	8.47	.575497	7
54	.410157	7.92	.985146	.55	.425011	8.47	.574989	6
55	.410632	7.90	.985113	.57	.425519	8.47	.574481	5
56	.411106	7.88	.985079	.57	.426027	8.45	.573973	4
57	.411579	7.88	.985045	.57	.426534	8.45	.573466	3
58	.412052	7.87	.985011	.55	.427041	8.43	.572959	2
59	.412524	7.87	.984978	.57	.427547	8.42	.572453	1
60	9.412996		9.984944		9.428052		10.571948	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

15° TABLE XVIII.—LOGARITHMIC SINES. 164°

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.412996	7.85	9.984944	.57	9.428052	8.43	10.571948	60
1	.413467	7.85	.984910	.57	.428558	8.40	.571442	59
2	.413938	7.83	.984876	.57	.429062	8.40	.570938	58
3	.414408	7.83	.984842	.57	.429566	8.40	.570434	57
4	.414878	7.82	.984808	.57	.430070	8.38	.569930	56
5	.415347	7.80	.984774	.57	.430573	8.37	.569427	55
6	.415815	7.80	.984740	.57	.431075	8.37	.568925	54
7	.416283	7.80	.984706	.57	.431577	8.37	.568423	53
8	.416751	7.77	.984672	.57	.432079	8.35	.567921	52
9	.417217	7.78	.984638	.58	.432580	8.33	.567420	51
10	.417684	7.77	.984603	.57	.433080	8.33	.566920	50
11	9.418150	7.75	9.984569	.57	9.433580	8.33	10.566420	49
12	.418615	7.73	.984535	.58	.434080	8.32	.565920	48
13	.419079	7.75	.984500	.57	.434579	8.32	.565421	47
14	.419544	7.72	.984466	.57	.435078	8.30	.564922	46
15	.420007	7.72	.984432	.58	.435576	8.28	.564424	45
16	.420470	7.72	.984397	.57	.436073	8.28	.563927	44
17	.420933	7.70	.984363	.58	.436570	8.28	.563430	43
18	.421395	7.70	.984328	.57	.437067	8.27	.562933	42
19	.421857	7.68	.984294	.58	.437563	8.27	.562437	41
20	.422318	7.67	.984259	.58	.438059	8.25	.561941	40
21	9.422778	7.67	9.984224	.57	9.438554	8.23	10.561446	39
22	.423238	7.65	.984190	.58	.439048	8.25	.560952	38
23	.423697	7.65	.984155	.58	.439543	8.22	.560457	37
24	.424156	7.65	.984120	.58	.440036	8.22	.559964	36
25	.424615	7.63	.984085	.58	.440529	8.22	.559471	35
26	.425073	7.62	.984050	.58	.441022	8.20	.558978	34
27	.425530	7.62	.984015	.57	.441514	8.20	.558486	33
28	425987	7.60	.983981	.58	.442006	8.18	.557994	32
29	.426443	7.60	.983946	.58	.442497	8.18	.557503	31
30	.426899	7.58	.983911	.60	.442988	8.18	.557012	30
31	9.427354	7.58	9.983875	.58	9.443479	8.15	10.556521	29
32	.427809	7.57	.983840	.58	.443968	8.17	.556032	28
33	.428263	7.57	.983805	.58	.444458	8.15	.555542	27
34	.428717	7.55	.983770	.58	.444947	8.13	.555053	26
35	.429170	7.55	.983735	.58	.445435	8.13	.554565	25
36	.429623	7.53	.983700	.60	.445923	8.13	.554077	24
37	.430075	7.53	.983664	.58	.446411	8.12	.553589	23
38	.430527	7.52	.983629	.58	.446898	8.10	.553102	22
39	.430978	7.52	.983594	.60	.447384	8.10	.552616	21
40	.431429	7.50	.983558	.58	.447870	8.10	.552130	20
41	9.431879	7.50	9.983523	.60	9.448356	8.08	10.551644	19
42	.432329	7.48	.983487	.58	.448841	8.08	.551159	18
43	.432778	7.47	.983452	.60	.449326	8.07	.550674	17
44	.433226	7.48	.983416	.58	.449810	8.07	.550190	16
45	.433675	7.45	.983381	.60	.450294	8.05	.549706	15
46	.434122	7.45	.983345	.60	.450777	8.05	.549223	14
47	.434569	7.45	.983309	.60	.451260	8.05	.548740	13
48	.435016	7.43	983273	.58	.451743	8.03	.548257	12
49	.435462	7.43	.983238	.60	.452225	8.02	.547775	11
50	.435908	7.42	.983202	.60	.452706	8.02	.547294	10
51	9.436353	7.42	9.983166	.60	9.453187	8.02	10.546813	9
52	.436798	7.40	.983130	.60	.453668	8.00	.546332	8
53	.437242	7.40	.983094	.60	.454148	8.00	.545852	7
54	.437686	7.38	.983058	.60	.454628	7.98	.545372	6
55	.438129	7.38	.983022	.60	.455107	7.98	.544893	5
56	.438572	7.37	.982986	.60	.455586	7.97	.544414	4
57	.439014	7.37	.982950	.60	.456064	7.97	.543936	3
58	.439456	7.35	.982914	.60	.456542	7.95	.543458	2
59	.439897	7.35	.982878	.60	.457019	7.95	.542981	1
60	9.440338		9.982842		9.457496		10.542504	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.440338	7.33	9.982842	.62	9.457496	7.95	10.542504	60
1	.440778	7.33	.982805	.60	.457973	7.93	.542027	59
2	.441218	7.33	.982769	.60	.458449	7.93	.541551	58
3	.441658	7.30	.982733	.62	.458925	7.92	.541075	57
4	.442096	7.32	.982696	.60	.459400	7.92	.540600	56
5	.442535	7.30	.982660	.60	.459875	7.90	.540125	55
6	.442973	7.28	.982624	.62	.460349	7.90	.539651	54
7	.443410	7.28	.982587	.60	.460823	7.90	.539177	53
8	.443847	7.28	.982551	.62	.461297	7.88	.538703	52
9	.444284	7.27	.982514	.62	.461770	7.87	.538230	51
10	.444720	7.25	.982477	.60	.462242	7.88	.537758	50
11	9.445155	7.25	9.982441	.62	9.462715	7.85	10.537285	49
12	.445590	7.25	.982404	.62	.463186	7.87	.536814	48
13	.446025	7.23	.982367	.60	.463658	7.83	.536342	47
14	.446459	7.23	.982331	.62	.464128	7.85	.535872	46
15	.446893	7.22	.982294	.62	.464599	7.83	.535401	45
16	.447326	7.22	.982257	.62	.465069	7.83	.534931	44
17	.447759	7.20	.982220	.62	.465539	7.82	.534461	43
18	.448191	7.20	.982183	.62	.466008	7.82	.533992	42
19	.448623	7.18	.982146	.62	.466477	7.80	.533523	41
20	.449054	7.18	.982109	.62	.466945	7.80	.533055	40
21	9.449485	7.17	9.982072	.62	9.467413	7.78	10.532587	39
22	.449915	7.17	.982035	.62	.467880	7.78	.532120	38
23	.450345	7.17	.981998	.62	.468347	7.78	.531653	37
24	.450775	7.15	.981961	.62	.468814	7.77	.531186	36
25	.451204	7.13	.981924	.63	.469280	7.77	.530720	35
26	.451632	7.13	.981886	.62	.469746	7.75	.530254	34
27	.452060	7.13	.981849	.62	.470211	7.75	.529789	33
28	.452488	7.12	.981812	.63	.470676	7.75	.529324	32
29	.452915	7.12	.981774	.62	.471141	7.73	.528859	31
30	.453342	7.10	.981737	.62	.471605	7.73	.528395	30
31	9.453768	7.10	9.981700	.63	9.472069	7.72	10.527931	29
32	.454194	7.08	.981662	.62	.472532	7.72	.527468	28
33	.454619	7.08	.981625	.63	.472995	7.70	.527005	27
34	.455044	7.08	.981587	.63	.473457	7.70	.526543	26
35	.455469	7.07	.981549	.62	.473919	7.70	.526081	25
36	.455893	7.05	.981512	.63	.474381	7.68	.525619	24
37	.456316	7.05	.981474	.63	.474842	7.68	.525158	23
38	.456739	7.05	.981436	.62	.475303	7.67	.524697	22
39	.457162	7.03	.981399	.63	.475763	7.67	.524237	21
40	.457584	7.03	.981361	.63	.476223	7.67	.523777	20
41	9.458006	7.02	9.981323	.63	9.476683	7.65	10.523317	19
42	.458427	7.02	.981285	.63	.477142	7.65	.522858	18
43	.458848	7.00	.981247	.63	.477601	7.63	.522399	17
44	.459268	7.00	.981209	.63	.478059	7.63	.521941	16
45	.459688	7.00	.981171	.63	.478517	7.63	.521483	15
46	.460108	6.98	.981133	.63	.478975	7.62	.521025	14
47	.460527	6.98	.981095	.63	.479432	7.62	.520568	13
48	.460946	6.97	.981057	.63	.479889	7.60	.520111	12
49	.461364	6.97	.981019	.63	.480345	7.60	.519655	11
50	.461782	6.95	.980981	.65	.480801	7.60	.519199	10
51	9.462199	6.95	9.980942	.63	9.481257	7.58	10.518743	9
52	.462616	6.93	.980904	.63	.481712	7.58	.518288	8
53	.463032	6.93	.980866	.65	.482167	7.57	.517833	7
54	.463448	6.93	.980827	.63	.482621	7.57	.517379	6
55	.463864	6.92	.980789	.65	.483075	7.57	.516925	5
56	.464279	6.92	.980750	.63	.483529	7.55	.516471	4
57	.464694	6.90	.980712	.65	.483982	7.55	.516018	3
58	.465108	6.90	.980673	.63	.484435	7.53	.515565	2
59	.465522	6.88	.980635	.65	.484887	7.53	.515113	1
60	9.465935		9.980596		9.485339		10.514661	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.465935	6.88	9.980596	.63	9.485339	7.53	10.514661	60
1	.466348	6.88	.980558	.65	.485791	7.52	.514209	59
2	.466761	6.87	.980519	.65	.486242	7.52	.513758	58
3	.467173	6.87	.980480	.63	.486693	7.50	.513307	57
4	.467585	6.85	.980442	.65	.487143	7.50	.512857	56
5	.467996	6.85	.980403	.65	.487593	7.50	.512407	55
6	.468407	6.83	.980364	.65	.488043	7.48	.511957	54
7	.468817	6.83	.980325	.65	.488492	7.48	.511508	53
8	.469227	6.83	.980286	.65	.488941	7.48	.511059	52
9	.469637	6.82	.980247	.65	.489390	7.47	.510610	51
10	.470046	6.82	.980208	.65	.489838	7.47	.510162	50
11	9.470455	6.80	9.980169	.65	9.490286	7.45	10.509714	49
12	.470863	6.80	.980130	.65	.490733	7.45	.509267	48
13	.471271	6.80	.980091	.65	.491180	7.45	.508820	47
14	.471679	6.78	.980052	.67	.491627	7.43	.508373	46
15	.472086	6.77	.980012	.65	.492073	7.43	.507927	45
16	.472492	6.77	.979973	.65	.492519	7.43	.507481	44
17	.472898	6.77	.979934	.65	.492965	7.42	.507035	43
18	.473304	6.77	.979895	.67	.493410	7.40	.506590	42
19	.473710	6.75	.979855	.65	.493854	7.42	.506146	41
20	.474115	6.73	.979816	.67	.494299	7.40	.505701	40
21	9.474519	6.73	9.979776	.65	9.494743	7.38	10.505257	39
22	.474923	6.73	.979737	.67	.495186	7.40	.504814	38
23	.475327	6.72	.979697	.65	.495630	7.38	.504370	37
24	.475730	6.72	.979658	.67	.496073	7.37	.503927	36
25	.476133	6.72	.979618	.65	.496515	7.37	.503485	35
26	.476536	6.70	.979579	.67	.496957	7.37	.503043	34
27	.476938	6.70	.979539	.67	.497399	7.37	.502601	33
28	.477340	6.68	.979499	.67	.497841	7.35	.502159	32
29	.477741	6.68	.979459	.65	.498282	7.33	.501718	31
30	.478142	6.67	.979420	.67	.498722	7.35	.501278	30
31	9.478542	6.67	9.979380	.67	9.499163	7.33	10.500837	29
32	.478942	6.67	.979340	.67	.499603	7.32	.500397	28
33	.479342	6.65	.979300	.67	.500042	7.32	.499958	27
34	.479741	6.65	.979260	.67	.500481	7.32	.499519	26
35	.480140	6.65	.979220	.67	.500920	7.32	.499080	25
36	.480539	6.63	.979180	.67	.501359	7.30	.498641	24
37	.480937	6.62	.979140	.67	.501797	7.30	.498203	23
38	.481334	6.62	.979100	.68	.502235	7.28	.497765	22
39	.481731	6.62	.979059	.67	.502672	7.28	.497328	21
40	.482128	6.62	.979019	.67	.503109	7.28	.496891	20
41	9.482525	6.60	9.978979	.67	9.503546	7.27	10.496454	19
42	.482921	6.58	.978939	.68	.503982	7.27	.496018	18
43	.483316	6.60	.978898	.67	.504418	7.27	.495582	17
44	.483712	6.58	.978858	.68	.504854	7.25	.495146	16
45	.484107	6.57	.978817	.67	.505289	7.25	.494711	15
46	.484501	6.57	.978777	.67	.505724	7.25	.494276	14
47	.484895	6.57	.978737	.68	.506159	7.23	.493841	13
48	.485289	6.55	.978696	.68	.506593	7.23	.493407	12
49	.485682	6.55	.978655	.67	.507027	7.23	.492973	11
50	.486075	6.53	.978615	.68	.507460	7.22	.492540	10
51	9.486467	6.55	9.978574	.68	9.507893	7.22	10.492107	9
52	.486860	6.52	.978533	.67	.508326	7.22	.491674	8
53	.487251	6.53	.978493	.68	.508759	7.20	.491241	7
54	.487643	6.52	.978452	.68	.509191	7.18	.490809	6
55	.488034	6.50	.978411	.68	.509622	7.20	.490378	5
56	.488424	6.50	.978370	.68	.510054	7.18	.489946	4
57	.488814	6.50	.978329	.68	.510485	7.18	.489515	3
58	.489204	6.48	.978288	.68	.510916	7.17	.489084	2
59	.489593	6.48	.978247	.68	.511346	7.17	.488654	1
60	9.489982		9.978206		9.511776		10.488224	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.489982	6.48	9.978206	.68	9.511776	7.17	10.488224	60
1	.490371	6.47	.978165	.68	.512206	7.15	.487794	59
2	.490759	6.47	.978124	.68	.512635	7.15	.487365	58
3	.491147	6.47	.978083	.68	.513064	7.15	.486936	57
4	.491535	6.45	.978042	.68	.513493	7.13	.486507	56
5	.491922	6.43	.978001	.70	.513921	7.13	.486079	55
6	.492308	6.45	.977959	.68	.514349	7.13	.485651	54
7	.492695	6.43	.977918	.68	.514777	7.12	.485223	53
8	.493081	6.42	.977877	.70	.515204	7.12	.484796	52
9	.493466	6.42	.977835	.68	.515631	7.10	.484369	51
10	.493851	6.42	.977794	.70	.516057	7.12	.483943	50
11	9.494236	6.42	9.977752	.68	9.516484	7.10	10.483516	49
12	.494621	6.40	.977711	.70	.516910	7.08	.483090	48
13	.495005	6.38	.977669	.68	.517335	7.10	.482665	47
14	.495388	6.40	.977628	.70	.517761	7.08	.482239	46
15	.495772	6.37	.977586	.70	.518186	7.07	.481814	45
16	.496154	6.38	.977544	.68	.518610	7.07	.481390	44
17	.496537	6.37	.977503	.70	.519034	7.07	.480966	43
18	.496919	6.37	.977461	.70	.519458	7.07	.480542	42
19	.497301	6.35	.977419	.70	.519882	7.05	.480118	41
20	.497682	6.35	.977377	.70	.520305	7.05	.479695	40
21	9.498064	6.33	9.977335	.70	9.520728	7.05	10.479272	39
22	.498444	6.35	.977293	.70	.521151	7.03	.478849	38
23	.498825	6.32	.977251	.70	.521573	7.03	.478427	37
24	.499204	6.33	.977209	.70	.521995	7.03	.478005	36
25	.499584	6.32	.977167	.70	.522417	7.02	.477583	35
26	.499963	6.32	.977125	.70	.522838	7.02	.477162	34
27	.500342	6.32	.977083	.70	.523259	7.02	.476741	33
28	.500721	6.30	.977041	.70	.523680	7.00	.476320	32
29	.501099	6.28	.976999	.70	.524100	7.00	.475900	31
30	.501476	6.30	.976957	.72	.524520	7.00	.475480	30
31	9.501854	6.28	9.976914	.70	9.524940	6.98	10.475060	29
32	.502231	6.27	.976872	.70	.525359	6.98	.474641	28
33	.502607	6.28	.976830	.72	.525778	6.98	.474222	27
34	.502984	6.27	.976787	.70	.526197	6.97	.473803	26
35	.503360	6.25	.976745	.72	.526615	6.97	.473385	25
36	.503735	6.25	.976702	.70	.527033	6.97	.472967	24
37	.504110	6.25	.976660	.72	.527451	6.95	.472549	23
38	.504485	6.25	.976617	.72	.527868	6.95	.472132	22
39	.504860	6.23	.976574	.70	.528285	6.95	.471715	21
40	.505234	6.23	.976532	.72	.528702	6.95	.471298	20
41	9.505608	6.22	9.976489	.72	9.529119	6.93	10.470881	19
42	.505981	6.22	.976446	.70	.529535	6.93	.470465	18
43	.506354	6.22	.976404	.72	.529951	6.92	.470049	17
44	.506727	6.20	.976361	.72	.530366	6.92	.469634	16
45	.507099	6.20	.976318	.72	.530781	6.92	.469219	15
46	.507471	6.20	.976275	.72	.531196	6.92	.468804	14
47	.507843	6.18	.976232	.72	.531611	6.90	.468389	13
48	.508214	6.18	.976189	.72	.532025	6.90	.467975	12
49	.508585	6.18	.976146	.72	.532439	6.90	.467561	11
50	.508956	6.17	.976103	.72	.532853	6.88	.467147	10
51	9.509326	6.17	9.976060	.72	9.533266	6.88	10.466734	9
52	.509696	6.15	.976017	.72	.533679	6.88	.466321	8
53	.510065	6.15	.975974	.73	.534092	6.87	.465908	7
54	.510434	6.15	.975930	.72	.534504	6.87	.465496	6
55	.510803	6.15	.975887	.72	.534916	6.87	.465084	5
56	.511172	6.13	.975844	.73	.535328	6.85	.464672	4
57	.511540	6.12	.975800	.72	.535739	6.85	.464261	3
58	.511907	6.13	.975757	.72	.536150	6.85	.463850	2
59	.512275	6.12	.975714	.73	.536561	6.85	.463439	1
60	9.512642		9.975670		9.536972		10.463028	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.512642	6.12	9.975670	.72	9.536972	6.83	10.463028	60
1	.513009	6.10	.975627	.73	.537382	6.83	.462618	59
2	.513375	6.10	.975583	.73	.537792	6.83	.462208	58
3	.513741	6.10	.975539	.72	.538202	6.82	.461798	57
4	.514107	6.08	.975496	.73	.538611	6.82	.461389	56
5	.514472	6.08	.975452	.73	.539020	6.82	.460980	55
6	.514837	6.08	.975408	.72	.539429	6.80	.460571	54
7	.515202	6.07	.975365	.73	.539837	6.80	.460163	53
8	.515566	6.07	.975321	.73	.540245	6.80	.459755	52
9	.515930	6.07	.975277	.73	.540653	6.80	.459347	51
10	.516294	6.05	.975233	.73	.541061	6.78	.458939	50
11	9.516657	6.05	9.975189	.73	9.541468	6.78	10.458532	49
12	.517020	6.03	.975145	.73	.541875	6.77	.458125	48
13	.517382	6.05	.975101	.73	.542281	6.78	.457719	47
14	.517745	6.03	.975057	.73	.542688	6.77	.457312	46
15	.518107	6.02	.975013	.73	.543094	6.75	.456906	45
16	.518468	6.02	.974969	.73	.543499	6.77	.456501	44
17	.518829	6.02	.974925	.75	.543905	6.75	.456095	43
18	.519190	6.02	.974880	.73	.544310	6.75	.455690	42
19	.519551	6.00	.974836	.73	.544715	6.73	.455285	41
20	.519911	6.00	.974792	.73	.545119	6.75	.454881	40
21	9.520271	6.00	9.974748	.75	9.545524	6.73	10.454476	39
22	.520631	5.98	.974703	.73	.545928	6.72	.454072	38
23	.520990	5.98	.974659	.75	.546331	6.73	.453669	37
24	.521349	5.97	.974614	.73	.546735	6.72	.453265	36
25	.521707	5.98	.974570	.75	.547138	6.70	.452862	35
26	.522066	5.97	.974525	.73	.547540	6.72	.452460	34
27	.522424	5.95	.974481	.75	.547943	6.70	.452057	33
28	.522781	5.95	.974436	.75	.548345	6.70	.451655	32
29	.523138	5.95	.974391	.73	.548747	6.70	.451253	31
30	.523495	5.95	.974347	.75	.549149	6.68	.450851	30
31	9.523852	5.93	9.974302	.75	9.549550	6.68	10.450450	29
32	.524208	5.93	.974257	.75	.549951	6.68	.450049	28
33	.524564	5.93	.974212	.75	.550352	6.67	.449648	27
34	.524920	5.92	.974167	.75	.550752	6.68	.449248	26
35	.525275	5.92	.974122	.75	.551153	6.65	.448847	25
36	.525630	5.90	.974077	.75	.551552	6.67	.448448	24
37	.525984	5.92	.974032	.75	.551952	6.65	.448048	23
38	.526339	5.90	.973987	.75	.552351	6.65	.447649	22
39	.526693	5.88	.973942	.75	.552750	6.65	.447250	21
40	.527046	5.90	.973897	.75	.553149	6.65	.446851	20
41	9.527400	5.88	9.973852	.75	9.553548	6.63	10.446452	19
42	.527753	5.87	.973807	.77	.553946	6.63	.446054	18
43	.528105	5.88	.973761	.75	.554344	6.62	.445656	17
44	.528458	5.87	.973716	.75	.554741	6.63	.445259	16
45	.528810	5.85	.973671	.77	.555139	6.62	.444861	15
46	.529161	5.87	.973625	.75	.555536	6.62	.444464	14
47	.529513	5.85	.973580	.75	.555933	6.60	.444067	13
48	.529864	5.85	.973535	.77	.556329	6.60	.443671	12
49	.530215	5.83	.973489	.75	.556725	6.60	.443275	11
50	.530565	5.83	.973444	.77	.557121	6.60	.442879	10
51	9.530915	5.83	9.973398	.77	9.557517	6.60	10.442483	9
52	.531265	5.82	.973352	.75	.557913	6.58	.442087	8
53	.531614	5.82	.973307	.77	.558308	6.58	.441692	7
54	.531963	5.82	.973261	.77	.558703	6.57	.441297	6
55	.532312	5.82	.973215	.77	.559097	6.57	.440903	5
56	.532661	5.80	.973169	.75	.559491	6.57	.440509	4
57	.533009	5.80	.973124	.77	.559885	6.57	.440115	3
58	.533357	5.78	.973078	.77	.560279	6.57	.439721	2
59	.533704	5.80	.973032	.77	.560673	6.55	.439327	1
60	9.534052		9.972986		9.561066		10.438934	0
′	*Cosine.*	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.534052	5.78	9.972986	.77	9.561066	6.55	10.438934	60
1	.534399	5.77	.972940	.77	.561459	6.53	.438541	59
2	.534745	5.78	.972894	.77	.561851	6.55	.438149	58
3	.535092	5.77	.972848	.77	.562244	6.53	.437756	57
4	.535438	5.75	.972802	.78	.562636	6.53	.437364	56
5	.535783	5.77	.972755	.77	.563028	6.52	.436972	55
6	.536129	5.75	.972709	.77	.563419	6.53	.436581	54
7	.536474	5.73	.972663	.77	.563811	6.52	.436189	53
8	.536818	5.75	.972617	.78	.564202	6.52	.435798	52
9	.537163	5.73	.972570	.77	.564593	6.50	.435407	51
10	.537507	5.73	.972524	.77	.564983	6.50	.435017	50
11	9.537851	5.72	9.972478	.78	9.565373	6.50	10.434627	49
12	.538194	5.73	.972431	.77	.565763	6.50	.434237	48
13	.538538	5.70	.972385	.78	.566153	6.48	.433847	47
14	.538880	5.72	.972338	.78	.566542	6.50	.433458	46
15	.539223	5.70	.972291	.77	.566932	6.47	.433068	45
16	.539565	5.70	.972245	.78	.567320	6.48	.432680	44
17	.539907	5.68	.972198	.78	.567709	6.48	.432291	43
18	.540249	5.68	.972151	.77	.568098	6.47	.431902	42
19	.540590	5.68	.972105	.78	.568486	6.45	.431514	41
20	.540931	5.68	.972058	.78	.568873	6.47	.431127	40
21	9.541272	5.68	9.972011	.78	9.569261	6.45	10.430739	39
22	.541613	5.67	.971964	.78	.569648	6.45	.430352	38
23	.541953	5.67	.971917	.78	.570035	6.45	.429965	37
24	.542293	5.65	.971870	.78	.570422	6.45	.429578	36
25	.542632	5.65	.971823	.78	.570809	6.43	.429191	35
26	.542971	5.65	.971776	.78	.571195	6.43	.428805	34
27	.543310	5.65	.971729	.78	.571581	6.43	.428419	33
28	.543649	5.63	.971682	.78	.571967	6.42	.428033	32
29	.543987	5.63	.971635	.78	.572352	6.43	.427648	31
30	.544325	5.63	.971588	.80	.572738	6.42	.427262	30
31	9.544663	5.62	9.971540	.78	9.573123	6.40	10.426877	29
32	.545000	5.63	.971493	.78	.573507	6.42	.426493	28
33	.545338	5.60	.971446	.80	.573892	6.40	.426108	27
34	.545674	5.62	.971398	.78	.574276	6.40	.425724	26
35	.546011	5.60	.971351	.80	.574660	6.40	.425340	25
36	.546347	5.60	.971303	.78	.575044	6.38	.424956	24
37	.546683	5.60	.971256	.80	.575427	6.38	.424573	23
38	.547019	5.58	.971208	.78	.575810	6.38	.424190	22
39	.547354	5.58	.971161	.80	.576193	6.38	.423807	21
40	.547689	5.58	.971113	.78	.576576	6.38	.423424	20
41	9.548024	5.58	9.971066	.80	9.576959	6.37	10.423041	19
42	.548359	5.57	.971018	.80	.577341	6.37	.422659	18
43	.548693	5.57	.970970	.80	.577723	6.35	.422277	17
44	.549027	5.55	.970922	.80	.578104	6.37	.421896	16
45	549360	5.55	.970874	.78	.578486	6.35	.421514	15
46	.549693	5.55	.970827	.80	.578867	6.35	.421133	14
47	550026	5.55	.970779	.80	.579248	6.35	.420752	13
48	.550359	5.55	.970731	.80	.579629	6.33	.420371	12
49	.550692	5.53	.970683	.80	.580009	6.33	.419991	11
50	.551024	5.53	.970635	.82	.580389	6.33	.419611	10
51	9.551356	5.52	9.970586	.80	9.580769	6.33	10.419231	9
52	.551687	5.52	.970538	.80	.581149	6.32	.418851	8
53	.552018	5.52	.970490	.80	.581528	6.32	.418472	7
54	.552349	5.52	.970442	.80	.581907	6.32	.418093	6
55	.552680	5.50	.970394	.82	.582286	6.32	.417714	5
56	.553010	5.52	.970345	.80	.582665	6.32	.417335	4
57	.553341	5.48	.970297	.80	.583044	6.30	.416956	3
58	.553670	5.50	.970249	.82	.583422	6.30	.416578	2
59	554000	5.48	.970200	.80	.583800	6.28	.416200	1
60	9 554329		9.970152		9.584177		10.415823	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.554329	5.48	9.970152	.82	9.584177	6.30	10.415823	60
1	.554658	5.48	.970103	.80	.584555	6.28	.415445	59
2	.554987	5.47	.970055	.82	.584932	6.28	.415068	58
3	.555315	5.47	.970006	.82	.585309	6.28	.414691	57
4	.555643	5.47	.969957	.80	.585686	6.27	.414314	56
5	.555971	5.47	.969909	.82	.586062	6.28	.413938	55
6	.556299	5.45	.969860	.82	.586439	6.27	.413561	54
7	.556626	5.45	.969811	.82	.586815	6.25	.413185	53
8	.556953	5.45	.969762	.80	.587190	6.27	.412810	52
9	.557280	5.43	.969714	.82	.587566	6.25	.412434	51
10	.557606	5.43	.969665	.82	.587941	6.25	.412059	50
11	9.557932	5.43	9.969616	.82	9.588316	6.25	10.411684	49
12	.558258	5.42	.969567	.82	.588691	6.25	.411309	48
13	.558583	5.43	.969518	.82	.589066	6.23	.410934	47
14	.558909	5.42	.969469	.82	.589440	6.23	.410560	46
15	.559234	5.40	.969420	.83	.589814	6.23	.410186	45
16	.559558	5.42	.969370	.82	.590188	6.23	.409812	44
17	.559883	5.40	.969321	.82	.590562	6.22	.409438	43
18	.560207	5.40	.969272	.82	.590935	6.22	.409065	42
19	.560531	5.40	.969223	.83	.591308	6.22	.408692	41
20	.560855	5.38	.969173	.82	.591681	6.22	.408319	40
21	9.561178	5.38	9.969124	.82	9.592054	6.20	10.407946	39
22	.561501	5.38	.969075	.83	.592426	6.22	.407574	38
23	.561824	5.37	.969025	.82	.592799	6.20	.407201	37
24	.562146	5.37	.968976	.83	.593171	6.18	.406829	36
25	.562468	5.37	.968926	.82	.593542	6.20	.406458	35
26	.562790	5.37	.968877	.83	.593914	6.18	.406086	34
27	.563112	5.35	.968827	.83	.594285	6.18	.405715	33
28	.563433	5.37	.968777	.82	.594656	6.18	.405344	32
29	.563755	5.33	.968728	.83	.595027	6.18	.404973	31
30	.564075	5.35	.968678	.83	.595398	6.17	.404602	30
31	9.564396	5.33	9.968628	.83	9.595768	6.17	10.404232	29
32	.564716	5.33	.968578	.83	.596138	6.17	.403862	28
33	.565036	5.33	.968528	.82	.596508	6.17	.403492	27
34	.565356	5.33	.968479	.83	.596878	6.15	.403122	26
35	.565676	5.32	.968429	.83	.597247	6.15	.402753	25
36	.565995	5.32	.968379	.83	.597616	6.15	.402384	24
37	.566314	5.30	.968329	.85	.597985	6.15	.402015	23
38	.566632	5.32	.968278	.83	.598354	6.13	.401646	22
39	.566951	5.30	.968228	.83	.598722	6.15	.401278	21
40	.567269	5.30	.968178	.83	.599091	6.13	.400909	20
41	9.567587	5.28	9.968128	.83	9.599459	6.13	10.400541	19
42	.567904	5.30	.968078	.85	.599827	6.12	.400173	18
43	.568222	5.28	.968027	.83	.600194	6.13	.399806	17
44	.568539	5.28	.967977	.83	.600562	6.12	.399438	16
45	.568856	5.27	.967927	.85	.600929	6.12	.399071	15
46	.569172	5.27	.967876	.83	.601296	6.12	.398704	14
47	.569488	5.27	.967826	.85	.601663	6.10	.398337	13
48	.569804	5.27	.967775	.83	.602029	6.10	.397971	12
49	.570120	5.25	.967725	.85	.602395	6.10	.397605	11
50	.570435	5.27	.967674	.83	.602761	6.10	.397239	10
51	9.570751	5.25	9.967624	.85	9.603127	6.10	10.396873	9
52	.571066	5.23	.967573	.85	.603493	6.08	.396507	8
53	.571380	5.25	.967522	.85	.603858	6.08	.396142	7
54	.571695	5.23	.967471	.83	.604223	6.08	.395777	6
55	.572009	5.23	.967421	.85	.604588	6.08	.395412	5
56	.572323	5.22	.967370	.85	.604953	6.07	.395047	4
57	.572636	5.23	.967319	.85	.605317	6.08	.394683	3
58	.572950	5.22	.967268	.85	.605682	6.07	.394318	2
59	.573263	5.20	.967217	.85	.606046	6.07	.393954	1
60	9.573575		9.967166		9.606410		10.393590	0
′	*Cosine.*	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.573575	5.22	9.967166	.85	9.606410	6.05	10.393590	60
1	.573888	5.20	.967115	.85	.606773	6.07	.393227	59
2	.574200	5.20	.967064	.85	.607137	6.05	.392863	58
3	.574512	5.20	.967013	.87	.607500	6.05	.392500	57
4	.574824	5.20	.966961	.85	.607863	6.03	.392137	56
5	.575136	5.18	.966910	.85	.608225	6.05	.391775	55
6	.575447	5.18	.966859	.85	.608588	6.03	.391412	54
7	.575758	5.18	.966808	.87	.608950	6.03	.391050	53
8	.576069	5.17	.966756	.85	.609312	6.03	.390688	52
9	.576379	5.17	.966705	.87	.609674	6.03	.390326	51
10	.576689	5.17	.966653	.85	.610036	6.02	.389964	50
11	9.576999	5.17	9.966602	.87	9.610397	6.03	10.389603	49
12	.577309	5.15	.966550	.85	.610759	6.02	.389241	48
13	.577618	5.15	.966499	.87	.611120	6.00	.388880	47
14	.577927	5.15	.966447	.87	.611480	6.02	.388520	46
15	.578236	5.15	.966395	.85	.611841	6.00	.388159	45
16	.578545	5.13	.966344	.87	.612201	6.00	.387799	44
17	.578853	5.15	.966292	.87	.612561	6.00	.387439	43
18	.579162	5.13	.966240	.87	.612921	6.00	.387079	42
19	.579470	5.12	.966188	.87	.613281	6.00	.386719	41
20	.579777	5.13	.966136	.85	.613641	5.98	.386359	40
21	9.580085	5.12	9.966085	.87	9.614000	5.98	10.386000	39
22	.580392	5.12	.966033	.87	.614359	5.98	.385641	38
23	.580699	5.10	.965981	.87	.614718	5.98	.385282	37
24	.581005	5.12	.965929	.88	.615077	5.97	.384923	36
25	.581312	5.10	.965876	.87	.615435	5.97	.384565	35
26	.581618	5.10	.965824	.87	.615793	5.97	.384207	34
27	.581924	5.08	.965772	.87	.616151	5.97	.383849	33
28	.582229	5.10	.965720	.87	.616509	5.97	.383491	32
29	.582535	5.08	.965668	.88	.616867	5.95	.383133	31
30	.582840	5.08	.965615	.87	.617224	5.97	.382776	30
31	9.583145	5.07	9.965563	.87	9.617582	5.95	10.382418	29
32	.583449	5.08	.965511	.88	.617939	5.93	.382061	28
33	.583754	5.07	.965458	.87	.618295	5.95	.381705	27
34	.584058	5.05	.965406	.88	.618652	5.93	.381348	26
35	.584361	5.07	.965353	.87	.619008	5.93	.380992	25
36	.584665	5.05	.965301	.88	.619364	5.93	.380636	24
37	.584968	5.07	.965248	.88	.619720	5.93	.380280	23
38	.585272	5.03	.965195	.87	.620076	5.93	.379924	22
39	.585574	5.05	.965143	.88	.620432	5.92	.379568	21
40	.585877	5.03	.965090	.88	.620787	5.92	.379213	20
41	9.586179	5.05	9.965037	.88	9.621142	5.92	10.378858	19
42	.586482	5.02	.964984	.88	.621497	5.92	.378503	18
43	.586783	5.03	.964931	.87	.621852	5.92	.378148	17
44	.587085	5.02	.964879	.88	.622207	5.90	.377793	16
45	.587386	5.03	.964826	.88	.622561	5.90	.377439	15
46	.587688	5.02	.964773	.88	.622915	5.90	.377085	14
47	.587989	5.00	.964720	.90	.623269	5.90	.376731	13
48	.588289	5.02	.964666	.88	.623623	5.88	.376377	12
49	.588590	5.00	.964613	.88	.623976	5.90	.376024	11
50	.588890	5.00	.964560	.88	.624330	5.88	.375670	10
51	9.589190	4.98	9.964507	.88	9.624683	5.88	10.375317	9
52	.589489	5.00	.964454	.90	.625036	5.87	.374964	8
53	.589789	4.98	.964400	.88	.625388	5.88	.374612	7
54	.590088	4.98	.964347	.88	.625741	5.87	.374259	6
55	.590387	4.98	.964294	.90	.626093	5.87	.373907	5
56	.590686	4.97	.964240	.88	.626445	5.87	.373555	4
57	.590984	4.97	.964187	.90	.626797	5.87	.373203	3
58	.591282	4.97	.964133	.88	.627149	5.87	.372851	2
59	.591580	4.97	.964080	.90	.627501	5.85	.372499	1
60	9.591878		9.964026		9.627852		10.372148	0
'	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.591878	4.97	9.964026	.90	9.627852	5.85	10.372148	60
1	.592176	4.95	.963972	.88	.628203	5.85	.371797	59
2	.592473	4.95	.963919	.90	.628554	5.85	.371446	58
3	.592770	4.95	.963865	.90	.628905	5.83	.371095	57
4	.593067	4.93	.963811	.90	.629255	5.85	.370745	56
5	.593363	4.93	.963757	.88	.629606	5.83	.370394	55
6	.593659	4.93	.963704	.90	.629956	5.83	.370044	54
7	.593955	4.93	.963650	.90	.630306	5.83	.369694	53
8	.594251	4.93	.963596	.90	.630656	5.82	.369344	52
9	.594547	4.92	.963542	.90	.631005	5.83	.368995	51
10	.594842	4.92	.963488	.90	.631355	5.82	.368645	50
11	9.595137	4.92	9.963434	.92	9.631704	5.82	10.368296	49
12	.595432	4.92	.963379	.90	.632053	5.82	.367947	48
13	.595727	4.90	.963325	.90	.632402	5.80	.367598	47
14	.596021	4.90	.963271	.90	.632750	5.82	.367250	46
15	.596315	4.90	.963217	.90	.633099	5.80	.366901	45
16	.596609	4.90	.963163	.92	.633447	5.80	.366553	44
17	.596903	4.88	.963108	.90	.633795	5.80	.366205	43
18	.597196	4.90	.963054	.92	.634143	5.78	.365857	42
19	.597490	4.88	.962999	.90	.634490	5.80	.365510	41
20	.597783	4.87	.962945	.92	.634838	5.78	.365162	40
21	9.598075	4.88	9.962890	.90	9.635185	5.78	10.364815	39
22	.598368	4.87	.962836	.92	.635532	5.78	.364468	38
23	.598660	4.87	.962781	.90	.635879	5.78	.364121	37
24	.598952	4.87	.962727	.92	.636226	5.77	.363774	36
25	.599244	4.87	.962672	.92	.636572	5.78	.363428	35
26	.599536	4.85	.962617	.92	.636919	5.77	.363081	34
27	.599827	4.85	.962562	.90	.637265	5.77	.362735	33
28	.600118	4.85	.962508	.92	.637611	5.75	.362389	32
29	.600409	4.85	.962453	.92	.637956	5.77	.362044	31
30	.600700	4.83	.962398	.92	.638302	5.75	.361698	30
31	9.600990	4.83	9.962343	.92	9.638647	5.75	10.361353	29
32	.601280	4.83	.962288	.92	.638992	5.75	.361008	28
33	.601570	4.83	.962233	.92	.639337	5.75	.360663	27
34	.601860	4.83	.962178	.92	.639682	5.75	.360318	26
35	.602150	4.82	.962123	.93	.640027	5.73	.359973	25
36	.602439	4.82	.962067	.92	.640371	5.75	.359629	24
37	.602728	4.82	.962012	.92	.640716	5.73	.359284	23
38	.603017	4.80	.961957	.92	.641060	5.73	.358940	22
39	.603305	4.82	.961902	.93	.641404	5.72	.358596	21
40	.603594	4.80	.961846	.92	.641747	5.73	.358253	20
41	9.603882	4.80	9.961791	.93	9.642091	5.72	10.357909	19
42	.604170	4.78	.961735	.92	.642434	5.72	.357566	18
43	.604457	4.80	.961680	.93	.642777	5.72	.357223	17
44	.604745	4.78	.961624	.92	.643120	5.72	.356880	16
45	.605032	4.78	.961569	.93	.643462	5.72	.356537	15
46	.605319	4.78	.961513	.92	.643806	5.70	.356194	14
47	.605606	4.77	.961458	.93	.644148	5.70	.355852	13
48	.605892	4.78	.961402	.93	.644490	5.70	.355510	12
49	.606179	4.77	.961346	.93	.644832	5.70	.355168	11
50	.606465	4.77	.961290	.92	.645174	5.70	.354826	10
51	9.606751	4.75	9.961235	.93	9.645516	5.68	10.354484	9
52	.607036	4.77	.961179	.93	.645857	5.70	.354143	8
53	.607322	4.75	.961123	.93	.646199	5.68	.353801	7
54	.607607	4.75	.961067	.93	.646540	5.68	.353460	6
55	.607892	4.75	.961011	.93	.646881	5.68	.353119	5
56	.608177	4.73	.960955	.93	.647222	5.67	.352778	4
57	.608461	4.73	.960899	.93	.647562	5.68	.352438	3
58	.608745	4.73	.960843	.95	.647903	5.67	.352097	2
59	.609029	4.73	.960786	.93	.648243	5.67	.351757	1
60	9.609313		9.960730		9.648583		10.351417	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.609313	4.73	9.960730	.93	9.648583	5.67	10.351417	60
1	.609597	4.72	.960674	.93	.648923	5.67	.351077	59
2	.609880	4.73	.960618	.95	.649263	5.65	.350737	58
3	.610164	4.72	.960561	.93	.649602	5.67	.350398	57
4	.610447	4.70	.960505	.95	.649942	5.65	.350058	56
5	.610729	4.72	.960448	.93	.650281	5.65	.349719	55
6	.611012	4.70	.960392	.95	.650620	5.65	.349380	54
7	.611294	4.70	.960335	.93	.650959	5.63	.349041	53
8	.611576	4.70	.960279	.95	.651297	5.65	.348703	52
9	.611858	4.70	.960222	.95	.651636	5.63	.348364	51
10	.612140	4.68	.960165	.93	.651974	5.63	.348026	50
11	9.612421	4.68	9.960109	.95	9.652312	5.63	10.347688	49
12	.612702	4.68	.960052	.95	.652650	5.63	.347350	48
13	.612983	4.68	.959995	.95	.652988	5.63	.347012	47
14	.613264	4.68	.959938	.93	.653326	5.62	.346674	46
15	.613545	4.67	.959882	.95	.653663	5.62	.346337	45
16	.613825	4.67	.959825	.95	.654000	5.62	.346000	44
17	.614105	4.67	.959768	.95	.654337	5.62	.345663	43
18	.614385	4.67	.959711	.95	.654674	5.62	.345326	42
19	.614665	4.65	.959654	.97	.655011	5.62	.344989	41
20	.614944	4.65	.959596	.95	.655348	5.60	.344652	40
21	9.615223	4.65	9.959539	.95	9.655684	5.60	10.344316	39
22	.615502	4.65	.959482	.95	.656020	5.60	.343980	38
23	.615781	4.65	.959425	.95	.656356	5.60	.343644	37
24	.616060	4.63	.959368	.97	.656692	5.60	.343308	36
25	.616338	4.63	.959310	.95	.657028	5.60	.342972	35
26	.616616	4.63	.959253	.97	.657364	5.58	.342636	34
27	.616894	4.63	.959195	.95	.657699	5.58	.342301	33
28	.617172	4.63	.959138	.97	.658034	5.58	.341966	32
29	.617450	4.62	.959080	.95	.658369	5.58	.341631	31
30	.617727	4.62	.959023	.97	.658704	5.58	.341296	30
31	9.618004	4.62	9.958965	.95	9.659039	5.57	10.340961	29
32	.618281	4.62	.958908	.97	.659373	5.58	.340627	28
33	.618558	4.60	.958850	.97	.659708	5.57	.340292	27
34	.618834	4.60	.958792	.97	.660042	5.57	.339958	26
35	.619110	4.60	.958734	.95	.660376	5.57	.339624	25
36	.619386	4.60	.958677	.97	.660710	5.55	.339290	24
37	.619662	4.60	.958619	.97	.661043	5.57	.338957	23
38	.619938	4.58	.958561	.97	.661377	5.55	.338623	22
39	.620213	4.58	.958503	.97	.661710	5.55	.338290	21
40	.620488	4.58	.958445	.97	.662043	5.55	.337957	20
41	9.620763	4.58	9.958387	.97	9.662376	5.55	10.337624	19
42	.621038	4.58	.958329	.97	.662709	5.55	.337291	18
43	.621313	4.57	.958271	.97	.663042	5.55	.336958	17
44	.621587	4.57	.958213	.98	.663375	5.53	.336625	16
45	.621861	4.57	.958154	.97	.663707	5.53	.336293	15
46	.622135	4.57	.958096	.97	.664039	5.53	.335961	14
47	.622409	4.55	.958038	.98	.664371	5.53	.335629	13
48	.622682	4.57	.957979	.97	.664703	5.53	.335297	12
49	.622956	4.55	.957921	.97	.665035	5.52	.334965	11
50	.623229	4.55	.957863	.98	.665366	5.53	.334634	10
51	9.623502	4.53	9.957804	.97	9.665698	5.52	10.334302	9
52	.623774	4.55	.957746	.98	.666029	5.52	.333971	8
53	.624047	4.53	.957687	.98	.666360	5.52	.333640	7
54	.624319	4.53	.957628	.97	.666691	5.50	.333309	6
55	.624591	4.53	.957570	.98	.667021	5.52	.332979	5
56	.624863	4.53	.957511	.98	.667352	5.50	.332648	4
57	.625135	4.52	.957452	.98	.667682	5.52	.332318	3
58	.625406	4.52	.957393	.97	.668013	5.50	.331987	2
59	.625677	4.52	.957335	.98	.668343	5.50	.331657	1
60	9.625948		9.957276		9.668673		10.331327	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	
0	9.625948	4.52	9.957276	.98	9.668673	5.48	10.331327	60
1	.626219	4.52	.957217	.98	.669002	5.50	.330998	59
2	.626490	4.50	.957158	.98	.669332	5.48	.330668	58
3	.626760	4.50	.957099	.98	.669661	5.50	.330339	57
4	.627030	4.50	.957040	.98	.669991	5.48	.330009	56
5	.627300	4.50	.956981	1.00	.670320	5.48	.329680	55
6	.627570	4.50	.956921	.98	.670649	5.47	.329351	54
7	.627840	4.48	.956862	.98	.670977	5.48	.329023	53
8	.628109	4.48	.956803	.98	.671306	5.48	.328694	52
9	.628378	4.48	.956744	1.00	.671635	5.47	.328365	51
10	.628647	4.48	.956684	.98	.671963	5.47	.328037	50
11	9.628916	4.48	9.956625	.98	9.672291	5.47	10.327709	49
12	.629185	4.47	.956566	1.00	.672619	5.47	.327381	48
13	.629453	4.47	.956506	.98	.672947	5.45	.327053	47
14	.629721	4.47	.956447	1.00	.673274	5.47	.326726	46
15	.629989	4.47	.956387	1.00	.673602	5.45	.326398	45
16	.630257	4.45	.956327	.98	.673929	5.47	.326071	44
17	.630524	4.47	.956268	1.00	.674257	5.45	.325743	43
18	.630792	4.45	.956208	1.00	.674584	5.45	.325416	42
19	.631059	4.45	.956148	.98	.674911	5.43	.325089	41
20	.631326	4.45	.956089	1.00	.675237	5.45	.324763	40
21	9.631593	4.43	9.956029	1.00	9.675564	5.43	10.324436	39
22	.631859	4.43	.955969	1.00	.675890	5.45	.324110	38
23	.632125	4.45	.955909	1.00	.676217	5.43	.323783	37
24	.632392	4.43	.955849	1.00	.676543	5.43	.323457	36
25	.632658	4.42	.955789	1.00	.676869	5.42	.323131	35
26	.632923	4.43	.955729	1.00	.677194	5.43	.322806	34
27	.633189	4.42	.955669	1.00	.677520	5.43	.322480	33
28	.633454	4.42	.955609	.98	.677846	5.42	.322154	32
29	.633719	4.42	.955548	1.00	.678171	5.42	.321829	31
30	.633984	4.42	.955488	1.00	.678496	5.42	.321504	30
31	9.634249	4.42	9.955428	1.00	9.678821	5.42	10.321179	29
32	.634514	4.40	.955368	1.02	.679146	5.42	.320854	28
33	.634778	4.40	.955307	1.00	.679471	5.40	.320529	27
34	.635042	4.40	.955247	1.02	.679795	5.42	.320205	26
35	.635306	4.40	.955186	1.00	.680120	5.40	.319880	25
36	.635570	4.40	.955126	1.02	.680444	5.40	.319556	24
37	.635834	4.38	.955065	1.00	.680768	5.40	.319232	23
38	.636097	4 38	.955005	1.02	.681092	5.40	.318908	22
39	.636360	4.38	.954944	1.02	.681416	5.40	.318584	21
40	.636623	4.38	.954883	1.00	.681740	5.38	.318260	20
41	9.636886	4.37	9.954823	1.02	9.682063	5.40	10.317937	19
42	.637148	4.38	.954762	1.02	.682387	5.38	.317613	18
43	.637411	4.37	.954701	1.02	.682710	5.38	.317290	17
44	.637673	4.37	.954640	1.02	.683033	5.38	.316967	16
45	.637935	4.37	.954579	1.02	.683356	5.38	.316644	15
46	.638197	4.35	.954518	1.02	.683679	5.37	.316321	14
47	.638458	4.37	.954457	1.02	.684001	5.38	.315999	13
48	.638720	4.35	.954396	1.02	.684324	5.37	.315676	12
49	.638981	4.35	.954335	1.02	.684646	5.37	.315354	11
50	.639242	4.35	.954274	1.02	.684968	5.37	.315032	10
51	9.639503	4.35	9.954213	1.02	9.685290	5.37	10.314710	9
52	.639764	4.33	.954152	1.03	.685612	5.37	.314388	8
53	.640024	4.33	.954090	1.02	.685934	5.35	.314066	7
54	.640284	4.33	.954029	1.02	.686255	5.37	.313745	6
55	.640544	4.33	.953968	1.03	.686577	5.35	.313423	5
56	.640804	4.33	.953906	1.02	.686898	5.35	.313102	4
57	.641064	4.33	.953845	1.03	.687219	5.35	.312781	3
58	.641324	4.32	.953783	1.02	.687540	5.35	.312460	2
59	.641583	4.32	.953722	1.03	.687861	5.35	.312139	1
60	9.641842		9.953660		9.688182		10.311818	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.641842	4.32	9.953660	1.02	9.688182	5.33	10.311818	60
1	.642101	4.32	.953599	1.03	.688502	5.32	.311498	59
2	.642360	4.30	.953537	1.03	.688823	5.33	.311177	58
3	.642618	4.32	.953475	1.03	.689143	5.33	.310857	57
4	.642877	4.30	.953413	1.02	.689463	5.33	.310537	56
5	.643135	4.30	.953352	1.03	.689783	5.33	.310217	55
6	.643393	4.28	.953290	1.03	.690103	5.33	.309897	54
7	.643650	4.30	.953228	1.03	.690423	5.32	.309577	53
8	.643908	4.28	.953166	1.03	.690742	5.33	.309258	52
9	.644165	4.30	.953104	1.03	.691062	5.32	.308938	51
10	.644423	4.28	.953042	1.03	.691381	5.32	.308619	50
11	9.644680	4.27	9.952980	1.03	9.691700	5.32	10.308300	49
12	.644936	4.28	.952918	1.05	.692019	5.32	.307981	48
13	.645193	4.28	.952855	1.03	.692338	5.30	.307662	47
14	.645450	4.27	.952793	1.03	.692656	5.32	.307344	46
15	.645706	4.27	.952731	1.03	.692975	5.30	.307025	45
16	.645962	4.27	.952669	1.05	.693293	5.32	.306707	44
17	.646218	4.27	.952606	1.03	.693612	5.30	.306388	43
18	.646474	4.25	.952544	1.05	.693930	5.30	.306070	42
19	.646729	4.25	.952481	1.03	.694248	5.30	.305752	41
20	.646984	4.27	.952419	1.05	.694566	5.28	.305434	40
21	9.647240	4.23	9.952356	1.03	9.694883	5.30	10.305117	39
22	.647494	4.25	.952294	1.05	.695201	5.28	.304799	38
23	.647749	4.25	.952231	1.05	.695518	5.30	.304482	37
24	.648004	4.23	.952168	1.03	.695836	5.28	.304164	36
25	.648258	4.23	.952106	1.05	.696153	5.28	.303847	35
26	.648512	4.23	.952043	1.05	.696470	5.28	.303530	34
27	.648766	4.23	.951980	1.05	.696787	5.27	.303213	33
28	.649020	4.23	.951917	1.05	.697103	5.28	.302897	32
29	.649274	4.22	.951854	1.05	.697420	5.27	.302580	31
30	.649527	4.23	.951791	1.05	.697736	5.28	.302264	30
31	9.649781	4.22	9.951728	1.05	9.698053	5.27	10.301947	29
32	.650034	4.22	.951665	1.05	.698369	5.27	.301631	28
33	.650287	4.20	.951602	1.05	.698685	5.27	.301315	27
34	.650539	4.22	.951539	1.05	.699001	5.25	.300999	26
35	.650792	4.20	.951476	1.07	.699316	5.27	.300684	25
36	.651044	4.22	.951412	1.05	.699632	5.25	.300368	24
37	.651297	4.20	.951349	1.05	.699947	5.27	.300053	23
38	.651549	4.18	.951286	1.07	.700263	5.25	.299737	22
39	.651800	4.20	.951222	1.05	.700578	5.25	.299422	21
40	.652052	4.20	.951159	1.05	.700893	5.25	.299107	20
41	9.652304	4.18	9.951096	1.07	9.701208	5.25	10.298792	19
42	.652555	4.18	.951032	1.07	.701523	5.23	.298477	18
43	.652806	4.18	.950968	1.05	.701837	5.25	.298163	17
44	.653057	4.18	.950905	1.07	.702152	5.23	.297848	16
45	.653308	4.17	.950841	1.05	.702466	5.25	.297534	15
46	.653558	4.17	.950778	1.07	.702781	5.23	.297219	14
47	.653808	4.18	.950714	1.07	.703095	5.23	.296905	13
48	.654059	4.17	.950650	1.07	.703409	5.22	.296591	12
49	.654309	4.15	.950586	1.07	.703722	5.23	.296278	11
50	.654558	4.17	.950522	1.07	.704036	5.23	.295964	10
51	9.654808	4.17	9.950458	1.07	9.704350	5.22	10.295650	9
52	.655058	4.15	.950394	1.07	.704663	5.22	.295337	8
53	.655307	4.15	.950330	1.07	.704976	5.23	.295024	7
54	.655556	4.15	.950266	1.07	.705290	5.22	.294710	6
55	.655805	4.15	.950202	1.07	.705603	5.22	.294397	5
56	.656054	4.13	.950138	1.07	.705916	5.20	.294084	4
57	.656302	4.15	.950074	1.07	.706228	5.22	.293772	3
58	.656551	4.13	.950010	1.08	.706541	5.22	.293459	2
59	.656799	4.13	.949945	1.07	.706854	5.20	.293146	1
60	9.657047		9.949881		9.707166		10.292834	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.657047	4.13	9.949881	1.08	9.707166	5.20	10.292834	60
1	.657295	4.12	.949816	1.07	.707478	5.20	.292522	59
2	.657542	4.13	.949752	1.07	.707790	5.20	.292210	58
3	.657790	4.12	.949688	1.08	.708102	5.20	.291898	57
4	.658037	4.12	.949623	1.08	.708414	5.20	.291586	56
5	.658284	4.12	.949558	1.07	.708726	5.18	.291274	55
6	.658531	4.12	.949494	1.08	.709037	5.20	.290963	54
7	.658778	4.12	.949429	1.08	.709349	5.18	.290651	53
8	.659025	4.10	.949364	1.07	.709660	5.18	.290340	52
9	.659271	4.10	.949300	1.08	.709971	5.18	.290029	51
10	.659517	4.10	.949235	1.08	.710282	5.18	.289718	50
11	9.659763	4.10	9.949170	1.08	9.710593	5.18	10.289407	49
12	.660009	4.10	.949105	1.08	.710904	5.18	.289096	48
13	.660255	4.10	.949040	1.08	.711215	5.17	.288785	47
14	.660501	4.08	.948975	1.08	.711525	5.18	.288475	46
15	.660746	4.08	.948910	1.08	.711836	5.17	.288164	45
16	.660991	4.08	.948845	1.08	.712146	5.17	.287854	44
17	.661236	4.08	.948780	1.08	.712456	5.17	.287544	43
18	.661481	4.08	.948715	1.08	.712766	5.17	.287234	42
19	.661726	4.07	.948650	1.10	.713076	5.17	.286924	41
20	.661970	4.07	.948584	1.08	.713386	5.17	.286614	40
21	9.662214	4.08	9.948519	1.08	9.713696	5.15	10.286304	39
22	.662459	4.07	.948454	1.10	.714005	5.15	.285995	38
23	.662703	4.05	.948388	1.08	.714314	5.17	.285686	37
24	.662946	4.07	.948323	1.10	.714624	5.15	.285376	36
25	.663190	4.05	.948257	1.08	.714933	5.15	.285067	35
26	.663433	4.07	.948192	1.10	.715242	5.15	.284758	34
27	.663677	4.05	.948126	1.10	.715551	5.15	.284449	33
28	.663920	4.05	.948060	1.08	.715860	5.13	.284140	32
29	.664163	4.05	.947995	1.10	.716168	5.15	.283832	31
30	.664406	4.03	.947929	1.10	.716477	5.13	.283523	30
31	9.664648	4.05	9.947863	1.10	9.716785	5.13	10.283215	29
32	.664891	4.03	.947797	1.10	.717093	5.13	.282907	28
33	.665133	4.03	.947731	1.10	.717401	5.13	.282599	27
34	.665375	4.03	.947665	1.08	.717709	5.13	.282291	26
35	.665617	4.03	.947600	1.12	.718017	5.13	.281983	25
36	.665859	4.02	.947533	1.10	.718325	5.13	.281675	24
37	.666100	4.03	.947467	1.10	.718633	5.12	.281367	23
38	.666342	4.02	.947401	1.10	.718940	5.13	.281060	22
39	.666583	4.02	.947335	1.10	.719248	5.12	.280752	21
40	.666824	4.02	.947269	1.10	.719555	5.12	.280445	20
41	9.667065	4.00	9.947203	1.12	9.719862	5.12	10.280138	19
42	.667305	4.02	.947136	1.10	.720169	5.12	.279831	18
43	.667546	4.00	.947070	1.10	.720476	5.12	.279524	17
44	.667786	4.02	.947004	1.12	.720783	5.10	.279217	16
45	.668027	4.00	.946937	1.10	.721089	5.12	.278911	15
46	.668267	3.98	.946871	1.12	.721396	5.10	.278604	14
47	.668506	4.00	.946804	1.10	.721702	5.12	.278298	13
48	.668746	4.00	.946738	1.12	.722009	5.10	.277991	12
49	.668986	3.98	.946671	1.12	.722315	5.10	.277685	11
50	.669225	3.98	.946604	1.10	.722621	5.10	.277379	10
51	9.669464	3.98	9.946538	1.12	9.722927	5.08	10.277073	9
52	.669703	3.98	.946471	1.12	.723232	5.10	.276768	8
53	.669942	3.98	.946404	1.12	.723538	5.10	.276462	7
54	.670181	3.97	.946337	1.12	.723844	5.08	.276156	6
55	.670419	3.98	.946270	1.12	.724149	5.08	.275851	5
56	.670658	3.97	.946203	1.12	.724454	5.10	.275546	4
57	.670896	3.97	.946136	1.12	.724760	5.08	.275240	3
58	.671134	3.97	.946069	1.12	.725065	5.08	.274935	2
59	.671372	3.95	.946002	1.12	.725370	5.07	.274630	1
60	9.671609		9.945935		9.725674		10.274326	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.671609	3.97	9.945935	1.12	9.725674	5.08	10.274326	60
1	.671847	3.95	.945868	1.13	.725979	5.08	.274021	59
2	.672084	3.95	.945800	1.12	.726284	5.07	.273716	58
3	.672321	3.95	.945733	1.12	.726588	5.07	.273412	57
4	.672558	3.95	.945666	1.13	.726892	5.05	.273108	56
5	.672795	3.95	.945598	1.12	.727197	5.07	.272803	55
6	.673032	3.93	.945531	1.12	.727501	5.07	.272499	54
7	.673268	3.95	.945464	1.13	.727805	5.07	.272195	53
8	.673505	3.93	.945396	1.13	.728109	5.05	.271891	52
9	.673741	3.93	.945328	1.12	.728412	5.07	.271588	51
10	.673977	3.93	.945261	1.13	.728716	5.07	.271284	50
11	9.674213	3.92	9.945193	1.13	9.729020	5.05	10.270980	49
12	.674448	3.93	.945125	1.12	.729323	5.05	.270677	48
13	.674684	3.92	.945058	1.13	.729626	5.05	.270374	47
14	.674919	3.93	.944990	1.13	.729929	5.07	.270071	46
15	.675155	3.92	.944922	1.13	.730233	5.03	.269767	45
16	.675390	3.90	.944854	1.13	.730535	5.05	.269465	44
17	.675624	3.92	.944786	1.13	.730838	5.05	.269162	43
18	.675859	3.92	.944718	1.13	.731141	5.05	.268859	42
19	.676094	3.90	.944650	1.13	.731444	5.03	.268556	41
20	.676328	3.90	.944582	1.13	.731746	5.03	.268254	40
21	9.676562	3.90	9.944514	1.13	9.732048	5.05	10.267952	39
22	.676796	3.90	.944446	1.15	.732351	5.03	.267649	38
23	.677030	3.90	.944377	1.13	.732653	5.03	.267347	37
24	.677264	3.90	.944309	1.13	.732955	5.03	.267045	36
25	.677498	3.88	.944241	1.15	.733257	5.02	.266743	35
26	.677731	3.88	.944172	1.13	.733558	5.03	.266442	34
27	.677964	3.88	.944104	1.13	.733860	5.03	.266140	33
28	.678197	3.88	.944036	1.15	.734162	5.02	.265838	32
29	.678430	3.88	.943967	1.13	.734463	5.02	.265537	31
30	.678663	3.87	.943899	1.15	.734764	5.03	.265236	30
31	9.678895	3.88	9.943830	1.15	9.735066	5.02	10.264934	29
32	.679128	3.87	.943761	1.13	.735367	5.02	.264633	28
33	.679360	3.87	.943693	1.15	.735668	5.02	.264332	27
34	.679592	3.87	.943624	1.15	.735969	5.00	.264031	26
35	.679824	3.87	.943555	1.15	.736269	5.02	.263731	25
36	.680056	3.87	.943486	1.15	.736570	5.00	.263430	24
37	.680288	3.85	.943417	1.15	.736870	5.02	.263130	23
38	.680519	3.85	.943348	1.15	.737171	5.00	.262829	22
39	.680750	3.87	.943279	1.15	.737471	5.00	.262529	21
40	.680982	3.85	.943210	1.15	.737771	5.00	.262229	20
41	9.681213	3.83	9.943141	1.15	9.738071	5.00	10.261929	19
42	.681443	3.85	.943072	1.15	.738371	5.00	.261629	18
43	.681674	3.85	.943003	1.15	.738671	5.00	.261329	17
44	.681905	3.83	.942934	1.17	.738971	5.00	.261029	16
45	.682135	3.83	.942864	1.15	.739271	4.98	.260729	15
46	.682365	3.83	.942795	1.15	.739570	5.00	.260430	14
47	.682595	3.83	.942726	1.17	.739870	4.98	.260130	13
48	.682825	3.83	.942656	1.15	.740169	4.98	.259831	12
49	.683055	3.82	.942587	1.17	.740468	4.98	.259532	11
50	.683284	3.83	.942517	1.15	.740767	4.98	.259233	10
51	9.683514	3.82	9.942448	1.17	9.741066	4.98	10.258934	9
52	.683743	3.82	.942378	1.17	.741365	4.98	.258635	8
53	.683972	3.82	.942308	1.15	.741664	4.97	.258336	7
54	.684201	3.82	.942239	1.17	.741962	4.98	.258038	6
55	.684430	3.80	.942169	1.17	.742261	4.97	.257739	5
56	.684658	3.82	.942099	1.17	.742559	4.98	.257441	4
57	.684887	3.80	.942029	1.17	.742858	4.97	.257142	3
58	.685115	3.80	.941959	1.17	.743156	4.97	.256844	2
59	.685343	3.80	.941889	1.17	.743454	4.97	.256546	1
60	9.685571		9.941819		9.743752		10.256248	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.685571	3.80	9.941819	1.17	9.743752	4.97	10.256248	60
1	.685799	3.80	.941749	1.17	.744050	4.97	.255950	59
2	.686027	3.78	.941679	1.17	.744348	4.95	.255652	58
3	.686254	3.80	.941609	1.17	.744645	4.97	.255355	57
4	.686482	3.78	.941539	1.17	.744943	4.95	.255057	56
5	.686709	3.78	.941469	1.18	.745240	4.97	.254760	55
6	.686936	3.78	.941398	1.17	.745538	4.95	.254462	54
7	.687163	3.77	.941328	1.17	.745835	4.95	.254165	53
8	.687389	3.78	.941258	1.18	.746132	4.95	.253868	52
9	.687616	3.78	.941187	1.17	.746429	4.95	.253571	51
10	.687843	3.77	.941117	1.18	.746726	4.95	.253274	50
11	9.688069	3.77	9.941046	1.18	9.747023	4.93	10.252977	49
12	.688295	3.77	.940975	1.17	.747319	4.95	.252681	48
13	.688521	3.77	.940905	1.18	.747616	4.95	.252384	47
14	.688747	3.75	.940834	1.18	.747913	4.93	.252087	46
15	.688972	3.77	.940763	1.17	.748209	4.93	.251791	45
16	.689198	3.75	.940693	1.18	.748505	4.93	.251495	44
17	.689423	3.75	.940622	1.18	.748801	4.93	.251199	43
18	.689648	3.75	.940551	1.18	.749097	4.93	.250903	42
19	.689873	3.75	.940480	1.18	.749393	4.93	.250607	41
20	.690098	3.75	.940409	1.18	.749689	4.93	.250311	40
21	9.690323	3.75	9.940338	1.18	9.749985	4.93	10.250015	39
22	.690548	3.73	.940267	1.18	.750281	4.92	.249719	38
23	.690772	3.73	.940196	1.18	.750576	4.93	.249424	37
24	.690996	3.73	.940125	1.18	.750872	4.92	.249128	36
25	.691220	3.73	.940054	1.20	.751167	4.92	.248833	35
26	.691444	3.73	.939982	1.18	.751462	4.92	.248538	34
27	.691668	3.73	.939911	1.18	.751757	4.92	.248243	33
28	.691892	3.72	.939840	1.20	.752052	4.92	.247948	32
29	.692115	3.73	.939768	1.18	.752347	4.92	.247653	31
30	.692339	3.72	.939697	1.20	.752642	4.92	.247358	30
31	9.692562	3.72	9.939625	1.18	9.752937	4.90	10.247063	29
32	.692785	3.72	.939554	1.20	.753231	4.92	.246769	28
33	.693008	3.72	.939482	1.20	.753526	4.90	.246474	27
34	.693231	3.70	.939410	1.18	.753820	4.92	.246180	26
35	.693453	3.72	.939339	1.20	.754115	4.90	.245885	25
36	.693676	3.70	.939267	1.20	.754409	4.90	.245591	24
37	.693898	3.70	.939195	1.20	.754703	4.90	.245297	23
38	.694120	3.70	.939123	1.18	.754997	4.90	.245003	22
39	.694342	3.70	.939052	1.20	.755291	4.90	.244709	21
40	.694564	3.70	.938980	1.20	.755585	4.88	.244415	20
41	9.694786	3.68	9.938908	1.20	9.755878	4.90	10.244122	19
42	.695007	3.70	.938836	1.22	.756172	4.88	.243828	18
43	.695229	3.68	.938763	1.20	.756465	4.90	.243535	17
44	.695450	3.68	.938691	1.20	.756759	4.88	.243241	16
45	.695671	3.68	.938619	1.20	.757052	4.88	.242948	15
46	.695892	3.68	.938547	1.20	.757345	4.88	.242655	14
47	.696113	3.68	.938475	1.22	.757638	4.88	.242362	13
48	.696334	3.67	.938402	1.20	.757931	4.88	.242069	12
49	.696554	3.68	.938330	1.20	.758224	4.88	.241776	11
50	.696775	3.67	.938258	1.22	.758517	4.88	.241483	10
51	9.696995	3.67	9.938185	1.20	9.758810	4.87	10.241190	9
52	.697215	3.67	.938113	1.22	.759102	4.88	.240898	8
53	.697435	3.65	.938040	1.22	.759395	4.87	.240605	7
54	.697654	3.67	.937967	1.20	.759687	4.87	.240313	6
55	.697874	3.67	.937895	1.22	.759979	4.88	.240021	5
56	.698094	3.65	.937822	1.22	.760272	4.87	.239728	4
57	.698313	3.65	.937749	1.22	.760564	4.87	.239436	3
58	.698532	3.65	.937676	1.20	.760856	4.87	.239144	2
59	.698751	3.65	.937604	1.22	.761148	4.85	.238852	1
60	9.698970		9.937531		9.761439		10.238561	0
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.698970	3.65	9.937531	1.22	9.761439	4.87	10.238561	60
1	.699189	3.63	.937458	1.22	.761731	4.87	.238269	59
2	.699407	3.65	.937385	1.22	.762023	4.85	.237977	58
3	.699626	3.63	.937312	1.23	.762314	4.87	.237686	57
4	.699844	3.63	.937238	1.22	.762606	4.85	.237394	56
5	.700062	3.63	.937165	1.22	.762897	4.85	.237103	55
6	.700280	3.63	.937092	1.22	.763188	4.85	.236812	54
7	.700498	3.63	.937019	1.22	.763479	4.85	.236521	53
8	.700716	3.62	.936946	1.23	.763770	4.85	.236230	52
9	.700933	3.63	.936872	1.22	.764061	4.85	.235939	51
10	.701151	3.62	.936799	1.23	.764352	4.85	.235648	50
11	9.701368	3.62	9.936725	1.22	9.764643	4.83	10.235357	49
12	.701585	3.62	.936652	1.23	.764933	4.85	.235067	48
13	.701802	3.62	.936578	1.22	.765224	4.83	.234776	47
14	.702019	3.62	.936505	1.23	.765514	4.85	.234486	46
15	.702236	3.60	.936431	1.23	.765805	4.83	.234195	45
16	.702452	3.62	.936357	1.22	.766095	4.83	.233905	44
17	.702669	3.60	.936284	1.23	.766285	4.83	.233615	43
18	.702885	3.60	.936210	1.23	.766675	4.83	.233325	42
19	.703101	3.60	.936136	1.23	.766965	4.83	.233035	41
20	.703317	3.60	.936062	1.23	.767255	4.83	.232745	40
21	9.703533	3.60	9.935988	1.23	9.767545	4.82	10.232455	39
22	.703749	3.58	.935914	1.23	.767834	4.83	.232166	38
23	.703964	3.58	.935840	1.23	.768124	4.83	.231876	37
24	.704179	3.60	.935766	1.23	.768414	4.82	.231586	36
25	.704395	3.58	.935692	1.23	.768703	4.82	.231297	35
26	.704610	3.58	.935618	1.25	.768992	4.82	.231008	34
27	.704825	3.58	.935543	1.23	.769281	4.83	.230719	33
28	.705040	3.57	.935469	1.23	.769571	4.82	.230429	32
29	.705254	3.58	.935395	1.25	.769860	4.80	.230140	31
30	.705469	3.57	.935320	1.23	.770148	4.82	.229852	30
31	9.705683	3.58	9.935246	1.25	9.770437	4.82	10.229563	29
32	.705898	3.57	.935171	1.23	.770726	4.82	.229274	28
33	.706112	3.57	.935097	1.25	.771015	4.80	.228985	27
34	.706326	3.55	.935022	1.23	.771303	4.82	.228697	26
35	.706539	3.57	.934948	1.25	.771592	4.80	.228408	25
36	.706753	3.57	.934873	1.25	.771880	4.80	.228120	24
37	.706967	3.55	.934798	1.25	.772168	4.82	.227832	23
38	.707180	3·55	.934723	1.23	.772457	4.80	.227543	22
39	.707393	3.55	.934649	1.25	.772745	4.80	.227255	21
40	.707606	3.55	.934574	1.25	.773033	4.80	.226967	20
41	9.707819	3.55	9.934499	1.25	9.773321	4.78	10.226679	19
42	.708032	3.55	.934424	1.25	.773608	4.80	.226392	18
43	.708245	3.55	.934349	1.25	.773896	4.80	.226104	17
44	.708458	3.53	.934274	1.25	.774184	4.78	.225816	16
45	.708670	3.53	.934199	1.27	.774471	4.80	.225529	15
46	.708882	3.53	.934123	1.25	.774759	4.78	.225241	14
47	.709094	3.53	.934048	1.25	.775046	4.78	.224954	13
48	.709306	3.53	.933973	1.25	.775333	4.80	.224667	12
49	.709518	3.53	.933898	1.27	.775621	4.78	.224379	11
50	.709730	3.52	.933822	1.25	.775908	4.78	.224092	10
51	9.709941	3.53	9.933747	1.27	9.776195	4.78	10.223805	9
52	.710153	3.52	.933671	1.25	.776482	4.77	.223518	8
53	.710364	3.52	.933596	1.27	.776768	4.78	.223232	7
54	.710575	3.52	.933520	1.25	.777055	4.78	.222945	6
55	.710786	3.52	933445	1.27	.777342	4.77	.222658	5
56	.710997	3.52	.933369	1.27	.777628	4.78	.222372	4
57	.711208	3.52	.933293	1.27	.777915	4.77	.222085	3
58	.711419	3.50	.933217	1.27	.778201	4.78	.221799	2
59	.711629	3.50	.933141	1.25	.778488	4.77	.221512	1
60	9.711839		9.933066		9.778774		10.221226	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.711839	3.52	9.933066	1.27	9.778774	4.77	10.221226	60
1	.712050	3.50	.932990	1.27	.779060	4.77	.220940	59
2	.712260	3.48	.932914	1.27	.779346	4.77	.220654	58
3	.712469	3.50	.932838	1.27	.779632	4.77	.220368	57
4	.712679	3.50	.932762	1.28	.779918	4.75	.220082	56
5	.712889	3.48	.932685	1.27	.780203	4.77	.219797	55
6	.713098	3.50	.932609	1.27	.780489	4.77	.219511	54
7	.713306	3.48	.932533	1.27	.780775	4.75	.219225	53
8	.713517	3.48	.932457	1.28	.781060	4.77	.218940	52
9	.713726	3.48	.932380	1.27	.781346	4.75	.218654	51
10	.713935	3.48	.932304	1.27	.781631	4.75	.218369	50
11	9.714144	3.47	9.932228	1.28	9.781916	4.75	10.218084	49
12	.714352	3.48	.932151	1.27	.782201	4.75	.217799	48
13	.714561	3.47	.932075	1.28	.782486	4.75	.217514	47
14	.714769	3.48	.931998	1.28	.782771	4.75	.217229	46
15	.714978	3.47	.931921	1.27	.783056	4.75	.216944	45
16	.715186	3.47	.931845	1.28	.783341	4.75	.216659	44
17	.715394	3.47	.931768	1.28	.783626	4.73	.216374	43
18	.715602	3.45	.931691	1.28	.783910	4.75	.216090	42
19	.715809	3.47	.931614	1.28	.784195	4.73	.215805	41
20	.716017	3.45	.931537	1.28	.784479	4.75	.215521	40
21	9.716224	3.47	9.931460	1.28	9.784764	4.73	10.215236	39
22	.716432	3.45	.931383	1.28	.785048	4.73	.214952	38
23	.716639	3.45	.931306	1.28	.785332	4.73	.214668	37
24	.716846	3.45	.931229	1.28	.785616	4.73	.214384	36
25	.717053	3.43	.931152	1.28	.785900	4.73	.214100	35
26	.717259	3.45	.931075	1.28	.786184	4.73	.213816	34
27	.717466	3.45	.930998	1.28	.786468	4.73	.213532	33
28	.717673	3.43	.930921	1.30	.786752	4.73	.213248	32
29	.717879	3.43	.930843	1.28	.787036	4.72	.212964	31
30	.718085	3.43	.930766	1.30	.787319	4.73	.212681	30
31	9.718291	3.43	9.930688	1.28	9.787603	4.72	10.212397	29
32	.718497	3.43	.930611	1.30	.787886	4.73	.212114	28
33	.718703	3.43	.930533	1.28	.788170	4.72	.211830	27
34	.718909	3.42	.930456	1.30	.788453	4.72	.211547	26
35	.719114	3.43	.930378	1.30	.788736	4.72	.211264	25
36	.719320	3.42	.930300	1.28	.789019	4.72	.210981	24
37	.719525	3.42	.930223	1.30	.789302	4.72	.210698	23
38	.719730	3.42	.930145	1.30	.789585	4.72	.210415	22
39	.719935	3.42	.930067	1.30	.789868	4.72	.210132	21
40	.720140	3.42	.929989	1.30	.790151	4.72	.209849	20
41	9.720345	3.40	9.929911	1.30	9.790434	4.70	10.209566	19
42	.720549	3.42	.929833	1.30	.790716	4.72	.209284	18
43	.720754	3.40	.929755	1.30	.790999	4.70	.209001	17
44	.720958	3.40	.929677	1.30	.791281	4.70	.208719	16
45	.721162	3.40	.929599	1.30	.791563	4.72	.208437	15
46	.721366	3.40	.929521	1.32	.791846	4.70	.208154	14
47	.721570	3.40	.929442	1.30	.792128	4.70	.207872	13
48	.721774	3.40	.929364	1.30	.792410	4.70	.207590	12
49	.721978	3.38	.929286	1.32	.792692	4.70	.207308	11
50	.722181	3.40	.929207	1.30	.792974	4.70	.207026	10
51	9.722385	3.38	9.929129	1.32	9.793256	4.70	10.206744	9
52	.722588	3.38	.929050	1.30	.793538	4.68	.206462	8
53	.722791	3.38	.928972	1.32	.793819	4.70	.206181	7
54	.722994	3.38	.928893	1.30	.794101	4.70	.205899	6
55	.723197	3.38	.928815	1.32	.794383	4.68	.205617	5
56	.723400	3.38	.928736	1.32	.794664	4.70	.205336	4
57	.723603	3.37	.928657	1.32	.794946	4.68	.205054	3
58	.723805	3.37	.928578	1.32	.795227	4.68	.204773	2
59	.724007	3.38	.928499	1.32	.795508	4.68	.204492	1
60	9.724210		9.928420		9.795789		10.204211	0
		D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

'	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	'
0	9.724210	3.37	9.928420	1.30	9.795789	4.68	10.204211	60
1	.724412	3.37	.928342	1.32	.796070	4.68	.203930	59
2	.724614	3.37	.928263	1.33	.796351	4.68	.203649	58
3	.724816	3.35	.928183	1.32	.796632	4.68	.203368	57
4	.725017	3.37	.928104	1.32	.796913	4.68	.203087	56
5	.725219	3.35	.928025	1.32	.797194	4.67	.202806	55
6	.725420	3.37	.927946	1.32	.797474	4.68	.202526	54
7	.725622	3.35	.927867	1.33	.797755	4.68	.202245	53
8	.725823	3.35	.927787	1.32	.798036	4.67	.201964	52
9	.726024	3.35	.927708	1.32	.798316	4.67	.201684	51
10	.726225	3.35	.927629	1.33	.798596	4.68	.201404	50
11	9.726426	3.33	9.927549	1.32	9.798877	4.67	10.201123	49
12	.726626	3.35	.927470	1.33	.799157	4.67	.200843	48
13	.726827	3.33	.927390	1.33	.799437	4.67	.200563	47
14	.727027	3.35	.927310	1.32	.799717	4.67	.200283	46
15	.727228	3.33	.927231	1.33	.799997	4.67	.200003	45
16	.727428	3 33	.927151	1.33	.800277	4.67	.199723	44
17	.727628	3.33	.927071	1.33	.800557	4.65	.199443	43
18	.727828	3.32	.926991	1.33	.800836	4.67	.199164	42
19	.728027	3.33	.926911	1.33	.801116	4.67	.198884	41
20	.728227	3.33	.926831	1.33	.801396	4.65	.198604	40
21	9.728427	3.32	9.926751	1.33	9.801675	4.67	10.198325	39
22	.728626	3.32	.926671	1.33	.801955	4.65	.198045	38
23	.728825	3.32	.926591	1.33	.802234	4.65	.197766	37
24	.729024	3.32	.926511	1.33	.802513	4.65	.197487	36
25	.729223	3.32	.926431	1.33	.802792	4.67	.197208	35
26	.729422	3.32	.926351	1.35	.803072	4.65	.196928	34
27	.729621	3.32	.926270	1.33	.803351	4.65	.196649	33
28	.729820	3.30	.926190	1.33	.803630	4.65	.196370	32
29	.730018	3.32	.926110	1.35	.803909	4.63	.196091	31
30	.730217	3.30	.926029	1.33	.804187	4.65	.195813	30
31	9.730415	3.30	9.925949	1.35	9.804466	4.65	10.195534	29
32	.730613	3.30	.925868	1.33	.804745	4.63	.195255	28
33	.730811	3.30	.925788	1.35	.805023	4 65	.194977	27
34	.731009	3.28	.925707	1.35	.805302	4.63	.194698	26
35	.731206	3.30	.925626	1.35	.805580	4.65	.194420	25
36	.731404	3.30	.925545	1.33	.805859	4 63	.194141	24
37	.731602	3.28	.925465	1.35	.806137	4.63	.193863	23
38	.731799	3.28	.925384	1.35	.806415	4.63	.193585	22
39	.731996	3.28	.925303	1.35	.806693	4.63	.193307	21
40	.732193	3.28	.925222	1.35	.806971	4.63	.193029	20
41	9.732390	3.28	9.925141	1.35	9.807249	4.63	10.192751	19
42	.732587	3.28	.925060	1.35	.807527	4.63	.192473	18
43	.732784	3.27	.924979	1:37	.807805	4.63	.192195	17
44	.732980	3.28	.924897	1.35	.808083	4.63	.191917	16
45	.733177	3.27	.924816	1.35	.808361	4.62	.191639	15
46	.733373	3.27	.924735	1.35	.808638	4.63	.191362	14
47	.733569	3.27	.924[illegible]54	1.37	.808916	4.62	.191084	13
48	.733765	3.27	.924[illegible]72	1.35	.809193	4.63	.190807	12
49	.733961	3.27	.924[illegible]91	1.37	.809471	4.62	.190529	11
50	.734157	3.27	.924[illegible]09	1.35	.809748	4.62	.190252	10
51	9.734353	3.27	9.924[illegible]	1.37	9.810025	4.62	10.189975	9
52	.734549	3.25	.924[illegible]	1.37	.810302	4.63	.189698	8
53	.734744	3.25	.924[illegible]4	1.35	.810580	4.62	.189420	7
54	.734939	3.27	.924[illegible]3	1.37	.810857	4.62	.189143	6
55	.735135	3.25	.924[illegible]1	1.37	.811134	4.60	.188866	5
56	.735330	3.25	.923[illegible]	1.37	.811410	4.62	.188590	4
57	.735525	3.28	.923[illegible]	1.37	.811687	4.62	.188313	3
58	.7357[illegible]	[illegible]	[illegible]	1.37	.811964	4.62	.188036	2
59	.7359[illegible]	[illegible]	[illegible]	1.37	.812241	4.60	.187759	1
60	9.73[illegible]		[illegible]		9.812517		10.187483	0
				D. 1".	Cotang.	D. 1".	Tang.	'

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.736109	3.23	9.923591	1.37	9.812517	4.62	10.187483	60
1	.736303	3.25	.923509	1.37	.812794	4.60	.187206	59
2	.736498	3.23	.923427	1.37	.813070	4.62	.186930	58
3	.736692	3.23	.923345	1.37	.813347	4.60	.186653	57
4	.736886	3.23	.923263	1.37	.813623	4.60	.186377	56
5	.737080	3.23	.923181	1.38	.813899	4.62	.186101	55
6	.737274	3.22	.923098	1.37	.814176	4.60	.185824	54
7	.737467	3.23	.923016	1.38	.814452	4.60	.185548	53
8	.737661	3.23	.922933	1.37	.814728	4.60	.185272	52
9	.737855	3.22	.922851	1.38	.815004	4.60	.184996	51
10	.738048	3.22	.922768	1.37	.815280	4.58	.184720	50
11	9.738241	3.22	9.922686	1.38	9.815555	4.60	10.184445	49
12	.738434	3.22	.922603	1.38	.815831	4.60	.184169	48
13	.738627	3.22	.922520	1.37	.816107	4.58	.183893	47
14	.738820	3.22	.922438	1.38	.816382	4.60	.183618	46
15	.739013	3.22	.922355	1.38	.816658	4.58	.183342	45
16	.739206	3.20	.922272	1.38	.816933	4.60	.183067	44
17	.739398	3.20	.922189	1.38	.817209	4.58	.182791	43
18	.739590	3.22	.922106	1.38	.817484	4.58	.182516	42
19	.739783	3.20	.922023	1.38	.817759	4.60	.182241	41
20	.739975	3.20	.921940	1.38	.818035	4.58	.181965	40
21	9.740167	3.20	9.921857	1.38	9.818310	4.58	10.181690	39
22	.740359	3.18	.921774	1.38	.818585	4.58	.181415	38
23	.740550	3.20	.921691	1.40	.818860	4.58	.181140	37
24	.740742	3.20	.921607	1.38	.819135	4.58	.180865	36
25	.740934	3.18	.921524	1.38	.819410	4.57	.180590	35
26	.741125	3.18	.921441	1.40	.819684	4.58	.180316	34
27	.741316	3.20	.921357	1.38	.819959	4.58	.180041	33
28	.741508	3.18	.921274	1.40	.820234	4.57	.179766	32
29	.741699	3.17	.921190	1.38	.820508	4.58	.179492	31
30	.741889	3.18	.921107	1.40	.820783	4.57	.179217	30
31	9.742080	3.18	9.921023	1.40	9.821057	4.58	10.178943	29
32	.742271	3.18	.920939	1.38	.821332	4.57	.178668	28
33	.742462	3.17	.920856	1.40	.821606	4.57	.178394	27
34	.742652	3.17	.920772	1.40	.821880	4.57	.178120	26
35	.742842	3.18	.920688	1.40	.822154	4.58	.177846	25
36	.743033	3.17	.920604	1.40	.822429	4.57	.177571	24
37	.743223	3.17	.920520	1.40	.822703	4.57	.177297	23
38	.743413	3.15	.920436	1.40	.822977	4.57	.177023	22
39	.743602	3.17	.920352	1.40	.823251	4.55	.176749	21
40	.743792	3.17	.920268	1.40	.823524	4.57	.176476	20
41	9.743982	3.15	9.920184	1.42	9.823798	4.57	10.176202	19
42	.744171	3.17	.920099	1.40	.824072	4.55	.175928	18
43	.744361	3.15	.920015	1.40	.824345	4.57	.175655	17
44	.744550	3.15	.919931	1.42	.824619	4.57	.175381	16
45	.744739	3.15	.919846	1.40	.824893	4.55	.175107	15
46	.744928	3.15	.919762	1.42	.825166	4.55	.174834	14
47	.745117	3.15	.919677	1.40	.825439	4.57	.174561	13
48	.745306	3.13	.919593	1.42	.825713	4.55	.174287	12
49	.745494	3.15	.919508	1.40	.825986	4.55	.174014	11
50	.745683	3.13	.919424	1.42	.826259	4.55	.173741	10
51	9.745871	3.15	9.919339	1.42	9.826532	4.55	10.173468	9
52	.746060	3.13	.919254	1.42	.826805	4.55	.173195	8
53	.746248	3.13	.919169	1.40	.827078	4.55	.172922	7
54	.746436	3.13	.919085	1.42	.827351	4.55	.172649	6
55	.746624	3.13	.919000	1.42	.827624	4.55	.172376	5
56	.746812	3.12	.918915	1.42	.827897	4.55	.172103	4
57	.746999	3.13	.918830	1.42	.828170	4.53	.171830	3
58	.747187	3.12	.918745	1.43	.828442	4.55	.171558	2
59	.747374	3.13	.918659	1.42	.828715	4.53	.171285	1
60	9.747562		9.918574		9.828987		10.171013	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

56°

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.747562	3.12	9.918574	1.42	9.828987	4.55	10.171013	60
1	.747749	3.12	.918489	1.42	.829260	4.53	.170740	59
2	.747936	3.12	.918404	1.43	.829532	4.55	.170468	58
3	.748123	3.12	.918318	1.42	.829805	4.53	.170195	57
4	.748310	3.12	.918233	1.43	.830077	4.53	.169923	56
5	.748497	3.10	.918147	1.42	.830349	4.53	.169651	55
6	.748683	3.12	.918062	1.43	.830621	4.53	.169379	54
7	.748870	3.10	.917976	1.42	.830893	4.53	.169107	53
8	.749056	3.12	.917891	1.43	.831165	4.53	.168835	52
9	.749243	3.10	.917805	1.43	.831437	4.53	.168563	51
10	.749429	3.10	.917719	1.42	.831709	4.53	.168291	50
11	9.749615	3.10	9.917634	1.43	9.831981	4.53	10.168019	49
12	.749801	3.10	.917548	1.43	.832253	4.53	.167747	48
13	.749987	3.08	.917462	1.43	.832525	4.52	.167475	47
14	.750172	3.10	.917376	1.43	.832796	4.53	.167204	46
15	.750358	3.08	.917290	1.43	.833068	4.52	.166932	45
16	.750543	3.10	.917204	1.43	.833339	4.53	.166661	44
17	.750729	3.08	.917118	1.43	.833611	4.52	.166389	43
18	.750914	3.08	.917032	1.43	.833882	4.53	.166118	42
19	.751099	3.08	.916946	1.45	.834154	4.52	.165846	41
20	.751284	3.08	.916859	1.43	.834425	4.52	.165575	40
21	9.751469	3.08	9.916773	1.43	9.834696	4.52	10.165304	39
22	.751654	3.08	.916687	1.45	.834967	4.52	.165033	38
23	.751839	3.07	.916600	1.43	.835238	4.52	.164762	37
24	.752023	3.08	.916514	1.45	.835509	4.52	.164491	36
25	.752208	3.07	.916427	1.43	.835780	4.52	.164220	35
26	.752392	3.07	.916341	1.45	.836051	4.52	.163949	34
27	.752576	3.07	.916254	1.45	.836322	4.52	.163678	33
28	.752760	3.07	.916167	1.43	.836593	4.52	.163407	32
29	.752944	3.07	.916081	1.45	.836864	4.50	.163136	31
30	.753128	3.07	.915994	1.45	.837134	4.52	.162866	30
31	9.753312	3.05	9.915907	1.45	9.837405	4.50	10.162595	29
32	.753495	3.07	.915820	1.45	.837675	4.52	.162325	28
33	.753679	3.07	.915733	1.45	.837946	4.50	.162054	27
34	.753862	3.07	.915646	1.45	.838216	4.52	.161784	26
35	.754046	3.05	.915559	1.45	.838487	4.50	.161513	25
36	.754229	3.05	.915472	1.45	.838757	4.50	.161243	24
37	.754412	3.05	.915385	1.47	.839027	4.50	.160973	23
38	.754595	3.05	.915297	1.45	.839297	4.52	.160703	22
39	.754778	3.03	.915210	1.45	.839568	4.50	.160432	21
40	.754960	3.05	.915123	1.47	.839838	4.50	.160162	20
41	9.755143	3.05	9.915035	1.45	9.840108	4.50	10.159892	19
42	.755326	3.03	.914948	1.47	.840378	4.50	.159622	18
43	.755508	3.03	.914860	1.45	.840648	4.48	.159352	17
44	.755690	3.03	.914773	1.47	.840917	4.50	.159083	16
45	.755872	3.03	.914685	1.45	.841187	4.50	.158813	15
46	.756054	3.03	.914598	1.47	.841457	4.50	.158543	14
47	.756236	3.03	.914510	1.47	.841727	4.48	.158273	13
48	.756418	3.03	.914422	1.47	.841996	4.50	.158004	12
49	.756600	3.03	.914334	1.47	.842266	4.48	.157734	11
50	.756782	3.02	.914246	1.47	.842535	4.50	.157465	10
51	9.756963	3.02	9.914158	1.47	9.842805	4.48	10.157195	9
52	.757144	3 03	.914070	1.47	.843074	4.48	156926	8
53	.757326	3.02	.913982	1.47	.843343	4.48	.156657	7
54	.757507	3.02	.913894	1.47	.843612	4.50	.156388	6
55	.757688	3.02	.913806	1.47	.843882	4.48	.156118	5
56	.757869	3.02	.913718	1.47	.844151	4.48	.155849	4
57	.758050	3.00	.913630	1.48	.844420	4.48	.155580	3
58	.758230	3.02	913541	1.47	.844689	4.48	.155311	2
59	.758411	3.00	.913453	1.47	.844958	4.48	.155042	1
60	9.758591		9.913365		9.845227		10.154773	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.758591	3.02	9.913365	1.48	9.845227	4.48	10.154773	60
1	.758772	3.00	.913276	1.48	.845496	4.47	.154504	59
2	.758952	3.00	.913187	1.47	.845764	4.48	.154236	58
3	.759132	3.00	.913099	1.48	.846033	4.48	.153967	57
4	.759312	3.00	.913010	1.47	.846302	4.47	.153698	56
5	.759492	3.00	.912922	1.48	.846570	4.48	.153430	55
6	.759672	3.00	.912833	1.48	.846839	4.48	.153161	54
7	.759852	2.98	.912744	1.48	.847108	4.47	.152892	53
8	.760031	3.00	.912655	1.48	.847376	4.47	.152624	52
9	.760211	2.98	.912566	1.48	.847644	4.48	.152356	51
10	.760390	2.98	.912477	1.48	.847913	4.47	.152087	50
11	9.760569	2.98	9.912388	1.43	9.848181	4.47	10.151819	49
12	.760748	2.98	.912299	1.48	.848449	4.47	.151551	48
13	.760927	2.98	.912210	1.48	.848717	4.48	.151283	47
14	.761106	2.98	.912121	1.50	.848986	4.47	.151014	46
15	.761285	2.98	.912031	1.48	.849254	4.47	.150746	45
16	.761464	2.97	.911942	1.48	.849522	4.47	.150478	44
17	.761642	2.98	.911853	1.50	.849790	4.45	.150210	43
18	.761821	2.97	.911763	1.48	.850057	4.47	.149943	42
19	.761999	2.97	.911674	1.50	.850325	4.47	.149675	41
20	.762177	2.98	.911584	1.48	.850593	4.47	.149407	40
21	9.762356	2.97	9.911495	1.50	9.850861	4.47	10.149139	39
22	.762534	2.97	.911405	1.50	.851129	4.45	.148871	38
23	.762712	2.95	.911315	1.48	.851396	4.47	.148604	37
24	.762889	2.97	.911226	1.50	.851664	4.45	.148336	36
25	.763067	2.97	.911136	1.50	.851931	4.47	.148069	35
26	.763245	2.95	.911046	1.50	.852199	4.45	.147801	34
27	.763422	2.97	.910956	1.50	.852466	4.45	.147534	33
28	.763600	2.95	.910866	1.50	.852733	4.47	.147267	32
29	.763777	2.95	.910776	1.50	.853001	4.45	.146999	31
30	.763954	2.95	.910686	1.50	.853268	4.45	.146732	30
31	9.764131	2.95	9.910596	1.50	9.853535	4.45	10.146465	29
32	.764308	2.95	.910506	1.52	.853802	4.45	.146198	28
33	.764485	2.95	.910415	1.50	.854069	4.45	.145931	27
34	.764662	2.93	.910325	1.50	.854336	4.45	.145664	26
35	.764838	2.95	.910235	1.52	.854603	4.45	.145397	25
36	.765015	2.93	.910144	1.50	.854870	4.45	.145130	24
37	.765191	2.93	.910054	1.52	.855137	4.45	.144863	23
38	.765367	2.95	.909963	1.50	.855404	4.45	.144596	22
39	.765544	2.93	.909873	1.52	.855671	4.45	.144329	21
40	.765720	2.93	.909782	1.52	.855938	4.43	.144062	20
41	9.765896	2.93	9.909691	1.50	9.856204	4.45	10.143796	19
42	.766072	2.92	.909601	1.52	.856471	4.43	.143529	18
43	.766247	2.93	.909510	1.52	.856737	4.45	.143263	17
44	.766423	2.92	.909419	1.52	.857004	4.43	.142996	16
45	.766598	2.93	.909328	1.52	.857270	4.45	.142730	15
46	.766774	2.92	.909237	1.52	.857537	4.43	.142463	14
47	.766949	2.92	.909146	1.52	.857803	4.43	.142197	13
48	.767124	2.93	.909055	1.52	.858069	4.45	.141931	12
49	.767300	2.92	.908964	1.52	.858336	4.43	.141664	11
50	.767475	2.90	.908873	1.53	.858602	4.43	.141398	10
51	9.767649	2.92	9.908781	1.52	9.858868	4.43	10.141132	9
52	.767824	2.92	.908690	1.52	.859134	4.43	.140866	8
53	.767999	2.90	.908599	1.53	.859400	4.43	.140600	7
54	.768173	2.92	.908507	1.52	.859666	4.43	.140334	6
55	.768348	2.90	.908416	1.53	.859932	4.43	.140068	5
56	.768522	2.92	.908324	1.52	.860198	4.43	.139802	4
57	.768697	2.90	.908233	1.53	.860464	4.43	.139536	3
58	.768871	2.90	.908141	1.53	.860730	4.42	.139270	2
59	.769045	2.90	.908049	1.52	.860995	4.43	.139005	1
60	9.769219		9.907958		9.861261		10.138739	0
	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

54°

	Sine.	D. 1".	Cosine.	D. 1".	Tang.	D. 1".	Cotang.	
0	9.769219	2.90	9.907958	1.53	9.861261	4.43	10.138739	
1	.769393	2.88	.907866	1.53	.861527	4.42	.138473	
2	.769566	2.90	.907774	1.53	.861792	4.43	.138208	
3	.769740	2.88	.907682	1.53	.862058	4.42	.137942	
4	.769913	2.90	.907590	1.53	.862323	4.43	.137677	5
5	.770087	2.88	.907498	1.53	.862589	4.42	.137411	5
6	.770260	2.88	.907406	1.53	.862854	4.42	.137146	5
7	.770433	2.88	.907314	1.53	.863119	4.43	.136881	5
8	.770606	2.88	.907222	1.55	.863385	4.42	.136615	5
9	.770779	2.88	.907129	1.53	.863650	4.42	.136350	51
10	.770952	2.88	.907037	1.53	.863915	4.42	.136085	50
11	9.771125	2.88	9.906945	1.55	9.864180	4.42	10.135820	49
12	.771298	2.87	.906852	1.53	.864445	4.42	.135555	48
13	.771470	2.88	.906760	1.55	.864710	4.42	.135290	47
14	.771643	2.87	.906667	1.53	.864975	4.42	.135025	46
15	.771815	2.87	.906575	1.55	.865240	4.42	.134760	45
16	.771987	2.87	.906482	1.55	.865505	4.42	.134495	44
17	.772159	2.87	.906389	1.55	.865770	4.42	.134230	43
18	.772331	2.87	.906296	1.53	.866035	4.42	.133965	42
19	.772503	2.87	.906204	1.55	.866300	4.40	.133700	41
20	.772675	2.87	.906111	1.55	.866564	4.42	.133436	40
21	9.772847	2.85	9.906018	1.55	9.866829	4.42	10.133171	39
22	.773018	2.87	.905925	1.55	.867094	4.40	.132906	38
23	.773190	2.85	.905832	1.55	.867358	4.42	.132642	37
24	.773361	2.87	.905739	1.57	.867623	4.40	.132377	36
25	.773533	2.85	.905645	1.55	.867887	4.42	.132113	35
26	.773704	2.85	.905552	1.55	.868152	4.40	.131848	34
27	.773875	2.85	.905459	1.55	.868416	4.40	.131584	33
28	.774046	2.85	.905366	1.57	.868680	4.42	.131320	32
29	.774217	2.85	.905272	1.55	.868945	4.40	.131055	31
30	.774388	2.83	.905179	1.57	.869209	4.40	.130791	30
31	9.774558	2.85	9.905085	1.55	9.869473	4.40	10.130527	29
32	.774729	2.83	.904992	1.57	.869737	4.40	.130263	28
33	.774899	2.85	.904898	1.57	.870001	4.40	.129999	27
34	.775070	2.83	.904804	1.55	.870265	4.40	.129735	26
35	.775240	2.83	.904711	1.57	.870529	4.40	.129471	25
36	.775410	2.83	.904617	1.57	.870793	4.40	.129207	24
37	.775580	2.83	.904523	1.57	.871057	4.40	.128943	23
38	.775750	2.83	.904429	1.57	.871321	4.40	.128679	22
39	.775920	2.83	.904335	1.57	.871585	4.40	.128415	21
40	.776090	2.82	.904241	1.57	.871849	4.38	.128151	20
41	9.776259	2.83	9.904147	1.57	9.872112	4.40	10.127888	19
42	.776429	2.82	.904053	1.57	.872376	4.40	.127624	18
43	.776598	2.83	.903959	1.58	.872640	4.38	.127360	17
44	.776768	2.82	.903864	1.57	.872903	4.40	.127097	16
45	.776937	2.82	.903770	1.57	.873167	4.38	.126833	15
46	.777106	2.82	.903676	1.58	.873430	4.40	.126570	14
47	.777275	2.82	.903581	1.57	.873694	4.38	.126306	13
48	.777444	2.82	.903487	1.58	.873957	4.38	.126043	12
49	.777613	2.80	.903392	1.57	.874220	4.40	.125780	11
50	.777781	2.82	.903298	1.58	.874484	4.38	.125516	10
51	9.777950	2.82	9.903203	1.58	9.874747	4.38	10.125253	9
52	.778119	2.80	.903108	1.57	.875010	4.38	.124990	8
3	.778287	2.80	.903014	1.58	.875273	4.40	.124727	7
4	.778455	2.82	.902919	1.58	.875537	4.38	.124463	6
5	.778624	2.80	.902824	1.58	.875800	4.38	.124200	5
6	.778792	2.80	.902729	1.58	.876063	4.38	.123937	4
7	.778960	2.80	.902634	1.58	.876326	4.38	.123674	3
	.779128	2.78	.902539	1.58	.876589	4.38	.123411	2
	.779295	2.80	.902444	1.58	.876852	4.37	.123148	1
	9.779463		9.902349		9.877114		10.122886	0
	Cosine.	D. 1".	Sine.	D. 1".	Cotang.	D. 1".	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.779463	2.80	9.902349	1.60	9.877114	4.38	10.122886	60
1	.779631	2.78	.902253	1.58	.877377	4.38	.122623	59
2	.779798	2.80	.902158	1.58	.877640	4.38	.122360	58
3	.779966	2.78	.902063	1.60	.877903	4.37	.122097	57
4	.780133	2.78	.901967	1.58	.878165	4.38	.121835	56
5	.780300	2.78	.901872	1.60	.878428	4.38	.121572	55
6	.780467	2.78	.901776	1.58	.878691	4.37	.121309	54
7	.780634	2.78	.901681	1.60	.878953	4.38	.121047	53
8	.780801	2.78	.901585	1.58	.879216	4.37	.120784	52
9	.780968	2.77	.901490	1.60	.879478	4.38	.120522	51
10	.781134	2.78	.901394	1.60	.879741	4.37	.120259	50
11	9.781301	2.78	9.901298	1.60	9.880003	4.37	10.119997	49
12	.781468	2.77	.901202	1.60	.880265	4.38	.119735	48
13	.781634	2.77	.901106	1.60	.880528	4.37	.119472	47
14	.781800	2.77	.901010	1.60	.880790	4.37	.119210	46
15	.781966	2.77	.900914	1.60	.881052	4.37	.118948	45
16	.782132	2.77	.900818	1.60	.881314	4.38	.118686	44
17	.782298	2.77	.900722	1.60	.881577	4.37	.118423	43
18	.782464	2.77	.900626	1.62	.881839	4.37	.118161	42
19	.782630	2.77	.900529	1.60	.882101	4.37	.117899	41
20	.782796	2.75	.900433	1.60	.882363	4.37	.117637	40
21	9.782961	2.77	9.900337	1.62	9.882625	4.37	10.117375	39
22	.783127	2.75	.900240	1.60	.882887	4.35	.117113	38
23	.783292	2.77	.900144	1.62	.883148	4.37	.116852	37
24	.783458	2.75	.900047	1.60	.883410	4.37	.116590	36
25	.783623	2.75	.899951	1.62	.883672	4.37	.116328	35
26	.783788	2.75	.899854	1.62	.883934	4.37	.116066	34
27	.783953	2.75	.899757	1.62	.884196	4.35	.115804	33
28	.784118	2.73	.899660	1.60	.884457	4.37	.115543	32
29	.784282	2.75	.899564	1.62	.884719	4.35	.115281	31
30	.784447	2.75	.899467	1.62	.884980	4.37	.115020	30
31	9.784612	2.73	9.899370	1.62	9.885242	4.37	10.114758	29
32	.784776	2.75	.899273	1.62	.885504	4.35	.114496	28
33	.784941	2.73	.899176	1.63	.885765	4.35	.114235	27
34	.785105	2.73	.899078	1.62	.886026	4.37	.113974	26
35	.785269	2.73	.898981	1.62	.886288	4.35	.113712	25
36	.785433	2.73	.898884	1.62	.886549	4.37	.113451	24
37	.785597	2.73	.898787	1.63	.886811	4.35	.113189	23
38	.785761	2.73	.898689	1.62	.887072	4.35	.112928	22
39	.785925	2.73	.898592	1.63	.887333	4.35	.112667	21
40	.786089	2.72	.898494	1.62	.887594	4.35	.112406	20
41	9.786252	2.73	9.898397	1.63	9.887855	4.35	10.112145	19
42	.786416	2.72	.898299	1.62	.888116	4.37	.111884	18
43	.786579	2.72	.898202	1.63	.888378	4.35	.111622	17
44	.786742	2.73	.898104	1.63	.888639	4.35	.111361	16
45	.786906	2.72	.898006	1.63	.888900	4.35	.111100	15
46	.787069	2.72	.897908	1.63	.889161	4.33	.110839	14
47	.787232	2.72	.897810	1.63	.889421	4.35	.110579	13
48	.787395	2.70	.897712	1.63	.889682	4.35	.110318	12
49	.787557	2.72	.897614	1.63	.889943	4.35	.110057	11
50	.787720	2.72	.897516	1.63	.890204	4.35	.109796	10
51	9.787883	2.70	9.897418	1.63	9.890465	4.33	10.109535	9
52	.788045	2.72	.897320	1.63	.890725	4.35	.109275	8
53	.788208	2.70	.897222	1.65	.890986	4.35	.109014	7
54	.788370	2.70	.897123	1.63	.891247	4.33	.108753	6
55	.788532	2.70	.897025	1.65	.891507	4.35	.108493	5
56	.788694	2.70	.896926	1.63	.891768	4.33	.108232	4
57	.788856	2.70	.896828	1.65	.892028	4.35	.107972	3
58	.789018	2.70	.896729	1.63	.892289	4.33	.107711	2
59	.789180	2.70	.896631	1.65	.892549	4.35	.107451	1
60	9.789342		9.896532		9.892810		10.107190	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.789342	2.70	9.896532	1.65	9.892810	4.33	10.107190	60
1	.789504	2.68	.896433	1.63	.893070	4.35	.106930	59
2	.789665	2.70	.896335	1.65	.893331	4.33	.106669	58
3	.789827	2.68	.896236	1.65	.893591	4.33	.106409	57
4	.789988	2.68	.896137	1.65	.893851	4.33	.106149	56
5	.790149	2.68	.896038	1.65	.894111	4.35	.105889	55
6	.790310	2.68	.895939	1.65	.894372	4.33	.105628	54
7	.790471	2.68	.895840	1.65	.894632	4.33	.105368	53
8	.790632	2.68	.895741	1.67	.894892	4.33	.105108	52
9	.790793	2.68	.895641	1.65	.895152	4.33	.104848	51
10	.790954	2.68	.895542	1.65	.895412	4.23	.104588	50
11	9.791115	2.67	9.895443	1.67	9.895672	4.33	10.104328	49
12	.791275	2.68	.895343	1.65	.895932	4.33	.104068	48
13	.791436	2.67	.895244	1.65	.896192	4.33	.103808	47
14	.791596	2.68	.895145	1.67	.896452	4.33	.103548	46
15	.791757	2.67	.895045	1.67	.896712	4.32	.103288	45
16	.791917	2.67	.894945	1.65	.896971	4.33	.103029	44
17	.792077	2.67	.894846	1.67	.897231	4.33	.102769	43
18	.792237	2.67	.894746	1.67	.897491	4.33	.102509	42
19	.792397	2.67	.894646	1.67	.897751	4.32	.102249	41
20	.792557	2.65	.894546	1.67	.898010	4.33	.101990	40
21	9.792716	2.67	9.894446	1.67	9.898270	4.33	10.101730	39
22	.792876	2.65	.894346	1.67	.898530	4.32	.101470	38
23	.793035	2.67	.894246	1.67	.898789	4.33	.101211	37
24	.793195	2.65	.894146	1.67	.899049	4.32	.100951	36
25	.793354	2.67	.894046	1.67	.899308	4.33	.100692	35
26	.793514	2.65	.893946	1.67	.899568	4.32	.100432	34
27	.793673	2.65	.893846	1.68	.899827	4.33	.100173	33
28	.793832	2.65	.893745	1.67	.900087	4.32	.099913	32
29	.793991	2.65	.893645	1.68	.900346	4.32	.099654	31
30	.794150	2.63	.893544	1.67	.900605	4.32	.099395	30
31	9.794308	2.65	9.893444	1.68	9.900864	4.33	10.099136	29
32	.794467	2.65	.893343	1.67	.901124	4.32	.098876	28
33	.794626	2.63	.893243	1.68	.901383	4.32	.098617	27
34	.794784	2.63	.893142	1.68	.901642	4.32	.098358	26
35	.794942	2.65	.893041	1.68	.901901	4.32	.098099	25
36	.795101	2.63	.892940	1.68	.902160	4.33	.097840	24
37	.795259	2.63	.892839	1.67	.902420	4.32	.097580	23
38	.795417	2.63	.892739	1.68	.902679	4.32	.097321	22
39	.795575	2.63	.892638	1.70	.902938	4.32	.097062	21
40	.795733	2.63	.892536	1.68	.903197	4.32	.096803	20
41	9.795891	2.63	9.892435	1.68	9.903456	4.30	10.096544	19
42	.796049	2.62	.892334	1.68	.903714	4.32	.096286	18
43	.796206	2.63	.892233	1.68	.903973	4.32	.096027	17
44	.796364	2.62	.892132	1.70	.904232	4.32	.095768	16
45	.796521	2.63	.892030	1.68	.904491	4.32	.095709	15
46	.796679	2.62	.891929	1.70	.904750	4.30	.095250	14
47	.796836	2.62	.891827	1.68	.905008	4.32	.094992	13
48	.796993	2.62	.891726	1.70	.905267	4.32	.094733	12
49	.797150	2.62	.891624	1.68	.905526	4.32	.094474	11
50	.797307	2.62	.891523	1.70	.905785	4.30	.094215	10
51	9.797464	2.62	9.891421	1.70	9.906043	4.32	10.093957	9
52	.797621	2.60	.891319	1.70	.906302	4.30	.093698	8
53	.797777	2.62	.891217	1.70	.906560	4.32	.093440	7
54	.797934	2.62	.891115	1.70	.906819	4.30	.093181	6
55	.798091	2.60	.891013	1.70	.907077	4.32	.092923	5
56	.798247	2.60	.890911	1.70	.907336	4.30	.092664	4
57	.798403	2.62	.890809	1.70	.907594	4.22	.092406	3
58	.798560	2.60	.890707	1.70	.907853	4.30	.092147	2
59	.798716	2.60	.890605	1.70	.908111	4.30	.091889	1
60	9.798872		9.890503		9.908369		10.091631	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.798872	2.60	9.890503	1.72	9.908369	4.32	10.091631	60
1	.799028	2.60	.890400	1.70	.908628	4.30	.091372	59
2	.799184	2.58	.890298	1.72	.908886	4.30	.091114	58
3	.799339	2.60	.890195	1.70	.909144	4.30	.090856	57
4	.799495	2.60	.890093	1.72	.909402	4.30	.090598	56
5	.799651	2.58	.889990	1.70	.909660	4.30	.090340	55
6	.799806	2.60	.889888	1.72	.909918	4.32	.090082	54
7	.799962	2.58	.889785	1.72	.910177	4.30	.089823	53
8	.800117	2.58	.889682	1.72	.910435	4.30	.089565	52
9	.800272	2.58	.889579	1.70	.910693	4.30	.089307	51
10	.800427	2.58	.889477	1.72	.910951	4.30	.089049	50
11	9.800582	2.58	9.889374	1.72	9.911209	4.30	10.088791	49
12	.800737	2.58	.889271	1.72	.911467	4.30	.088533	48
13	.800892	2.58	.889168	1.73	.911725	4.28	.088275	47
14	.801047	2.57	.889064	1.72	.911982	4.30	.088018	46
15	.801201	2.58	.888961	1.72	.912240	4.30	.087760	45
16	.801356	2.58	.888858	1.72	.912498	4.30	.087502	44
17	.801511	2.57	.888755	1.73	.912756	4.30	.087244	43
18	.801665	2.57	.888651	1.72	.913014	4.28	.086986	42
19	.801819	2.57	.888548	1.73	.913271	4.30	.086729	41
20	.801973	2.58	.888444	1.72	.913529	4.30	.086471	40
21	9.802128	2.57	9.888341	1.73	9.913787	4.28	10.086213	39
22	.802282	2.57	.888237	1.72	.914044	4.30	.085956	38
23	.802436	2.55	.888134	1.73	.914302	4.30	.085698	37
24	.802589	2.57	.888030	1.73	.914560	4.28	.085440	36
25	.802743	2.57	.887926	1.73	.914817	4.30	.085183	35
26	.802897	2.55	.887822	1.73	.915075	4.28	.084925	34
27	.803050	2.57	.887718	1.73	.915332	4.30	.084668	33
28	.803204	2.55	.887614	1.73	.915590	4.28	.084410	32
29	.803357	2.57	.887510	1.73	.915847	4.28	.084153	31
30	.803511	2.55	.887406	1.73	.916104	4.30	.083896	30
31	9.803664	2.55	9.887302	1.73	9.916362	4.28	10.083638	29
32	.803817	2.55	.887198	1.75	.916619	4.30	.083381	28
33	.803970	2.55	.887093	1.73	.916877	4.28	.083123	27
34	.804123	2.55	.886989	1.73	.917134	4.28	.082866	26
35	.804276	2.55	.886885	1.75	.917391	4.28	.082609	25
36	.804428	2.55	.886780	1.73	.917648	4.30	.082352	24
37	.804581	2.55	.886676	1.75	.917906	4.28	.082094	23
38	.804734	2.53	.886571	1.75	.918163	4.28	.081837	22
39	.804886	2.55	.886466	1.73	.918420	4.28	.081580	21
40	.805039	2.53	.886362	1.75	.918677	4.28	.081323	20
41	9.805191	2.53	9.886257	1.75	9.918934	4.28	10.081066	19
42	.805343	2.53	.886152	1.75	.919191	4.28	.080809	18
43	.805495	2.53	.886047	1.75	.919448	4.28	.080552	17
44	.805647	2.53	.885942	1.75	.919705	4.28	.080295	16
45	.805799	2.53	.885837	1.75	.919962	4.28	.080038	15
46	.805951	2.53	.885732	1.75	.920219	4.28	.079781	14
47	.806103	2.52	.885627	1.75	.920476	4.28	.079524	13
48	.806254	2.53	.885522	1.77	.920733	4.28	.079267	12
49	.806406	2.52	.885416	1.75	.920990	4.28	.079010	11
50	.806557	2.53	.885311	1.77	.921247	4.27	.078753	10
51	9.806709	2.52	9.885205	1.75	9.921503	4.28	10.078497	9
52	.806860	2.52	.885100	1.77	.921760	4.28	.078240	8
53	.807011	2.53	.884994	1.75	.922017	4.28	.077983	7
54	.807163	2.52	.884889	1.77	.922274	4.27	.077726	6
55	.807314	2.52	.884783	1.77	.922530	4.28	.077470	5
56	.807465	2.50	.884677	1.75	.922787	4.28	.077213	4
57	.807615	2.52	.884572	1.77	.923044	4.27	.076956	3
58	.807766	2.52	.884466	1.77	.923300	4.28	.076700	2
59	.807917	2.50	.884360	1.77	.923557	4.28	.076443	1
60	9.808067		9.884254		9.923814		10.076186	0
′	*Cosine.*	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

COSINES, TANGENTS, AND COTANGENTS.

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	
0	9.808067	2.52	9.884254	1.77	9.923814	4.27	10.076186	60
1	.808218	2.50	.884148	1.77	.924070	4.28	.075930	59
2	.808368	2.52	.884042	1.77	.924327	4.27	.075673	58
3	.808519	2.50	.883936	1.78	.924583	4.28	.075417	57
4	.808669	2.50	.883829	1.77	.924840	4.27	.075160	56
5	.808819	2.50	.883723	1.77	.925096	4.27	.074904	55
6	.808969	2.50	.883617	1.78	.925352	4.28	.074648	54
7	.809119	2.50	.883510	1.77	.925609	4.27	.074391	53
8	.809269	2.50	.883404	1.78	.925865	4.28	.074135	52
9	.809419	2.50	.883297	1.77	.926122	4.27	.073878	51
10	.809569	2.48	.883191	1.78	.926378	4.27	.073622	50
11	9.809718	2.50	9.883084	1.78	9.926634	4.27	10.073366	49
12	.809868	2.48	.882977	1.77	.926890	4.28	.073110	48
13	.810017	2.50	.882871	1.78	.927147	4.27	.072853	47
14	.810167	2.48	.882764	1.78	.927403	4.27	.072597	46
15	.810316	2.48	.882657	1.78	.927659	4.27	.072341	45
16	.810465	2.48	.882550	1.78	.927915	4.27	.072085	44
17	.810614	2.48	.882443	1.78	.928171	4.27	.071829	43
18	.810763	2.48	.882336	1.78	.928427	4.28	.071573	42
19	.810912	2.48	.882229	1.80	.928684	4.27	.071316	41
20	.811061	2.48	.882121	1.78	.928940	4.27	.071060	40
21	9.811210	2.47	9.882014	1.78	9.929196	4.27	10.070804	39
22	.811358	2.48	.881907	1.80	.929452	4.27	.070548	38
23	.811507	2.47	.881799	1.78	.929708	4.27	.070292	37
24	.811655	2.48	.881692	1.80	.929964	4.27	.070036	36
25	.811804	2.47	.881584	1.78	.930220	4.25	.069780	35
26	.811952	2.47	.881477	1.80	.930475	4.27	.069525	34
27	.812100	2.47	.881369	1.80	.930731	4.27	.069269	33
28	.812248	2.47	.881261	1.80	.930987	4.27	.069013	32
29	.812396	2.47	.881153	1.78	.931243	4.27	.068757	31
30	.812544	2.47	.881046	1.80	.931499	4.27	.068501	30
31	9.812692	2.47	9.880938	1.80	9.931755	4.25	10.068245	29
32	.812840	2.47	.880830	1.80	.932010	4.27	.067990	28
33	.812988	2.45	.880722	1.82	.932266	4.27	.067734	27
34	.813135	2.47	.880613	1.80	.932522	4.27	.067478	26
35	.813283	2.45	.880505	1.80	.932778	4.25	.067222	25
36	.813430	2.47	.880397	1.80	.933033	4.27	.066967	24
37	.813578	2.45	.880289	1.82	.933289	4.27	.066711	23
38	.813725	2.45	.880180	1.80	.933545	4.25	.066455	22
39	.813872	2.45	.880072	1.82	.933800	4.27	.066200	21
40	.814019	2.45	.879963	1.80	.934056	4.25	.065944	20
41	9.814166	2.45	9.879855	1.82	9.934311	4.27	10.065689	19
42	.814313	2.45	.879746	1.82	.934567	4.25	.065433	18
43	.814460	2.45	.879637	1.80	.934822	4.27	.065178	17
44	.814607	2.43	.879529	1.82	.935078	4.25	.064922	16
45	.814753	2.45	.879420	1.82	.935333	4.27	.064667	15
46	.814900	2.43	.879311	1.82	.935589	4.25	.064411	14
47	.815046	2.45	.879202	1.82	.935844	4.27	.064156	13
48	.815193	2.43	.879093	1.82	.936100	4.25	.063900	12
49	.815339	2.43	.878984	1.82	.936355	4.27	.063645	11
50	.815485	2.45	.878875	1.82	.936611	4.25	.063389	10
51	9.815632	2.43	9.878766	1.83	9.936866	4.25	10.063134	9
52	.815778	2.43	.878656	1.82	.937121	4.27	.062879	8
53	.815924	2.42	.878547	1.82	.937377	4.25	.062623	7
54	.816069	2.43	.878438	1.83	.937632	4.25	.062368	6
55	.816215	2.43	.878328	1.82	.937887	4.25	.062113	5
56	.816361	2.43	.878219	1.83	.938142	4.27	.061858	4
57	.816507	2.42	.878109	1.83	.938398	4.25	.061602	3
58	.816652	2.43	.877999	1.82	.938653	4.25	.061347	2
59	.816798	2.42	.877890	1.83	.938908	4.25	.061092	1
60	9.816943		9.877780		9.939163		10.060837	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.816943	2.42	9.877780	1.83	9.939163	4.25	10.060837	60
1	.817088	2.42	.877670	1.83	.939418	4.25	.060582	59
2	.817233	2.43	.877560	1.83	.939673	4.25	.060327	58
3	.817379	2.42	.877450	1.83	.939928	4.25	.060072	57
4	.817524	2.40	.877340	1.83	.940183	4.27	.059817	56
5	.817668	2.42	.877230	1.83	.940439	4.25	.059561	55
6	.817813	2.42	.877120	1.83	.940694	4.25	.059306	54
7	.817958	2.42	.877010	1.85	.940949	4.25	.059051	53
8	.818103	2.40	.876899	1.83	.941204	4.25	.058796	52
9	.818247	2.42	.876789	1.85	.941459	4.23	.058541	51
10	.818392	2.40	.876678	1.83	.941713	4.25	.058287	50
11	9.818536	2.42	9.876568	1.85	9.941968	4.25	10.058032	49
12	.818681	2.40	.876457	1.83	.942223	4.25	.057777	48
13	.818825	2.40	.876347	1.85	.942478	4.25	.057522	47
14	.818969	2.40	.876236	1.85	.942733	4.25	.057267	46
15	.819113	2.40	.876125	1.85	.942988	4.25	.057012	45
16	.819257	2.40	.876014	1.83	.943243	4.25	.056757	44
17	.819401	2.40	.875904	1.85	.943498	4.23	.056502	43
18	.819545	2.40	.875793	1.85	.943752	4.25	.056248	42
19	.819689	2.38	.875682	1.85	.944007	4.25	.055993	41
20	.819832	2.40	.875571	1.87	.944262	4.25	.055738	40
21	9.819976	2.40	9.875459	1.85	9.944517	4.23	10.055483	39
22	.820120	2.38	.875348	1.85	.944771	4.25	.055229	38
23	.820263	2.38	.875237	1.85	.945026	4.25	.054974	37
24	.820406	2.40	.875126	1.87	.945281	4.23	.054719	36
25	.820550	2.38	.875014	1.85	.945535	4.25	.054465	35
26	.820693	2.38	.874903	1.87	.945790	4.25	.054210	34
27	.820836	2.38	.874791	1.85	.946045	4.23	.053955	33
28	.820979	2.38	.874680	1.87	.946299	4.25	053701	32
29	.821122	2.38	.874568	1.87	.946554	4.23	.053446	31
30	.821265	2.37	.874456	1.87	.946808	4.25	.053192	30
31	9.821407	2.38	9.874344	1.87	9.947063	4.25	10.052937	29
32	.821550	2.38	.874232	1.85	.947318	4.23	.052682	28
33	.821693	2.37	.874121	1.87	.947572	4.25	.052428	27
34	.821835	2.37	.874009	1.88	.947827	4.23	.052173	26
35	.821977	2.38	.873896	1.87	.948081	4.23	.051919	25
36	.822120	2.37	.873784	1.87	.948335	4.25	.051665	24
37	.822262	2.37	.873672	1.87	.948590	4.23	.051410	23
38	.822404	2.37	.873560	1.87	.948844	4.25	.051156	22
39	.822546	2.37	.873448	1.88	.949099	4.23	.050901	21
40	.822688	2.37	.873335	1.87	.949353	4.25	.050647	20
41	9.822830	2.37	9.873223	1.88	9.949608	4.23	10.050392	19
42	.822972	2.37	.873110	1.87	.949862	4.23	.050138	18
43	.823114	2.35	.872998	1.88	.950116	4.25	.049884	17
44	.823255	2.37	.872885	1.88	.950371	4.23	.049629	16
45	.823397	2.37	.872772	1.88	.950625	4.23	.049375	15
46	.823539	2.35	.872659	1.87	.950879	4.23	.049121	14
47	.823680	2.35	.872547	1.88	.951133	4.25	.048867	13
48	.823821	2.37	.872434	1.88	.951388	4.23	.048612	12
49	.823963	2.35	.872321	1.88	.951642	4.23	.048358	11
50	.824104	2.35	.872208	1.88	.951896	4.23	.048104	10
51	9.824245	2.35	9.872095	1.90	9.952150	4.25	10.047850	9
52	.824386	2.35	.871981	1.88	.952405	4.23	.047595	8
53	.824527	2.35	.871868	1.88	.952659	4.23	.047341	7
54	.824668	2.33	.871755	1.90	.952913	4.23	.047087	6
55	.824808	2.35	.871641	1.88	.953167	4.23	.046833	5
56	.824949	2.35	.871528	1.90	.953421	4.23	.046579	4
57	.825090	2.33	.871414	1.88	.953675	4.23	.046325	3
58	.825230	2.35	.871301	1.90	.953929	4.23	.046071	2
59	.825371	2.33	.871187	1.90	.954183	4.23	.045817	1
60	9.825511		9.871073		9.954437		10.045563	0
′	*Cosine.*	*D. 1″.*	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.825511	2.33	9.871073	1.88	9.954437	4.23	10.045563	60
1	.825651	2.33	.870960	1.90	.954691	4.25	.045309	59
2	.825791	2.33	.870846	1.90	.954946	4.23	.045054	58
3	.825931	2.33	.870732	1.90	.955200	4.23	.044800	57
4	.826071	2.33	.870618	1.90	.955454	4.23	.044546	56
5	.826211	2.33	.870504	1.90	.955708	4.22	.044292	55
6	.826351	2.33	.870390	1.90	.955961	4.23	.044039	54
7	.826491	2.33	.870276	1.92	.956215	4.23	.043785	53
8	.826631	2.32	.870161	1.90	.956469	4.23	.043531	52
9	.826770	2.33	.870047	1.90	.956723	4.23	.043277	51
10	.826910	2.32	.869933	1.92	.956977	4.23	.043023	50
11	9.827049	2.33	9.869818	1.90	9.957231	4.23	10.042769	49
12	.827189	2.32	.869704	1.92	.957485	4.23	.042515	48
13	.827328	2.32	.869589	1.92	.957739	4.23	.042261	47
14	.827467	2.32	.869474	1.90	.957993	4.23	.042007	46
15	.827606	2.32	.869360	1.92	.958247	4.22	.041753	45
16	.827745	2.32	.869245	1.92	.958500	4.23	.041500	44
17	.827884	2.32	.869130	1.92	.958754	4.23	.041246	43
18	.828023	2.32	.869015	1.92	.959008	4.23	.040992	42
19	.828162	2.32	.868900	1.92	.959262	4.23	.040738	41
20	.828301	2.30	.868785	1.92	.959516	4.22	.040484	40
21	9.828439	2.32	9.868670	1.92	9.959769	4.23	10.040231	39
22	.828578	2.30	.868555	1.92	.960023	4.23	.039977	38
23	.828716	2.32	.868440	1.93	.960277	4.22	.039723	37
24	.828855	2.30	.868324	1.92	.960530	4.23	.039470	36
25	.828993	2.30	.868209	1.93	.960784	4.23	.039216	35
26	.829131	2.30	.868093	1.92	.961038	4.23	.038962	34
27	.829269	2.30	.867978	1.93	.961292	4.22	.038708	33
28	.829407	2.30	.867862	1.92	.961545	4.23	.038455	32
29	.829545	2.30	.867747	1.93	.961799	4.22	.038201	31
30	.829683	2.30	.867631	1.93	.962052	4.23	.037948	30
31	9.829821	2.30	9.867515	1.93	9.962306	4.23	10.037694	29
32	.829959	2.30	.867399	1.93	.962560	4.22	.037440	28
33	.830097	2.28	.867283	1.93	.962813	4.23	.037187	27
34	.830234	2.30	.867167	1.93	.963067	4.22	.036933	26
35	.830372	2.28	.867051	1.93	.963320	4.23	.036680	25
36	.830509	2.28	.866935	1.93	.963574	4.23	.036426	24
37	.830646	2.30	.866819	1.93	.963828	4.22	.036172	23
38	.830784	2.28	.866703	1.95	.964081	4.23	.035919	22
39	.830921	2.28	.866586	1.93	.964335	4.22	.035665	21
40	.831058	2.28	.866470	1.95	.964588	4.23	.035412	20
41	9.831195	2.28	9.866353	1.93	9.964842	4.22	10.035158	19
42	.831332	2.28	.866237	1.95	.965095	4.23	.034905	18
43	.831469	2.28	.866120	1.93	.965349	4.22	.034651	17
44	.831606	2.27	.866004	1.95	.965602	4.22	.034398	16
45	.831742	2.28	.865887	1.95	.965855	4.23	.034145	15
46	.831879	2.27	.865770	1.95	.966109	4.22	.033891	14
47	.832015	2.28	.865653	1.95	.966362	4.23	.033638	13
48	.832152	2.27	.865536	1.95	.966616	4.22	.033384	12
49	.832288	2.28	.865419	1.95	.966869	4.23	.033131	11
50	.832425	2.27	.865302	1.95	.967123	4.22	.032877	10
51	9·832561	2.27	9.865185	1.95	9.967376	4.22	10.032624	9
52	.832697	2.27	.865068	1.97	.967629	4.23	.032371	8
53	.832833	2.27	.864950	1.95	.967883	4.22	.032117	7
54	.832969	2.27	.864833	1.95	.968136	4.22	.031864	6
55	.833105	2.27	.864716	1.97	.968389	4.23	.031611	5
56	.833241	2.27	.864598	1.95	.968643	4.22	.031357	4
57	.833377	2.25	.864481	1.97	.968896	4.22	.031104	3
58	.833512	2.27	.864363	1.97	.969140	4.23	.030851	2
59	.833648	2.25	.864245	1.97	.969403	4.22	.030597	1
60	9.833783		9.864127		9.969656		10.030344	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.833783	2.27	9.864127	1.95	9.969656	4.22	10.030344	60
1	.833919	2.25	.864010	1.97	.969909	4.22	.030091	59
2	.834054	2.25	.863892	1.97	.970162	4.23	.029838	58
3	.834189	2.27	.863774	1.97	.970416	4.22	.029584	57
4	.834325	2.25	.863656	1.97	.970669	4.22	.029331	56
5	.834460	2.25	.863538	1.98	.970922	4.22	.029078	55
6	.834595	2.25	.863419	1.97	.971175	4.23	.028825	54
7	.834730	2.25	.863301	1.97	.971429	4.22	.028571	53
8	.834865	2.23	.863183	1.98	.971682	4.22	.028318	52
9	.834999	2.25	.863064	1.97	.971935	4.22	.028065	51
10	.835134	2.25	.862946	1.98	.972188	4.22	.027812	50
11	9.835269	2.23	9.862827	1.97	9.972441	4.23	10.027559	49
12	.835403	2.25	.862709	1.98	.972695	4.22	.027305	48
13	.835538	2.23	.862590	1.98	.972948	4.22	.027052	47
14	.835672	2.25	.862471	1.97	.973201	4.22	.026799	46
15	.835807	2.23	.862353	1.98	.973454	4.22	.026546	45
16	.835941	2.23	.862234	1.98	.973707	4.22	.026293	44
17	.836075	2.23	.862115	1.98	.973960	4.22	.026040	43
18	.836209	2.23	.861996	1.98	.974213	4.22	.025787	42
19	.836343	2.23	.861877	1.98	.974466	4.23	.025534	41
20	.836477	2.23	.861758	2.00	.974720	4.22	.025280	40
21	9.836611	2.23	9.861638	1.98	9.974973	4.22	10.025027	39
22	.836745	2.22	.861519	1.98	.975226	4.22	.024774	38
23	.836878	2.23	.861400	2.00	.975479	4.22	.024521	37
24	.837012	2.23	.861280	1.98	.975732	4.22	.024268	36
25	.837146	2.22	.861161	2.00	.975985	4.22	.024015	35
26	.837279	2.22	.861041	1.98	.976238	4.22	.023762	34
27	.837412	2.23	.860922	2.00	.976491	4.22	.023509	33
28	.837546	2.22	.860802	2.00	.976744	4.22	.023256	32
29	.837679	2.22	.860682	2.00	.976997	4.22	.023003	31
30	.837812	2.22	.860562	2.00	.977250	4.22	.022750	30
31	9.837945	2.22	9.860442	2.00	9.977503	4.22	10.022497	29
32	.838078	2.22	.860322	2.00	.977756	4.22	.022244	28
33	.838211	2.22	.860202	2.00	.978009	4.22	.021991	27
34	.838344	2.22	.860082	2.00	.978262	4.22	.021738	26
35	.838477	2.22	.859962	2.00	.978515	4.22	.021485	25
36	.838610	2.20	.859842	2.02	.978768	4.22	.021232	24
37	.838742	2.22	.859721	2.00	.979021	4.22	.020979	23
38	.838875	2.20	.859601	2.02	.979274	4.22	.020726	22
39	.839007	2.22	.859480	2.00	.979527	4.22	.020473	21
40	.839140	2.20	.859360	2.02	.979780	4.22	.020220	20
41	9.839272	2.20	9.859239	2.00	9.980033	4.22	10.019967	19
42	.839404	2.20	.859119	2.02	.980286	4.20	.019714	18
43	.839536	2.20	.858998	2.02	.980538	4.22	.019462	17
44	.839668	2.20	.858877	2.02	.980791	4.22	.019209	16
45	.839800	2.20	.858756	2.02	.981044	4.22	.018956	15
46	.839932	2.20	.858635	2.02	.981297	4.22	.018703	14
47	.840064	2.20	.858514	2.02	.981550	4.22	.018450	13
48	.840196	2.20	.858393	2.02	.981803	4.22	.018197	12
49	.840328	2.18	.858272	2.02	.982056	4.22	.017944	11
50	.840459	2.20	.858151	2.03	.982309	4.22	.017691	10
51	9.840591	2.18	9.858029	2.02	9.982562	4.20	10.017438	9
52	.840722	2.20	.857908	2.03	.982814	4.22	.017186	8
53	.840854	2.18	.857786	2.02	.983067	4.22	.016933	7
54	.840985	2.18	.857665	2.03	.983320	4.22	.016680	6
55	.841116	2.18	.857543	2.02	.983573	4.22	.016427	5
56	.841247	2.18	.857422	2.03	.983826	4.22	.016174	4
57	.841378	2.18	.857300	2.03	.984079	4.22	.015921	3
58	.841509	2.18	.857178	2.03	.984332	4.20	.015668	2
59	.841640	2.18	.857056	2.03	.984584	4.22	.015416	1
60	9.841771		9.856934		9.984837		10.015163	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

′	Sine.	D. 1″.	Cosine.	D. 1″.	Tang.	D. 1″.	Cotang.	′
0	9.841771	2.18	9.856934	2.03	9.984837	4.22	10.015163	60
1	.841902	2.18	.856812	2.03	.985090	4.22	.014910	59
2	.842033	2.17	.856690	2.03	.985343	4.22	.014657	58
3	.842163	2.18	.856568	2.03	.985596	4.20	.014404	57
4	.842294	2.17	.856446	2.05	.985848	4.22	.014152	56
5	.842424	2.18	.856323	2.03	.986101	4.22	.013899	55
6	.842555	2.17	.856201	2.05	.986354	4.22	.013646	54
7	.842685	2.17	.856078	2.03	.986607	4.22	.013393	53
8	.842815	2.18	.855956	2.05	.986860	4.20	.013140	52
9	.842946	2.17	.855833	2.03	.987112	4.22	.012888	51
10	.843076	2.17	.855711	2.05	.987365	4.22	.012635	50
11	9.843206	2.17	9.855588	2.05	9.987618	4.22	10.012382	49
12	.843336	2.17	.855465	2.05	.987871	4.20	.012129	48
13	.843466	2.15	.855342	2.05	.988123	4.22	.011877	47
14	.843595	2.17	.855219	2.05	.988376	4.22	.011624	46
15	.843725	2.17	.855096	2.05	.988629	4.22	.011371	45
16	.843855	2.15	.854973	2.05	.988882	4.20	.011118	44
17	.843984	2.17	.854850	2.05	.989134	4.22	.010866	43
18	.844114	2.15	.854727	2.07	.989387	4.22	.010613	42
19	.844243	2.15	.854603	2.05	.989640	4.22	.010360	41
20	.844372	2.17	.854480	2.07	.989893	4.20	.010107	40
21	9.844502	2.15	9.854356	2.05	9.990145	4.22	10.009855	39
22	.844631	2.15	.854233	2.07	.990398	4.22	.009602	38
23	.844760	2.15	.854109	2.05	.990651	4.20	.009349	37
24	.844889	2.15	.853986	2.07	.990903	4.22	.009097	36
25	.845018	2.15	.853862	2.07	.991156	4.22	.008844	35
26	.845147	2.15	.853738	2.07	.991409	4.22	.008591	34
27	.845276	2.15	.853614	2.07	.991662	4.20	.008338	33
28	.845405	2.13	.853490	2.07	.991914	4.22	.008086	32
29	.845533	2.15	.853366	2.07	.992167	4.22	.007833	31
30	.845662	2.13	.853242	2.07	.992420	4.20	.007580	30
31	9.845790	2.15	9.853118	2.07	9.992672	4.22	10.007328	29
32	.845919	2.13	.852994	2.08	.992925	4.22	.007075	28
33	.846047	2.13	.852869	2.07	.993178	4.22	.006822	27
34	.846175	2.15	.852745	2.08	.993431	4.20	.006569	26
35	.846304	2.13	.852620	2.07	.993683	4.22	.006317	25
36	.846432	2.13	.852496	2.08	.993936	4.22	.006064	24
37	.846560	2.13	.852371	2.07	.994189	4.20	.005811	23
38	.846688	2.13	.852247	2.08	.994441	4.22	.005559	22
39	.846816	2.13	.852122	2.08	.994694	4.22	.005306	21
40	.846944	2.12	.851997	2.08	.994947	4.20	.005053	20
41	9.847071	2.13	9.851872	2.08	9.995199	4.22	10.004801	19
42	.847199	2.13	.851747	2.08	.995452	4.22	.004548	18
43	.847327	2.12	.851622	2.08	.995705	4.20	.004295	17
44	.847454	2.13	.851497	2.08	.995957	4.22	.004043	16
45	.847582	2.12	.851372	2.10	.996210	4.22	.003790	15
46	.847709	2.12	.851246	2.08	.996463	4.20	.003537	14
47	.847836	2.13	.851121	2.08	.996715	4.22	.003285	13
48	.847964	2.12	.850996	2.10	.996968	4.22	.003032	12
49	.848091	2.12	.850870	2.08	.997221	4.20	.002779	11
50	.848218	2.12	.850745	2.10	.997473	4.22	.002527	10
51	9.848345	2.12	9.850619	2.10	9.997726	4.22	10.002274	9
52	.848472	2.12	.850493	2.08	.997979	4.20	.002021	8
53	.848599	2.12	.850368	2.10	.998231	4.22	.001769	7
54	.848726	2.10	.850242	2.10	.998484	4.22	.001516	6
55	.848852	2.12	.850116	2.10	.998737	4.20	.001263	5
56	.848979	2.12	.849990	2.10	.998989	4.22	.001011	4
57	.849106	2.10	.849864	2.10	.999242	4.22	.000758	3
58	.849232	2.12	.849738	2.12	.999495	4.20	.000505	2
59	.849359	2.10	.849611	2.10	.999747	4.22	.000253	1
60	9.849485		9.849485		10.000000		10.000000	0
′	Cosine.	D. 1″.	Sine.	D. 1″.	Cotang.	D. 1″.	Tang.	′

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